Introduction to Aerospace Engineering

Exams

TU Delft

Answers Exam AE1101 11-11-11

Problem 1

MC- question:

B) In the early nineteen-fifties aircraft flew at higher altitudes and had pressure cabins

Problem 2:

a. load carrying, protection, attachment of systems
b. we are looking for the best P/W ratios, which means: e.g. a very strong, but heavy material could be less interesting than a weaker but much lighter material (compensate by adding more material - volume)
c. applied material, shape of the beam, thickness of the girder and web plate, joining method

3a) $m = 1088\,kg \Rightarrow W = m \cdot g = 1088 \cdot 9.81 = 10673.28\,N$

$L = W = 10673.28\,N$

$\frac{L}{D} = 12.5 \Rightarrow \frac{L}{12.5} = D = \frac{10673.28}{12.5} = 853.8624\,N$

$T = D = 853.8624\,N$ $(= 213.4656\,N$ per engine$)$

$P_a = T \cdot V \Rightarrow V = 491\,km/hr = 136.39\,m/s$

$P_a = T \cdot V = 853.8624 \cdot 136.39 = 116457.34\,W$ (one engine 29114.33 per eng)

$P_a = \eta\, P_{br} \Rightarrow P_{br} = \frac{P_a}{\eta} = \frac{116457.34}{0.67} = 173816.9\,W$

4 engines $\Rightarrow P_{br_{1eng}} = \frac{P_{br}}{4} = 43454\,W = 43{,}5\,kW$

$$\Rightarrow \boxed{P_{br_{1eng}} = 43.5\,kW}$$

(other method → via $C_L \to C_D$ but not necessary)

b)

$$M_{tot} = M_{ac_w} + L_{w_1} \cdot l_w + M_{ac_w} - L_{w_2} \cdot l_w - L_H \cdot l_H$$

$$C_{m_{tot}} = 2 \cdot \tfrac{1}{2} C_{m_{ac_w}} + \frac{C_{L_{w_1}} l_w S_1}{S c} - \frac{C_{L_{w_2}} \cdot l_w \cdot S_2}{S c} - \frac{C_{L_H} \cdot l_H \cdot S_H}{S c}$$

$$= C_{m_{ac_w}} + \tfrac{1}{2} C_{L_{w_1}} \frac{l_w}{c} - \tfrac{1}{2} C_{L_{w_2}} \frac{l_w}{c} - C_{L_H} \cdot V_H$$

$$C_{m_\alpha} = 0 + \tfrac{1}{2} C_{L_{\alpha_w}} \cdot \frac{l_w}{c} - \tfrac{1}{2} C_{L_{\alpha_w}} \frac{l_w}{c} \cdot \frac{d\alpha_2}{d\alpha} - C_{L_H} \cdot V_H \cdot \frac{d\alpha_H}{d\alpha}$$

$$= \tfrac{1}{2} C_{L_{\alpha_w}} \frac{l_w}{c} - \tfrac{1}{2} C_{L_{\alpha_w}} \frac{l_w}{c} \cdot 0.90 - C_{L_H} V_H \cdot 0.87$$

$$C_{m_\alpha} = 0.05\, C_{L_{\alpha_w}} \cdot \frac{l_w}{c} - 0.87\, C_{L_H} V_H \qquad V_H = \frac{S_H l_H}{S \bar{c}}$$

qed.

c) stable $\Rightarrow$ $C_{m_\alpha} < 0$

$$C_{m_\alpha} = 0.05\, C_{L_{\alpha w}} \frac{l_w}{c} - 0.87\, C_{L_{\alpha H}} \cdot V_H < 0$$

$$\Leftrightarrow \quad 0.87\, C_{L_{\alpha H}} V_H > 0.05\, C_{L_{\alpha w}} \frac{l_w}{c}$$

$$0.87\, C_{L_{\alpha H}} \cdot \frac{S_H\, l_H}{S \cdot \cancel{c}} > 0.05\, C_{L_{\alpha w}} \frac{l_w}{\cancel{c}}$$

$$0.87 \cdot 0.095 \cdot \frac{S_H \cdot 6.0}{16.20} > 0.05 \cdot 0.10 \cdot \frac{3.00}{1}$$

$$0.0306111\, S_H > 0.015$$

$$S_H > 0.4900 \text{ m}^2$$

$$\boxed{S_H \text{ minimally } 0.49 \text{ m}^2}$$

or:

$$\Rightarrow V_H > \frac{0.05}{0.87} \frac{C_{L_{\alpha w}}}{C_{L_{\alpha H}}} \cdot \frac{l_w}{c}$$

$$> 0.1871$$

$$S_H = \frac{V_H \cdot S \cdot \bar{c}}{l_H} = 0.49 \text{ m}^2$$

d) This means there will be distance between centre of gravity and neutral point. So there will be a static margin. This allows the user a larger range of center of gravity positions of the aircraft including payload. Also aircraft will be more stable.

preload. key item

option 2

Problem 4

naam / *name*	
studienummer / *student number*	
vak / *course*	
code / *code*	datum / *date*
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Answers Problem 4

a) Determine the Mach number

$M_\infty = \frac{V}{a}$ $\quad V = 720 \text{ km/hr} = 720/3.6 = 200 \text{ m/s}$

$$\left.\begin{array}{l} a = \sqrt{\gamma R T} \\ \gamma = 1.4 \\ R = 287.05 \\ T = 281.66 \end{array}\right\} a = 336.4 \text{ m/s}$$

$$M_\infty = \frac{200}{336.4} = \underline{\underline{0.594}}$$

b) Calculate which part of the wing is supersonic

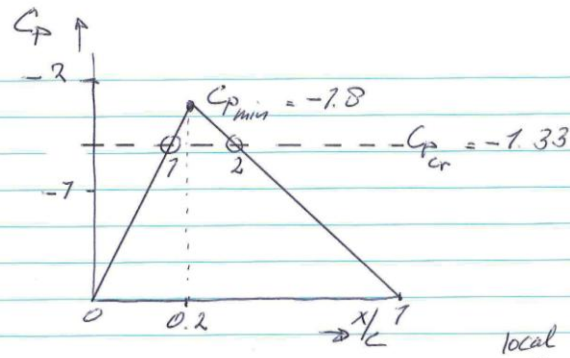
You can do this in two ways: either calculate the $C_{p_{cr}}$ and transfer this to $p - p_\infty$ or transfer $p - p_\infty$ into C_p's. We will do the latter here.

$C_p = \frac{p - p_\infty}{q_\infty}$ $\quad q_\infty = \frac{1}{2}\rho V^2 = \frac{1}{2} * 1.1117 * 200^2$

$= 22234 \text{ N/m}^2$

$$C_{p_{cr}} = \frac{2}{1.4 * 0.594^2}\left[\left(\frac{2 + 0.4 * 0.594^2}{2.4}\right)^{3.5} - 1\right]$$

$= -1.33$



At stations 1 and 2 the local Mach number is 1

1^{st} sonic when $\frac{-2 * 10^5 * \frac{x}{c}}{22\,234} = -1.33$

$$\Rightarrow x/c = 0.15$$

station 2: $\frac{5 * 10^4 \left(-1 + \frac{x}{c}\right)}{22\,234} = -1.33$

$$\Rightarrow x/c = 0.41$$

Supersonic when $0.15 < x/c < 0.41$

c) Calculate the drag force on the wing.

$D = C_D \cdot \frac{1}{2} \rho V_\infty^2 \cdot S$

$C_D = C_{D_p} + \frac{C_L^2}{\pi A e}$

To calculate C_{D_p} we need the angle of attack. We have the angle of the wing $(= 4°)$

We know $\alpha_{profile} = \alpha_{wing} - \alpha_i$

The induced angle of attack is:

$$\alpha_i = \frac{C_L}{\pi A e}$$

elliptic distribution: $e = 1$

$$\left.\begin{array}{l} A = \dfrac{b^2}{S} \\ b = 20 \\ S' = 50\,m^2 \end{array}\right\} \quad A = \frac{20^2}{50} = 8$$

$$C_L = ? \quad \Rightarrow \quad W.g = C_L \cdot \tfrac{1}{2} \rho V_\infty^2 \cdot S'$$

$$C_L = \frac{9.8 * 56719.4}{1/2 * 1.1117 * 200^2 * 50} = 0.5$$

$$\alpha_i = \frac{0.5}{\pi \cdot 8.1} * 57.3 = 1.14^\circ$$

$$\alpha_{profile} = 4^\circ - 1.14^\circ = 2.86^\circ$$

$$C_{dp} = 0.006 + 0.5 * 10^{-4} * 2.86 = 0.00614$$

$$C_D = 0.00614 + \frac{C_L^2}{\pi A . e} = 0.00614 + \frac{0.5^2}{\pi . 8.1} = 0.0161$$

$$D = 0.0161 * 22\,234 * 50$$

$$= 17\,888\ N$$

d) T in station 1 of problem 4b?

Use the 2^{nd} form of the isentropic relations:

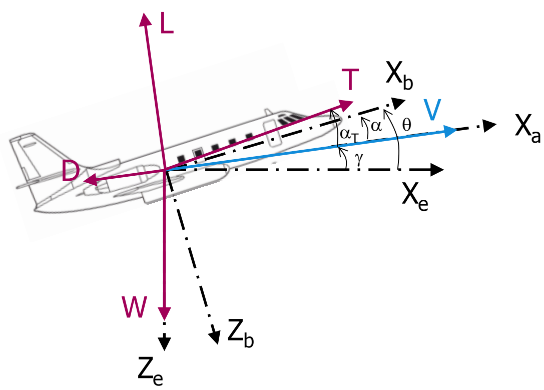
$$\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2, \text{ with } M_1 = 1 \text{ we find } \frac{T_0}{T_1} = 1.2$$

$$T_0 = 301\ K \quad T_1 = \frac{T_0}{1.2} = 250.8\ K$$

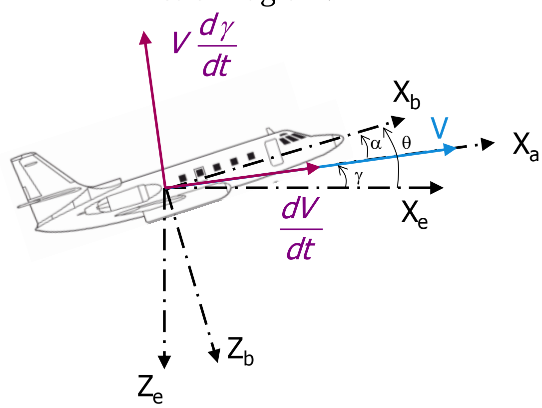
Problem 5

a.

Free Body Diagram:



Kinetic Diagram:



b.

$$\sum F_{//V} : \frac{W}{g}\frac{dV}{dt} = T\cos\alpha_T - D - W\sin\gamma$$

$$\sum F_{\perp V} : \frac{W}{g}V\frac{d\gamma}{dt} = L - W\cos\gamma + T\sin\alpha_T$$

Assumption: $\alpha_T \cong 0$ (thrust in direction of airspeed vector)

$$\frac{W}{g}\frac{dV}{dt} = T - D - W\sin\gamma$$

$$\frac{W}{g}V\frac{d\gamma}{dt} = L - W\cos\gamma$$

c.

Horizontal: $\gamma = 0$ (and thus $d\gamma/dt = 0$)

Steady: $dV/dt = 0$;

$$\frac{W}{g}\cdot 0 = T - D - W\sin 0$$

$$\frac{W}{g}V\cdot 0 = L - W\cos 0$$

Thus:

$$L = W$$

$$T = D$$

d.

Maximum endurance means maximum time in the air. Hence, the Fuel Flow (F) should be minimal.

$$F = c_T T$$

$$F = c_T D$$

$$F_{\min} \Rightarrow D_{\min}$$

This last step can be made because c_T is given to be a constant

$$D = \frac{L}{L} D = \frac{D}{L} W = \frac{C_D}{C_L} W$$

$$D_{\min} \Rightarrow \left(\frac{C_L}{C_D} \right)_{\max}$$

The optimal C_L/C_D ratio can be found by taking the derivative of the function and setting it equal to zero. (Please note that various other (correct) methods can be used from this point to find the solution)

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

$$\frac{d}{dC_L} \left(\frac{C_L}{C_D} \right) = 0$$

$$\frac{C_D \cdot 1 - C_L \cdot \dfrac{dC_D}{dC_L}}{C_D^2} = 0$$

$$\frac{dC_D}{dC_L} = \frac{C_D}{C_L}$$

$$\frac{2C_L}{\pi A e} = \frac{C_{D_0} + \dfrac{C_L^2}{\pi A e}}{C_L}$$

$$C_L = \sqrt{C_{D_0} \pi A e} = \sqrt{0.015 \cdot \pi \cdot 6.36 \cdot 0.67} = 0.45$$

e.

Correct answer: **C**. The endurance is maximal at minimum fuel flow

$$F = c_T \frac{C_D}{C_L} W$$

All parameters in the equation (max C_L/C_D, W, c_T) are independent of the altitude so the performance in terms of endurance will remain the same.

NOTE: other performance parameters such as maximum range, minimum airspeed etc. will change with altitude.