Introduction to Aerospace Engineering

Lecture slides

Launch of STS-122 on February 7, 2008 [NASA].

Part of the lecture material for this chapter originates from B.A.C. Ambrosius, R.J. Hamann and K.F. Wakker.

References to “”Introduction to Flight” by J.D. Anderson will be given in footnotes where relevant. AAAAAAAAAAAAA

9 - 10.

Rocket motion and launchers

TU Delft

Other material to be studied in addition to this presentation:
“Introduction to flight” (Anderson): pp. 728-729; section 9.10.

Overview

- What is a launcher
- Different types of launchers
- Tsiolkovsky's equation
- Fundamentals of rocket motion
- Ideal single-stage launcher
- Real single-stage launcher (gravity, atmosphere)
- Launch constraints

- Multi-stage launchers: treated in course ae2-104 (2^{nd} BSc year)

Introduction

Why we need high-thrust ?

- Huge velocity change: 0.48 km/s at Earth surface (max) → 7.9 km/s in LEO
- Bring satellite to minimum altitude above atmosphere (> 200 km)
- Overcome gravity pull and drag losses

Saturn-5 F-1 rocket engine

T = 700 ton
m = 3 ton/s

Introduction (cnt'd)

Role of low thrust ?

- Only effective after initial launch to LEO
- Novel techniques (Ion, plasma, solar sail, etc.)
- Much higher efficiency
- Acceleration too small to counteract Earth's gravitational pull
- No role for launch
- Major role for orbit maintenance and interplanetary trajectories

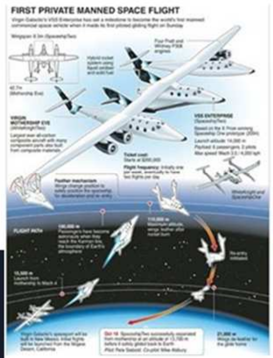
Examples: Deep Space 1 (1998-2001; Isp = 3100 s; F = 92 mN [http://nmp.nasa.gov/ds1]) or SMART-1 (2003-2006; Isp = 1640 s; F = 70 mN [http://www.esa.int/esaMI/SMART-1]).

Introduction (cnt'd)

Multi-stage launchers (some examples)

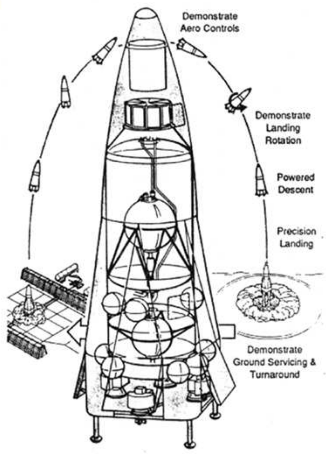
Introduction (cnt'd)

Sub-orbital launchers and sounding rockets



Introduction (cnt'd)

Single-stage launchers and (future) hypersonic hybrids



Introduction (cnt’d)

Air-launched

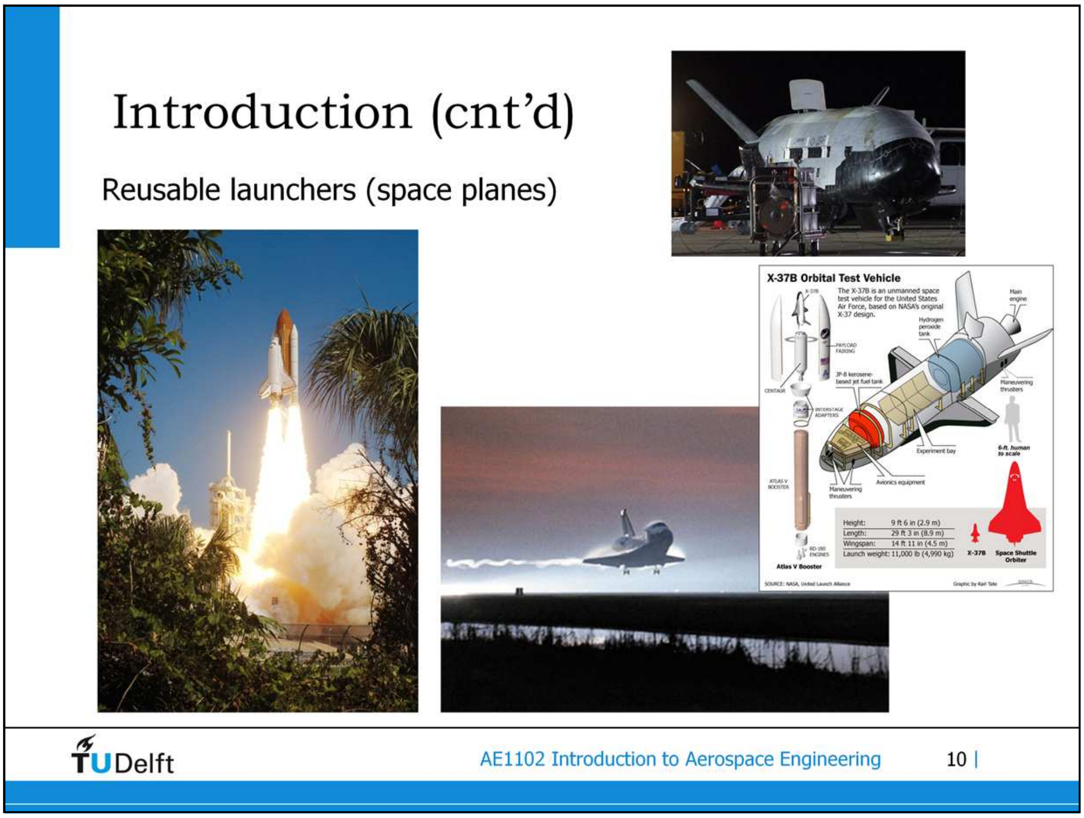
Pegasus [Orbital Sciences Corp.]

Payload mass: 455 kg

Total mass: 19653 kg

Payload fraction: 0.023

To overcome the drag of the lower part of the atmosphere, the launcher can be taken to altitude by a carrier aircraft (also compare with SpaceShipOne). OSC’s Pegasus is taken to 12 km before it is released. Depending on the technical requirements, it is a 3- or 4-stage launcher. Status April 2007: 37 launches, of which 34 successful [http://astroprofspage.com/archives/860].



Main advantage of reusable launchers: save operational costs. Drawback: maintenance. In spite of all efforts, the Space Transportation System STS (a.k.a. Space Shuttle) was the only reusable launcher until now. Status December 2011: Program terminated; 135 missions, of which 133 successful (Challenger destroyed during launch in January 1986, Columbia destroyed during re-entry in 2003) [http://www.nasa.gov/mission_pages/shuttle/shuttlemissions/list_main.html].

SpaceX writes history on Dec. 8, 2010

SpaceX writes history on Dec. 8, 2010

Introduction (cnt'd)

Pioneers with liquid-fueled rockets

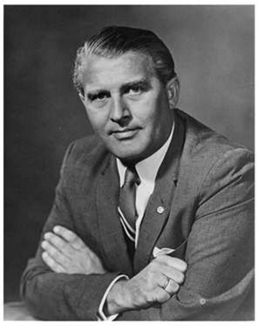
Robert H. Goddard, 1926

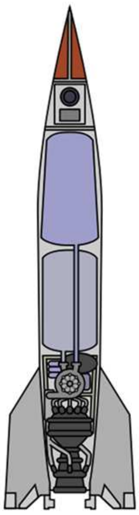
Sergei Korolev, GIRD-X, 1933

Independent of each other, liquid-fueled technology was developed both in the USA and in the former Soviet Union. Goddard and Korolev are among the founding fathers of modern spaceflight.

Introduction (cnt'd)

Pioneers with liquid-fueled rockets (cnt'd)





data:

- single stage
- liquid propellants (alcohol and cryogenic oxygen)
- steel structure
- 13000 kg lift-off mass
- 8800 kg propellants
- 250 kN thrust
- 1000 kg warhead
- range several hundred km
- "payload" fraction: 0.077

Wernher von Braun et al, V-2, 1942

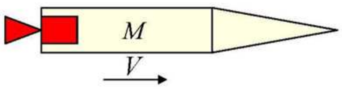
Nazi Germany developed its V-2 (”Vergeltungswaffen-2”); also propelled by liquid propellants. After the war, Wernher von Braun and colleagues continued this development in the USA, whereas other colleagues stepped to the Russian side. Ultimately, this lead to the development of actual space launchers, with crucial contributions to the Moon race and other projects. Devestating and unreliable as the V-2 was, it was a real revolution.

Fundamentals

Newton’s second Law of Motion:

- The time derivative of the impulse of a (point) body relative to an inertial coordinate frame equals the resulting force the body is subjected to.

Consider (closed) system of launcher and propellant:

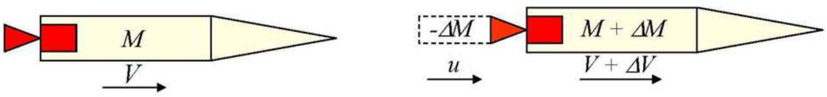


The total impulse is represented by parameter I.

$$I = \text{constant} \quad \text{or} \quad \frac{dI}{dt} = 0$$

Sir Isaac Newton (1643-1727) postulated 3 so-called Laws of Motion (cf. lectures 51 and 52). His second Law is elementary for describing rocket performance. It is crucial to consider the system of launch vehicle and expelled propellant as a whole; as a consequence the thrust of a rocket engine is an internal one.

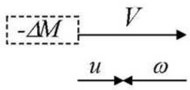
Ideal rocket



$$I_t = M V \; ; \qquad I_{t+\Delta t} = (M + \Delta M)(V + \Delta V) - u\,\Delta M$$

with

$$u = V - w$$



so

$$I_{t+\Delta t} = (M + \Delta M)(V + \Delta V) - (V - w)\Delta M$$

Total impulse considered before (left) and after (right) release of propellant mass ΔM (which has a negative sign in this convention). The vehicle has a velocity "V" (or "V+ΔV") w.r.t. an inertial reference frame, whereas the propellant is expelled with (relative) velocity "w" ("ω" in the sketch; sorry for the confusion in notations). So, "u" is the velocity of the propellant w.r.t. inertial frame.

Ideal rocket (cnt'd)

so

$$\frac{\Delta I}{\Delta t} = \frac{I_{t+\Delta t} - I_t}{\Delta t} = \frac{(MV + M\Delta V + \Delta M\, V + \Delta M\, \Delta V - V\,\Delta M + w\Delta M) - (M\,V)}{\Delta t} =$$

$$= M\,\frac{\Delta V}{\Delta t} + w\,\frac{\Delta M}{\Delta t} + \frac{\Delta M\,\Delta V}{\Delta t}$$

linearization and $\Delta t \Rightarrow 0$:

$$\frac{dI}{dt} = M\,\frac{dV}{dt} + w\frac{dM}{dt}$$

=0; Newton's second Law (NO external force)!

or

$$M\,\frac{dV}{dt} = -\,w\,\frac{dM}{dt}$$

and

$$dV = -w\,\frac{dM}{M}$$

Linearization: ignore terms "small to the power 2". Δt -> 0 introduces derivative w.r.t. time.

Ideal rocket (cnt'd)

result of burning propellant (exhaust velocity w is constant):

$$\int_{V_{begin}}^{V_{end}} dV = \int_{M_{begin}}^{M_{end}} -w \frac{dM}{M}$$

$$\Leftrightarrow \quad V_{end} - V_{begin} = -w\,(\ln M_{end} - \ln M_{begin}) = w \ln \frac{M_{begin}}{M_{end}}$$

or

$$\Delta V = w \ln \frac{M_{begin}}{M_{end}} = I_{sp}\, g_0 \ln \Lambda$$

$$(w = I_{sp}\, g_0\, ; \quad M_{begin} / M_{end} = \Lambda)$$

"Tsiolkovsky's equation"

or: "the rocket equation".

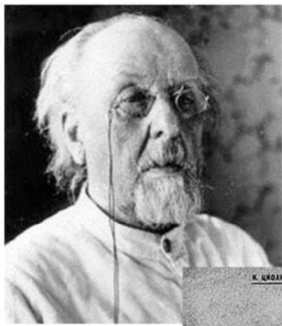
NB: Definition w = Isp*g0

In reality, exhaust velocity "w" is not constant but depends on the degree of expansion on the nozzle (under-expansion at low altitudes, over-expansion at high altitudes). However, to first order, "w" can be considered as constant. See next 3 sheets.

The specific impulse I_{sp} is by definition related to the exhaust velocity through the acceleration at sea level g_0 (9.81 m/s^2). For a given amount of propellant (*i.e.* parameter Λ), it is a direct measure for the efficiency of burning this propellant (the higher I_{sp}, the larger the achieved ΔV).

Tsiolkovsky

Konstantin Tsiolkovsky
- Russian scientist 1857-1935
- high school math teacher
- deaf
- father of theoretical astronautics



Tsiolkovsky's equation:
- velocity gain depends on masses and exhaust velocity
- velocity gain is independent of burn program

Tsiolkovsky is another founding father of spaceflight. The burn program (*i.e.* fast, slow, irregular, …) does not affect the achievable velocity increase for an ideal rocket engine; this is merely driven by the amount of propellant and the specific impulse.

Ideal rocket (cnt'd)

$$M\frac{dV}{dt} = -w\frac{dM}{dt} = m\,w$$

where

$$m = -\frac{dM}{dt}$$

Definition:

m = expelled (gaseous) mass per unit of time, or mass flow [kg/s]

Solidification Principle (impulsive thrust):

F = M a = M dV/dt = m w (M = instantaneous mass of rocket)

The Solidification Principle brings us back to Newtonian mechanics, albeit that we have to refer to the instantaneous mass of the launch vehicle (*i.e.* time-depending).

Ideal rocket (cnt'd)

Consider a vehicle with dry mass (*i.e.* construction + payload mass) of 1000 kg, and 4000 kg of propellant. The specific impulse is equal to 300 s, and the propellant mass flow is equal to 50 kg/s (and constant).

Questions:

1. What is the thrust?
2. What is the gain in velocity?

Answers: see footnotes below **(BUT TRY YOURSELF FIRST !!)**

Answers: **DID YOU TRY??**

1. F = 147150 N
2. ΔV = 4736.6 m/s

Ideal rocket (cnt'd)

burn time:

$$t_b = \frac{M_{begin} - M_{end}}{m} = \frac{M_{begin}}{m}\left(1 - \frac{M_{end}}{M_{begin}}\right) = \frac{I_{sp}}{\Psi_0}\left(1 - \frac{1}{\Lambda}\right)$$

where

$$\Lambda = \frac{M_{begin}}{M_{end}}; \quad \Psi_0 = \frac{F}{M_{begin}\, g_0}; \quad m = \frac{F}{g_0 I_{sp}}$$

Definition: Ψ_0: Initial thrust-to-weight ratio (or thrust load)

Note: impulsive shot: $t_b \rightarrow 0$; $\Psi_0 \rightarrow \infty$

Parameter Ψ_0 is the so-called thrust-to-weight ratio.

In an impulsive shot, the total ΔV is reached instantaneously, so the vehicle is still at the same position and the acceleration is infinitely large.

Ideal rocket (cnt'd)

traveled path:

$$s_e = \int_{t_{begin}}^{t_{end}} V\,dt = -\int_{t_{begin}}^{t_{end}} w \ln\frac{M}{M_{begin}}\,dt$$

m = -dM/dt, m and w constant:

$$s_e = \frac{w}{m}\int_{M_{begin}}^{M_{end}} \ln\frac{M}{M_{begin}}\,dM = \frac{w}{m}\int_{M_{begin}}^{M_{end}} (\ln M - \ln M_{begin})\,dM$$

integration yields:

$$s_e = \frac{w}{m}\left(M_{end}\ln\frac{M_{end}}{M_{begin}} - M_{end} + M_{begin}\right) = \frac{w(M_{begin} - M_{end})}{m}\left(1 + \frac{M_{end}}{M_{begin} - M_{end}}\ln\frac{M_{end}}{M_{begin}}\right)$$

Straightforward integrations.

Ideal rocket (cnt'd)

traveled path (cnt'd):

$$s_e = w\, t_b \left(1 - \frac{\ln \Lambda}{\Lambda - 1} \right) =$$

$$= \frac{g_0 \, I_{sp}^{\;2}}{\Psi_0} \left(1 - \frac{\ln \Lambda + 1}{\Lambda} \right)$$

- end velocity is independent of burn time
- traveled distance is dependent on burn time
- impulsive shot: $s_e \rightarrow 0$

Straightforward substitution of parameters.

Impulsive shot: infinite acceleration, vehicle still at same position -> s_e =0 indeed.

Ideal rocket (cnt'd)

Example 1:

Consider a vehicle with dry mass (*i.e.* construction + payload mass) of 1000 kg, and 4000 kg of propellant. The specific impulse is equal to 300 s, and the propellant mass flow is equal to 50 kg/s (and constant).

Questions:

1. What is the thrust?
2. What is the acceleration of the vehicle at the beginning of the thrust interval?
3. What is the acceleration of the vehicle at the end of the thrust interval?
4. What is the gain in velocity?

Answers: see footnotes below **(TRY YOURSELF FIRST !!)**

Answers: **(DID YOU TRY FIRST??)**

1. $F = 147150$ N
2. $a_{begin} = 29.43$ m/s^2 = 3 g_0
3. $a_{end} = 147.15$ m/s^2 = 15 g_0
4. $\Delta V = 4736.6$ m/s

Ideal rocket (cnt'd)

Example 2:

Consider a rocket with dry mass (*i.e.* construction + payload mass) of 1000 kg. It is to take the payload to escape velocity, so ΔV = 11.2 km/s. The specific impulse is equal to 300 s.

Questions:

1. How much propellant mass is required?
2. If the (constant) thrust were to be delivered over 2 minutes of time, what would be the thrust? What would be the acceleration at burnout? What would be the acceleration at ignition?
3. If the (constant) thrust were to be delivered over 8 minutes of time, what would be the thrust? What would be the acceleration at burnout? What would be the acceleration at ignition?

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

Answers: **(DID TOU TRY??)**

1. $M_{propellant}$ = 43954 kg (or: 97.8% of initial mass….)
2. T = 1077973 N; $a_{burnout}$ = 1078 m/s^2 = 110 g_0; $a_{lignition}$ = 24 m/s^2 = 2.4 g_0
3. T = 269493 N; $a_{burnout}$ = 269.5 m/s^2 = 27 $g_{0;}$ $a_{lignition}$ = 6 m/s^2 = 0.6 g_0

Ideal rocket: some graphs

Reference case:

Data:

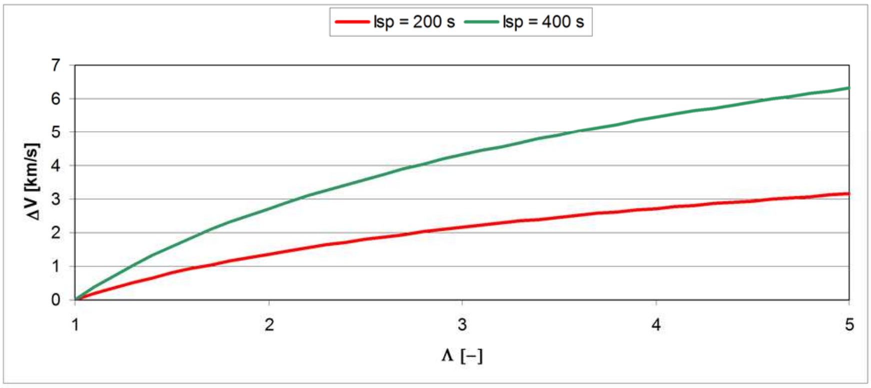
- specific impulse I_{sp} = 200 s
- Ψ_0 = 1.5
- Λ = 5

Results:

- propellant mass = 80 % of total launch mass
- end velocity = 3157.7 m/s
- burn time = 106.7 s
- traveled distance = 125.1 km

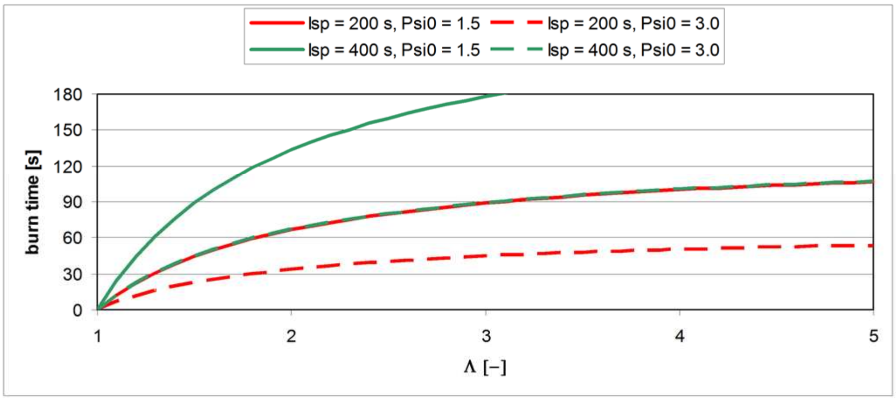
Verify these numbers yourself!

Ideal rocket: illustrations (cnt'd)



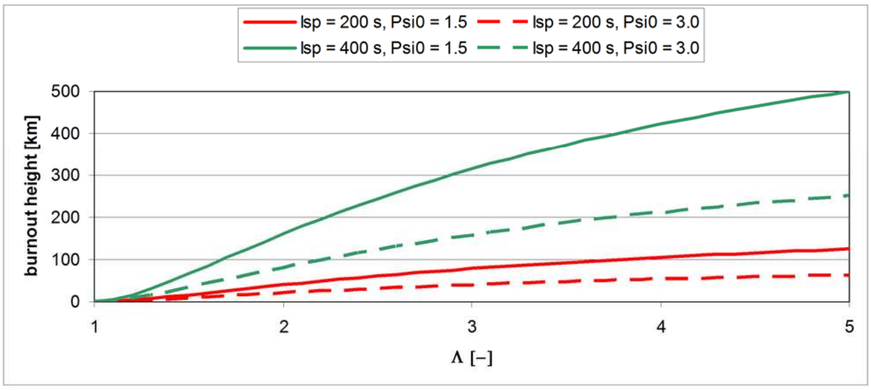
The rocket with I_{sp} equal to 400 s reaches a ΔV which is twice as large as that for the rocket with I_{sp} of 200 s (Tsiolkovsky!). The curve becomes less steep, because more propellant has to be taken on board, which also has to be accelerated in the first phase of the flight….. not effective -> multi-stage launchers. The values in this curve are still way below what is required for a LEO orbit, let alone an escape orbit… and Λ is already larger than 5!!

Ideal rocket: illustrations (cnt'd)



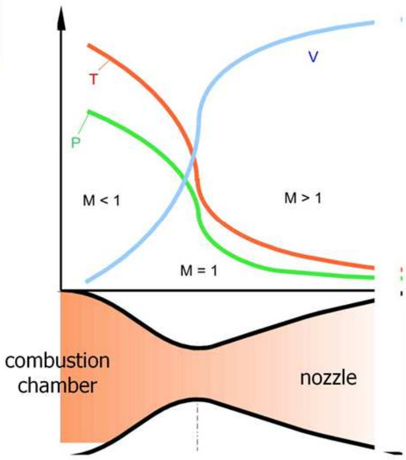
The factor of exactly 2 is also visible in this plot. Lower value for I_{sp} can be compensated for by higher propellant mass flow -> overlap of curves. Issue: is acceleration enough to lift of from launch platform (in particular, for longer burn times)?

Ideal rocket: illustrations (cnt'd)



Recognize the factor I_{sp}^2 (here: $2^2 = 4$) between the curves.

Thrust



[Wikipedia, 2009]

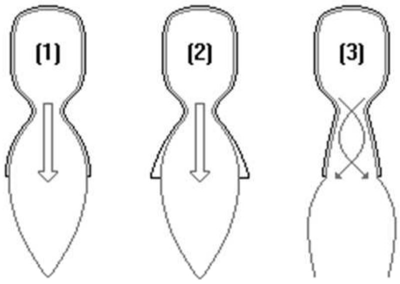
Some notes:

- propellants burned in combustion chamber
- $p_{chamber} >> p_{atm}$
- Laval nozzle
- velocity Mach 1 in throat
- expansion: pressure decreases in nozzle >
- velocity increases in nozzle (supersonic)

In an ideal nozzle, the gasses would be expanded until (local) atmospheric pressure (which is altitude-dependent….).

Thrust (cnt'd)

[http://www.braeunig.us/space/propuls.htm#Isp, 2009]



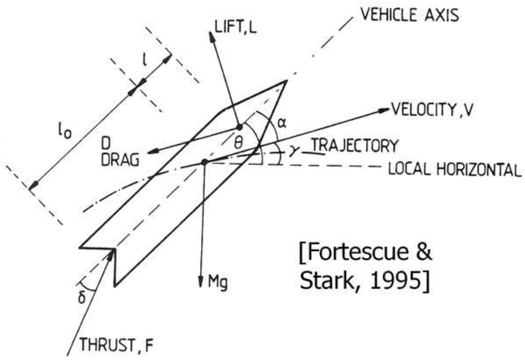
Some notes (2):

- case (1): ideal expansion, $p_{exit} = p_{atm}$
- case (2); over-expansion, $p_{exit} < p_{atm}$
- case (3): under-expansion, $p_{exit} > p_{atm}$
- In practice: thrust rocket engine = impulse component $\dot{m}w$ + pressure component $(p_{exit}-p_{atm})A_{nozzle}$
- $w \rightarrow w_{eff}$, and $I_{sp} \rightarrow I_{sp,eff}$
- *e.g.* Space Shuttle Solid Rocket Booster: $I_{sp,sealevel}$ = 242 s, $I_{sp,vacuum}$ = 268.6 s
- In this course: indices "effective" deleted, but….

See previous sheet.

Real flight

Real flight:
- direction of motion not only vertical
- angle of incidence
- gravitational field
- atmospheric drag
- thrust misalignment



[Fortescue & Stark, 1995]

equation of motion:

$$M\frac{d\overline{V}}{dt} = \overline{F} - M\,\overline{g} - \overline{D}$$

Note: all terms are time-dependent!

where

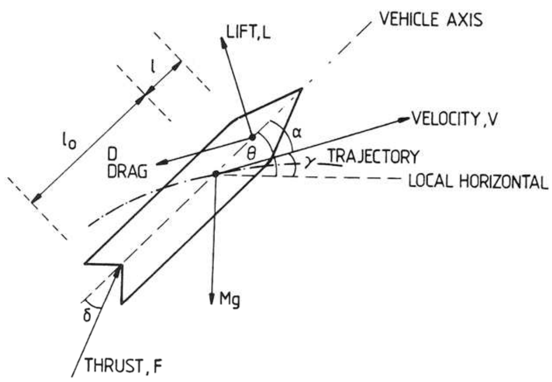
$\overline{F}$ = thrust [N]

$\overline{g}$ = gravitational acceleration [m/s^2]

$\overline{D}$ = atmospheric drag [N]

Note: vector notations.

Real flight (cnt'd)



along velocity direction:

$$M\frac{dV}{dt} = F\cos(\alpha+\delta) - M\,g\sin(\gamma) - D$$

perfect vertical flight:

$$M\frac{dV}{dt} = F - M\,g - D$$

To minimize drag losses, one tries to pass the (dense) atmosphere as rapidly as possible -> vertical flight (or carrier aircraft -> Pegasus).

The (integrated) drag effect is proportional to (integrated) dynamic pressure: $\frac{1}{2}\rho V^2$.

Vertical flight (incl. gravity)

perfect vertical flight, end velocity:

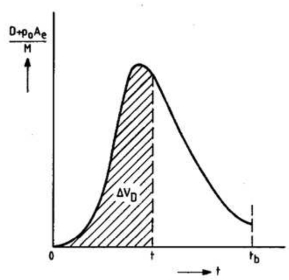
$$dV = -w\frac{dM}{M} - g\,dt - \frac{D}{M}dt$$

integration:

$$V_{end} = w\ln\Lambda - \int_0^{t_{burn}} g\,dt - \int_0^{t_{burn}} \frac{D}{M}dt$$

or

$$V_{end} = V_{end,ideal} - \Delta V_g - \Delta V_D$$



ΔV_g : gravity loss

ΔV_D : drag loss

homogeneous gravity field, no atmosphere:

$$V_{end} = w\ln\Lambda - g_0\,t_b$$

Parameter g_0 is gravitational acceleration at sea level: 9.81 m/s^2. In reality, g is depending on altitude ($g = -\mu/r^2$).

Gravitational acceleration: 9.81 m/s^2 (at sea level), 9.65 m/s^2 (at 50 km altitude), 9.50 m/s^2 (100 km altitude), 8.43 m/s^2 (500 km altitude) -> errors ranging from few percent to 15%.

Vertical flight (incl. gravity) (cnt'd)

end velocity:

$$V_{end} = w \ln \Lambda - g_0 t_b$$

or

$$V_{end} = g_0 I_{sp} \left[\ln \Lambda - \frac{1}{\Psi_0} \left(1 - \frac{1}{\Lambda} \right) \right]$$

gravity loss:

$$\Delta V_g = \frac{g_0 I_{sp}}{\Psi_0} \left(1 - \frac{1}{\Lambda} \right)$$

impulsive shot:

$$t_b \Rightarrow 0 \quad \text{so} \quad \Psi_0 \Rightarrow \infty \quad \text{so} \quad \Delta V_g \Rightarrow 0$$

Impulsive shot: in infinitely small time, so no gravity losses by definition.

Vertical flight (incl. gravity) (cnt'd)

mechanical load:

at launch:

$$\frac{a_{begin}}{g_0} = \frac{1}{g_0}\frac{dV}{dt} = \frac{F}{M_{begin}\, g_0} - 1 = \Psi_0 - 1$$

at burnout:

$$\frac{a_{end}}{g_0} = \frac{1}{g_0}\frac{dV}{dt} = \frac{F}{M_{end}\, g_0} - 1 = \frac{F}{M_{begin}\, g_0}\frac{M_{begin}}{M_{end}} - 1 = \Psi_0\,\Lambda - 1$$

Thrust acts in one way, but gravity in the other -> subtract value "1".

Typical requirements: acceleration at launch large enough to lift off ("we have.….") from platform, and acceleration at burnout not too large to crush the vehicle (note: tanks depleted, so total mass much smaller than initial mass).

Vertical flight (incl. gravity) (cnt'd)

burnout height:

$$h_b = \int_0^{t_b} V\,dt = \frac{w}{m}\int_{M_{begin}}^{M_{end}} \ln\frac{M}{M_0}\,dM - \int_0^{t_b} g_0\,t\,dt =$$

$$= w\,t_b\left(1-\frac{\ln\Lambda}{\Lambda-1}\right) - g_0\frac{t_b^{\,2}}{2} =$$

$$= \frac{g_0\,I_{sp}^{\,2}}{\Psi_0}\left[\left(1-\frac{\ln\Lambda+1}{\Lambda}\right)-\frac{1}{2\Psi_0}\left(1-\frac{1}{\Lambda}\right)^2\right]$$

1st part is identical to burnout height for ideal rocket, without gravity losses, but of course gravity reduces performance. Effect: similar to high-school expression ½*a*t^2 -> ½*g_0*t_b^2.

Impulsive shot: gravity loss reduced to zero, since burn time is zero and burnout height is zero by definition.

Vertical flight (incl. gravity) (cnt'd)

optimum burnout height:

$$\frac{\partial h_b}{\partial \Psi_0} = 0 \quad \Rightarrow \quad \Psi_{0,opt} = \frac{(\Lambda - 1)^2}{\Lambda(\Lambda - 1) - \Lambda \ln \Lambda}$$

$$1 \leq \Lambda \leq \infty \quad \Rightarrow \quad 1 \leq \Psi_{0,opt} \leq 2$$

$$h_{b,\max} = \frac{g_0 I_{sp}^{\ 2}}{2} \left[\frac{\Lambda - \ln \Lambda - 1}{\Lambda - 1} \right]^2$$

TUDelft

Derive yourself. For a given propellant (combination of fuel and oxydizer -> given I_{sp}) and a given ratio M_{begin}/M_{end} (realistic assumption, since one will not fly a launcher with 99% propellant…), what would be the optimal thrust to get burnout altitude as high as possible? -> translates to parameter Ψ_0, or burn time t_b. Here: optimization after Ψ_0 is pursued. As expected, $h_{b,max}$ increases with increasing value for I_{sp} and increasing value for Λ. Question: why optimize burnout height, and not total height (*i.e.* including coasting)?

Vertical flight (incl. gravity) (cnt'd)

coasting (i.e. after burnout, no thrust):

$$M_{end} \frac{dV}{dt} = -M_{end}\, g_0$$

integration: $V - V_{end} = -g_0\, t$

coast time: $t_{coast} = \frac{V_{end}}{g_0}$

total time: $t_{total} = t_{burn} + t_{coast} = \frac{w}{g_0} \ln \Lambda = I_{sp} \ln \Lambda$

Equivalent to high-school equations: V(t) = V(0) + a*t. Here: $V(t_{coast})=0$, $V(0)=V_{end}$ (*i.e.* of burn phase), and $a=-g_0$. Then: summation of time intervals.

Vertical flight (incl. gravity) (cnt'd)

coasting (cnt'd):

distance from point of burnout:

$$\Delta h_{coast} = \int_0^{t_{coast}} V\,dt = V_{end}\,t_{coast} - g_0 \frac{t_{coast}^{\ 2}}{2} = \frac{V_{end}^{\ 2}}{2\,g_0}$$

total altitude:

$$h_{total} = h_{burnout} + \Delta h_{coast} = w\,t_b \left(1 - \frac{\Lambda \ln \Lambda}{\Lambda - 1} \right) + \frac{w^2 \ln^2 \Lambda}{2\,g_0} =$$

$$= \frac{g_0\,I_{sp}^{\ 2}}{\Psi_0} \left(\frac{1}{2} \Psi_0 \ln^2 \Lambda - \ln \Lambda - \frac{1}{\Lambda} + 1 \right)$$

Again: equivalent to high-school rules: s(t) = s(0) + V(0)*t + ½*a*t^2. Here: t -> t_{coast}, V(0)->V_{end} (of burn phase), a->-g_0. Summation of terms.

Vertical flight (incl. gravity) (cnt'd)

coasting (cnt'd):

impulsive shot ($t_b \rightarrow 0$; $\Psi_0 \rightarrow \infty$):

$$h_{total,\max} = \frac{1}{2} g_0 \, I_{sp}^{\;2} \ln^2 \Lambda$$

Impulsive shot: no gravity losses in propelled phase. Absolute limit to performance.

Questions

- What is the energy that I need to launch a satellite to an 800 km orbit?
- How much propellant is needed to achieve this?
- What would be the optimal thrust to maximize payload mass?
- What would be the optimal thrust load to achieve maximum (culmination) altitude?

Questions – 1

Consider a lander hovering 10 m above the surface of the Moon, in search for a suitable landing spot. Given: μ_{moon} = 4906×10^9 m^3/s^2, R_{moon} = 1738×10^3 m; dry mass of vehicle (*i.e.* structure + payload) = 500 kg; I_{sp} = 300 s; g_0 = 9.81 m/s^2.

Questions:

1. Compute the gravitational acceleration that the vehicle experiences (g_{moon}).
2. Derive an equation for the propellant mass as a function of time.
3. Compute how much propellant would be needed for a maximum hover period of 1 second.
4. Idem, for a hover period of 1 minute.
5. Idem, for a hover period of 10 minutes.

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

Answers: **(DID YOU TRY??)**

1. g = 1.624 m/s^2
2. $M_{propellant}$ = M_{dry} * (exp((g_{moon}*t)/(I_{sp}*g_0)) – 1)
3. $M_{propellant}$ = 0.28 kg
4. $M_{propellant}$ = 16.8 kg
5. $M_{propellant}$ = 196.2 kg

Questions - 2

Consider a lander hovering 10 m above the surface of the Mars, in search for a suitable landing spot. Given: $\mu_{mars} = 42810 \times 10^9$ m^3/s^2, $R_{mars} = 3402 \times 10^3$ m; dry mass of vehicle (*i.e.* structure + payload) = 500 kg; I_{sp} = 300 s; g_0 = 9.81 m/s^2.

Questions:

1. Compute the gravitational acceleration that the vehicle experiences.
2. Derive an equation for the propellant mass as a function of time.
3. Compute how much propellant would be needed for a maximum hover period of 1 second.
4. Idem, for a hover period of 1 minute.
5. Idem, for a hover period of 10 minutes.

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**
Compare your results with those for the Moon case (previous sheet)

Answers: **(DID YOU TRY??)**

1. g = 3.699 m/s^2
2. $M_{propellant} = M_{dry} * (\exp((g_{mars}*t)/(I_{sp}*g_0)) - 1)$
3. $M_{propellant}$ = 0.63 kg
4. $M_{propellant}$ = 39.2 kg
5. $M_{propellant}$ = 562.9 kg

Vertical flight (ideal vs. real): Example

Data:

- specific impulse I_{sp} = 200 s
- Ψ_0 = 1.5
- Λ = 5

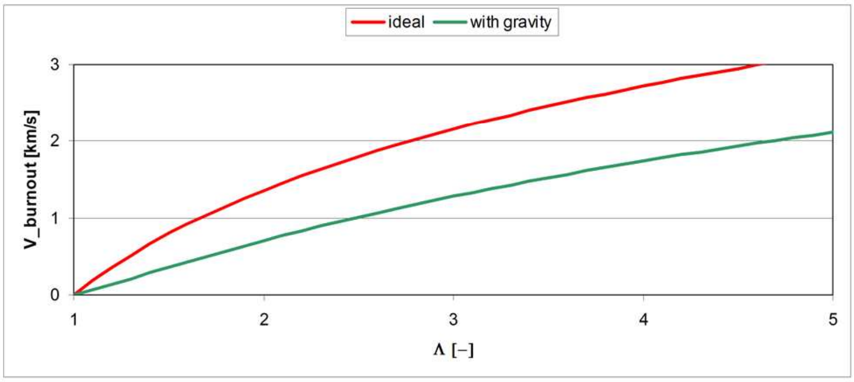
Results:

	ideal	with gravity
Burnout velocity [m/s]	3157.7	2111.3
Burn time [s]	106.7	106.7
Burnout height [km]	125.1	69.3
Culmination height [km]	-	296.5
Culmination height for impulsive shot [km]	-	508.2

Verify these numbers yourself!

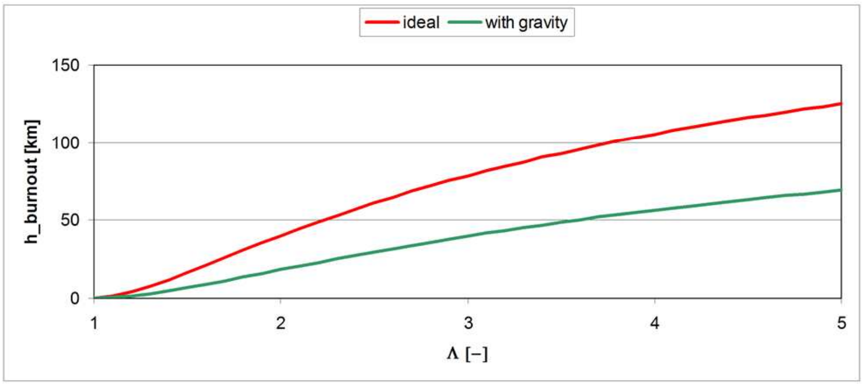
Question: why is there no value for the culmination height in case of an ideal launcher?

Vertical flight (incl. gravity): graphs



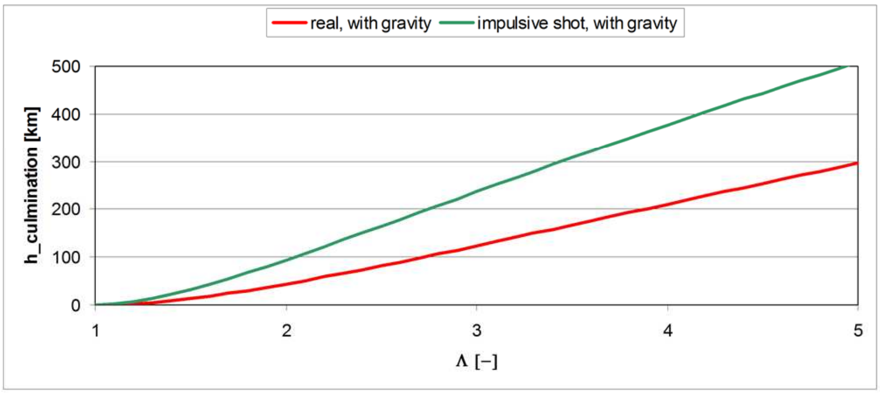
I_{sp} = 200 s; Ψ_0 = 1.5. Significant difference.

Vertical flight (incl. gravity): graphs



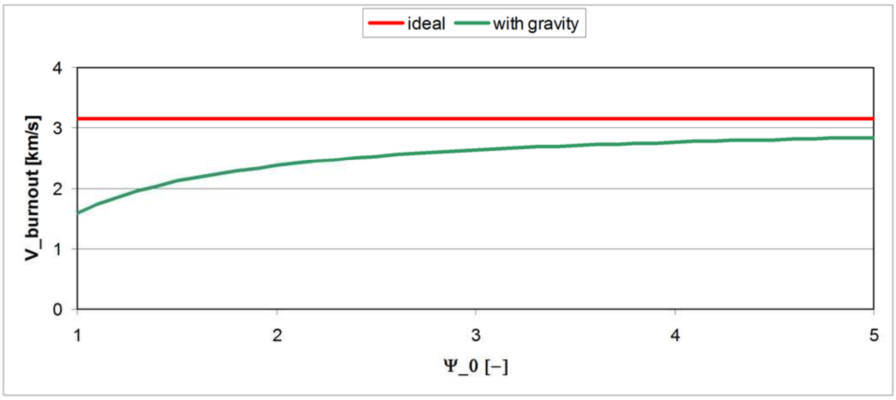
I_{sp} = 200 s; Ψ_0 = 1.5. Significant difference.

Vertical flight (incl. gravity): graphs



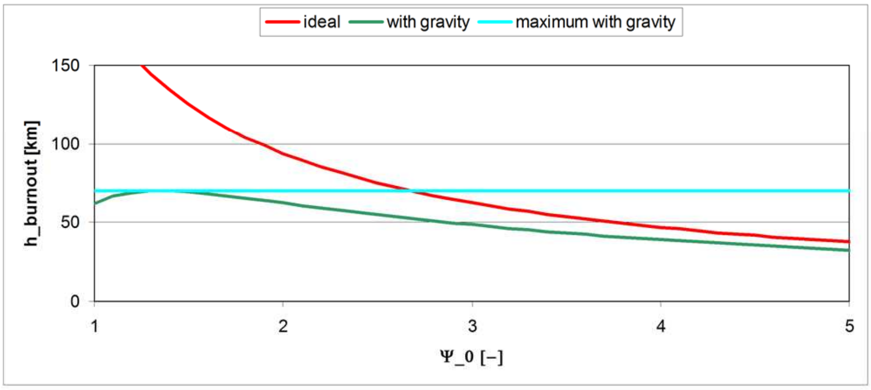
I_{sp} = 200 s; Ψ_0 = 1.5

Vertical flight (incl. gravity): graphs



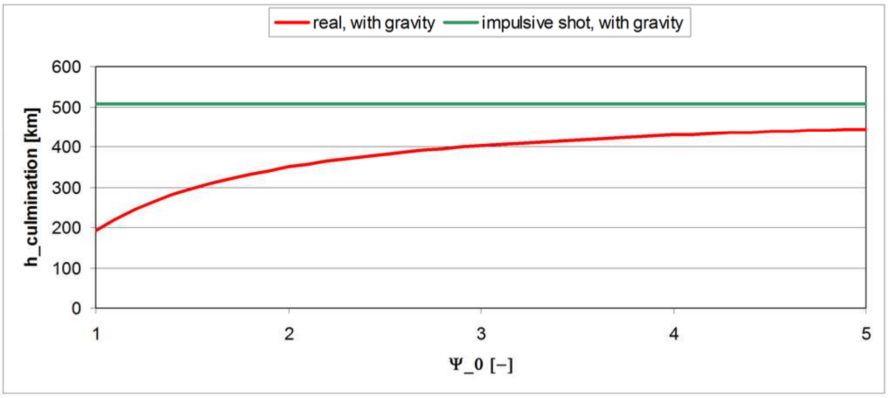
I_{sp} = 200 s; Λ = 1.5

Vertical flight (incl. gravity): graphs



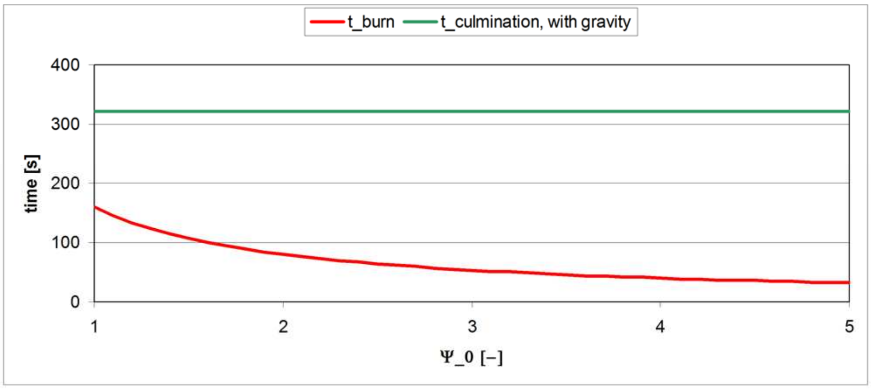
I_{sp} = 200 s; Λ = 1.5

Vertical flight (incl. gravity): graphs



I_{sp} = 200 s; Λ = 1.5 (*i.e.* propellant mass is half of dry mass -> very conservative). The higher the value for Ψ_0, the better an impulsive shot is approximated (of course).

Vertical flight (incl. gravity): graphs

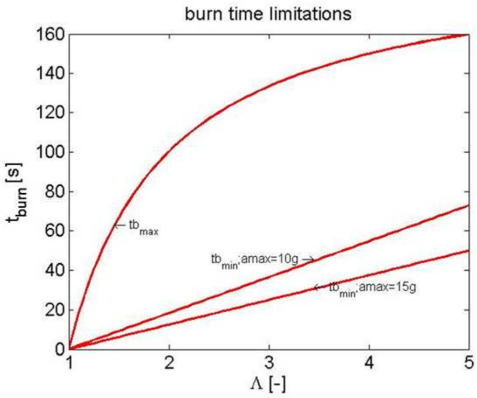


I_{sp} = 200 s; Λ = 1.5. Time until culmination point is independent of value of Ψ_0 (cf. sheet 45).

Vertical flight (incl. gravity) (cnt'd)

Requirements:

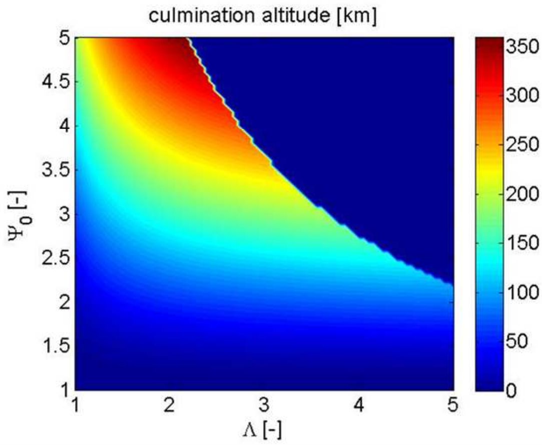
- for lifting off from launch pad: $F > M_0 g_0$ or $\Psi_0 > 1$
- for burn time: $t_b < I_{sp} (1 - 1/\Lambda)$
- mechanical load:
  $\Psi_0 < (1/\Lambda) (a_{end,max}/g_0 + 1)$
  or:
  $t_b > I_{sp} (\Lambda - 1) / (a_{end,max}/g_0 + 1)$



The launcher must be subjected to various constraints.

Vertical flight (incl. gravity) (cnt'd)

Culmination altitudes of single-stage launchers, for I_{sp} = 200 s. Maximum acceleration = 10g. Question: what happens if I_{sp} is increased to 400 s?



The values for Λ and Ψ_0 are larger than 1 for well-designed rockets. The maximum allowed acceleration limits the value for Ψ_0. As for the effect of I_{sp}, where does it appear in the equations? So?

Sounding rocket: example

Véronique (French sounding rocket, 1959-1975):

- payload mass: 59 kg
- structural mass: 207 kg
- propellant mass: 1064 kg
- initial mass: 1330 kg
- thrust: 39142 N
- specific impulse: 184 s

- g_0: 9.81 m/s^2

France is the lead country in Europe in the area of launcher development.

Sounding rocket : example (cnt'd)

Results (NB. Including gravity, but NO drag):

- mass ratio: $\Lambda = 5$
- thrust load: $\Psi_0 = 3$
- burn time: $t_b = 49$ s
- exhaust velocity: $w = 1805$ m/s
- acceleration at launch: $a_0/g_0 = 2$
- acceleration at burnout: $a_{end}/g_0 = 14$
- velocity at burnout: $V_{end} = 2423.8$ m/s
- altitude at burnout: $h_{burnout} = 41.1$ km
- coast time: $t_c = 247.1$ s
- height gain during coast: $\Delta h_{coast} = 299.4$ km
- total flight time: $t_{total} = 296.1$ s
- altitude culmination point: 340.6 km

Verify numbers yourself.

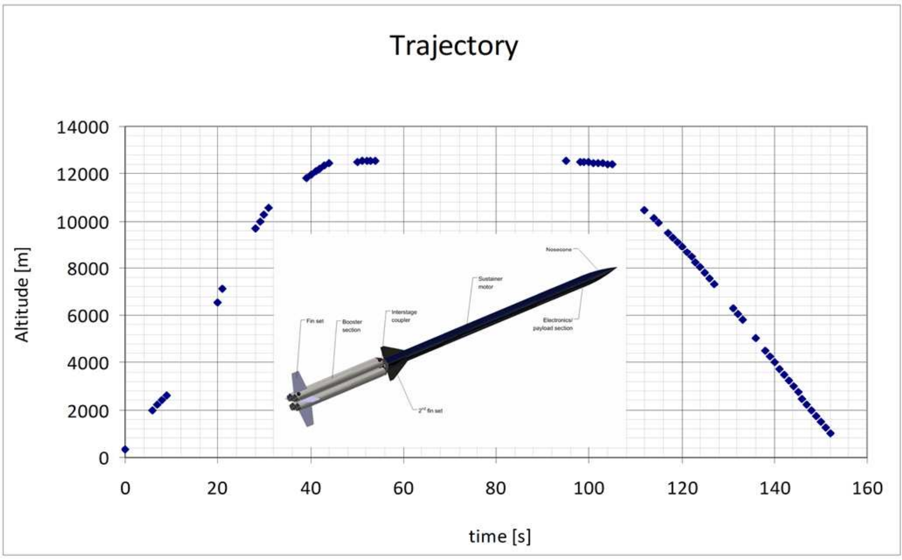
Sounding rocket: example (cnt'd)

Comparison of theoretic data and real flight data Véronique:

	ideal	with gravity, vacuum	real (with gravity and atmosphere)
velocity in burnout point [km/s]	2.905	2.424	1.875
height of burnout point [km]	52.9	41.1	35
culmination altitude [km]	-	340	219

Clear illustration of effects of gravity and atmosphere.

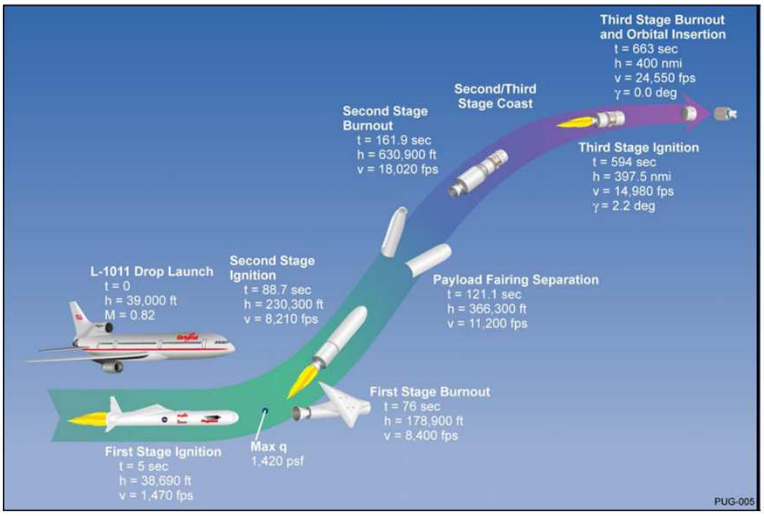
Example: launch trajectory of Stratos



Stratos is a project of DARE (”Delft Aerospace Rocket Engineering”), which was launched on March 17, 2009, from Kiruna (Sweden) and set a new height record for amateur launchers at 12.55 km. If a certain height is the target, how does one design the launcher to achieve this?

Launcher example

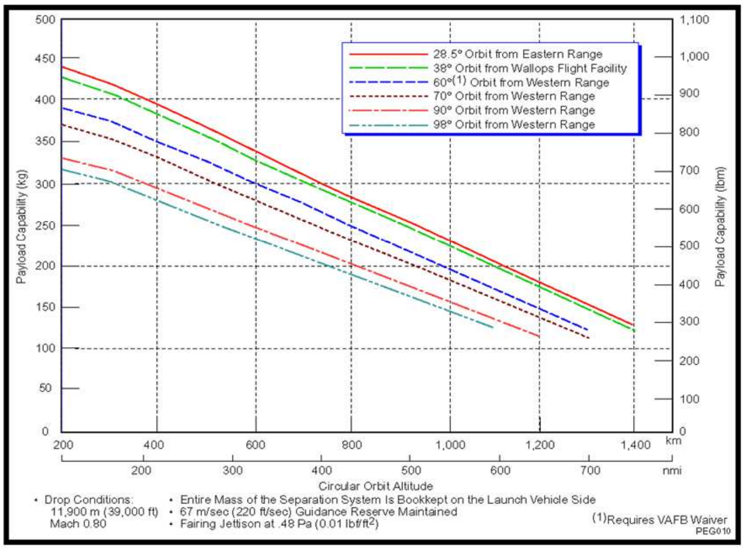
Pegasus XL mission profile [OSC, 2007]



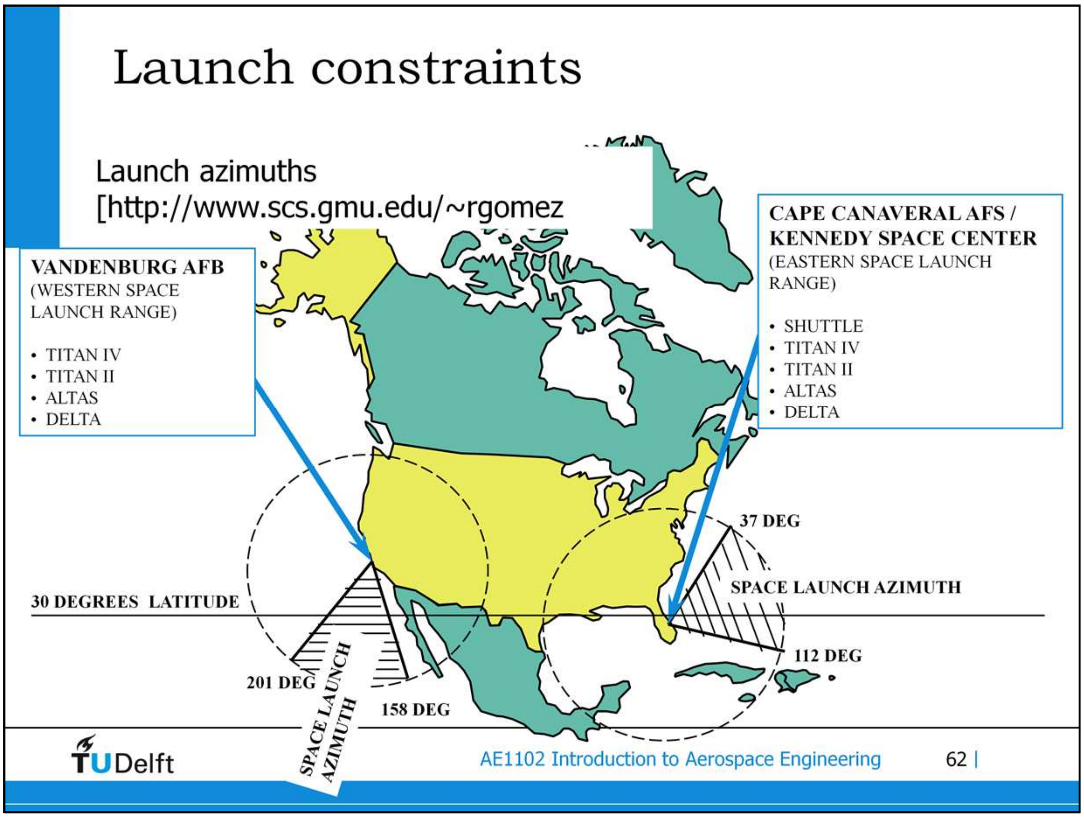
Sorry for the English metrics. “fps” = “feet per second”; “ft” = “feet”; “nmi” = “nautical mile”

Launcher example (cnt'd)

Payload capacity Pegasus XL [OSC, 2007].



The amount of payload that can be delivered to orbit is dependent on inclination (*i.e.* the positive effect of Earth rotation).



The launch azimuth is measured from direction North, positive in clock-wise direction. Potential hazards to populated areas result in limitations in launch azimuth (indicated by the grey areas in this plot), which has consequences, in turn, for the performance of launch vehicles (*i.e.* the profit from Earth rotation can be less than optimal).

Further reading

- Koelle, D.E., Cost Analysis of Present Expendable Launch Vehicles as contribution to Low Cost Access to Space Study. In: (2nd ed.), *Technical Note TCS-TN-147 (96)*, TransCostSystems, Ottobrun, Germany (December 1966).
- Parkinson, R.C., Total System Costing of Risk in a Launch Vehicle. In: *44th International Astronautical Congress* (2nd ed.), *AA-6.1-93-735* (16–22 Oct., 1993) Graz, Austria .
- Isakowitz, S.J.. In: (2nd ed.), *International Reference Guide to Space Launch Systems*, American Institute for Aeronautics and Astronautics, Washington DC (1991).
- "ESA Launch Vehicle Catalogue", *European Space Agency, Paris, Revision 8: December 1997*.
- http://www.orbital.com   info on Pegasus, Taurus and Minotaur
- users.commkey.net/Braeunig/space/specs/pegasus.htm
- http://arianespace.com/english/leader_launches/html
- http://www.boeing.com/defence-space/space/delta/record.htm)