

Introduction to Aerospace Engineering

Lecture slides



Selection of material & structure

Fatigue & durability

Faculty of Aerospace Engineering
23-12-2011

Learning objectives

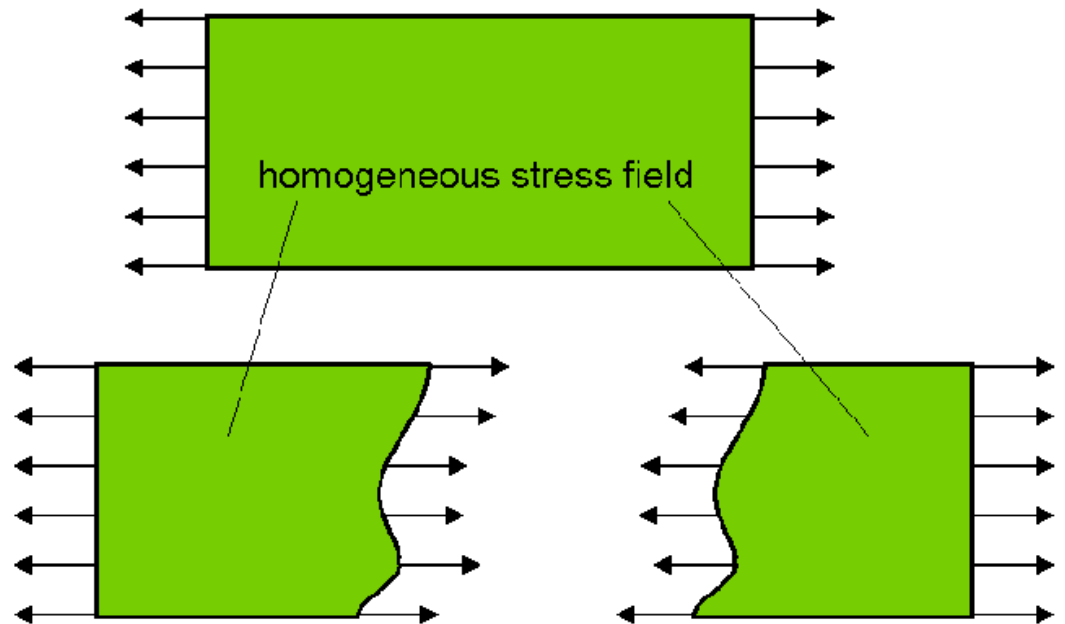
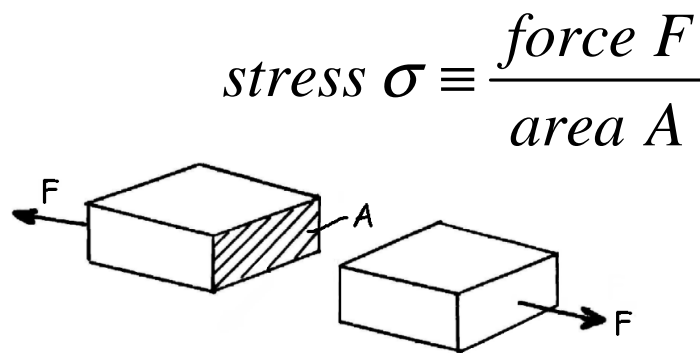
Student should be able to...

- Explain the meaning of stress & strain concentrations
- Give the definition of fatigue
- Explain & work with S-N curves
- Explain why stress concentration factors can not be applied for cracks

Stress-strain concentrations

Flat panel without notch

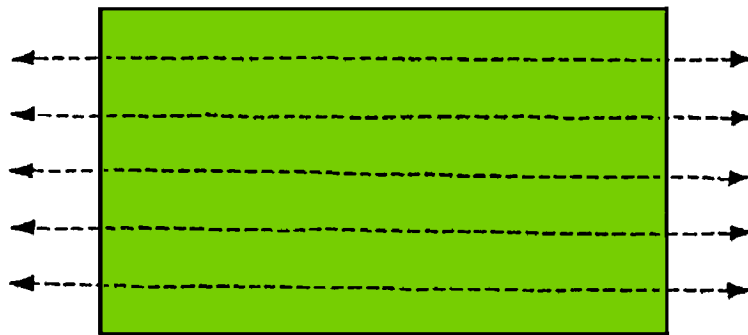
- The normal stresses are the same everywhere



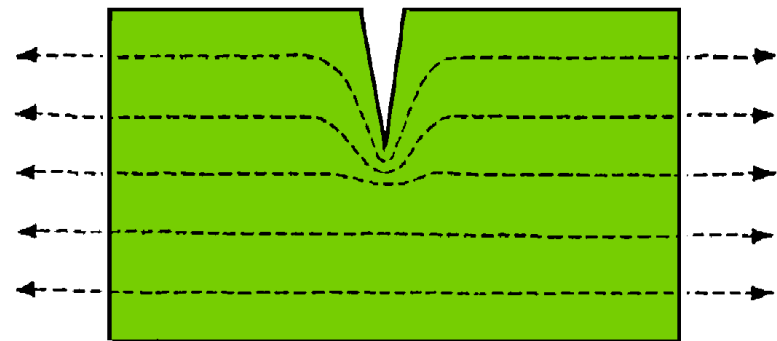
Stress-strain concentrations

Flat panel with notch

- A notch creates a disturbance in the stress flow



Homogeneous stress field



Stress field with disturbance

- Notch → not only decrease in cross section (increase average σ)
→ also concentration of stress (K_t)

Stress-strain concentrations

Flat panel with notch

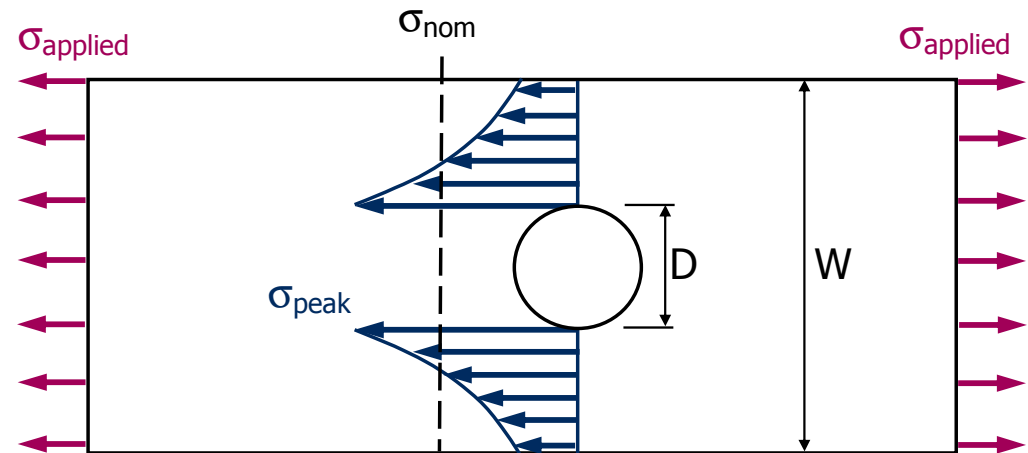
- Definition

- Nominal stress = average stress in the net-section higher than σ_{applied}

$$\sigma_{\text{nom}} = \frac{W}{W - D} \sigma_{\text{applied}}$$

- Stress concentration factor K_t

$$K_t = \frac{\sigma_{\text{peak}}}{\sigma_{\text{nom}}}$$



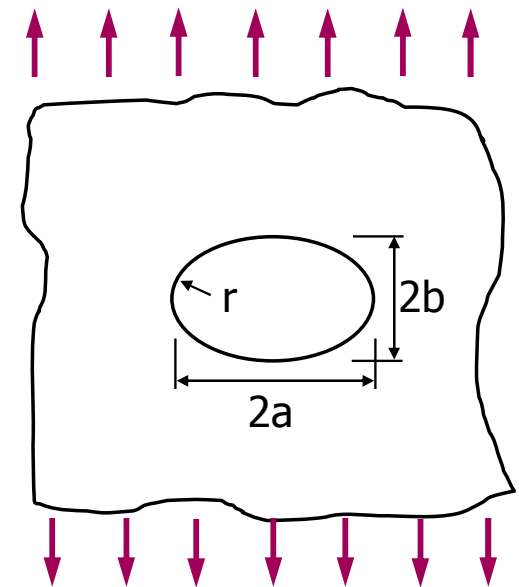
Stress-strain concentrations

Flat panel with notch

- Consider an isotropic infinite sheet with an elliptical hole

$$K_t = \frac{\sigma_{peak}}{\sigma_{nom}} = 1 + 2\frac{a}{b} = 1 + 2\sqrt{\frac{a}{r}}$$

- For a circular hole: $K_t = 3$, independent of hole dimensions!

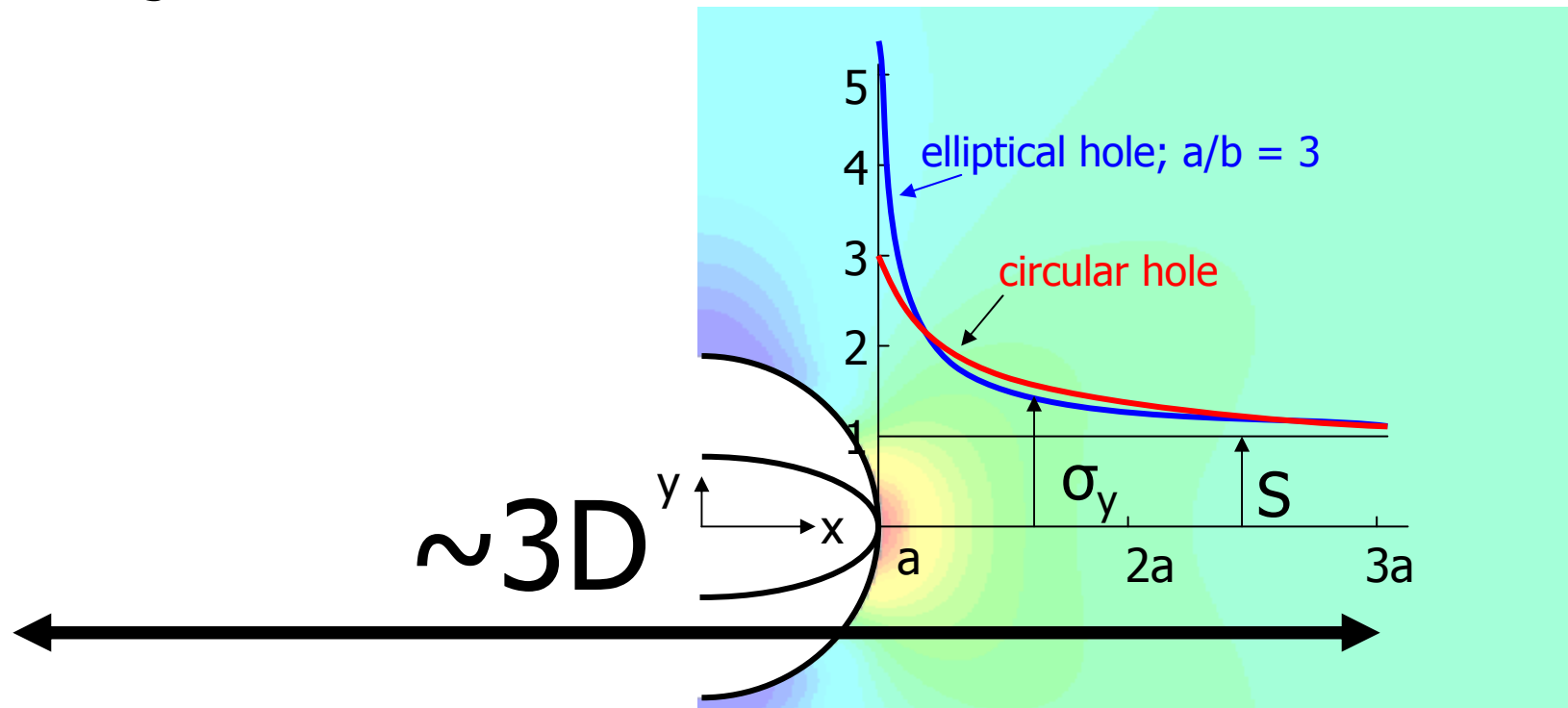


- NB: Linear elastic & anisotropic materials (e.g. composites) can be very sensitive for notches

Stress-strain concentrations

Saint Venant's principle

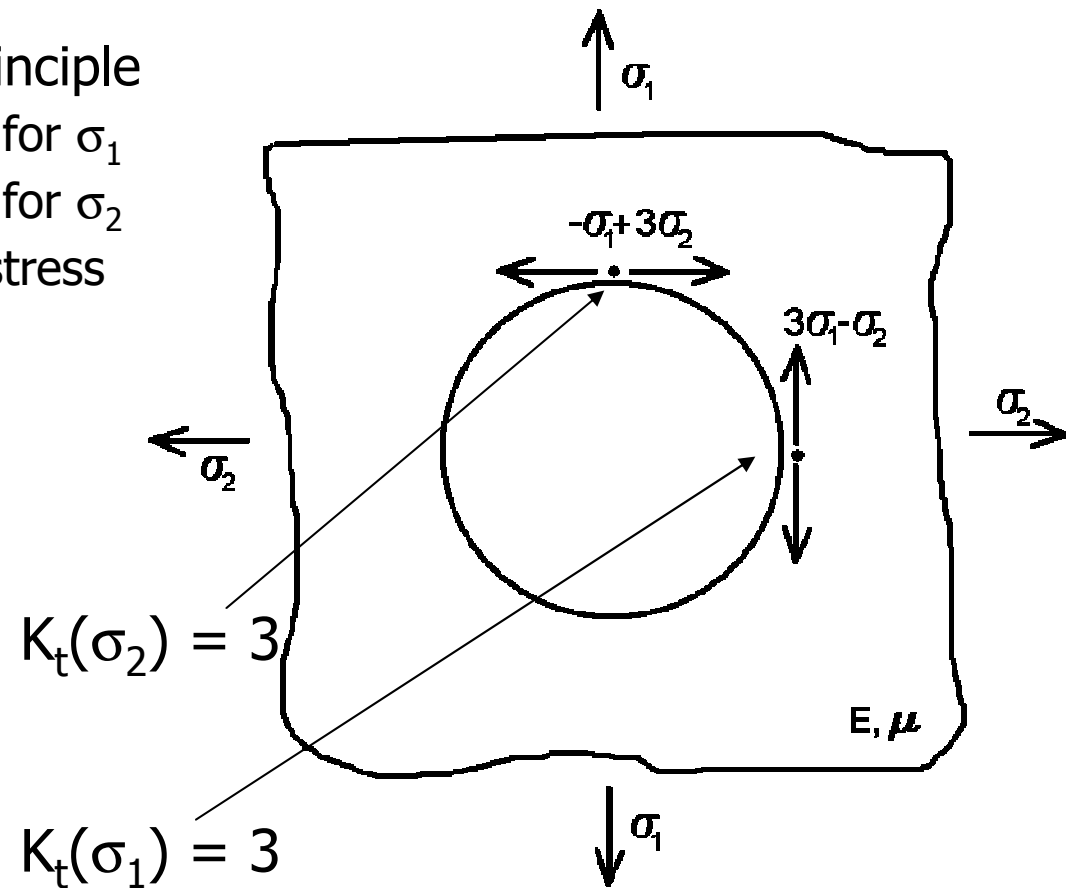
- Disturbance is limited to the direct neighbourhood of the notch causing the disturbance!



Stress-strain concentrations

Plane stress - superposition

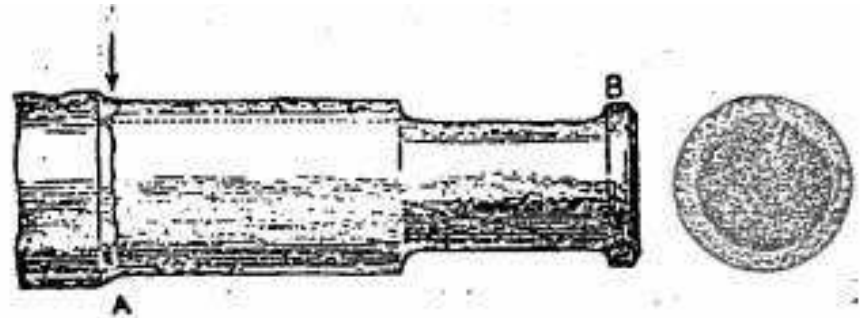
- Use superposition principle
 1. Determine stresses for σ_1
 2. Determine stresses for σ_2
 3. Superimpose both stress systems



Stress-strain concentrations

Versailles Railway Accident (1842)

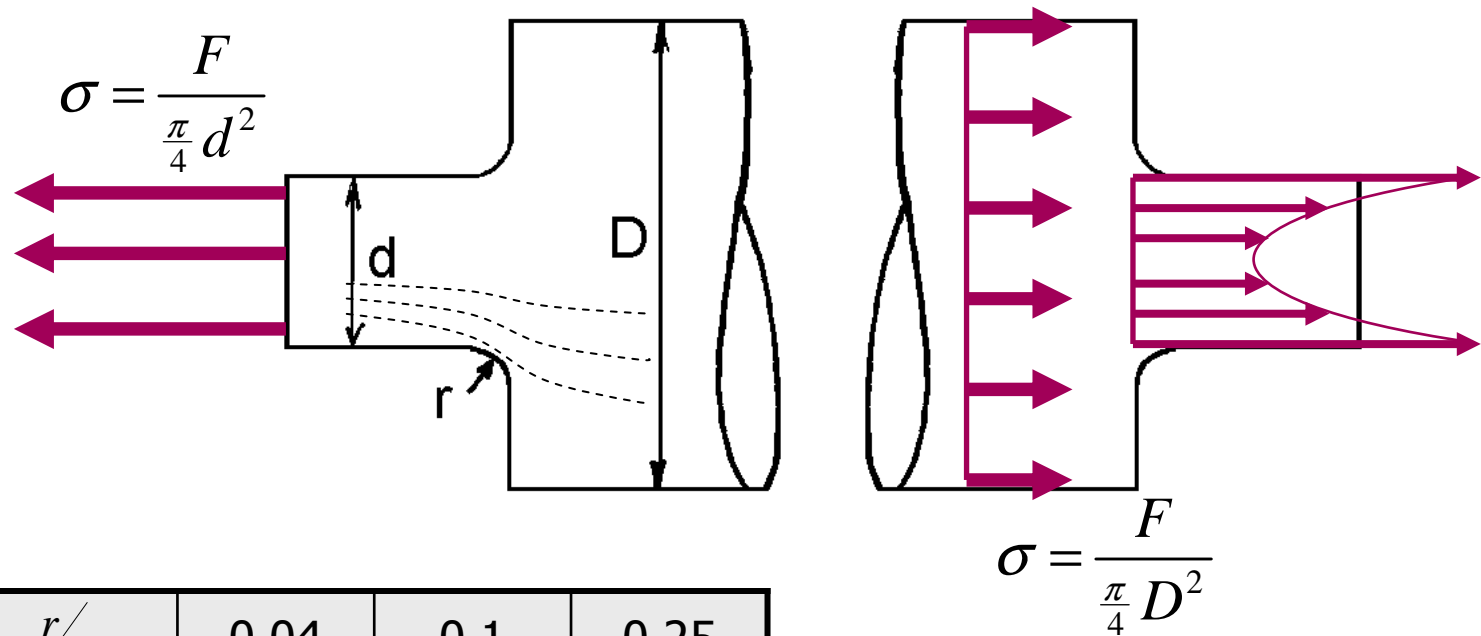
- Story
 - Long train: 17 carriages, 3 locomotives, and 1500 passengers.
 - Front axle of first locomotive failed. Second locomotive smashed into first; boiler thrown into air and burst; fire started
 - Approx. 60 fatalities.
- Investigation by W.J.M. Rankine:
 - Fatigue and effect of stress concentrations (stress raisers)



Stress-strain concentrations

Axle with 'reinforcement'

- Thickness increase to strengthen the axle locally implies weakening

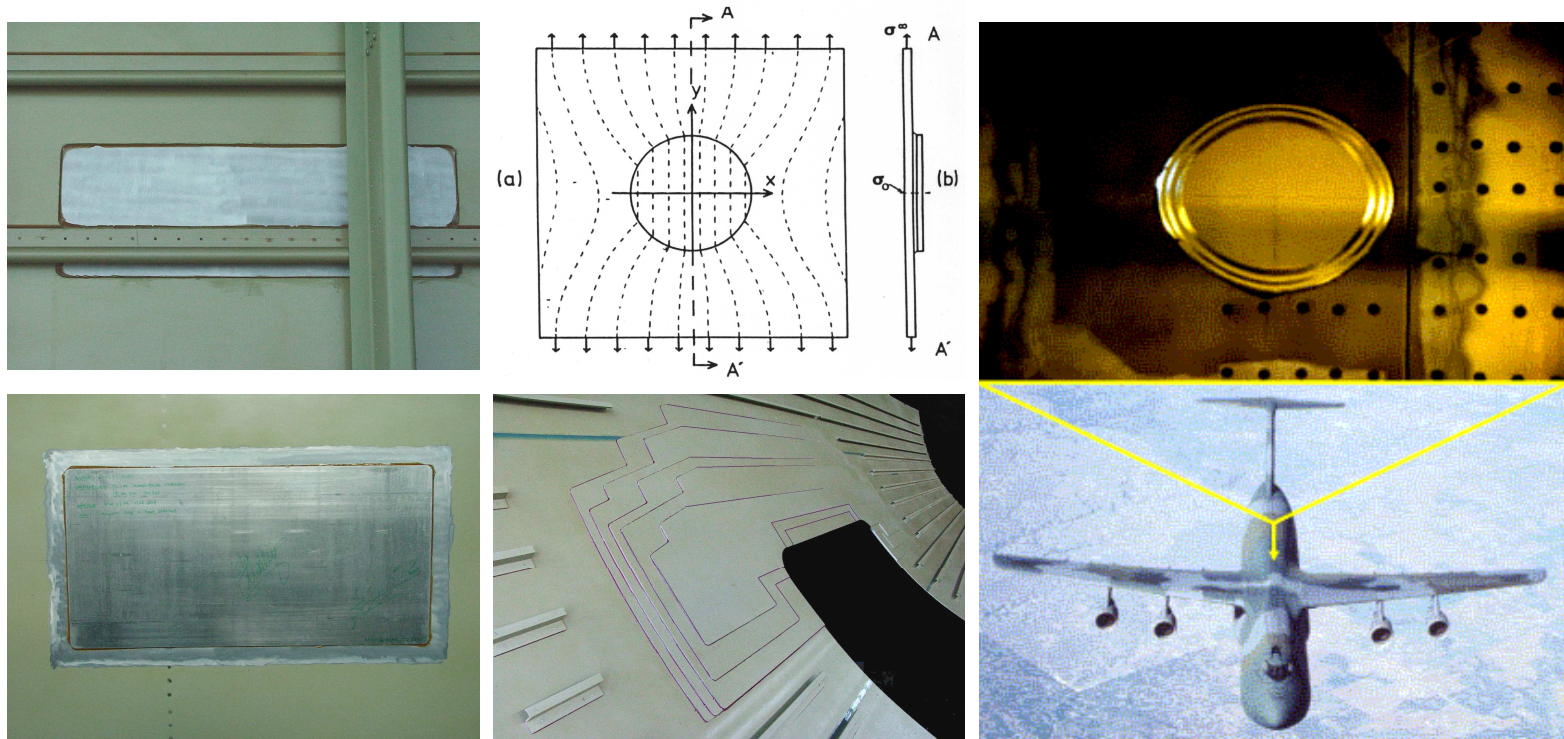


$D/d=2$	r/D	0.04	0.1	0.25
	K_t	3	2	1.5

Stress-strain concentrations

Adding material to reinforce (repair)

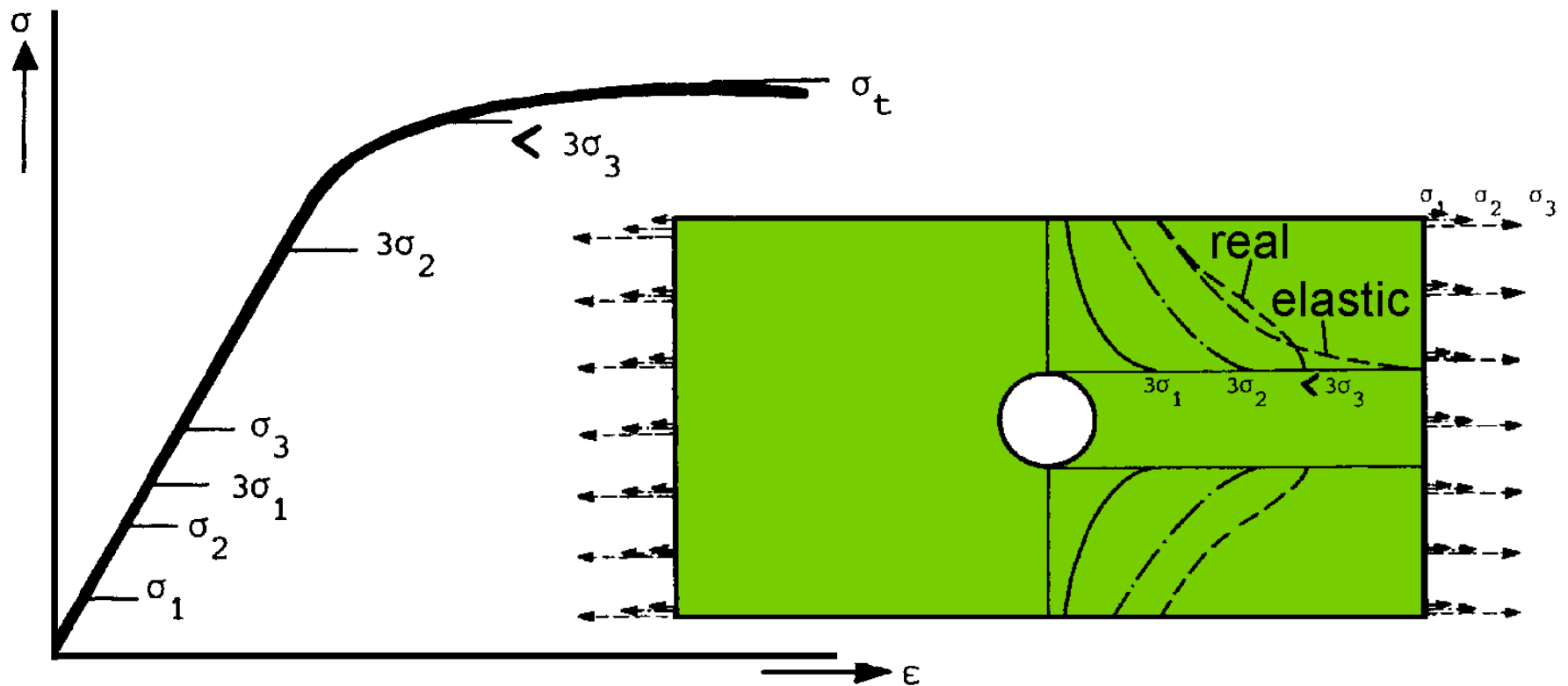
- Repair/reinforcement may 'attract stress' because of increased stiffness → Stress concentration



Stress-strain concentrations

Effect of K_t on ultimate strength

- Plasticity at notch reduces peak stress



Stress-strain concentrations

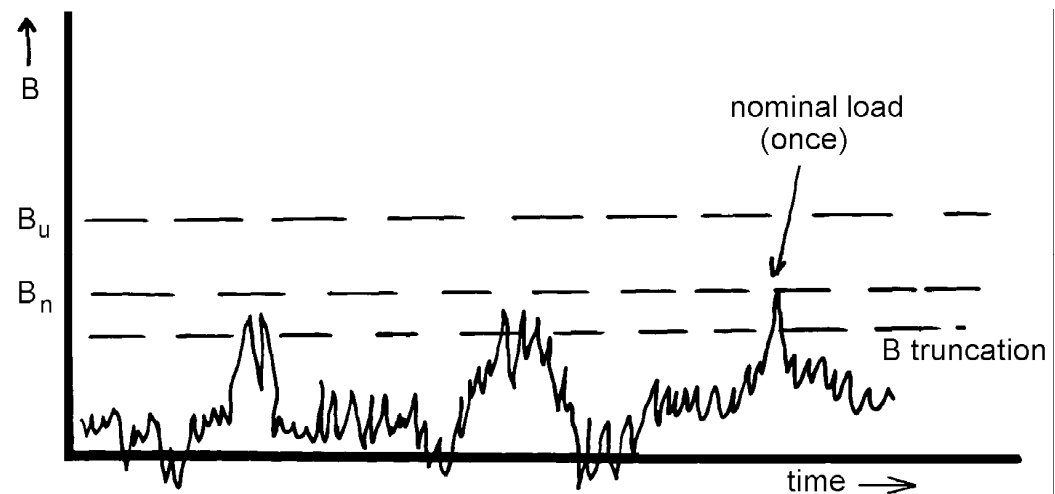
Effect of K_t on ultimate strength

- Concentration of stress & strain
 - A stronger, but more brittle material *can not be loaded as high* as the weaker, more ductile material
- Ductile materials are less notch sensitive under tensile loading
 - Composites do not yield \Rightarrow peak stresses are not leveled off (notch sensitive)
- The strength of the ductile material can be used much better

Fatigue

Definition

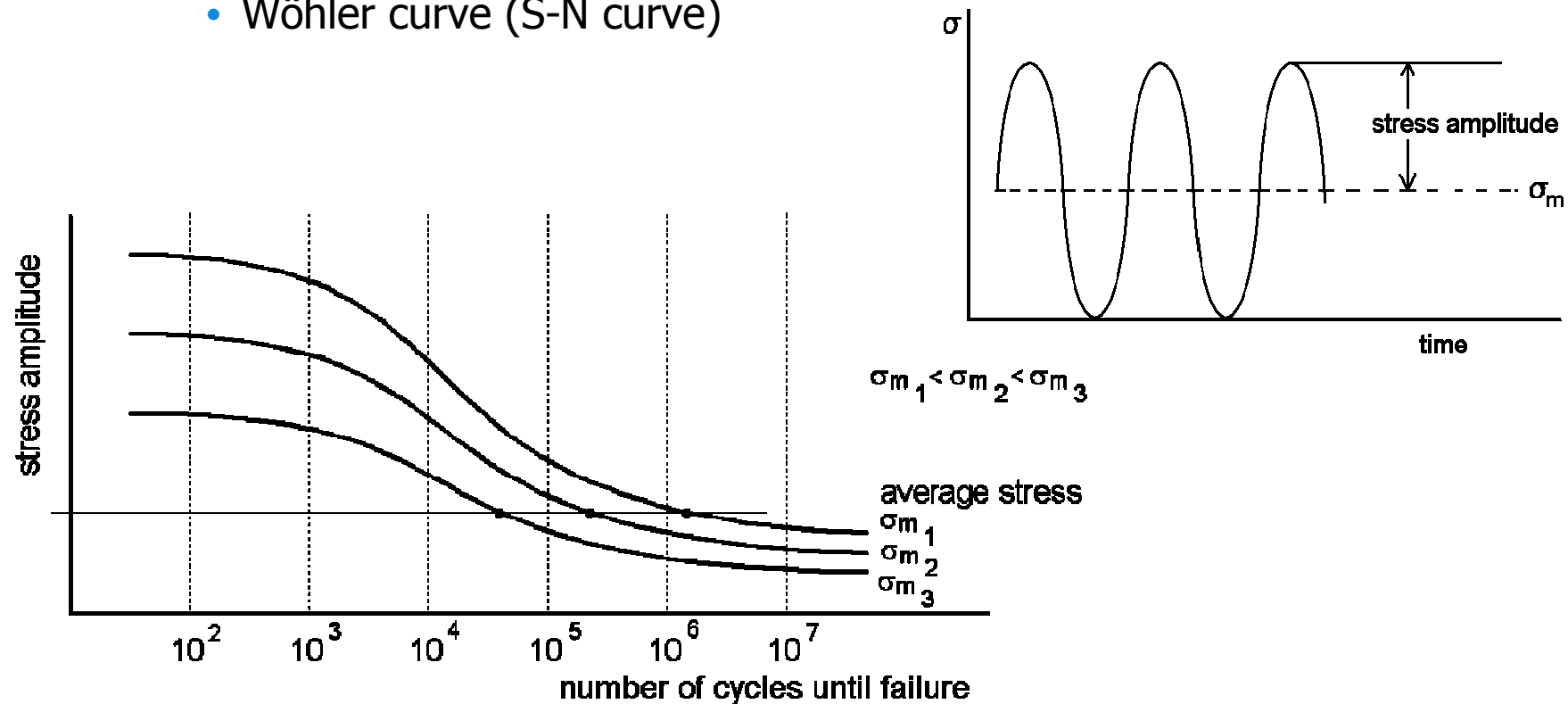
- Damage phenomenon induced by large number of load cycles below ultimate strength resulting in permanent deterioration of material or structure causing a reduction in load bearing capability
 - Load cycles are often related to number of flights, ranging from constant amplitude (fuselage pressurization) to arbitrary spectrum loading (wing loading)



Fatigue

Un-notched panels

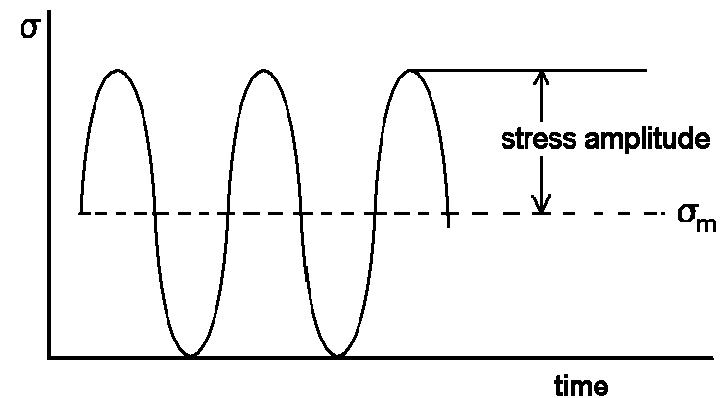
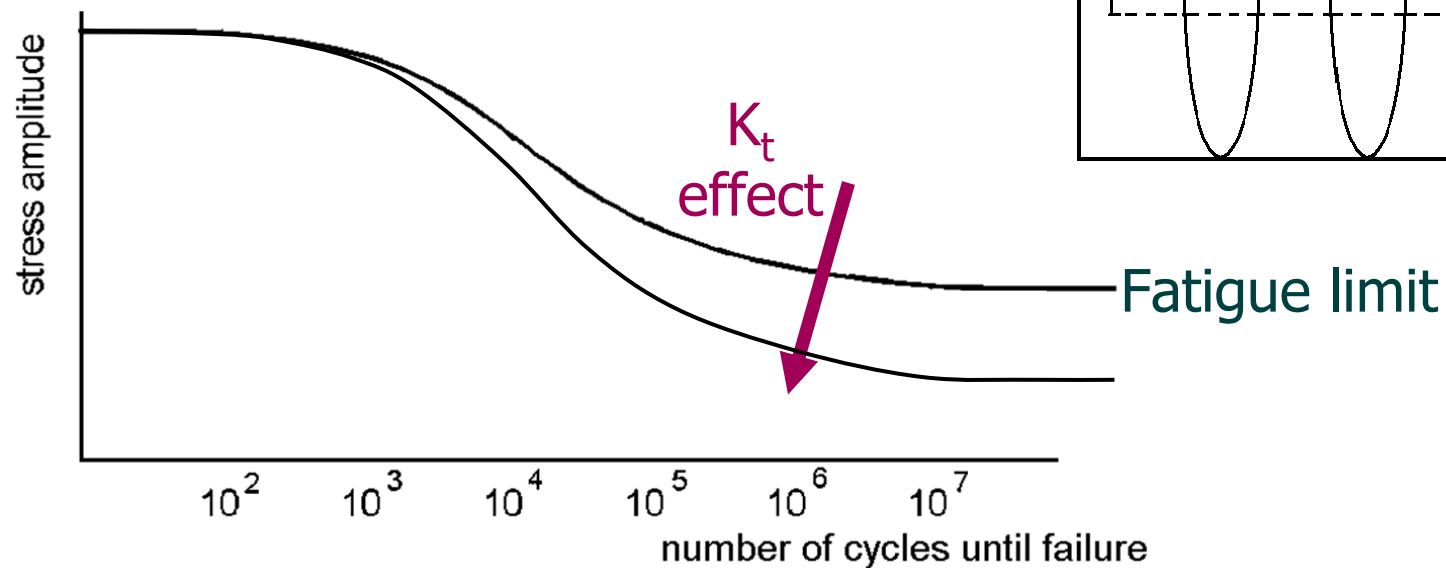
- Fatigue life plotted against flights/load cycles
 - Wöhler curve (S-N curve)



Fatigue

Notched panels

- Fatigue life plotted against flights/load cycles
 - Wöhler curve (S-N curve)
 - Stress concentration reduces fatigue limit



Fatigue

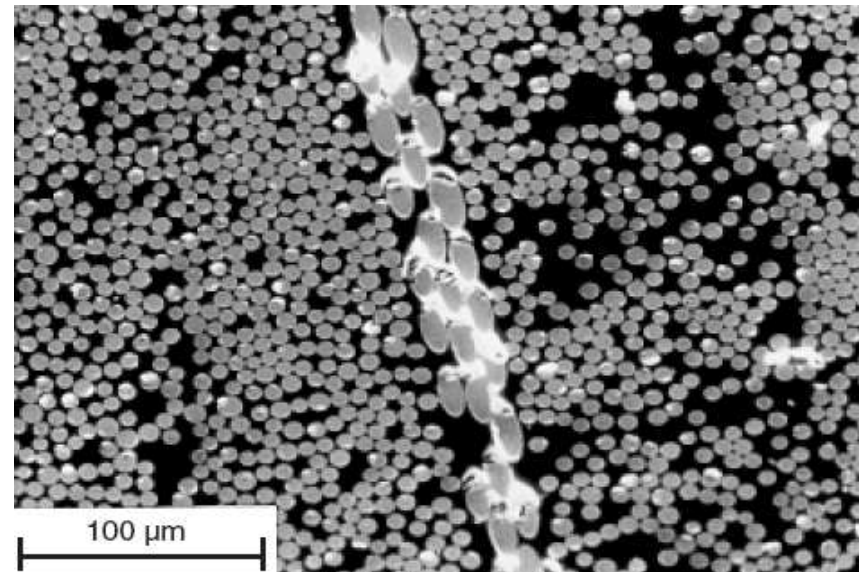
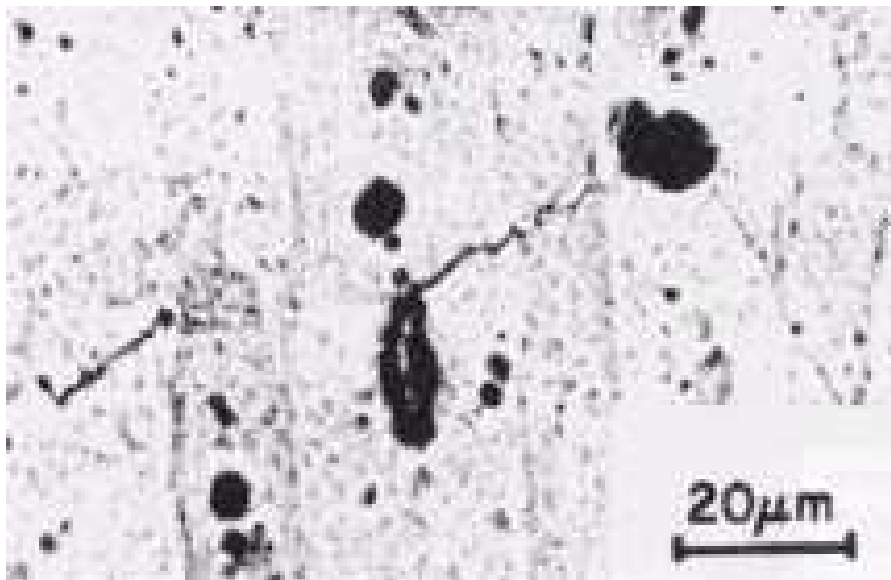
S-N curves

- Initiation & growth of small cracks (<1 mm) = 80-90% of total life
Remainder of life \equiv fast growth to failure (!)
- Experimental strength justification based on S-N fatigue (*safe life*)
 - Experimental life \approx 3 - 4 times required life
- Thickness of fatigue critical parts is greater than required for static strength
- Means to increase life:
 - Damage tolerance approach (inspection/repair/replacement)
 - Avoid stress concentrations and damage initiators in design
 - Less fatigue sensitive materials

Fatigue

Characteristics features

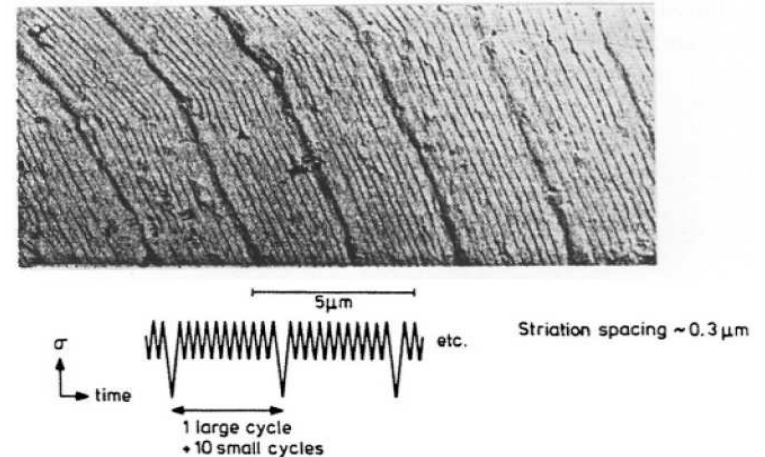
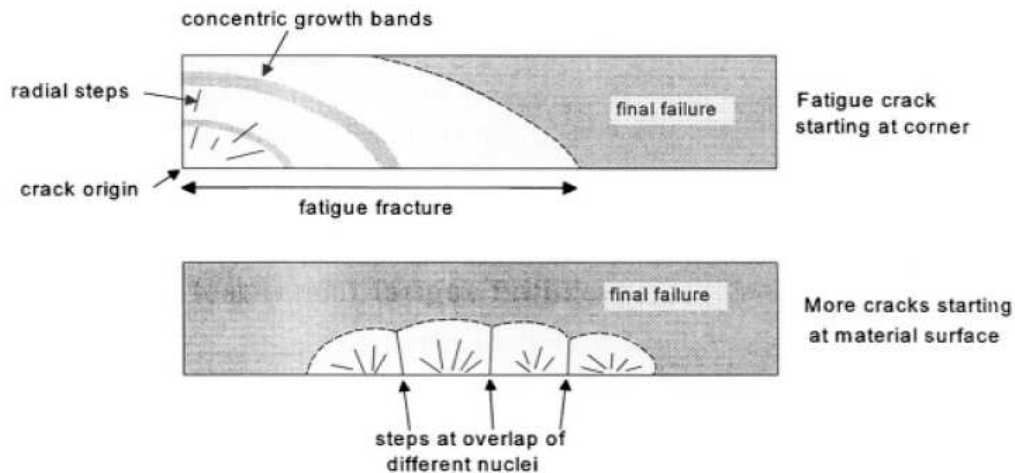
- Nucleation of micro cracks at inclusions in metals
- Nucleation of micro cracks at interface between fibre and resin



Fatigue

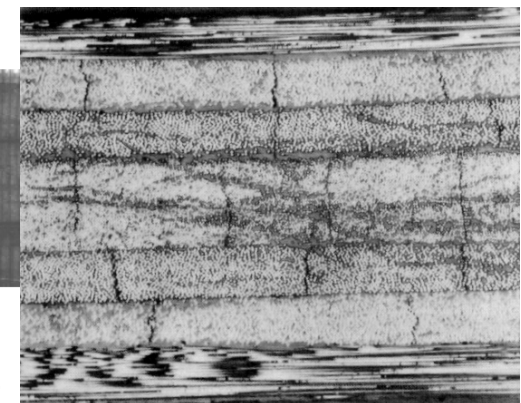
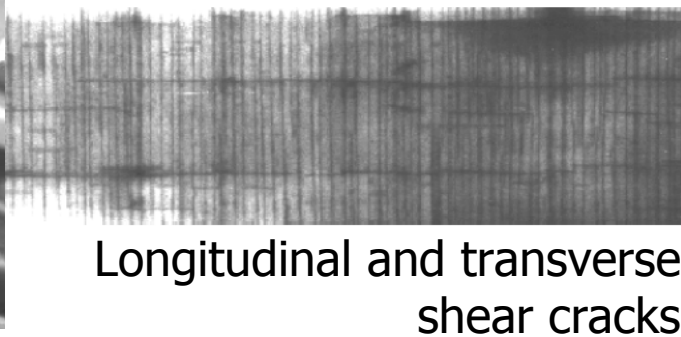
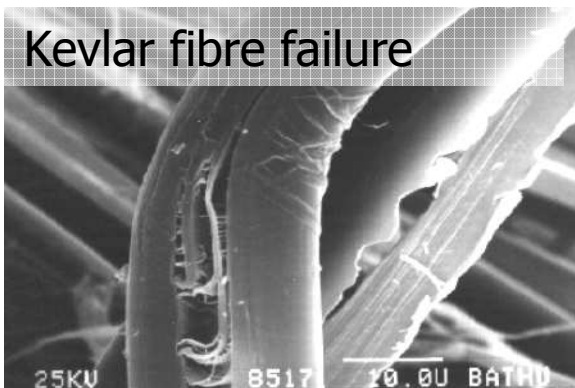
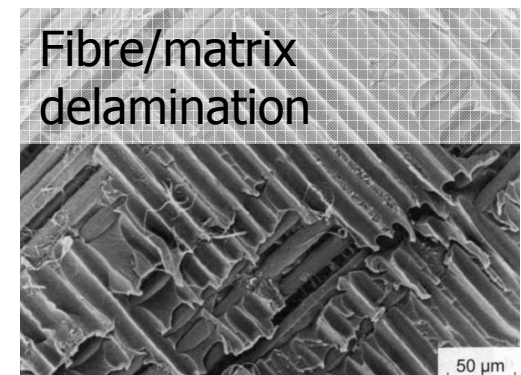
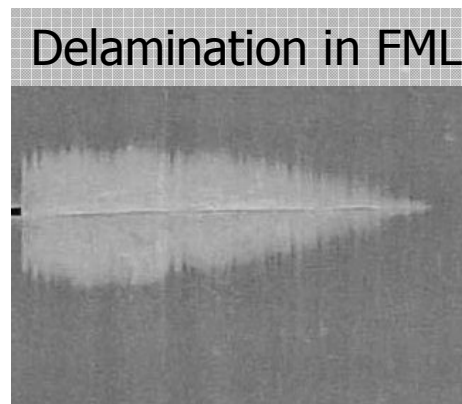
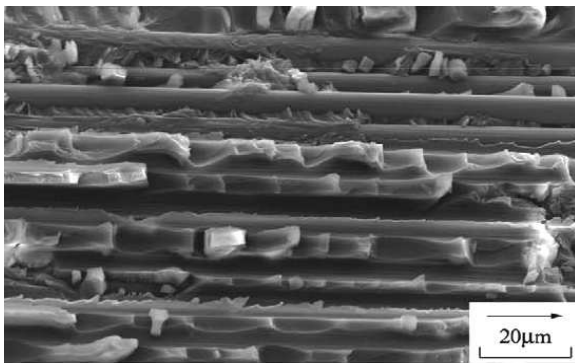
Characteristic features of fatigue failures in metals

- Macroscopic
 - No macro-plastic deformation
 - Growth bands
 - Growth direction \perp main stress
 - Radial steps
 - Number of fatigue nuclei
- Microscopic
 - Striations



Fatigue

Characteristics features of failure in composites



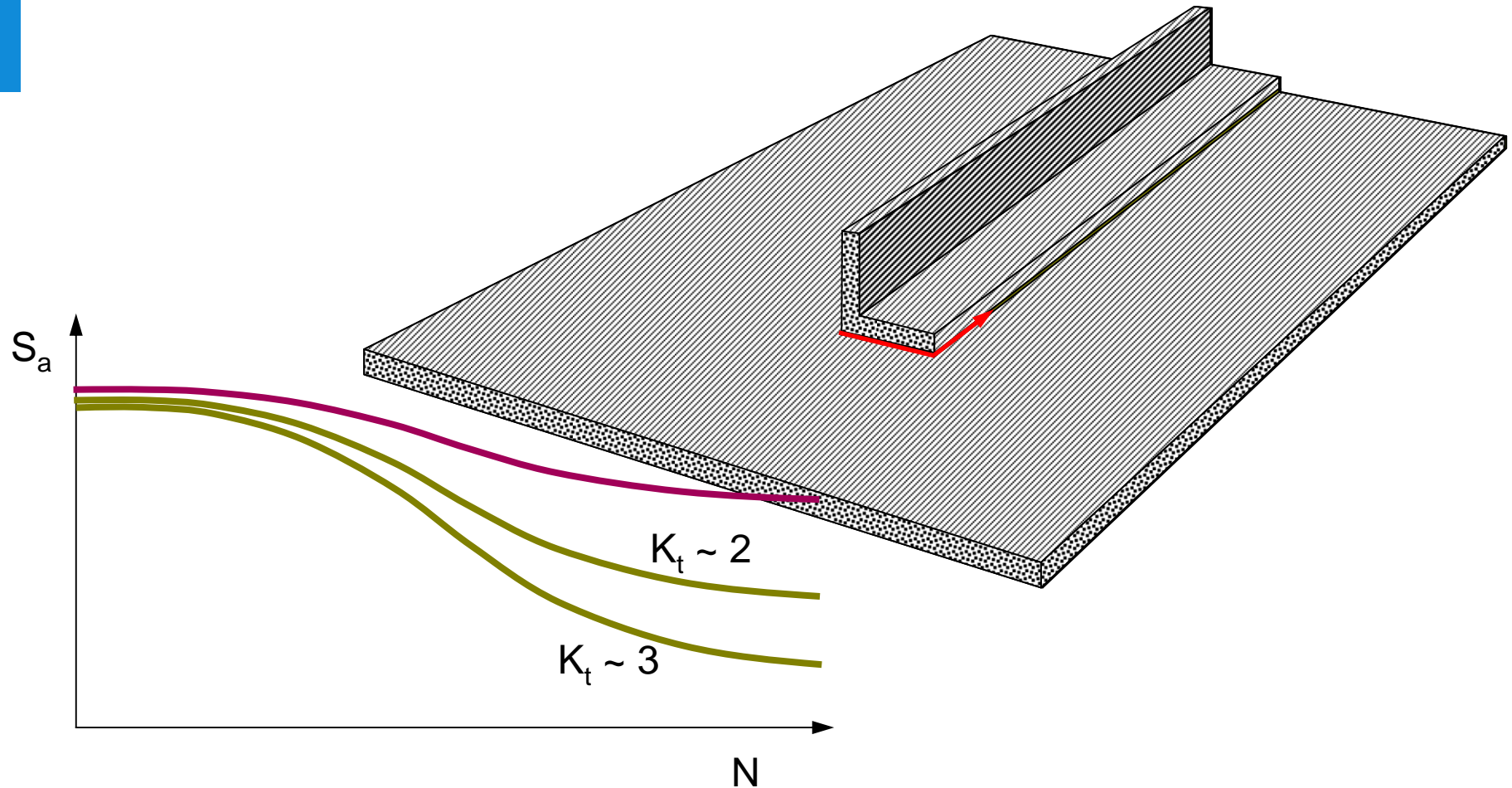
Fatigue

Characteristics features

- Fatigue appears different in metals and composites
 - Metal sensitive to tension-tension fatigue
 - Composites sensitive to compression-tension fatigue
- Static strength requirement for composites (0.35% max strain) often covers fatigue related aspects
 - The fact that fatigue does not show up in most of current applications does not mean that the phenomenon does not exist !
 - (see fatigue studies in composite wind turbine blades)

Fatigue

Fatigue is a structural phenomenon!



Damage tolerance

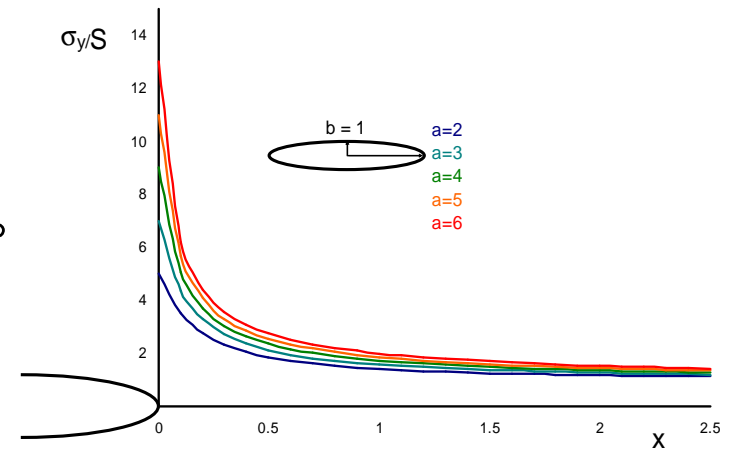
Residual strength in presence of damage

- Once damage has been initiated, how much strength is left?
 - Important aspect in design of damage tolerant structures !

- Stress concentration factor

$$K_t = \frac{\sigma_{peak}}{\sigma_{nom}} = 1 + 2 \frac{a}{b} = 1 + 2 \sqrt{\frac{a}{r}}$$

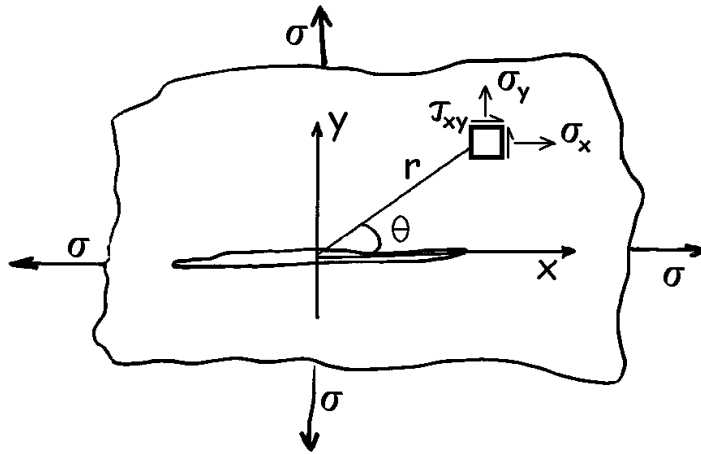
- Crack or delamination: $r \rightarrow 0 \Rightarrow K_t \rightarrow \infty$
 - Concept is meaningless!
- Introduction of stress intensity factor...



Damage tolerance

Stress Intensity Factor

- Parameter describing the singularity in elastic field at the crack tip



$$\sigma_x = \frac{S\sqrt{\pi a}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) - S$$

$$\sigma_y = \frac{S\sqrt{\pi a}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{S\sqrt{\pi a}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \sin \frac{3\theta}{2}$$

- Rewrite as $\sigma_{i,j} = K f_{i,j}(r, \theta)$
 $K = S \sqrt{\pi a}$

Damage tolerance

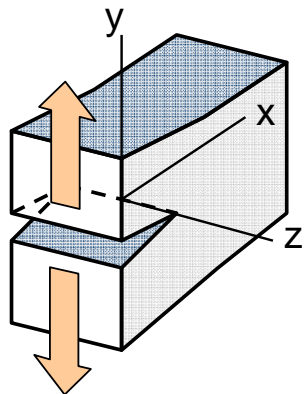
Stress Intensity Factor



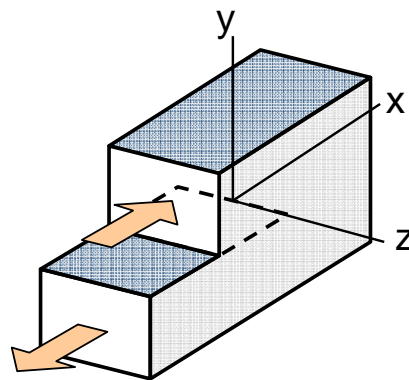
Damage tolerance

Stress Intensity Factor

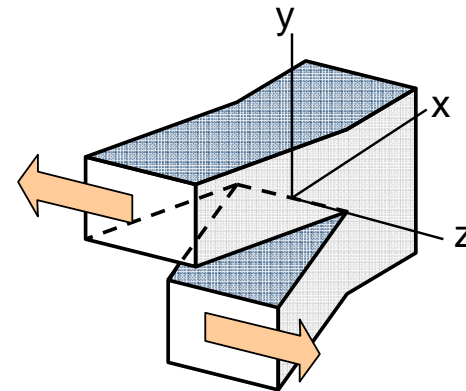
- Three modes to load a crack tip



• Mode I: Tension



Mode II: Shear



Mode III: transverse shear

- Critical K in mode I is K_{Ic} and is called fracture toughness = **material property indicating the sensitivity for cracks under static loading**
- (see lecture on material properties)

Damage tolerance

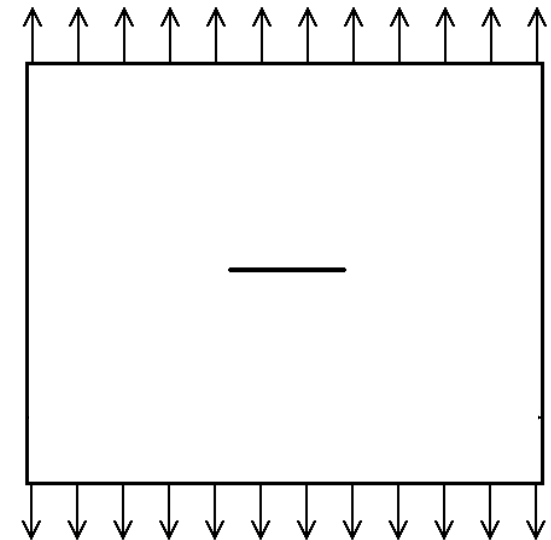
Fracture toughness

- Fracture toughness dependent on material property (ductility!)
- Related critical stress dependent on geometry

$$\sigma_{crit} = \frac{K_{Ic}}{\sqrt{\pi a_{crit}}}$$

- Example: assume $\sigma_{crit} = 1/2\sigma_{0.2}$

Alloy	$\sigma_{0.2}$ (MPa)	σ_c (MPa)	K_{Ic} (MPa \sqrt{m})	a_c (mm)
2024-T3	360	180	40	15.7
7075-T6	470	235	27	4.2
Ti-6Al-4V	1020	510	50	3.1
4340 steel	1660	830	58	1.55



Summary

Fatigue & durability

- Stress Concentration Factors K_t
- Fatigue
- Cracks \Rightarrow Stress Intensity Factor K