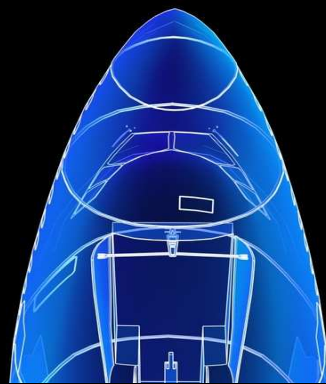
Introduction to Aerospace Engineering

Lecture slides

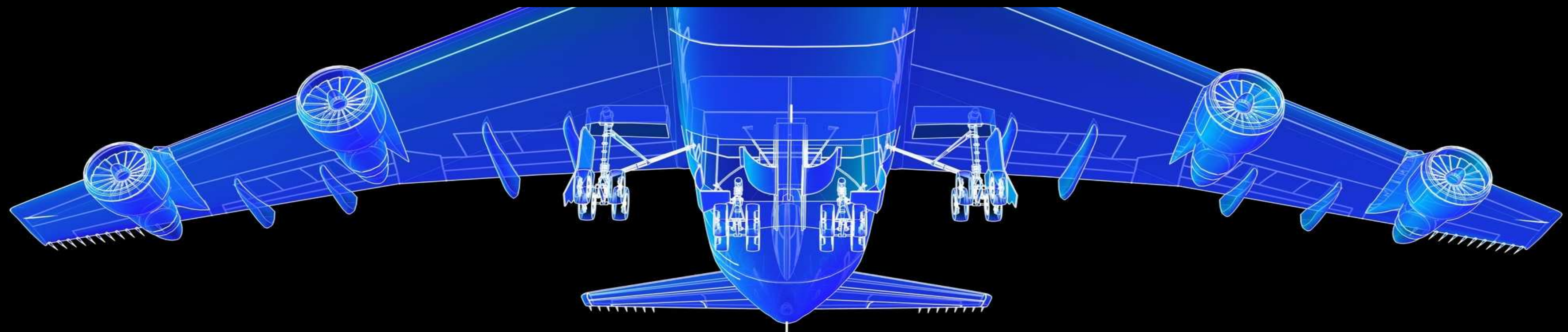


Aircraft & spacecraft loads

Translating loads to stresses

Faculty of Aerospace Engineering

29-11-2011



TU Delft Delft University of Technology

Learning objectives

Student should be able to...

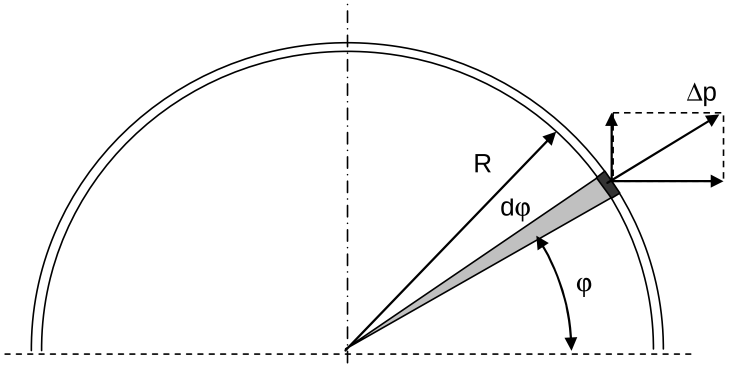
- Calculate the stresses in
  - Fuselage shell due to pressurization (case I)
  - Fuselage shells due to applied torsional load (case II)
  - Wing spars due to applied bending loads (case III)

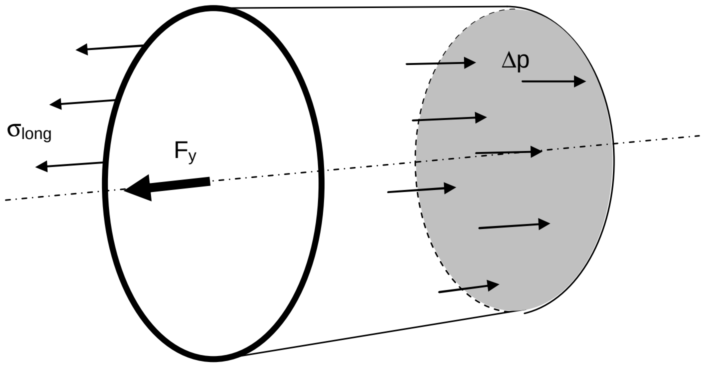
Fuselage structure

Pressurization (case I)

- Revisit stresses in pressure vessel

$$\sigma_{circ} = \frac{pR}{t} \quad ; \quad \sigma_{long} = \frac{pR}{2t}$$





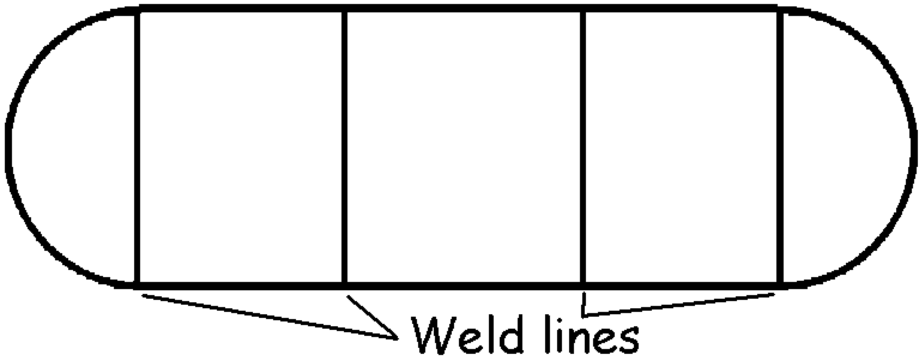
Fuselage structure

Pressurization (case I)

- With $\sigma_{circ} = 2\sigma_{long}$ this is

$$\varepsilon_{circ} = \frac{\sigma_{circ}}{E}\left(1 - \frac{\nu}{2}\right)$$

$$\varepsilon_{long} = \frac{\sigma_{circ}}{E}\left(\frac{1}{2} - \nu\right)$$



- For a metallic pressure vessel with $\nu \approx 0.3$ this means that

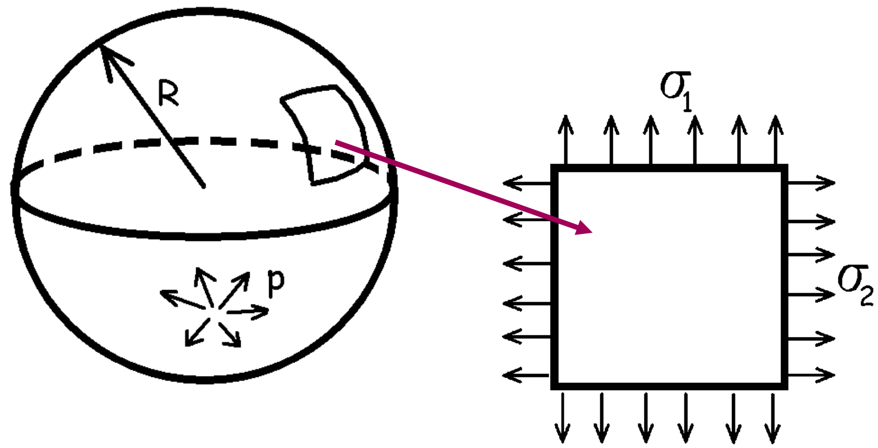
$\varepsilon_{circ} = 4.25\ \varepsilon_{long}$

Fuselage structure

Pressurization (case I)

- For a sphere under pressure

$$\sigma_1 = \sigma_2 = \frac{pR}{2t}$$



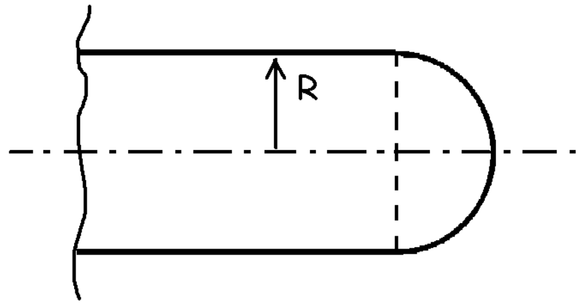
- The strain in a sphere is equal in all directions

$$\varepsilon_{sphere} = \frac{pR}{2t}\frac{1}{E}(1-\nu)$$

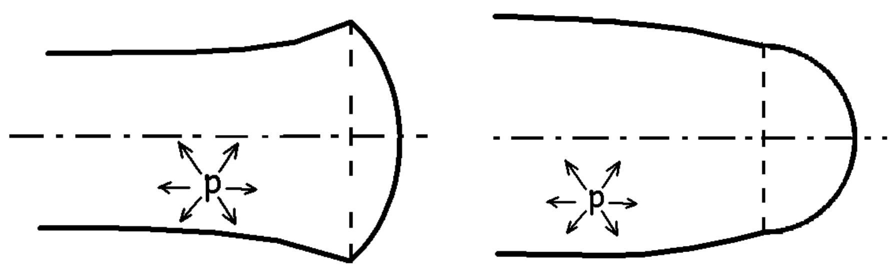
Fuselage structure

Pressurization (case I)

- What ratio $t_{sphere}/t_{cylinder}$ is needed to connect cylinder to sphere in a pressure vessel?



- Avoid discontinuities in ε_{circ} to avoid these deformation mismatches



- Thus $\varepsilon_{circ} = \varepsilon_{sphere}$!

Fuselage structure

Pressurization (case I)

- Derivation

$$\varepsilon_{cylinder} = \varepsilon_{sphere} \quad \Rightarrow \quad \frac{pR_{cylinder}}{t_{cylinder}} \frac{1}{E} \left(1 - \frac{\nu}{2}\right) = \frac{pR_{sphere}}{2t_{sphere}} \frac{1}{E} (1 - \nu)$$

- Thus

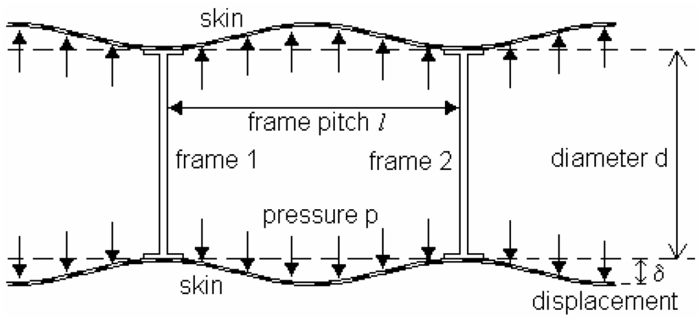
$$\frac{R_{sphere}}{R_{cylinder}} = \frac{2t_{sphere} \left(1 - \nu/2\right)}{t_{cylinder} (1 - \nu)} = 1$$

- With $R_{cylinder}$ = R_{sphere} this means $t_{sphere} \approx 0.4\ t_{cylinder}$

Fuselage structure

Pressurization (case I)

- Aircraft pressure cabin
  - Role frames & stringers (5 - 10%)
  - Other disturbances (doublers around cut-outs)



  - Fracture of fuselage could be disastrous: ‘exploding balloon’
  - No limit load (formal definition: once in the lifetime of the aircraft), “limit” load occurs every flight during pressurization of the cabin
    $\Rightarrow$ safety factor is 2

Fuselage structure

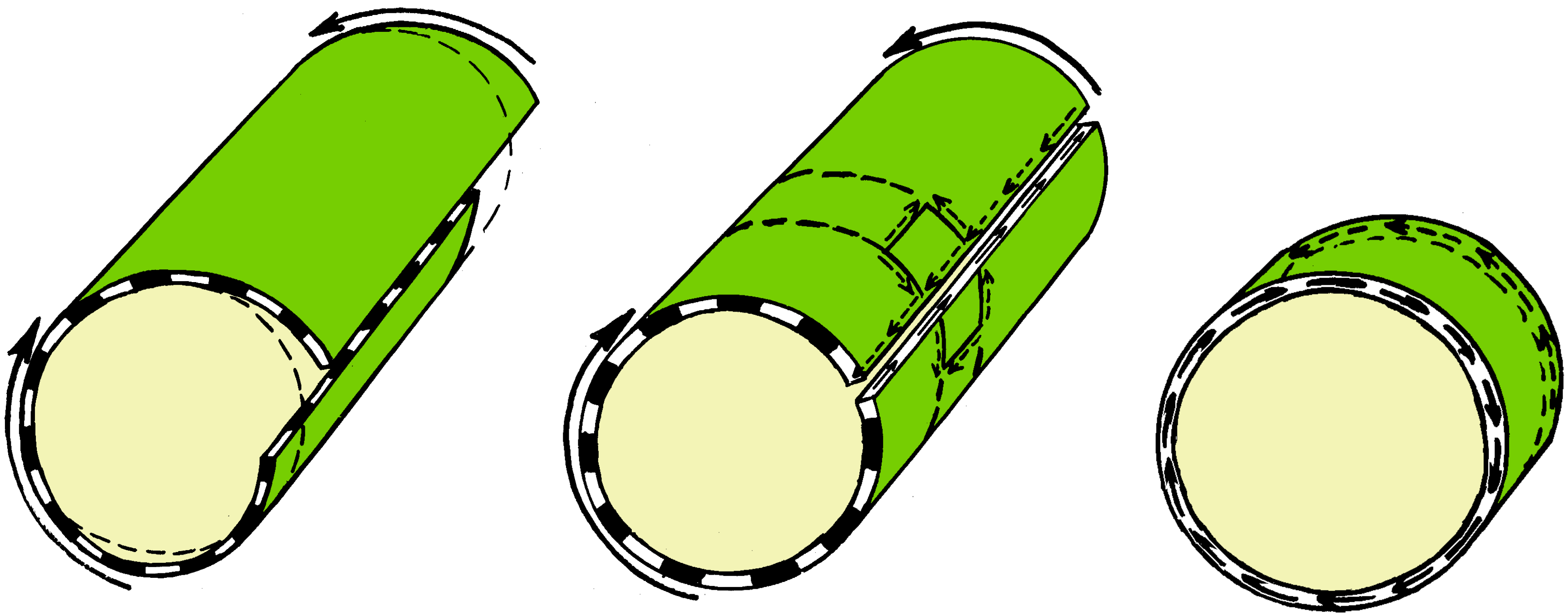
Pressurization (case I)

- Aircraft pressure cabin
  - Design pressure differential $p_d = p_{cabin} - p_h$
  - Cabin altitude: p_{cabin} related to altitude of 2400 - 3000 m
  - Safety factor j=2: $p_{ult} = 2\ p_d = 2(p_{cabin} - p_h)$
- Example
  - Pressure differential is 45 kPa
  - Aircraft altitude of h =9050 m $\Rightarrow$ cabin altitude of 2440 m
  - Aircraft altitude of h =10350 m $\Rightarrow$ cabin altitude of 3050 m

Fuselage structure

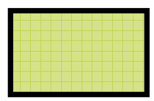
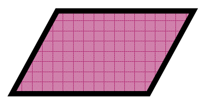
Torsional loading (case II)

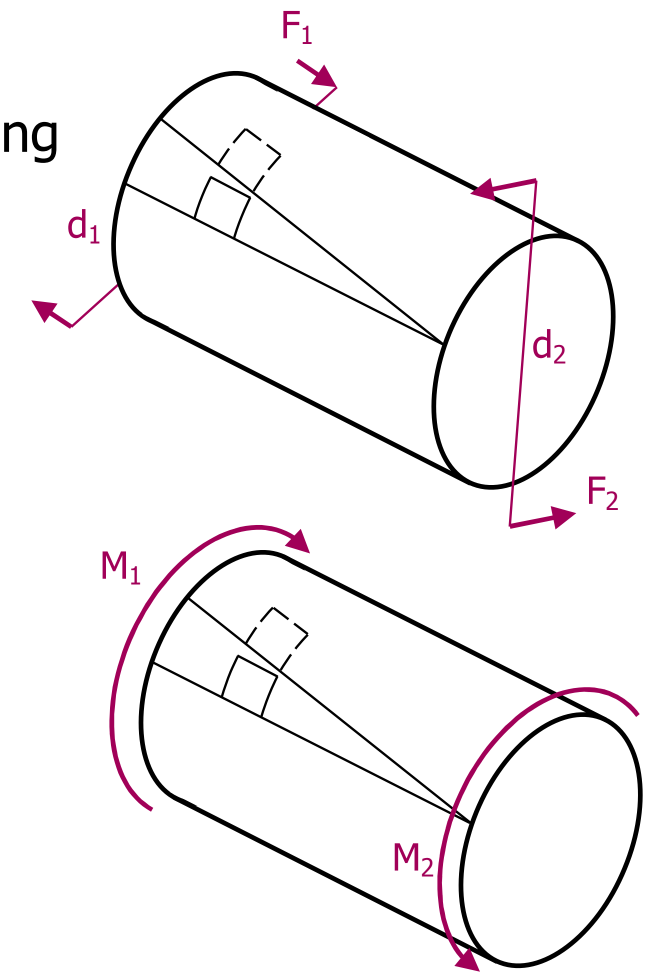
- The axial stresses in a tube under torsion



Fuselage structure

Torsional loading (case II)

- Function of cylinder against torsional loading
  - Rectangular shape 
  - Deforms like 



- Equilibrium if $F_1d_1 = F_2d_2 = F_nd_n$

$\Rightarrow M_1 = M_2 = M_T$

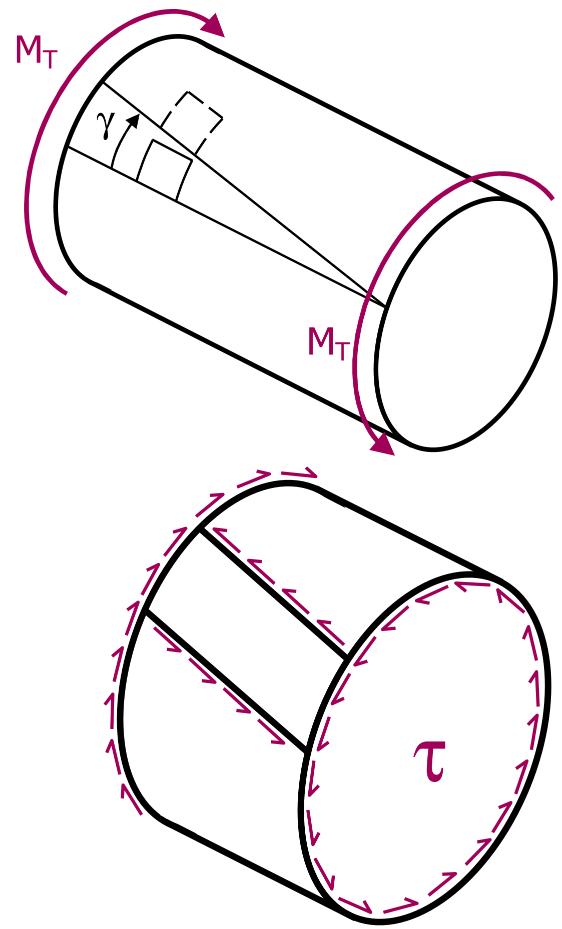
Fuselage structure

Torsional loading (case II)

- Torsion causes shear stress & strain

$$\gamma = \frac{\tau}{G}$$

- Equilibrium: $\tau_{outside} = \tau_{cross\ section}$



Fuselage structure

Torsional loading (case II)

- Torsion causes shear stress

$$\gamma = \frac{\tau}{G}$$

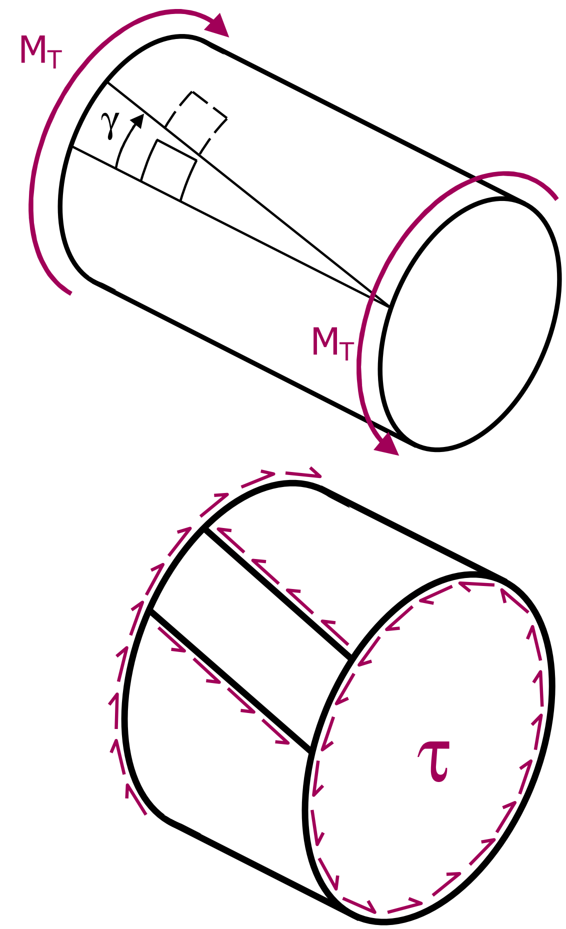
- Equilibrium: $\tau_{outside} = \tau_{cross\ section}$

- Torsional moment

$$M_T = \tau \underbrace{2\pi r}_{\text{circumference circle}} t\, r = 2\tau \underbrace{\pi r^2}_{\text{area circle}} t$$

circumference circle — area circle

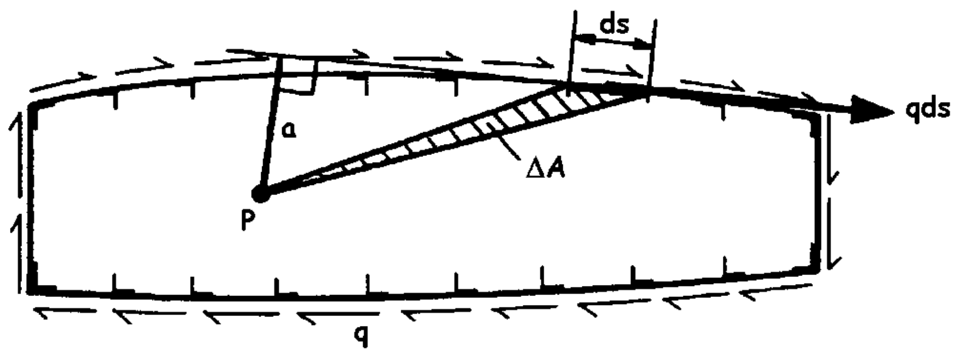
- Define $q = \tau\, t \;\Rightarrow\; M_T = 2qA$



Fuselage & wing structure

Torsional loading (case II)

- Thin walled box, stringers not loaded, torsion moment MT



- Moment around an arbitrary point: $q\, ds\, a = \Delta M_T$
- with $a\, ds = 2\Delta A$
- All 'fractions' together: $M_T = 2qA \Rightarrow q = \dfrac{M_T}{2A}$

Fuselage & wing structure

Torsional loading (case II)

- For a thin walled closed box (enclosed area A) the shear flow is

$$q = \frac{M_T}{2A}$$

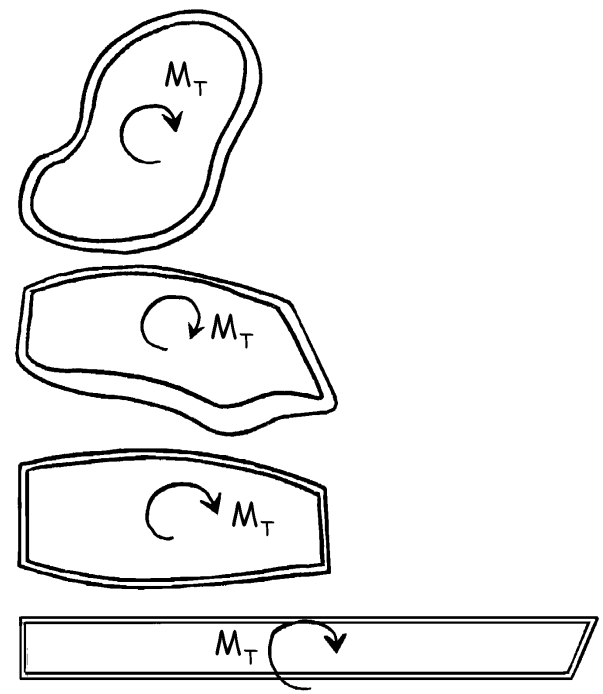
- All cross sections have the same shear-flow q
- A torsion box does not need to have a circular cross section

enclosed area ≡ A

enclosed area ≡ A

enclosed area ≡ A

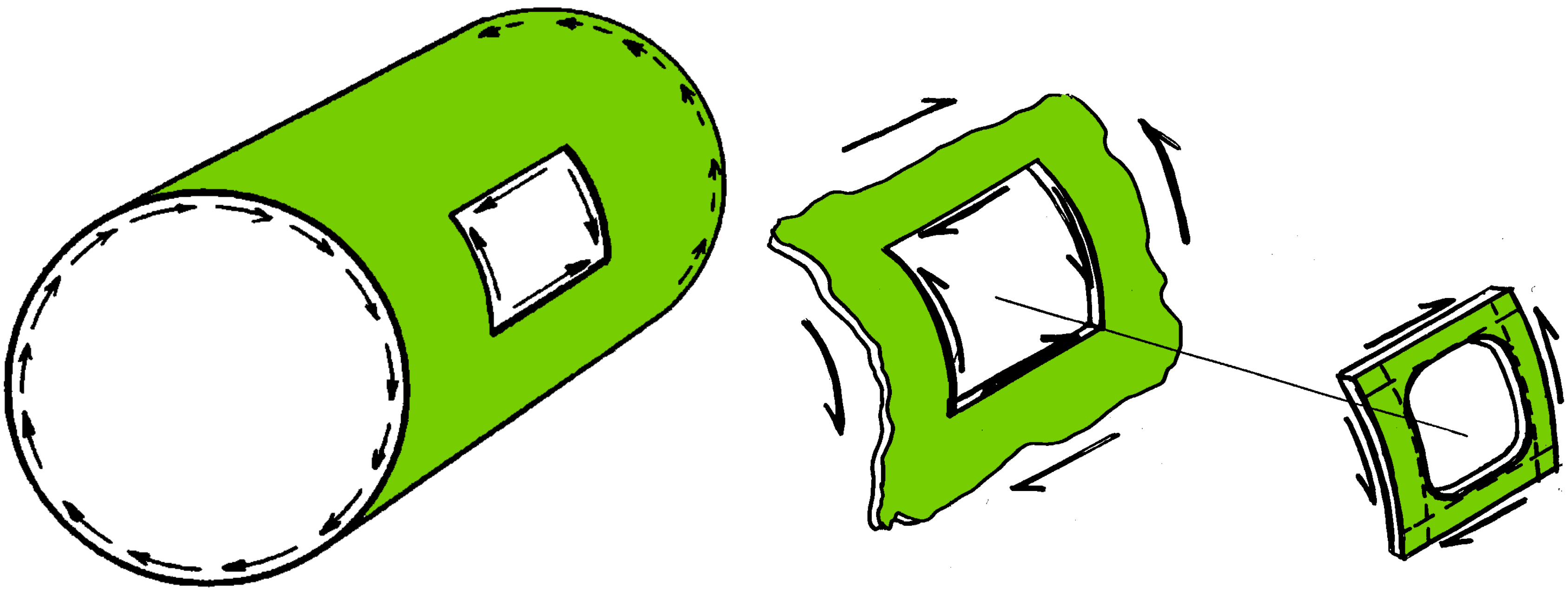
enclosed area ≡ A



Fuselage & wing structure

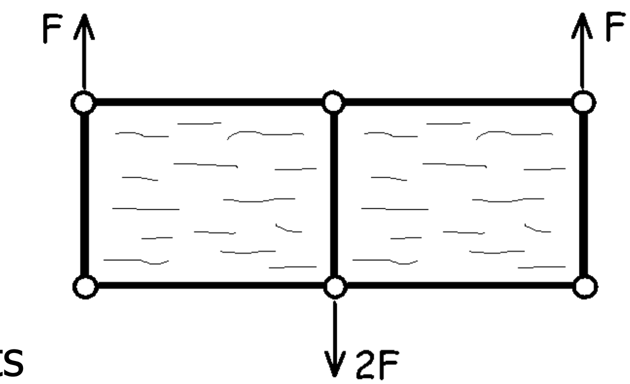
Torsional loading (case II)

- The torsion box will not "function" at a cut-out
- A stiff frame with rigid corners around the cut-out is needed



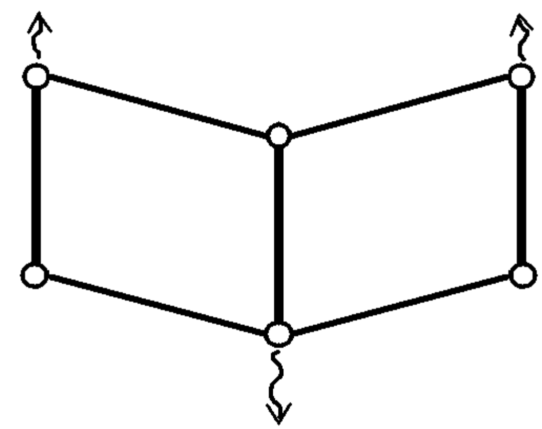
Wing structure

Bending of wing spar



- Consider elementary spar
  - Diamond shaped deformation is resisted by sheets
  - Frame exercises a shear stress on the sheets
  - As reaction: sheets exercise shear stress on the frame

  - Assumption: bars of frame very stiff, no deformation

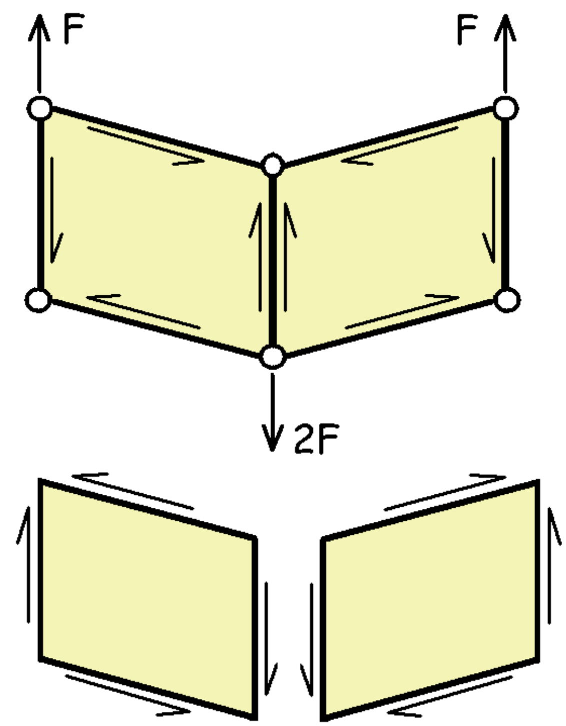


- Without sheets: Frame is not functioning

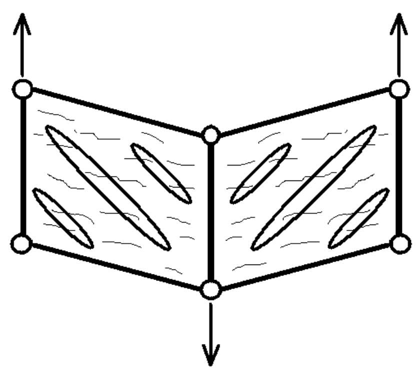
Wing structure

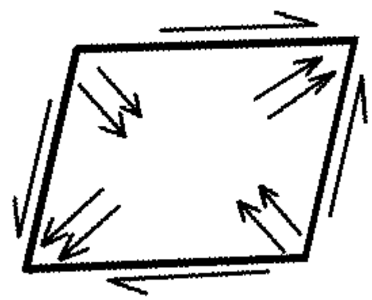
Bending of wing spar

- If frame functions, sheets in equilibrium



- High forces: pleat formation
  - However: Structure still functions !



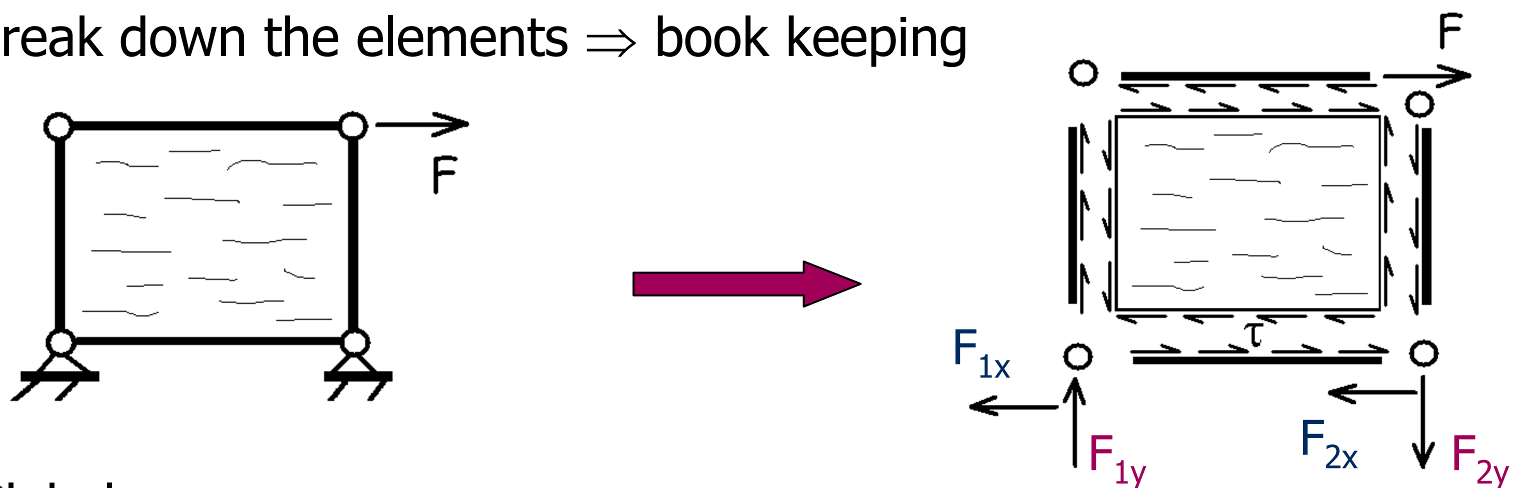


- Resistance to shear is the resistance to tension and compression under 45 degrees. Relationship: E, ν, G

Wing structure

Bending of wing spar

- Break down the elements ⇒ book keeping

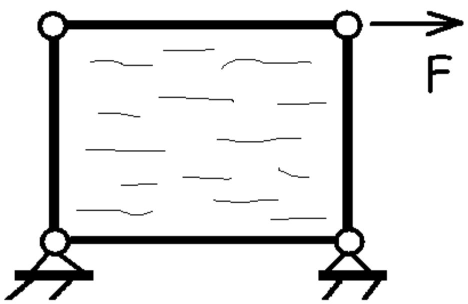


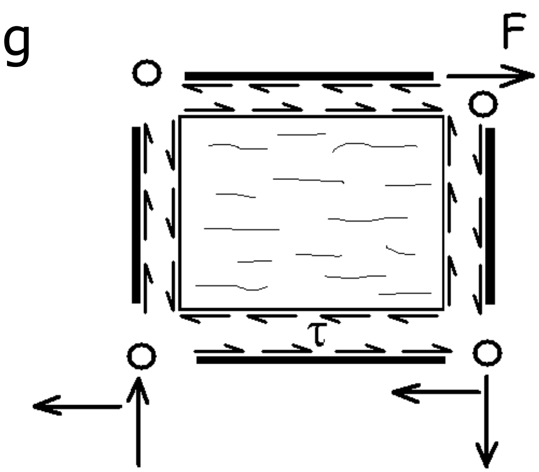
- Global
  - Horizontal equilibrium: $F - F_{1x} - F_{2x} = 0$
  - Vertical equilibrium: $F_{1y} - F_{2y} = 0$
  - Moment equilibrium: $F\,h + F_{1y}\,w = 0$
- Local
  - Equilibrium in elements …

Wing structure

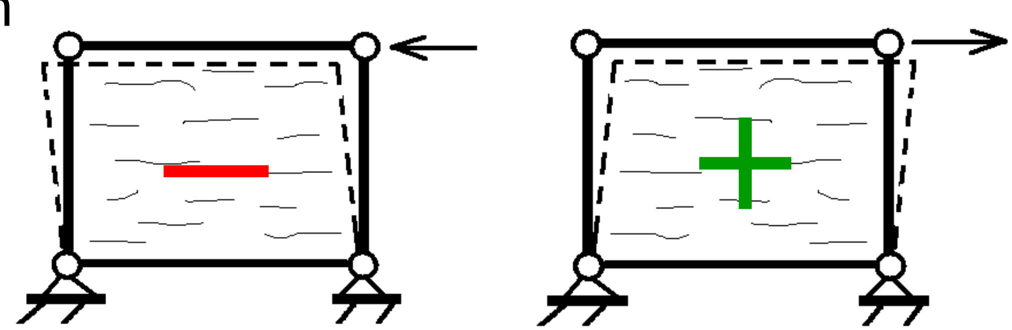
Bending of wing spar

- Break down the elements ⇒ book keeping





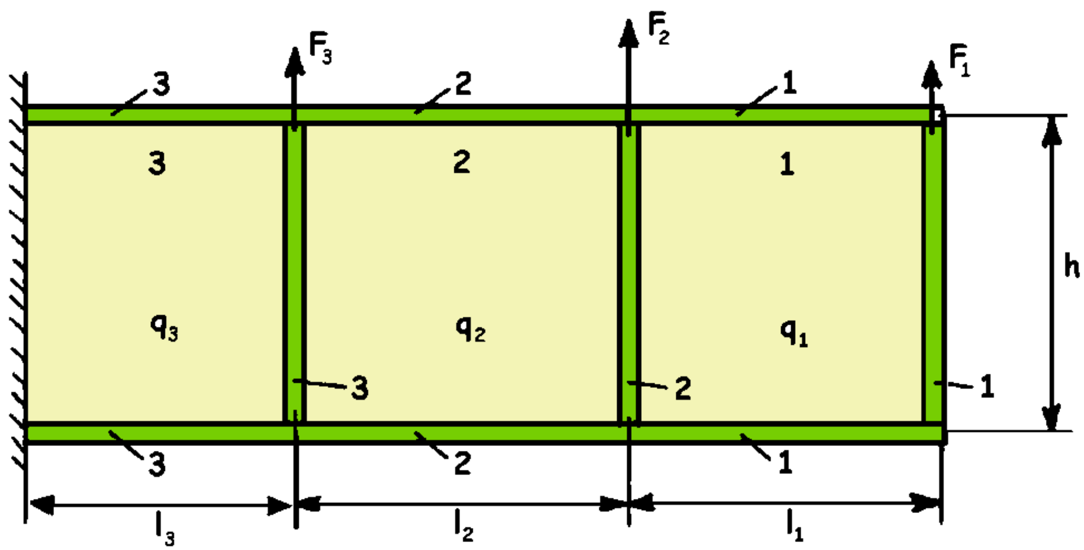
- For shear there is physically no difference but we need a sign convention



Wing structure

Bending of wing spar

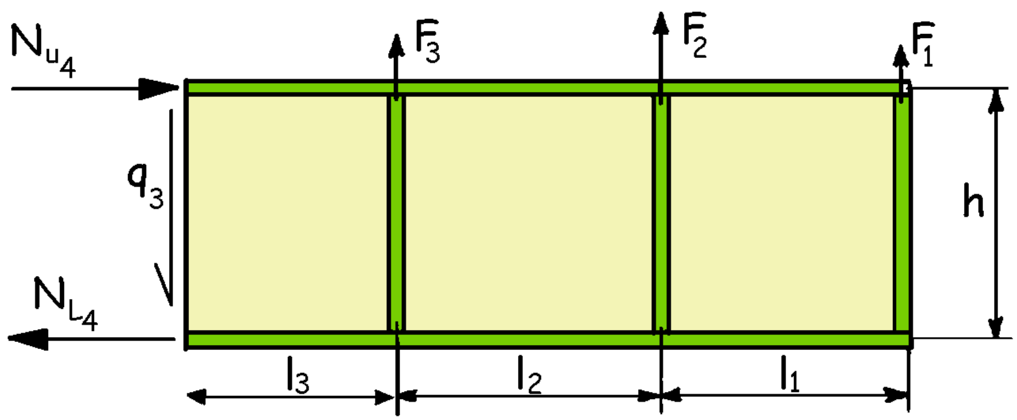
- In bending, the function of spar webs (shear) is essential! The webs have to be supported on the upper and lower side by caps in order to realize equilibrium
  - webs transfer (external) transverse forces into shear-flows
  - caps transfer shear-flows in normal forces



Wing structure

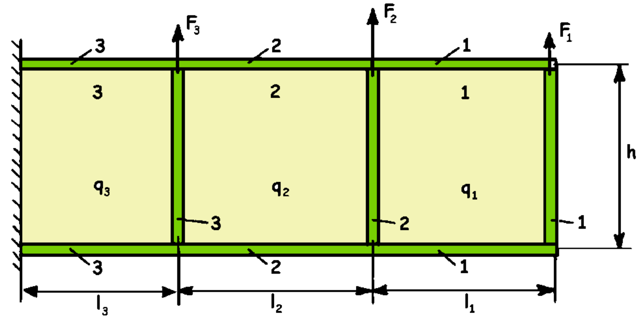
Bending of wing spar

- Global equilibrium (always check!)
  - Horizontal: $N_{U_4} - N_{L_4} = 0$ √
  - Vertical: $F_1 + F_2 + F_3 - q_3 h = 0$ √
  - Moment: $N_{U_4} h - F_1 \left(l_1 + l_2 + l_3 \right) - F_2 \left(l_2 + l_3 \right) - F_3 l_3 = 0$ √



Wing structure

Bending of wing spar



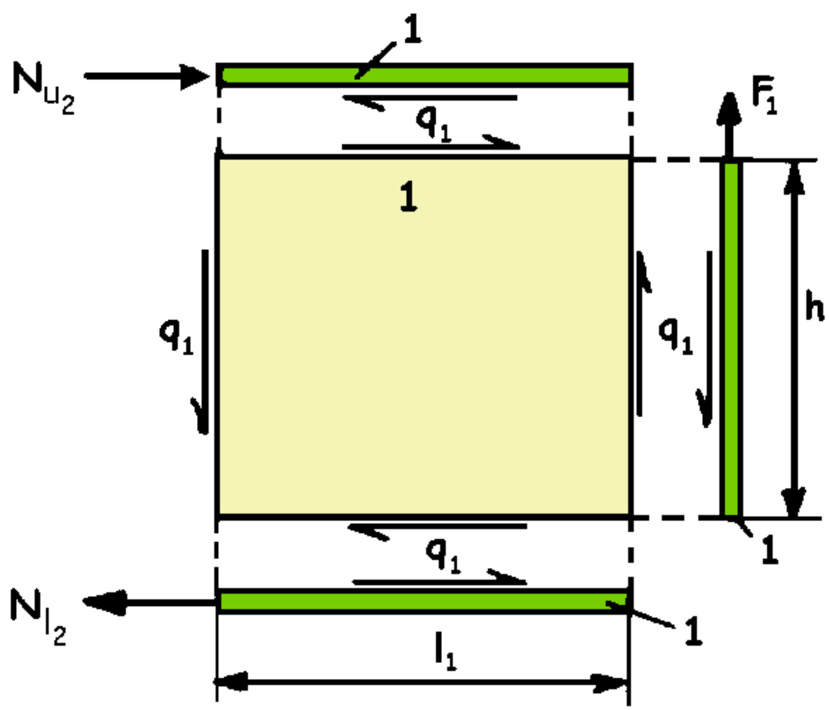
- Web plate 1 and upper and lower cap 1

- Equilibrium of forces in cap 1

$$F_1 - q_1 h = 0$$

$$N_{U_2} = q_1 l_1 = \frac{F_1}{h} l_1$$

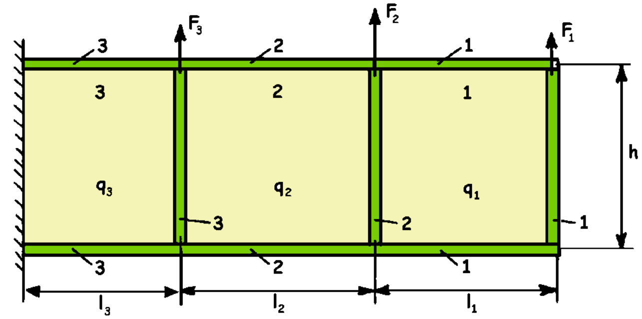
$$N_{L_2} = q_1 l_1 = \frac{F_1}{h} l_1$$



- q_1 is shear flow in web plate 1

Wing structure

Bending of wing spar



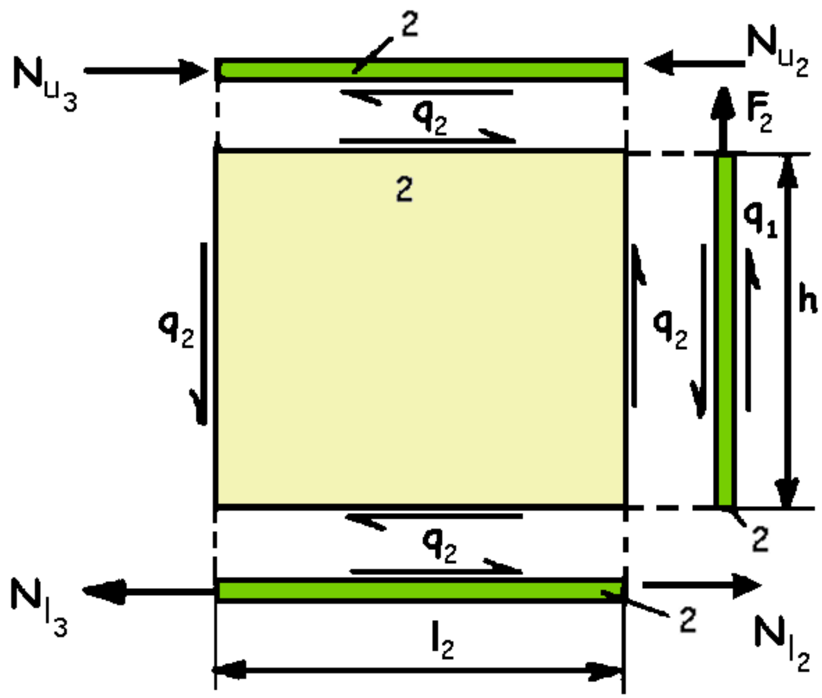
- Web plate 2 and upper and lower cap 2

- Equilibrium of forces in cap 2

$$F_2 = q_2 h - q_1 h = q_2 h - F_1$$

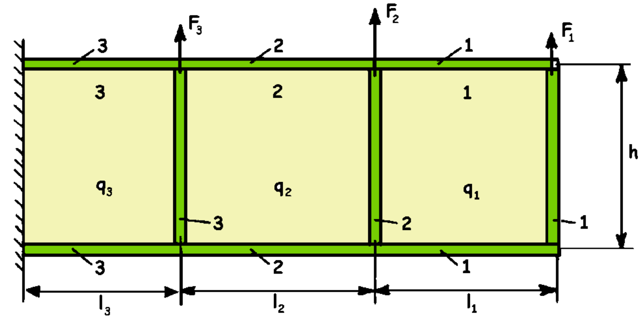
$$N_{U_3} = N_{U_2} + q_2 l_2 = \frac{F_1}{h} l_1 + \frac{F_1 + F_2}{h} l_2$$

$$N_{L_3} = \ldots = \frac{F_1}{h} l_1 + \frac{F_1 + F_2}{h} l_2$$



Wing structure

Bending of wing spar



- Web plate 3 and upper and lower cap 3

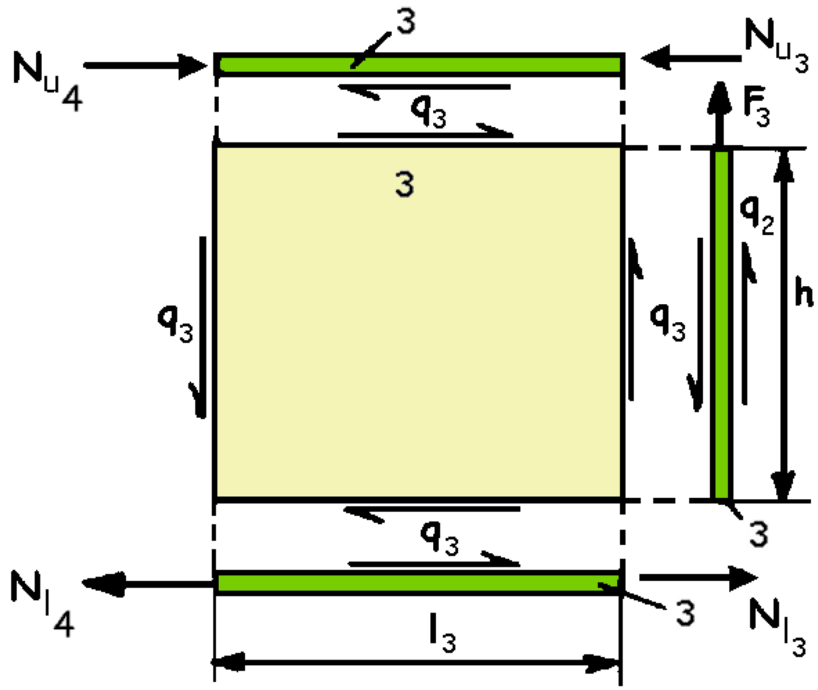
- Equilibrium of forces in cap 3

$$F_3 = q_3 h - q_2 h = q_3 h - F_1 - F_2$$

$$N_{U_4} = N_{U_3} + q_3 l_3$$

$$= \frac{F_1}{h} l_1 + \frac{F_1 + F_2}{h} l_2 + \frac{F_1 + F_2 + F_3}{h} l_3$$

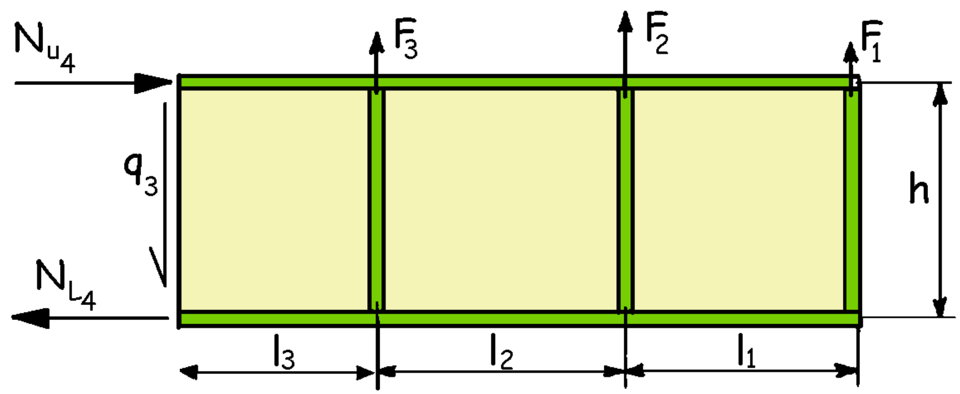
$$N_{L_4} = \ldots = \frac{F_1}{h} l_1 + \frac{F_1 + F_2}{h} l_2 + \frac{F_1 + F_2 + F_3}{h} l_3$$



Wing structure

Bending of wing spar

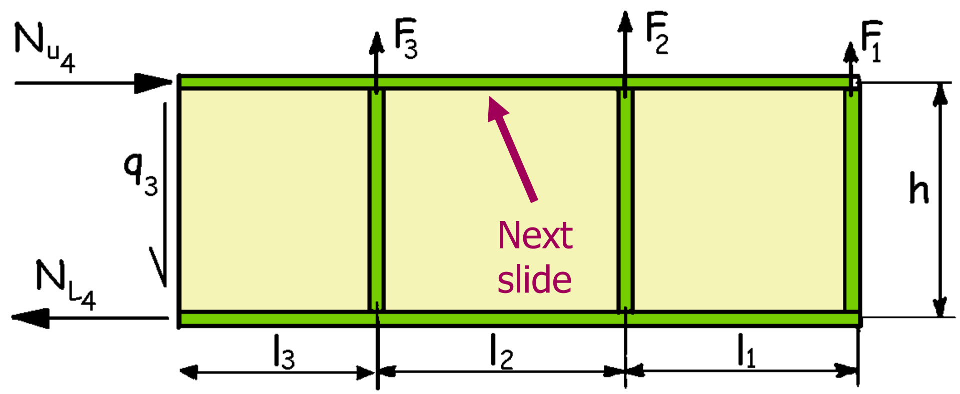
- Shear flow in the webs: $q_n = \frac{D_n}{h}$
- Transverse force building up: $D_n = F_1 + F_2 + \ldots + F_n = \sum_1^n F_n$



Wing structure

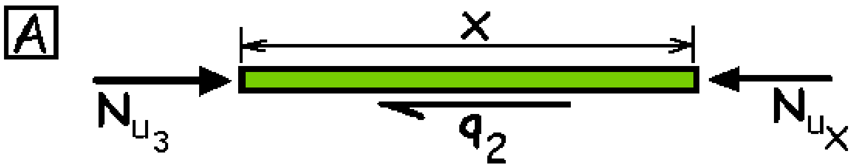
Bending of wing spar

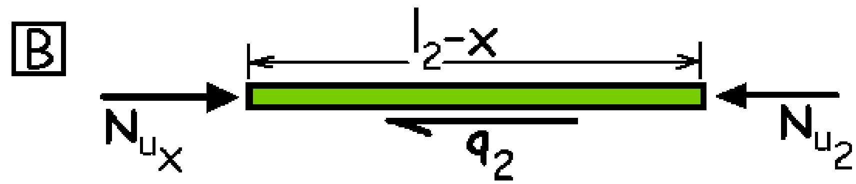
- Shear flow in the webs: $q_n = \frac{D_n}{h}$
- Transverse force building up: $D_n = F_1 + F_2 + \ldots + F_n = \sum_1^n F_n$
- Normal force in the caps at the location of stringers: $N_{m+1} = \frac{1}{h}\sum_{n=1}^{m} D_n l_n$



Wing structure

Bending of wing spar





- Equilibrium in A:

$$N_{U_3} - N_{U_x} - q_2 x = 0 \quad \longrightarrow \quad N_{U_x} = \frac{D_2}{h}\left(l_2 - x\right) + \frac{D_1}{h} l_1$$

- Equilibrium in B:

$$N_{U_x} - N_{U_2} - q_2\left(l_2 - x\right) = 0 \quad \longrightarrow \quad N_{U_x} = \frac{D_2}{h}\left(l_2 - x\right) + \frac{D_1}{h} l_1$$

- Normal force increases linearly from outboard to inboard !

Wing structure

Bending of wing spar

- The bending moment at location x in the spar is:

$$N_{U_x} h \equiv M_x = D_2 \left(l_2 - x \right) + D_1 l_1$$

- We see that that for l_2

$$\frac{dM_x}{dx} = -D_2$$

- This is valid on every location x on the spar:

$$\frac{dM_x}{dx} = -D_x$$

Wing structure

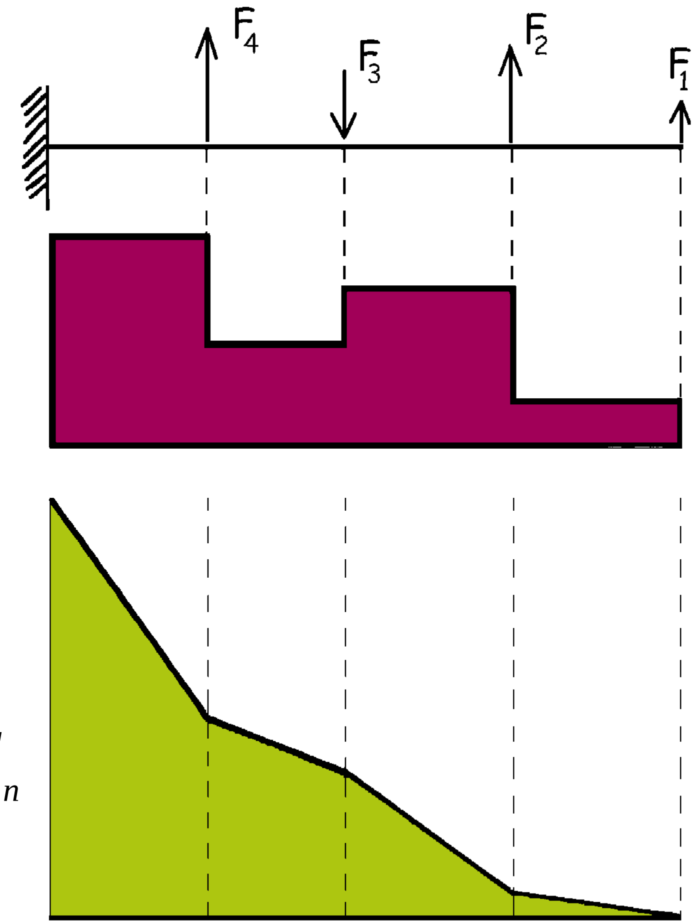
Bending of wing spar

- External forces F_n
- Transverse shear forces known at every location

$$q_n = \frac{D_n}{h} \qquad D_n = \sum_1^n F_n$$

- Normal forces in caps known at every location

$$N = \frac{M}{h} \qquad M_{n+1} = \sum_n^m D_n l_n$$



Wing structure

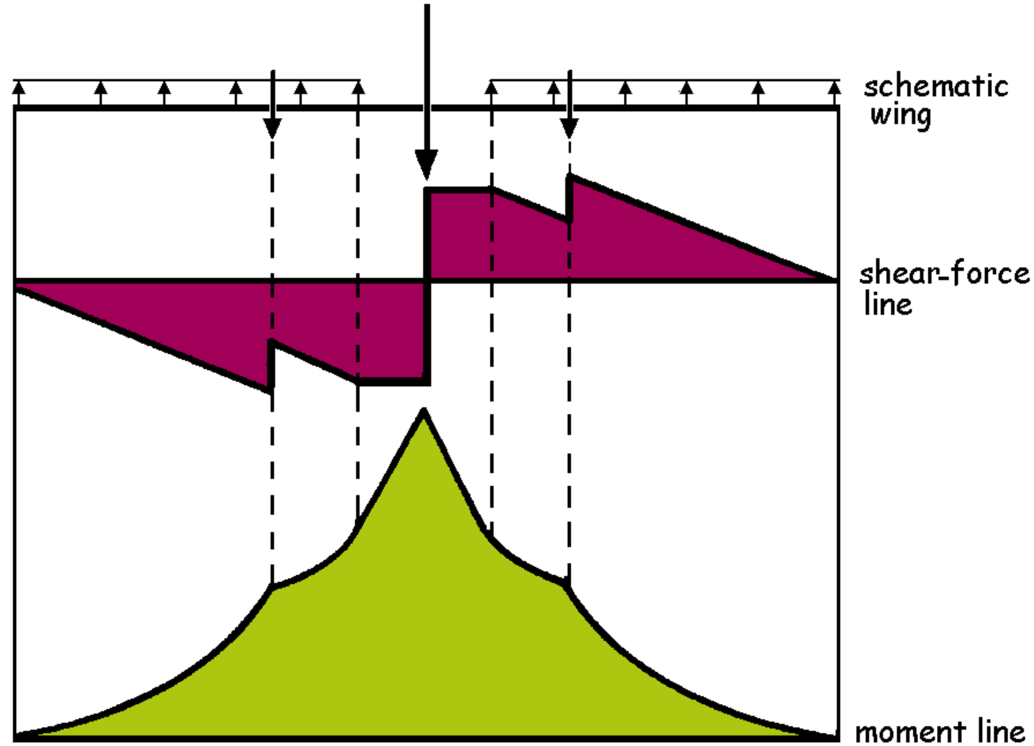
Lift, engine and fuselage weight

- Shear stress in webs

$$\tau_x = \frac{q_x}{t_x} = \frac{D_x}{ht_x}$$

- Normal stress in spar caps

$$\sigma_x = \pm\frac{N_x}{A_x} = \pm\frac{M_x}{hA_x}$$



- In a spar hA_x is called the moment of resistance (W)

Summary

Loads to stresses

- Three cases studied
  - Fuselage shell due to pressurization
  - Fuselage shells due to applied torsional load
  - Wing spars due to applied bending loads