Flight and Orbital Mechanics

Lecture slides

Flight and Orbital Mechanics

Lecture hours 1,2 – Unsteady Climb

Mark Voskuijl

Semester 1 - 2012

Content

- Introduction
- How does a pilot perform a climb?
- Equations of motion
- Story – Crash Boeing 727 (1974)
- Analytical solution
- Example exam question
- Summary

Introduction

Question

What is the most efficient way (minimum time) to go from take-off at sea-level to Mach 1.5 at 15,000 m?

A. Climb at airspeed for max γ. At 15,000 m accelerate to Mach 2

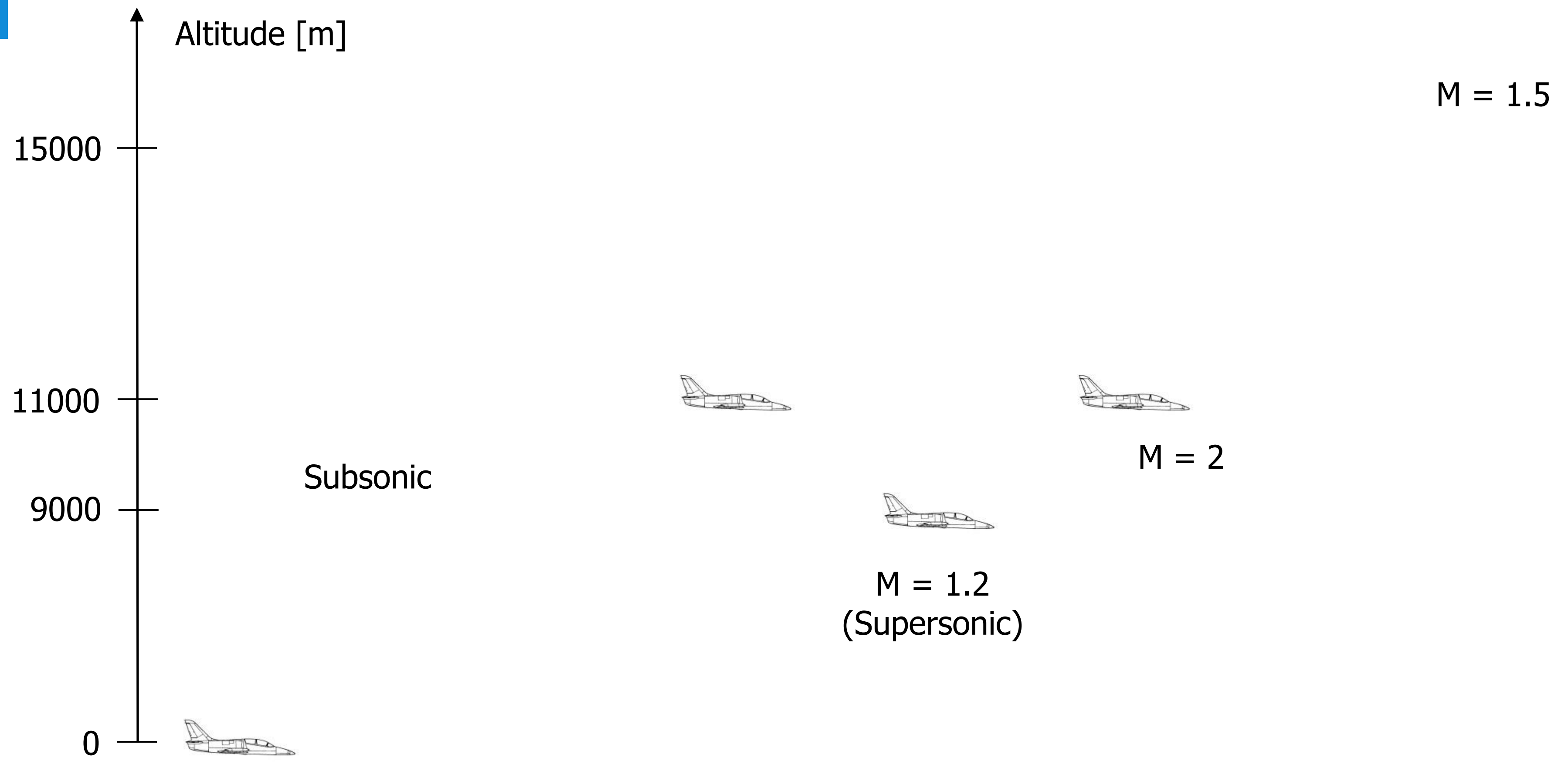
B. Climb at airspeed for max RC. At 15,000 m accelerate to Mach 2

√ **C. Climb at airspeed for max RC to 11,000 m. Descent and accelerate to Mach 1.2 at 9,000 m, climb and accelerate to Mach 2 at 11,000 m. Decelerated climb to 15,000 m, Mach 1.5**

D. Accelerate at sea-level to Mach 1.5, climb to 15,000 m at airspeed for max RC

Introduction

Solution



Introduction

Difference with AE1102 – Flight mechanics

Typical problem AE1102

- What is the maximum rate of climb of Aircraft X at a given altitude?

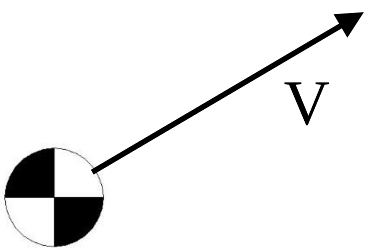
Typical problem AE2104

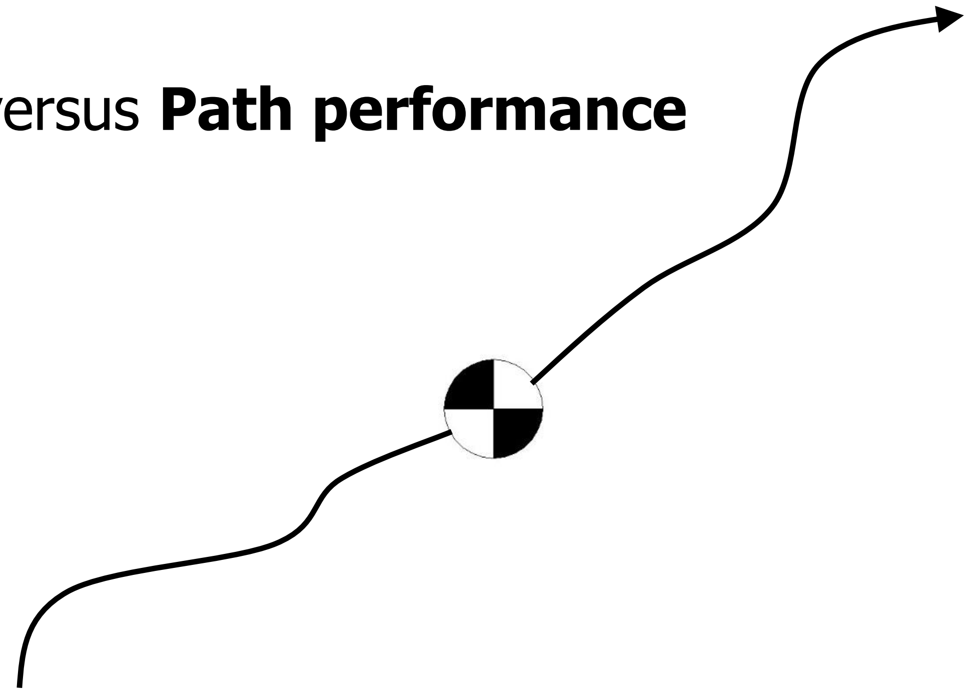
- What is the minimum time to climb from altitude A to altitude B for Aircraft X?

Introduction

Difference with AE1102 – Flight mechanics

Point performance versus **Path performance**





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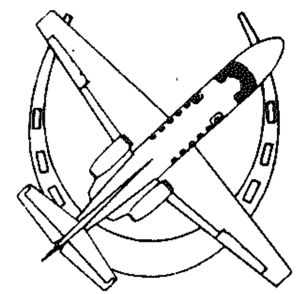
How does a pilot perform a climb?

Flight Planning Guide

(Unit 550-0557 and On)



Cessna Aircraft Company

Citation Marketing Division

P.O. Box 7706

Wichita, Kansas 67277

How does a pilot perform a climb?

MAXIMUM RATE CLIMB PERFORMANCE

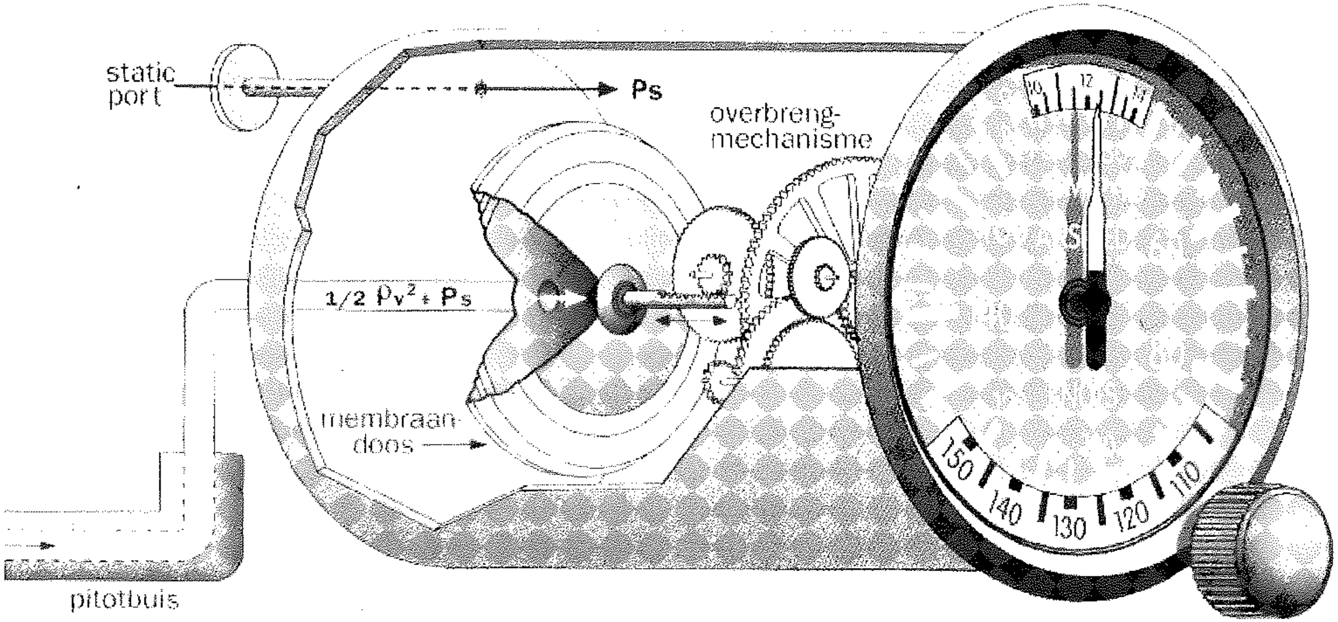
190 KIAS at Sea Level
Time, Distance and Fuel
Standard Day

T.O. Wt. × 1,000 Lbs.	13.3	12.5	11.5	10.5	13.3	12.5	11.5	10.5
Pressure Altitude	5,000 Feet				10,000 Feet			
Min.	2	2	2	2	4	4	3	3
N.M.	5	5	4	3	11	10	8	7
Lbs.	63	58	52	48	125	116	105	95
Pressure Altitude	15,000 Feet				21,000 Feet			
Min.	6	5	5	4	8	8	7	6
N.M.	18	16	14	12	28	25	22	19
Lbs.	188	174	158	142	267	246	222	200
Pressure Altitude	25,000 Feet				29,000 Feet			
Min.	11	10	9	8	14	13	11	10
N.M.	37	33	28	24	49	43	37	32
Lbs.	324	297	267	240	387	353	316	282
Pressure								

How does a pilot perform a climb?

How does a pilot perform a climb?

Airspeed indicator



How does a pilot perform a climb?

Airspeed indicator

For low Mach numbers:

$$p_t - p_s = \tfrac{1}{2}\rho V^2$$

One equation, two unknowns… assume sea level conditions:

$$p_t - p_s = \tfrac{1}{2}\rho_0 V_E^2$$

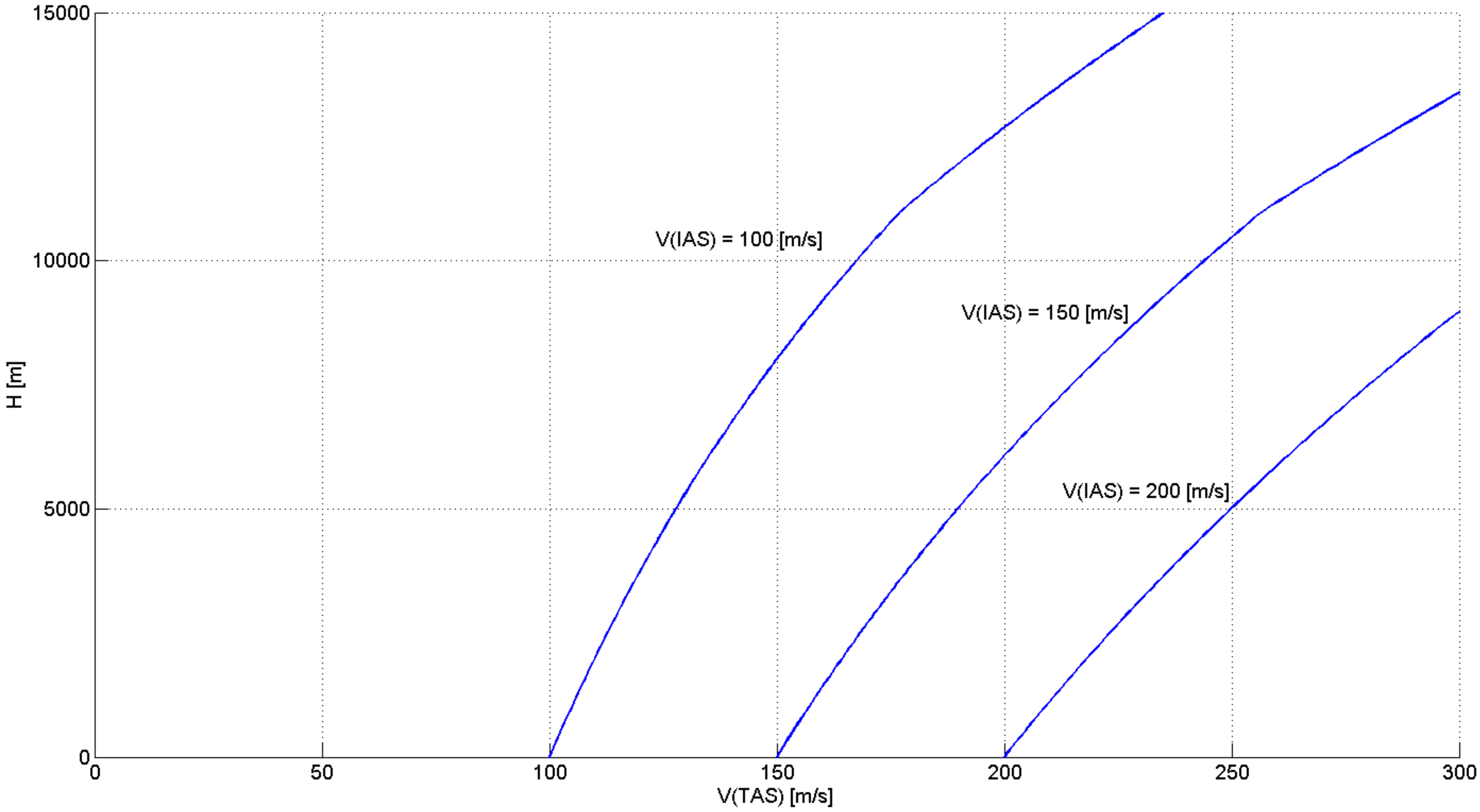
Relationship true airspeed – equivalent airspeed:

$$\tfrac{1}{2}\rho V^2 = \tfrac{1}{2}\rho_0 V_E^2 \Rightarrow V = \sqrt{\frac{\rho_0}{\rho}} V_E$$

The indicated airspeed is almost the same as the equivalent airspeed (instrument errors)

$$V_E \approx V_I$$

Large difference!!!



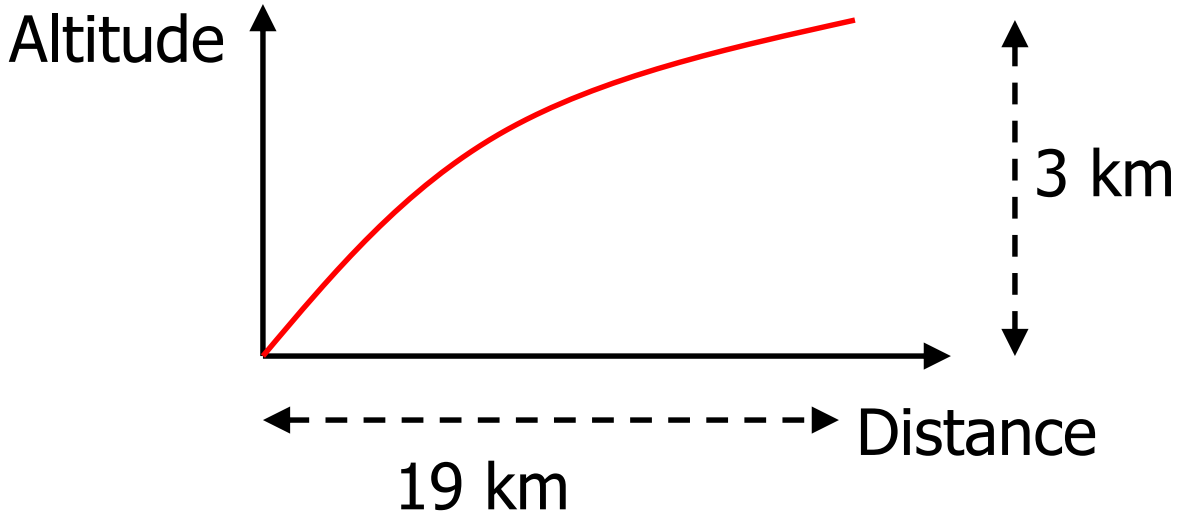
How does a pilot perform a climb

Summary

- Climb at **constant indicated airspeed** and constant power setting
- **True airspeed** is therefore **increasing**
- The climb is **unsteady**

$$\boxed{\frac{dV}{dt} \neq 0}$$

Typical climb profile



Conclusion: very faint curvature; the climb is almost a straight line

So, the climb is **quasi-rectilinear**

$$\boxed{\frac{d\gamma}{dt} \approx 0}$$

Summary

- A typical climb is performed at **constant indicated airspeed and** at a constant power setting**.** Therefore, the **true airspeed** is actually **increasing**. Since airspeed is not constant, it is an **unsteady climbing** flight

- The climb is almost a straight line. It is therefore a **quasi-rectilinear** flight

$$\boxed{\frac{dV}{dt} \neq 0} \qquad \boxed{\frac{d\gamma}{dt} \approx 0}$$

Is this a problem?

- Pilot flies at minimum airspeed

$$V_{\min} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L_{\max}}}}$$

- Minimum airspeed depends on altitude
- Equation for minimum equivalent airspeed

$$V_{e,\min} = \sqrt{\frac{\rho}{\rho_0}} V_{\min}$$

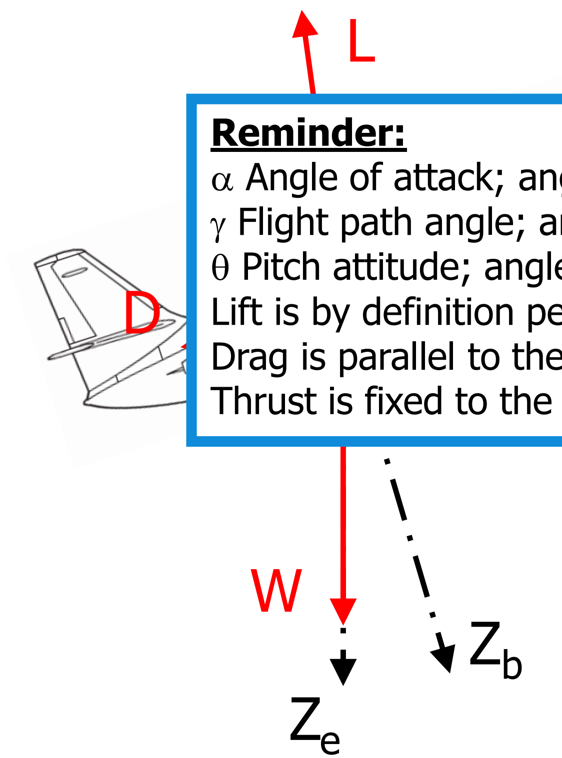
- Minimum airspeed seen by pilot is independent of altitude!

Content

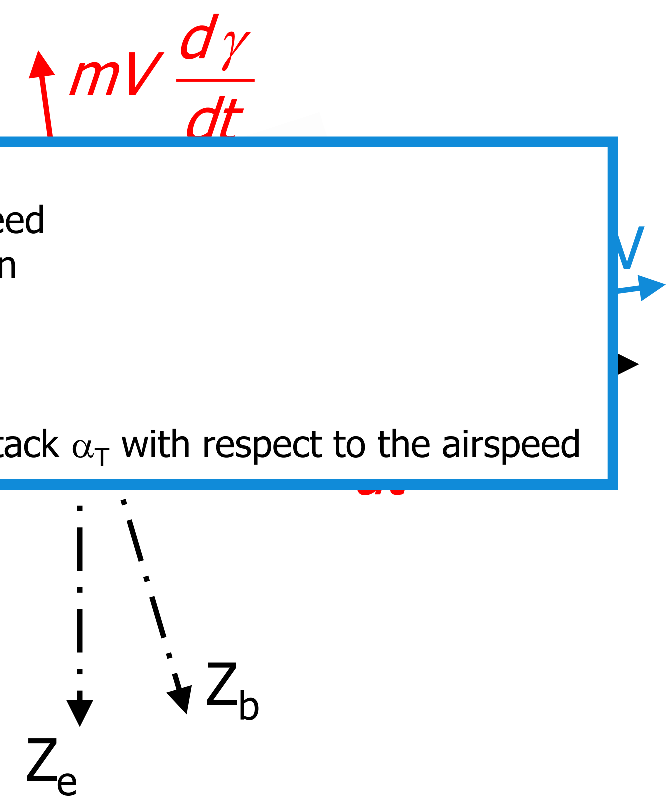
- Introduction
- How does a pilot perform a climb?
- **Equations of motion**
- Story – Crash B727, 1974
- Analytical solution
- Example exam question
- Summary

Equations of motion

Free Body Diagram



Kinetic Diagram



Reminder:

α Angle of attack; angle between aircraft body axis and airspeed

γ Flight path angle; angle between airspeed vector and horizon

θ Pitch attitude; angle between horizon and aircraft body axis

Lift is by definition perpendicular to airspeed

Drag is parallel to the airspeed

Thrust is fixed to the aircraft and therefore has an angle of attack α_T with respect to the airspeed

Equations of motion

$$\vec{F} = m\vec{a}$$

$$\sum F_{//V} : T \overset{=1}{\cos\alpha_T} - D - W\sin\gamma = m\frac{dV}{dt}$$

$$\sum F_{\perp V} : L - W \overset{=1}{\cos\gamma} + T \overset{=0}{\sin\alpha_T} = mV \overset{\cong 0}{\frac{d\gamma}{dt}}$$

Unsteady quasi-rectilinear climbing flight

Extra assumption: $\alpha_T = 0$,

Approximation: $\cos\gamma \cong 1$, $\sin\gamma \neq 0$

Equations of motion

Unsteady climb

$$\boxed{\begin{aligned} &T - D - W\sin\gamma = m\frac{dV}{dt} \\ &L = W \end{aligned}}$$

Rewrite to power equation by multiplying with airspeed

$$TV - DV - WV\sin\gamma = mV\frac{dV}{dt}$$

$$\boxed{P_a - P_r = \frac{W}{g}V\frac{dV}{dt} + W\frac{dH}{dt}}$$

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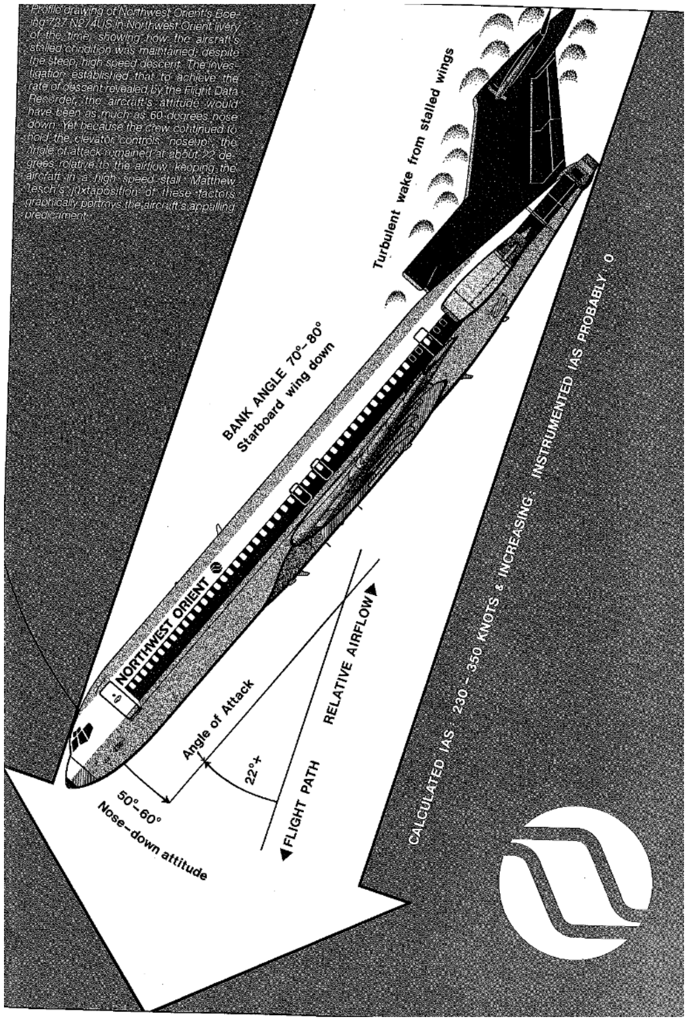
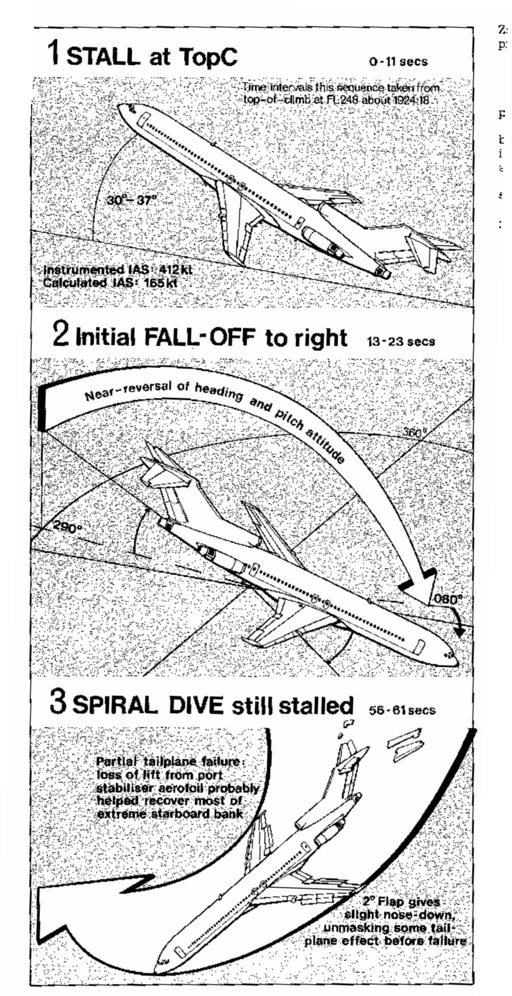
Story – Crash Boeing 727, 1974





Story – Crash Boeing 727, 1974

- B727 Northwest Orient
- 1 Dec 1974
- Flight J.F. Kennedy Airport – Buffalo
- Checklist: Pitot heaters off

- 16000 ft IAS = 305 kts RC = 2500 ft/min
- >16000 ft IAS > 340 kts RC = 5000 ft/min

- Comment pilot “We ‘re light”

Story – Crash Boeing 727, 1974

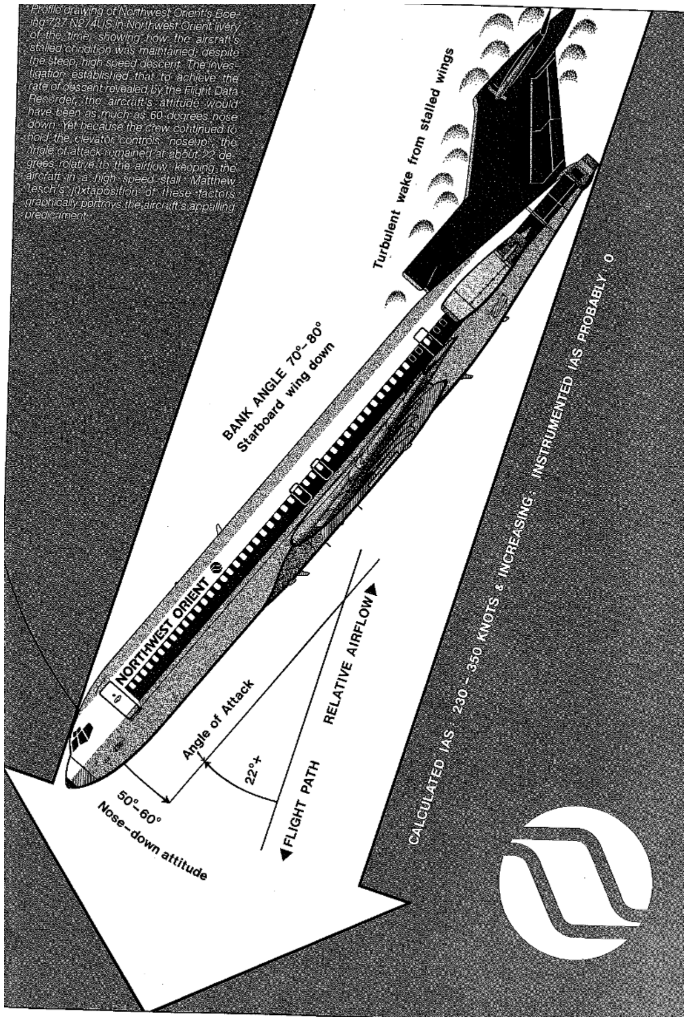
- 23000 ft IAS = 405 kts RC = 6500 ft/min

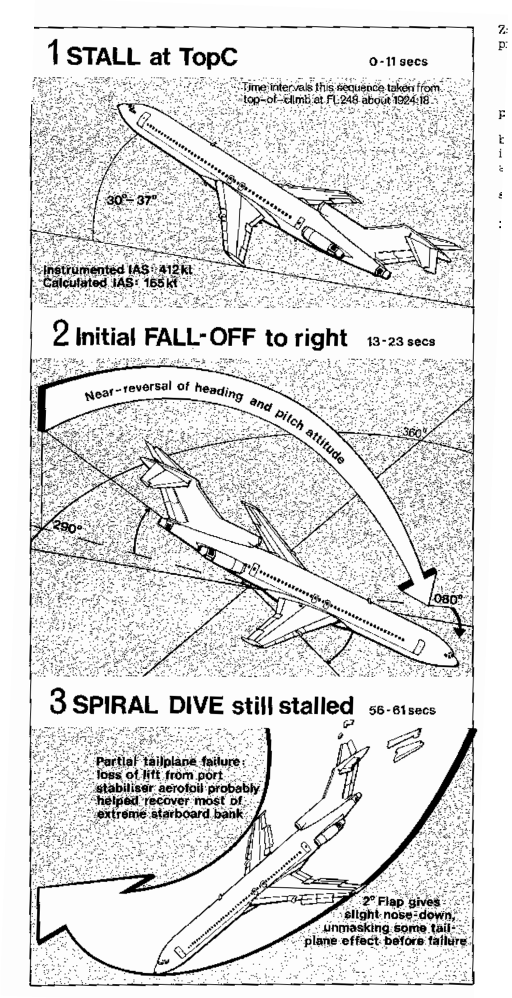
- Overspeed horn: "Pull back and let her climb"
- Stick shaker: "Mach Buffet"

"Guess we'll have to pull her up further"

- 24800 ft stall @ $\theta = 30°$
- Still pulling → Deep stall
- Horizontal stabiliser damaged

Story – Crash Boeing 727, 1974





Story – Crash Boeing 727, 1974

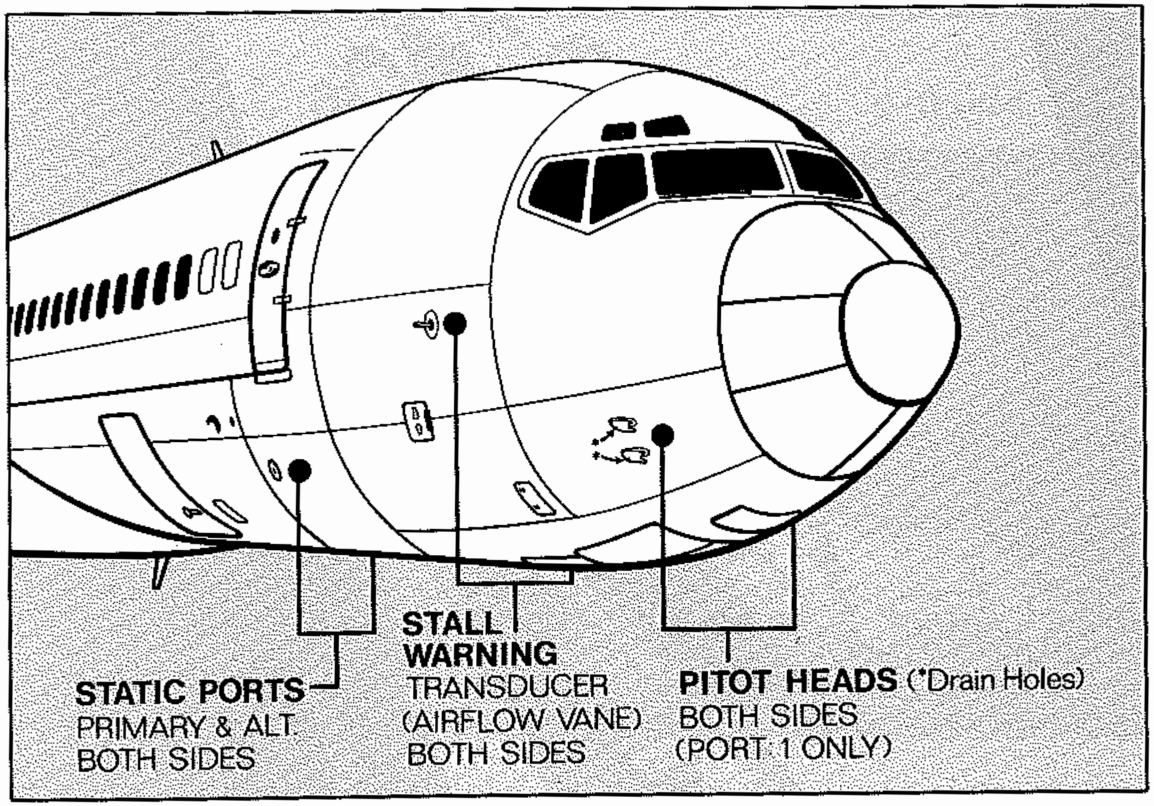
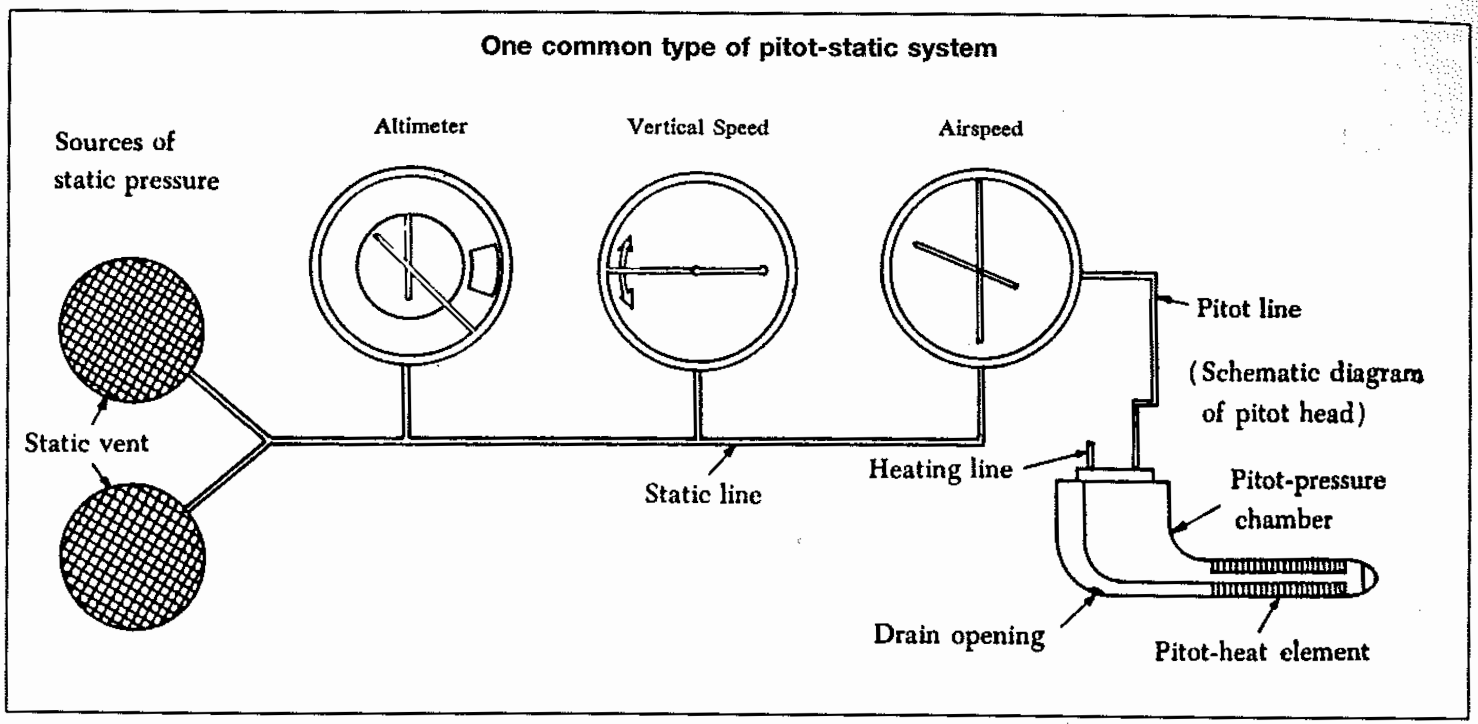


Diagram of Boeing 727's nose area, showing position of external sensors for pitot-static and stall warning systems. The stall warning transducer is a pivoting miniature aerofoil which responds to the angle of airflow past either side of the aircraft's nose. (Matthew Tesch)

Story – Crash Boeing 727, 1974



Schematic diagram of typical pitot-static system driving an aircraft's altimeter, vertical speed indicator and airspeed indicator. Blockage of the pitot head by ice would have a negligible effect on altimeter readings as this instrument senses static pressure only.

Content

- Introduction
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Analytical solution

What is the difference between the climb performance in steady flight and in unsteady flight?

Analytical solution

Steady Climb

Equation of motion // V

$$0 = T - D - W\sin\gamma$$

Multiply with airspeed

$$0 = P_a - P_r - WV\sin\gamma$$

$$\frac{P_a - P_r}{W} = V\sin\gamma = RC_{st}$$

$$\boxed{RC_{st} = \frac{P_a - P_r}{W}}$$

Unsteady climb

Equation of motion // V

$$\frac{W}{g}\frac{dV}{dt} = T - D - W\sin\gamma$$

Multiply with airspeed

$$\frac{W}{g}V\frac{dV}{dt} = P_a - P_r - WV\sin\gamma$$

$$\frac{P_a - P_r}{W} = V\sin\gamma + \frac{V}{g}\frac{dV}{dt}$$

$$RC_{st} = RC + \frac{V}{g}\frac{dV}{dt}$$

Intermezzo

$$\frac{V}{g}\frac{dV}{dt} = \frac{V}{g}\frac{dV}{dh}\frac{dh}{dt}$$

Analytical solution

Unsteady climb continued

$$RC_{st} = RC + \frac{V}{g}\frac{dV}{dh}RC$$

result

$$\boxed{\frac{RC}{RC_{st}} = \frac{1}{1+\dfrac{V}{g}\dfrac{dV}{dh}}}$$

What does this actually mean physically?

Analytical solution

- The **excess power** is partially used to **accelerate** (**kinetic energy**) and partially to **climb** (**potential energy**)

- The rate of climb in an unsteady climb is therefore smaller than in a steady climb.

$$\boxed{\frac{RC}{RC_{st}} = \frac{1}{1+\frac{V}{g}\frac{dV}{dh}}; \qquad \frac{dV}{dh} > 0}$$

This is called the **kinetic energy correction factor**

Content

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- Summary

Example exam question

An aircraft carries out a quasi-rectilinear symmetrical climbing flight at constant EAS in the troposphere of the ISA

Calculate at flight altitude H = 10 km the ratio between the actual rate of climb in the unsteady climbing flight and the rate of climb in the steady climbing flight at an instantaneous Mach number of M = 0.8 (attention M is not constant)

Carefully derive the “kinetic energy correction factor” for this flight using the equations of motion in unsteady flight.

Given for the troposphere (ISA):

$$\rho = \rho_0 \left[1 + \frac{\lambda H}{T_0} \right]^{-\left(\frac{g_0}{R\lambda} + 1 \right)} ; \quad dp = -\rho g_0 dH; \quad p = \rho R T$$

$$\frac{dT}{dH} = -0.0065 \text{ [K/m]}$$

Solution (1/2)

$$\frac{RC}{RC_{st}} = \frac{1}{1+\frac{V}{g_0}\frac{dV}{dH}}$$

In an exam you will be most likely be asked to derive this ratio first by deriving the equations of motion

Constant equivalent airspeed

$$V = V_E\sqrt{\frac{\rho_0}{\rho}} \quad \Rightarrow 1+\frac{V}{g_0}\frac{dV}{dH} = \ldots \frac{V_E^2}{g_0}\frac{d\left(\frac{\rho_0}{\rho}\right)}{dH}$$

So the derivative in the ki… …ospheric properties

Gas law:

$$p = \rho RT$$

$$\frac{d\left(\frac{\rho_0}{\rho}\right)}{dH} = \frac{d\left(\frac{\rho_0 RT}{p}\right)}{dH} = \frac{\rho_0 pR\frac{d\ldots}{dH} \ldots}{p^2} \ldots \frac{\ldots}{p^2}\frac{dp}{dH}$$

My graphical calculator says d(ρ_0/ρ)/dH equals: ….

0 points

Hydrostatic equation

$$\frac{dp}{dH} = -\rho g_0$$

$$\frac{d\left(\frac{\rho_0}{\rho}\right)}{dH} = \frac{\rho_0 R}{p}\frac{dT}{dH} + \frac{\rho_0 g_0}{p}$$

Solution (2/2)

So the correction factor becomes

$$1+\frac{V_E^2}{2g}\rho_0\left[\frac{R}{p}\frac{dT}{dH}+\frac{g_0}{p}\right]=1+\frac{V_E^2}{2}\frac{\rho_0}{p}\left[\frac{R}{g_0}\frac{dT}{dH}+1\right]$$

Instantaneous Mach number is given; so rewrite:

$$V_E^2=V^2\frac{\rho}{\rho_0}=M^2\gamma RT\frac{\rho}{\rho_0}=M^2\gamma p\frac{1}{\rho_0}$$

$$1+\frac{V_E^2}{2}\frac{\rho_0}{p}\left[\frac{R}{g_0}\frac{dT}{dH}+1\right]=1+\frac{M^2\gamma}{2}\left[\frac{R}{g_0}\frac{dT}{dH}+1\right]$$

Fill in the given values:

$$1+\frac{M^2\gamma}{2}\left[\frac{R}{g}\frac{dT}{dH}+1\right]=1+\frac{0.8^2\cdot 1.4}{2}\left[\frac{287.05}{9.81}\cdot -0.0065+1\right]=1.36$$

$$\frac{RC}{RC_{st}}=\frac{1}{1.36}=0.73$$

So, the actual rate of climb in an unsteady climb is actually only 73% of the maximum achievable rate of climb in steady flight (for this flight condition)

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- Analytical solution
- Example exam question
- **Summary**

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- The climb is almost a straight line. It is therefore a **quasi-rectilinear** flight

- Corresponding equations of motion:

$$\boxed{\begin{aligned} &T - D - W\sin\gamma = m\frac{dV}{dt} \\ &L = W \end{aligned}}$$

Summary

- The rate of climb in an unsteady climb is smaller than in a steady climb because part of the excess energy is used to accelerate

$$\boxed{\frac{RC}{RC_{st}} = \frac{1}{1+\frac{V}{g}\frac{dV}{dh}}; \qquad \frac{dV}{dh} > 0}$$

- You must be able to derive this ratio from the equations of motion
- You must be able to calculate this ratio (see example exam question)
- For more background information: read Ruijgrok – Elements of airplane performance section 14.2

Questions?