Flight and Orbital Mechanics

Lecture slides

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Flight and Orbital Mechanics

Lecture hours 5, 6, 7 – Turning performance

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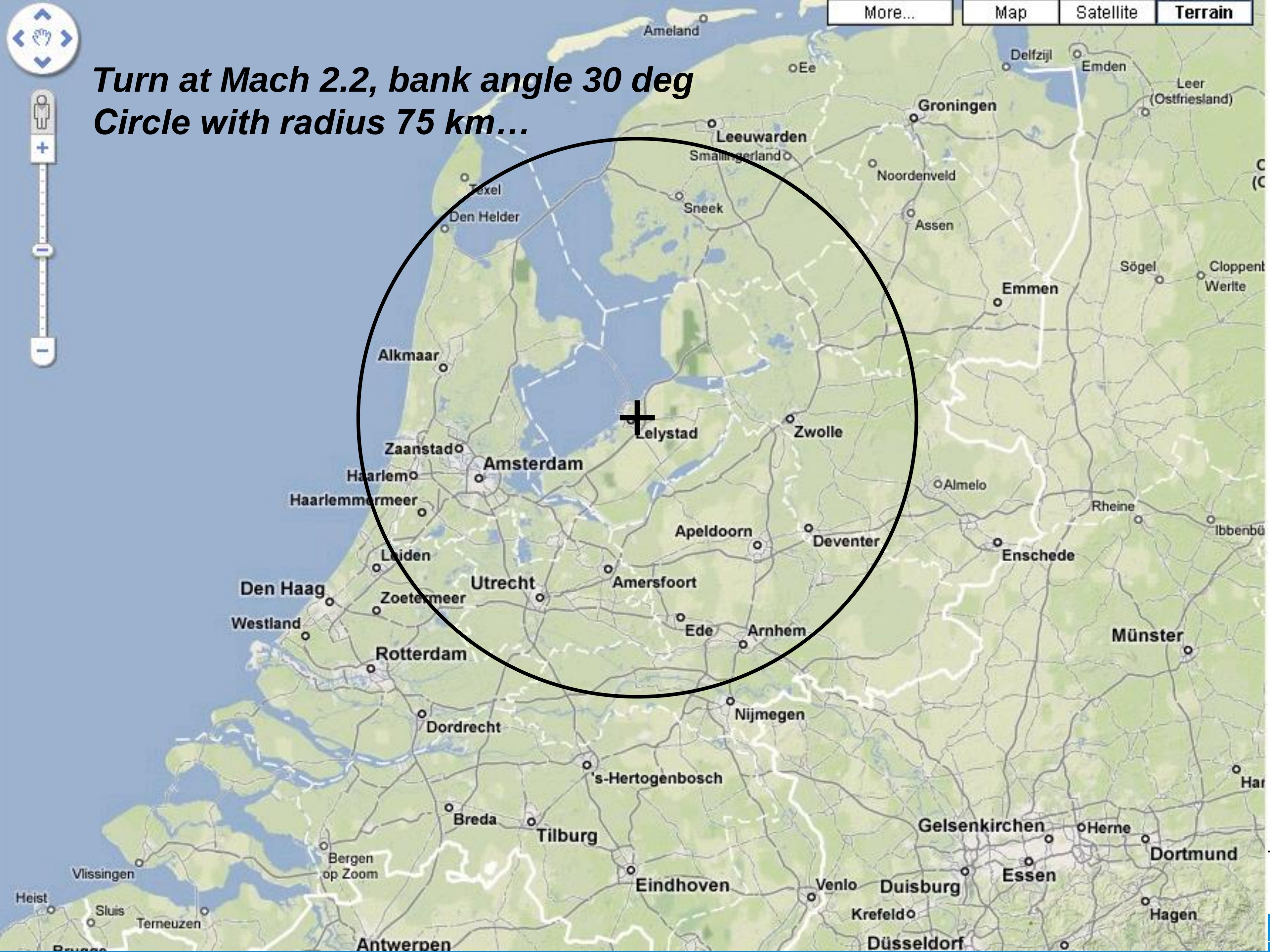
Semester 1 - 2012

Question

Lockheed F-104 Starfighter
Top speed Mach 2.2 (at 10 km altitude)

*How large would the **turning radius** be if an aircraft performs a horizontal steady turn at this airspeed, with a bank angle of 30 deg?*

Turn at Mach 2.2, bank angle 30 deg
Circle with radius 75 km…



Content

- Aim
- How to fly a turn
- Equations of motion
- Load factor and performance diagrams
- Turning performance
- Example calculations
- Advanced manoeuvres
- Altitude effects
- Summary
- Example exam question

Content

- **Aim**
- How to fly a turn
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Aim

Calculate turning performance of a given aircraft

- Minimum turn radius (tightest turn)
- Minimum time to turn (fastest turn)
- Maximum load factor (steepest turn)
- Rate one/two/three turn (civil operations)
- Assumption: sustained turns

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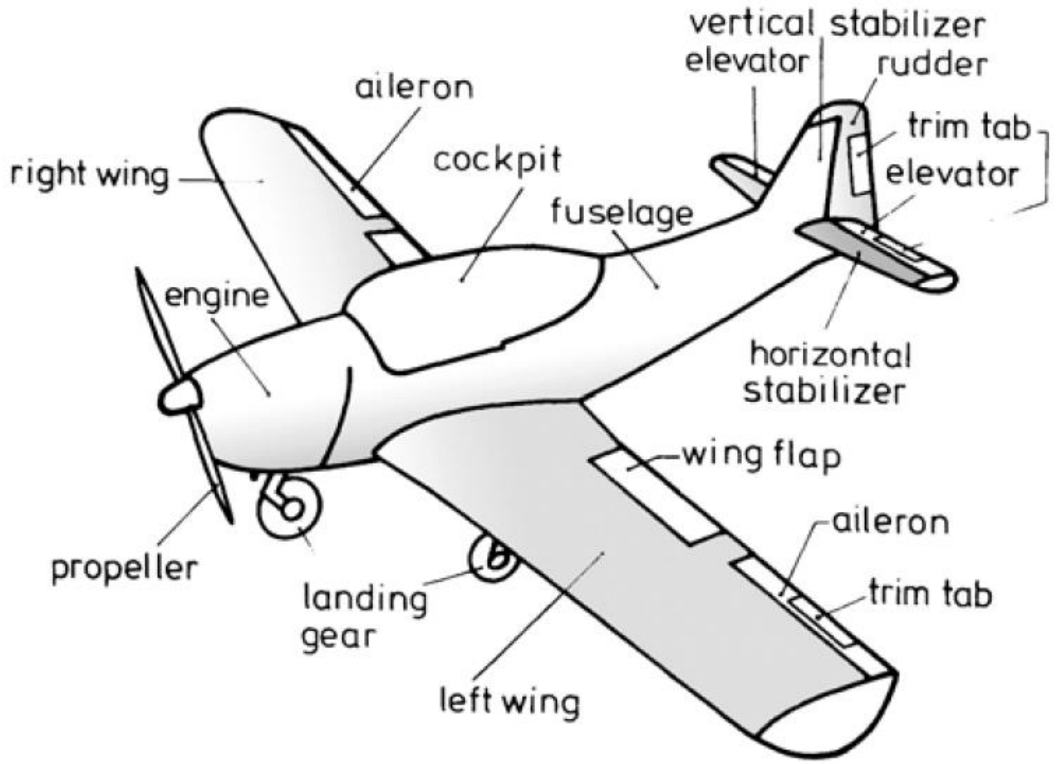
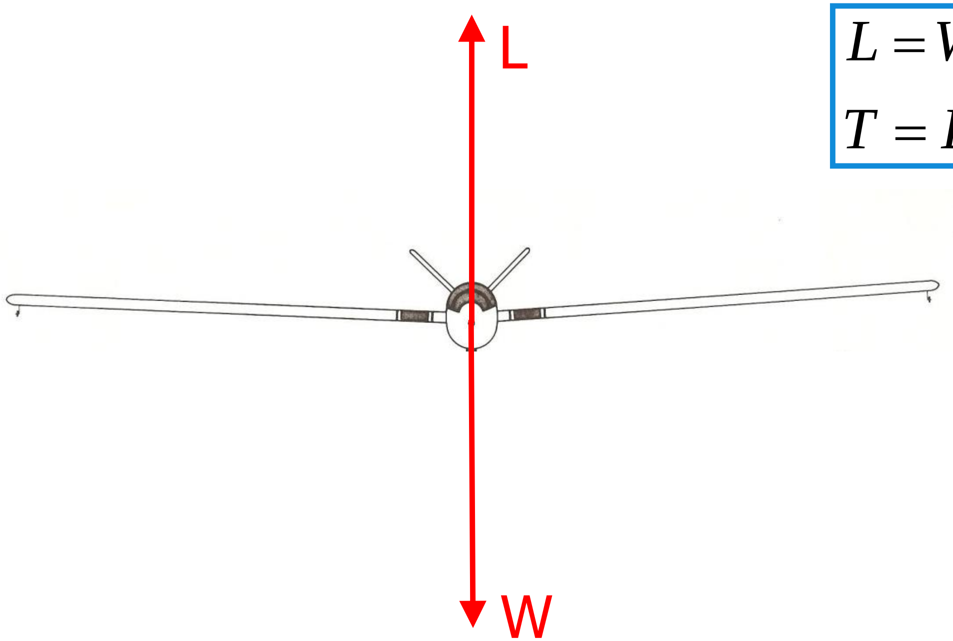
How to fly a turn?



How to fly a turn

Steady horizontal flight



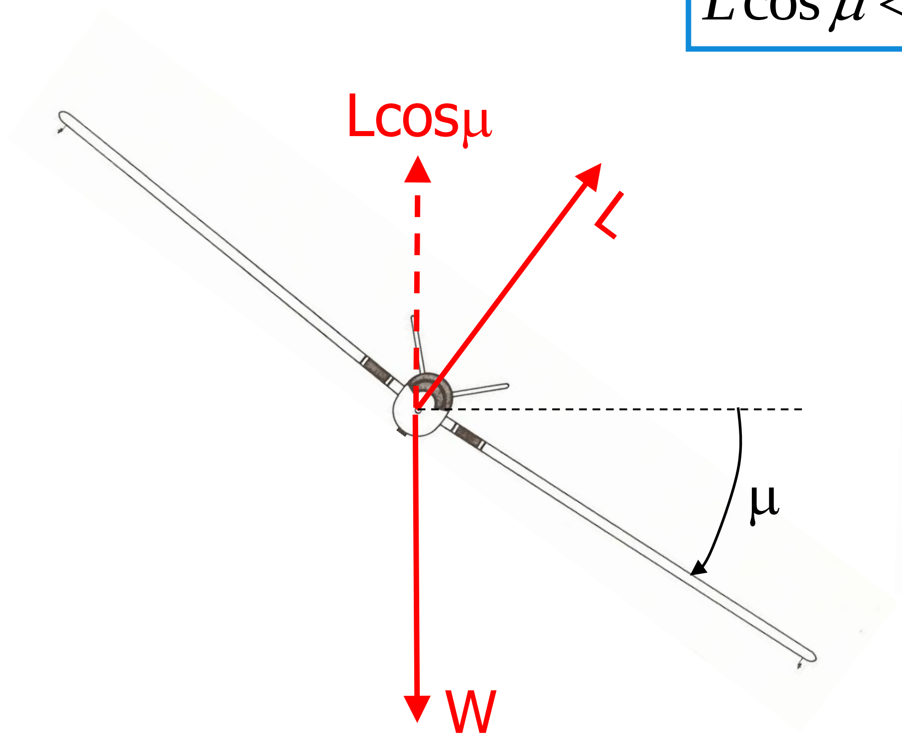
$$L = W$$

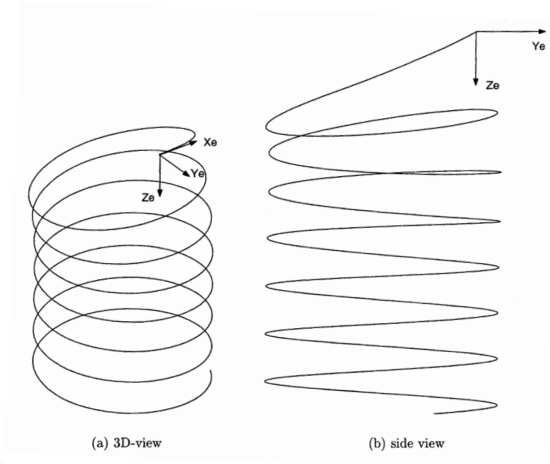
$$T = D$$

How to fly a turn

Roll the aircraft

$$L\cos\mu < W$$





<u>Only roll:</u>
Vertical component of the lift is smaller than the weight

How to fly a turn

Roll the aircraft

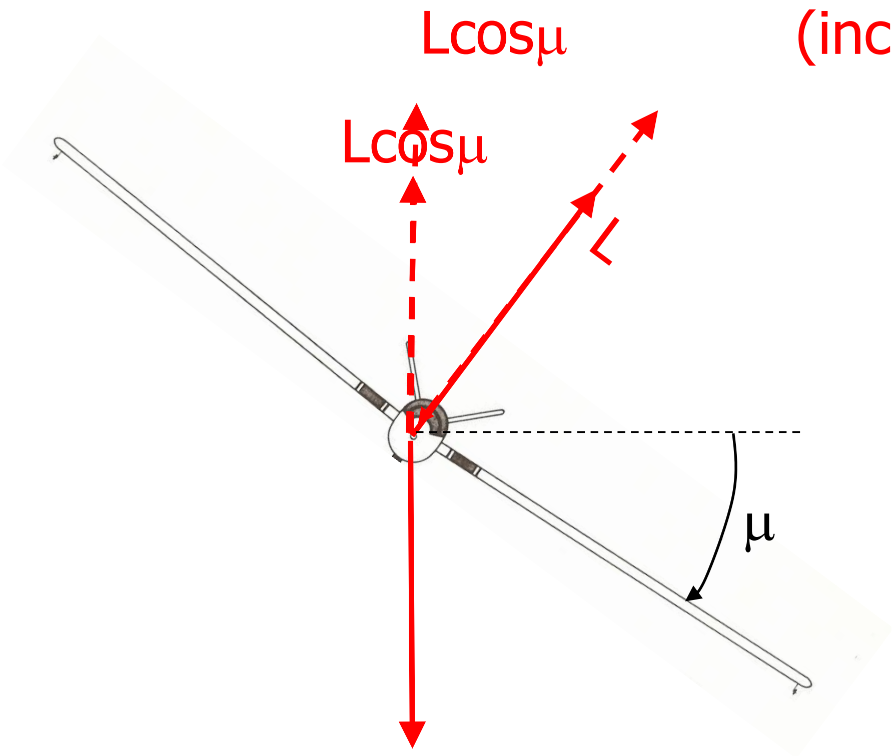
(increase angle of attack)

$L > W$

$L\cos\mu = W$

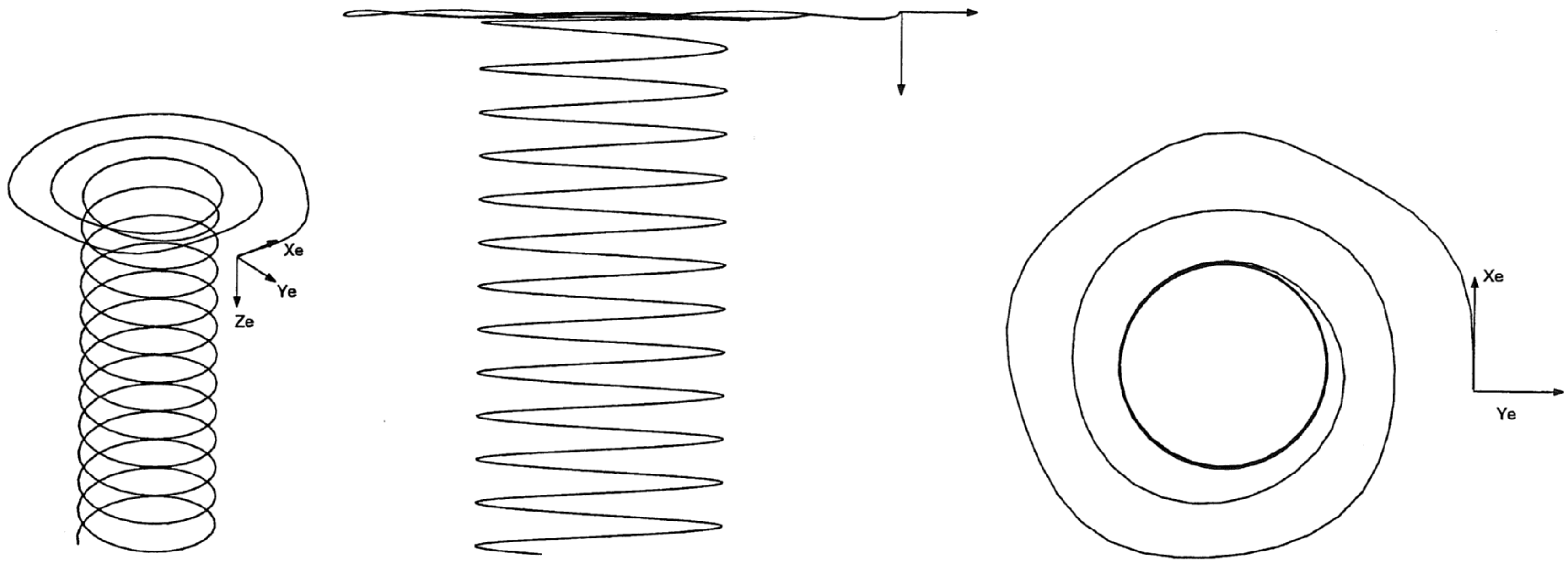
$D\uparrow$

$D > T$



How to fly a turn

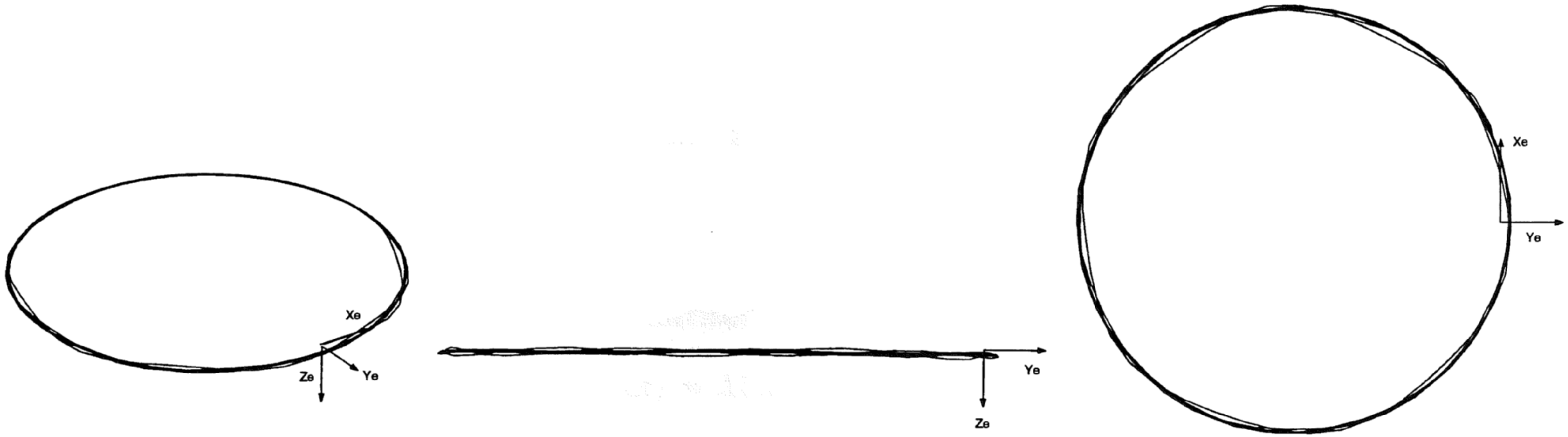
Roll the aircraft and increase the pitch



(a) 3D-view (b) side view (c) top view

How to fly a turn

1. Roll the aircraft to start turning
2. Nose up to maintain altitude
3. Increase the thrust to maintain airspeed



(a) 3D-view (b) side view (c) top view

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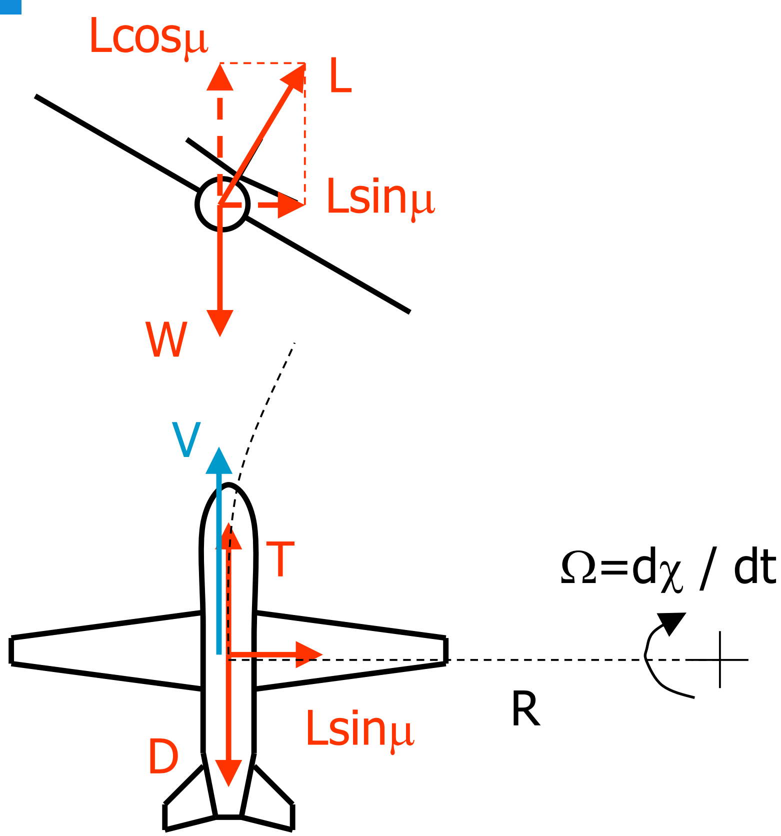
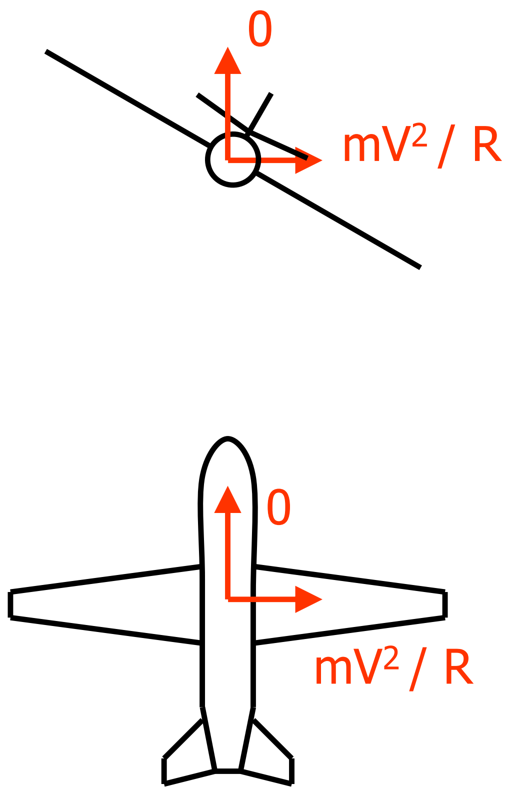
Equations of motion

Horizontal sustained turn

Free Body Diagram



Kinetic Diagram



Eq. of motion

$$\vec{F} = m\vec{a}$$

$$0 = T - D$$

$$\frac{WV^2}{gR} = L\sin\mu$$

$$0 = L\cos\mu - W$$

$$T = D$$

$$\frac{WV^2}{gR} = L\sin\mu$$

$$L\cos\mu = W$$

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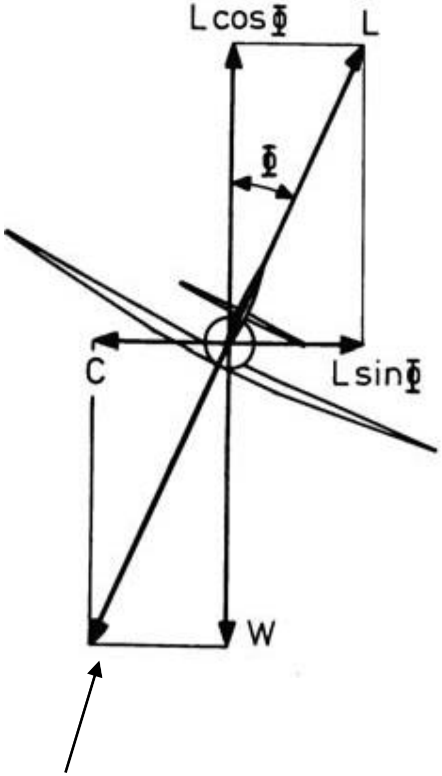
Load factor and performance diagrams

Load factor



Apparent weight
(C is a virtual / fictitious force)

Load factor:

$$n \equiv \frac{L}{W}$$

$$L = nW$$

Load factor during steady horizontal turn:

$$W = L\cos\mu$$

$$n_{turn} = \frac{L}{L\cos\mu} = \frac{1}{\cos\mu}$$

$$n_{turn,30^\circ} \approx 1.15$$

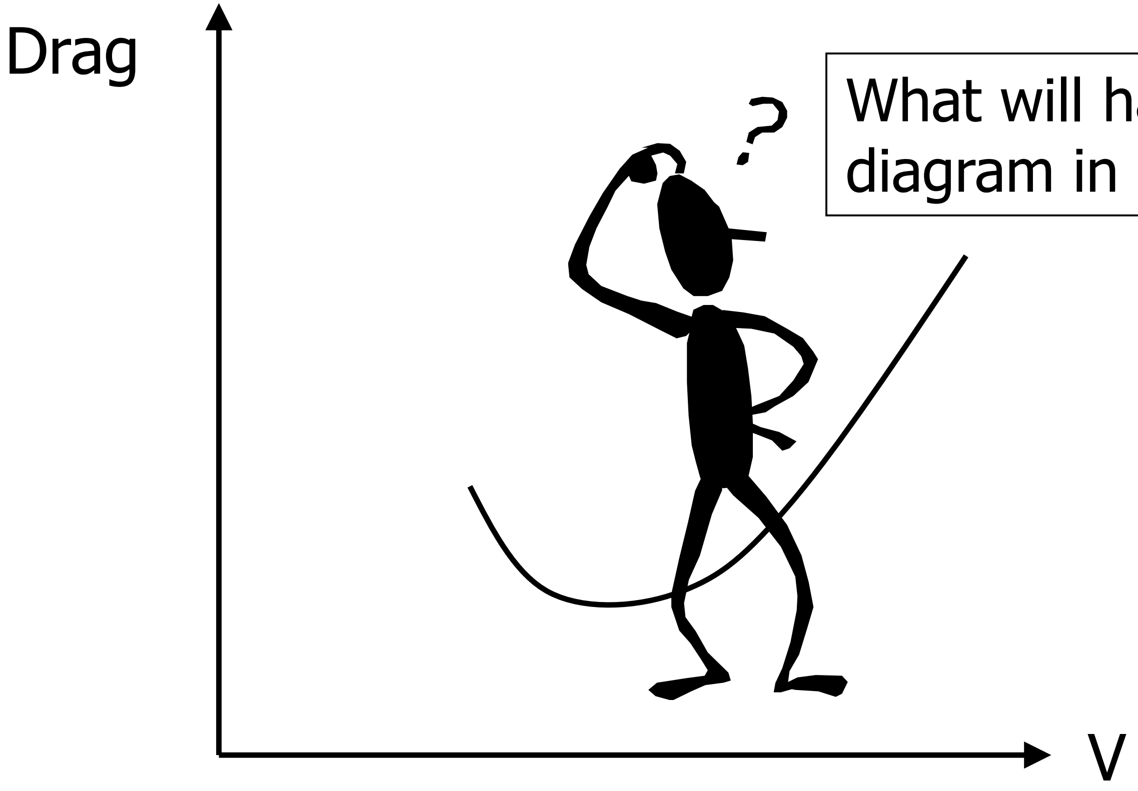
$$n_{turn,45^\circ} \approx 1.41$$

$$n_{turn,60^\circ} \approx 2$$

Video 1 F22 high g turn
http://www.youtube.com/watch?v=n34RwIUlnAo&feature=related
Video 2 (pilot's perspective)
http://www.youtube.com/watch?v=5LMVVey5bCM&feature=related

Load factor and performance diagrams

Performance diagram



What will happen to this diagram in a turn?

Load factor and performance diagrams

Performance diagram

Airspeed

$$L = nW$$

$$C_L \tfrac{1}{2} \rho V^2 S = nW$$

$$V = \sqrt{\frac{nW}{S} \frac{2}{\rho} \frac{1}{C_L}}$$

$$V = function(n, C_L)$$

Aerodynamic drag

$$D = D \frac{L}{L} = D \frac{nW}{L}$$

$$D = \frac{C_D}{C_L} nW$$

$$D = function(n, C_L)$$

Power required

$$P_r = DV$$

$$P_r = \frac{C_D}{C_L} nW \sqrt{\frac{nW}{S} \frac{2}{\rho} \frac{1}{C_L}}$$

$$P_r = \sqrt{\frac{n^3 W^3}{S} \frac{2}{\rho} \frac{C_D^2}{C_L^3}}$$

$$P_r = function(n, C_L)$$

Note:

$$T = D$$

$$P_a = P_r$$

Conclusion:
V, D, P_r are functions of lift coefficient and load factor
(In symmetric flight, they only depend on the lift coefficient)

Load factor and performance diagrams

Performance diagram

Effect of load factor (n) on airspeed (V)
(at constant angle of attack)

$$V = \sqrt{\frac{nW}{S}\frac{2}{\rho}\frac{1}{C_L}} \propto \sqrt{n}$$

Effect of load factor (n) on aerodynamic drag (D)
(at constant angle of attack)

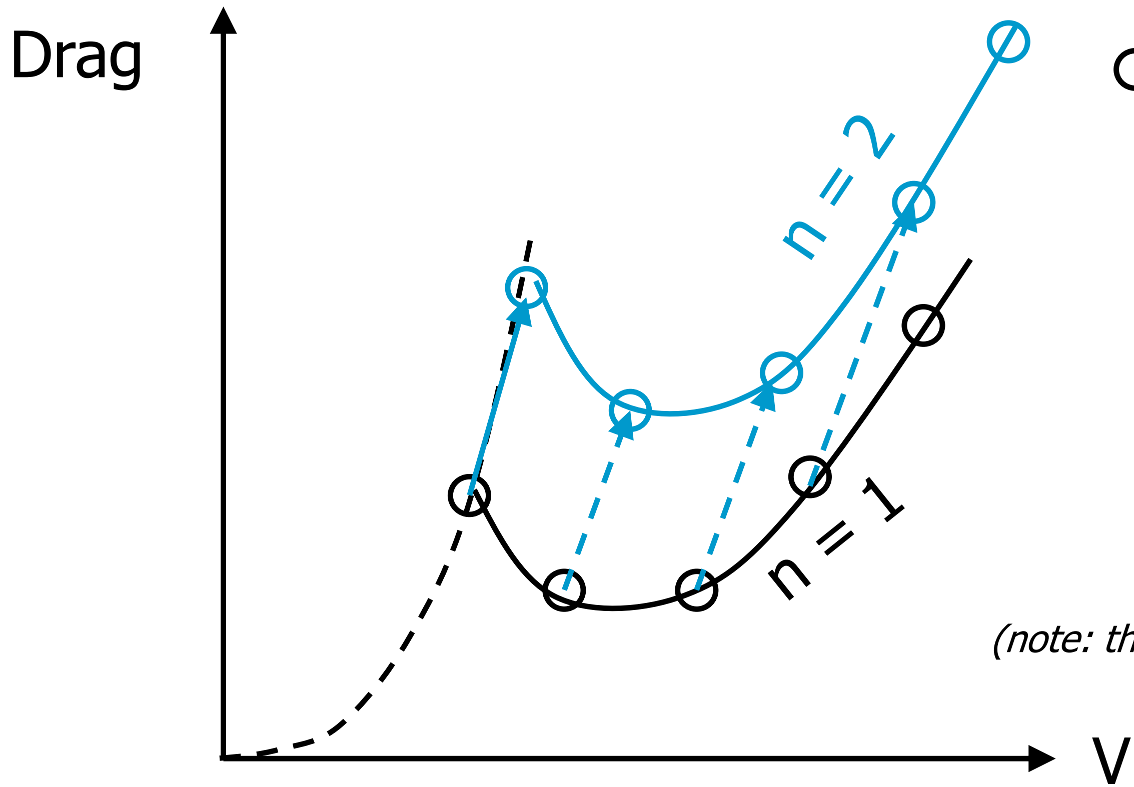
$$D = \frac{C_D}{C_L} nW \propto n$$

Effect of load factor (n) on Power required (P_r)
(at constant angle of attack)

$$P_r = DV \propto n\sqrt{n}$$

Load factor and performance diagrams

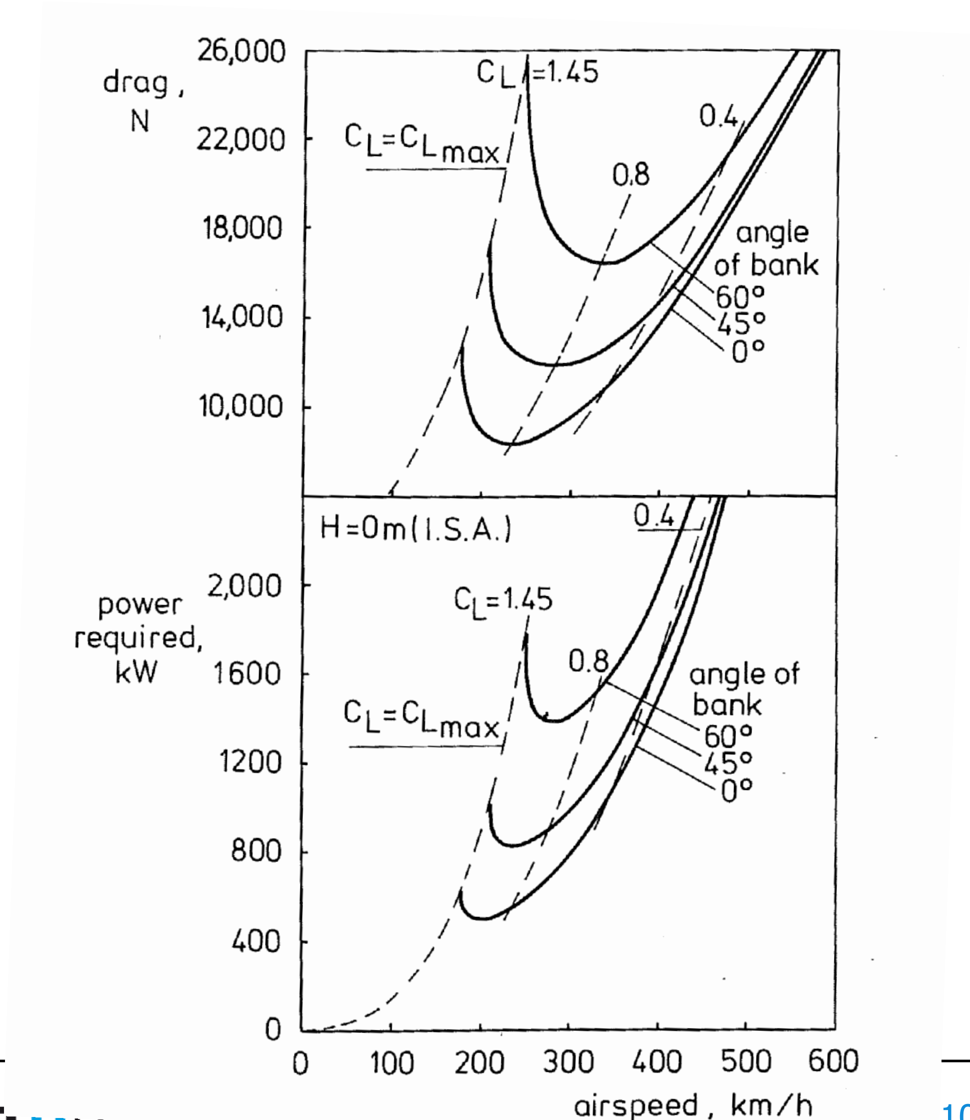
Performance diagram



○ Each point relates to 1 angle of attack (C_L)

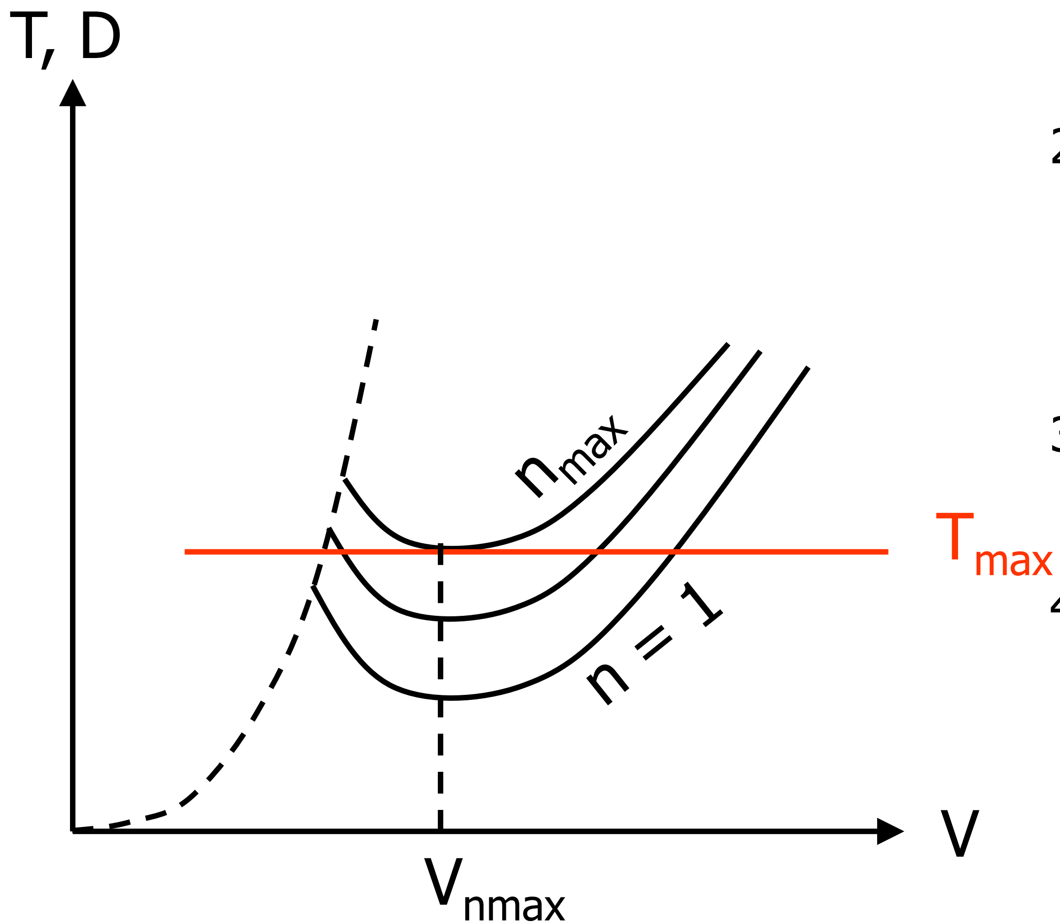
(note: this graph is purely qualitative)

Load factor and performance diagrams



Load factor and performance diagrams

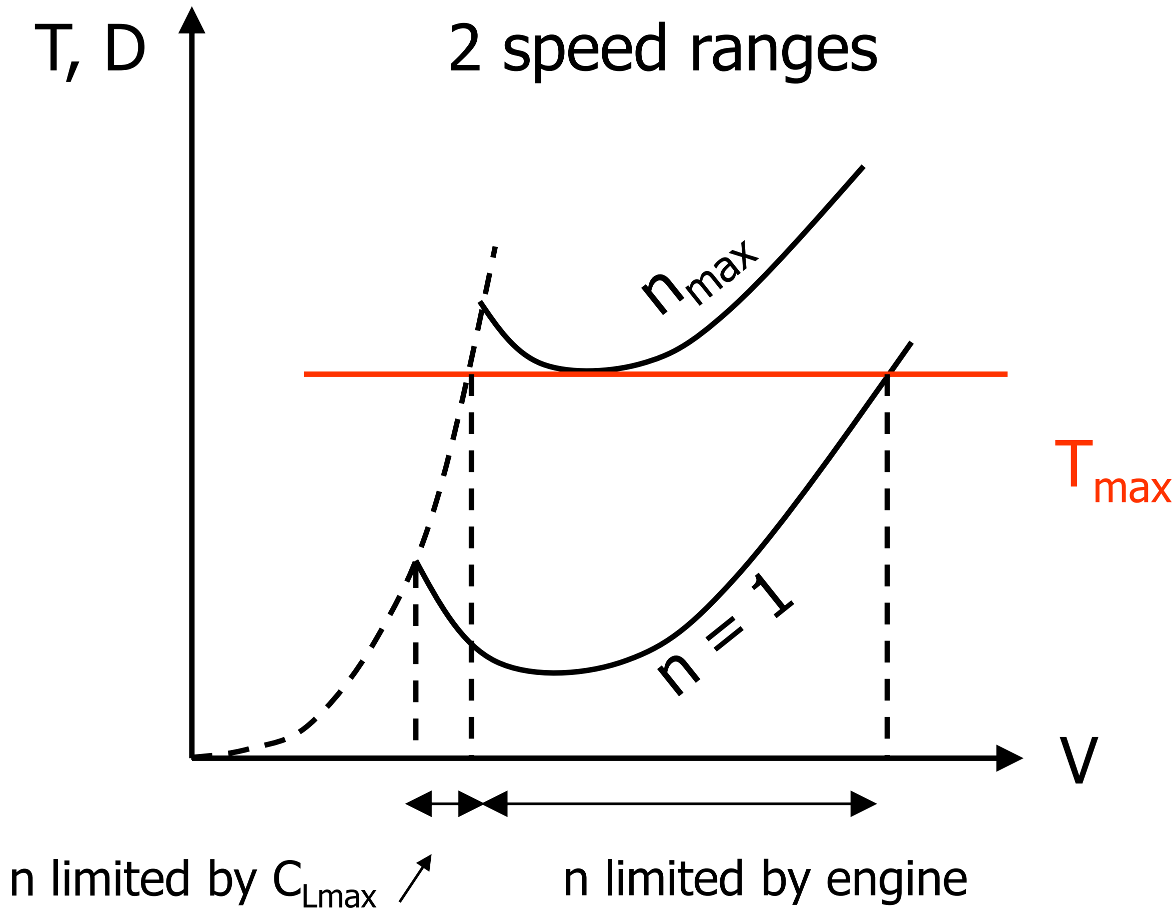
Performance diagram



1. V_{min} increases when n increases
2. V_{min} first aerodynamically limited then thrust limited (safety: regulation for stick force/g)
3. V_{max} decreases when n increases
4. At n_{max} $V_{min} = V_{max}$

Load factor and performance diagrams

Performance diagram



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Turning performance

- Maximum load factor (steepest turn)
- Minimum turn radius (tightest turn)
- Minimum time to turn (fastest turn)
- Rate one/two/three turn (civil operations)

Assumption: sustained turns

Turning performance

- **Maximum load factor (steepest turn)**
- Minimum turn radius (tightest turn)
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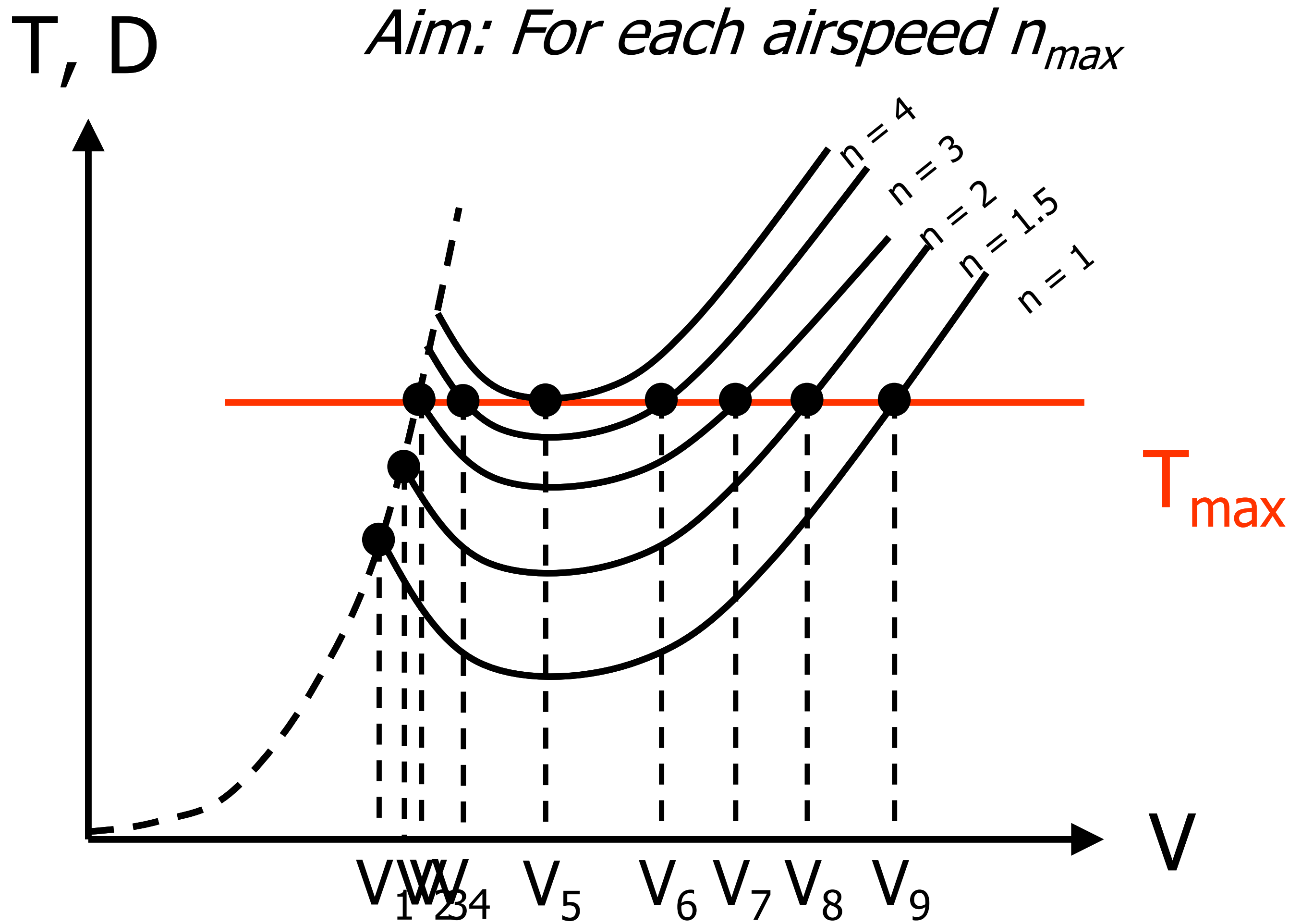
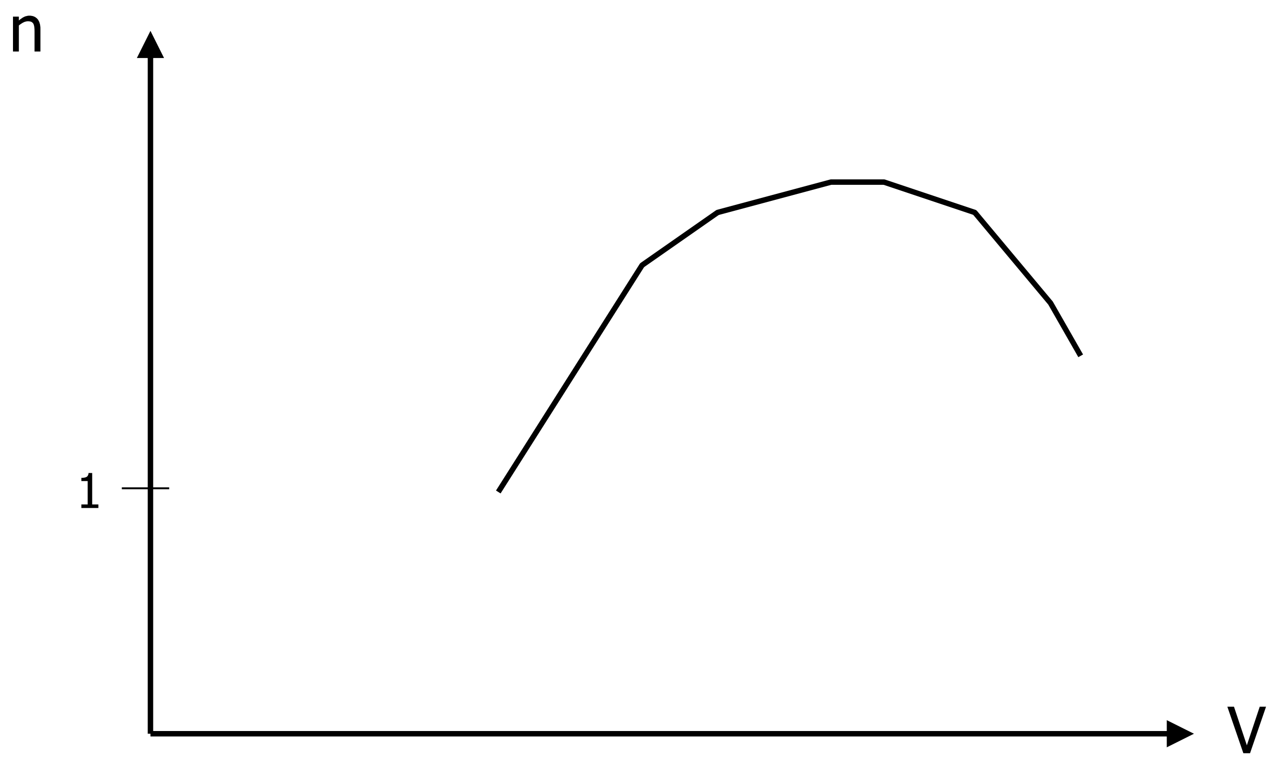
Steepest turn



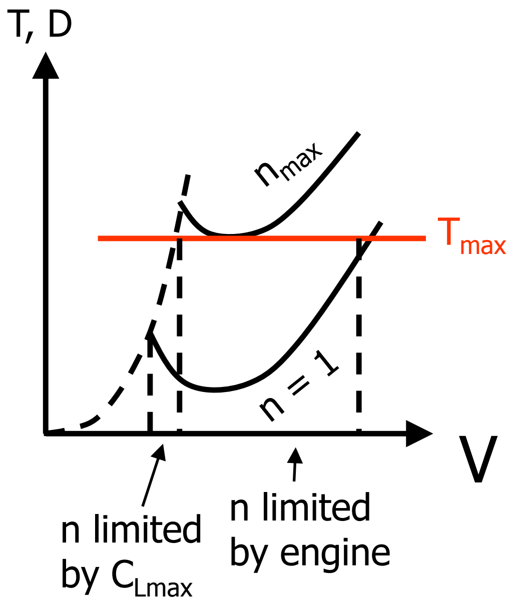
Steepest turn

Achievable load factor as function of airspeed



Steepest turn

Calculation of $n_{max}(V)$



- Range A: $C_L = C_{L\max}$

$$L = nW \rightarrow n = \frac{C_{L\max}}{\frac{W}{S}\frac{2}{\rho}\frac{1}{V^2}} \propto V^2$$

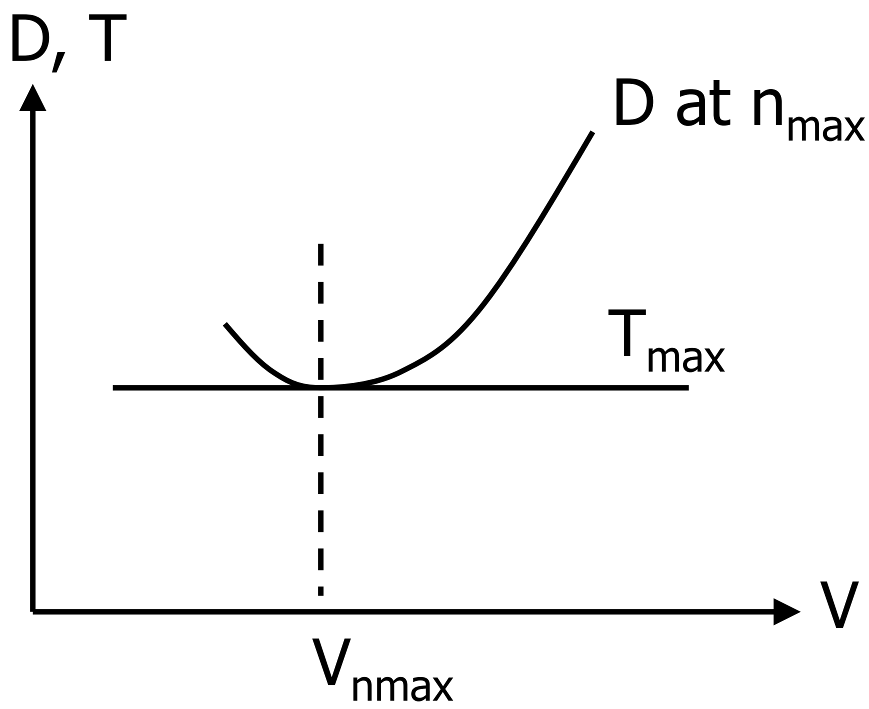
- Range B: $T = T_{\max}$ Vary C_L

$$T = D = \frac{C_D}{C_L} nW \rightarrow n = \frac{T}{W}\frac{C_L}{C_D}$$

$$V = \sqrt{\frac{nW}{S}\frac{2}{\rho}\frac{1}{C_L}} \quad \text{at every } C_L \rightarrow (n, V)$$

Steepest turn

Solution for jet aircraft



- V_{nmax} at D_{min}
- $D = \frac{C_D}{C_L} nW$

$$\rightarrow \left(\frac{C_L}{C_D} \right)_{\max}$$

$$\rightarrow C_L = \sqrt{C_{D_0} \pi A e}$$

Note: You must be able to derive the final step

Steepest turn

Solution for jet aircraft

$$T = D = \frac{C_D}{C_L} nW$$

(A numerical example will follow later)

$$n_{\max} = \frac{T_{\max}}{W}\left(\frac{C_L}{C_D}\right)_{\max}$$

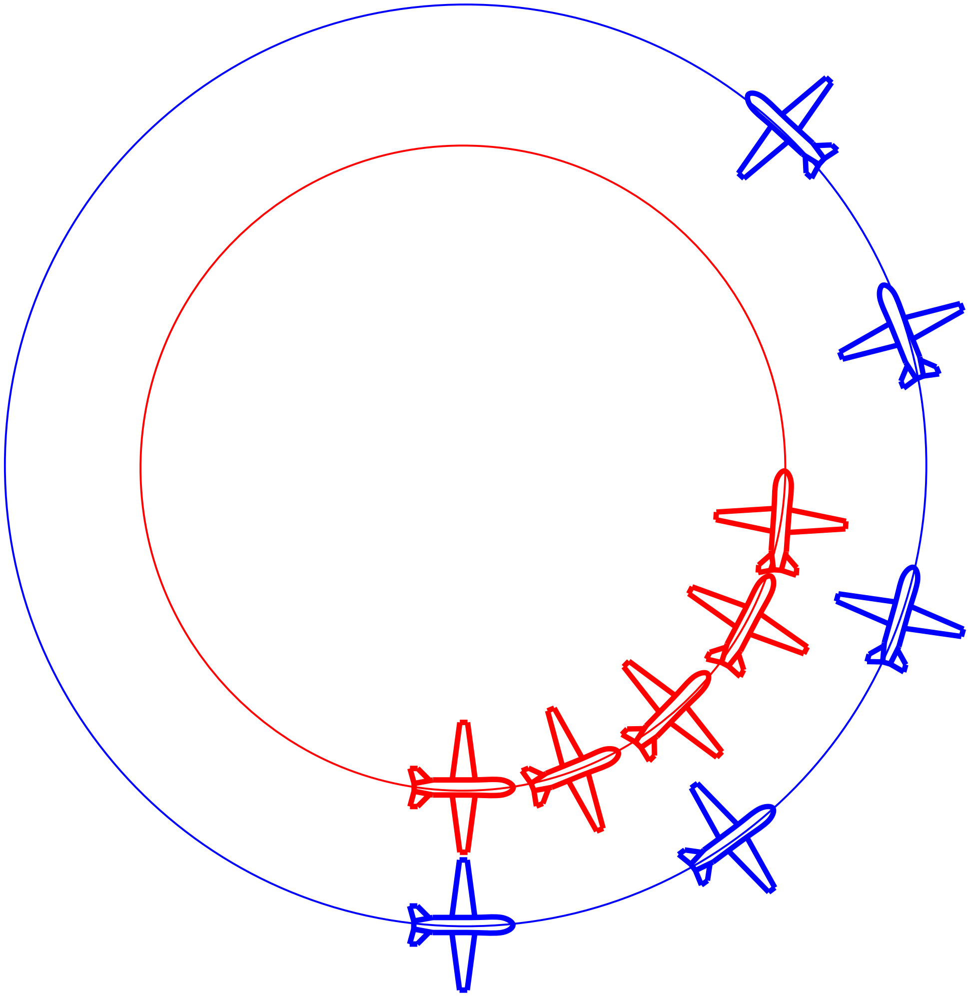
$$C_{L_{opt}} = \sqrt{C_{D_0} \pi A e}$$

$$V_{n_{\max}} = \sqrt{\frac{n_{\max} W}{S} \frac{2}{\rho} \frac{1}{C_{L_{opt}}}}$$

Turning performance

- Maximum load factor (steepest turn)
- **Minimum turn radius (tightest turn)**
- Minimum time to turn (fastest turn)
- Rate one/two/three turn (civil operations)
- Assumption: sustained turns

Do not confuse turn radius with time to turn!



Minimum turn radius (tightest turn)

First we must have an equation for the turn radius

Using the equilibrium equations:

$$L\sin\mu = \frac{W}{g}\frac{V^2}{R}$$

$$W = L\cos\mu$$

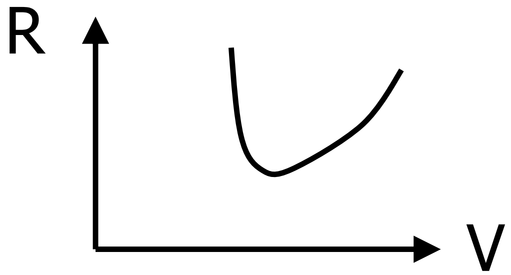
$$n \equiv \frac{L}{W} = \frac{1}{\cos\mu}$$

$$\cos^2 x + \sin^2 x = 1 \Rightarrow \sin\mu = \sqrt{1-\frac{1}{n^2}}$$

$$nW\sqrt{1-\frac{1}{n^2}} = \frac{W}{g}\frac{V^2}{R} \Rightarrow R = \frac{V^2}{g\sqrt{n^2-1}}$$

Minimum turn radius

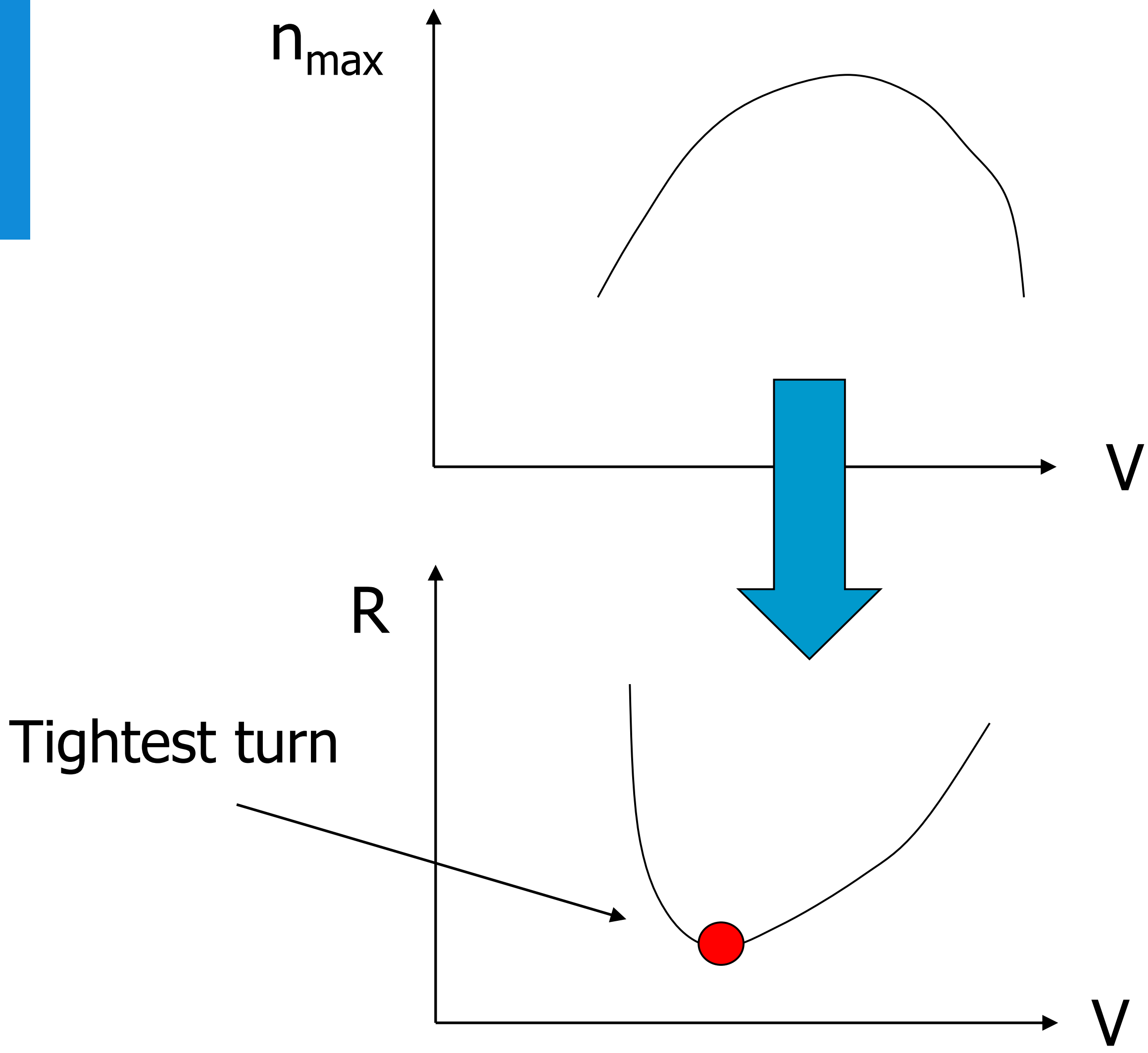
$$R = \frac{V^2}{g\sqrt{n^2 - 1}}$$



For a given n, R decreases when V decreases. So, tighter turns are possible at lower airspeeds

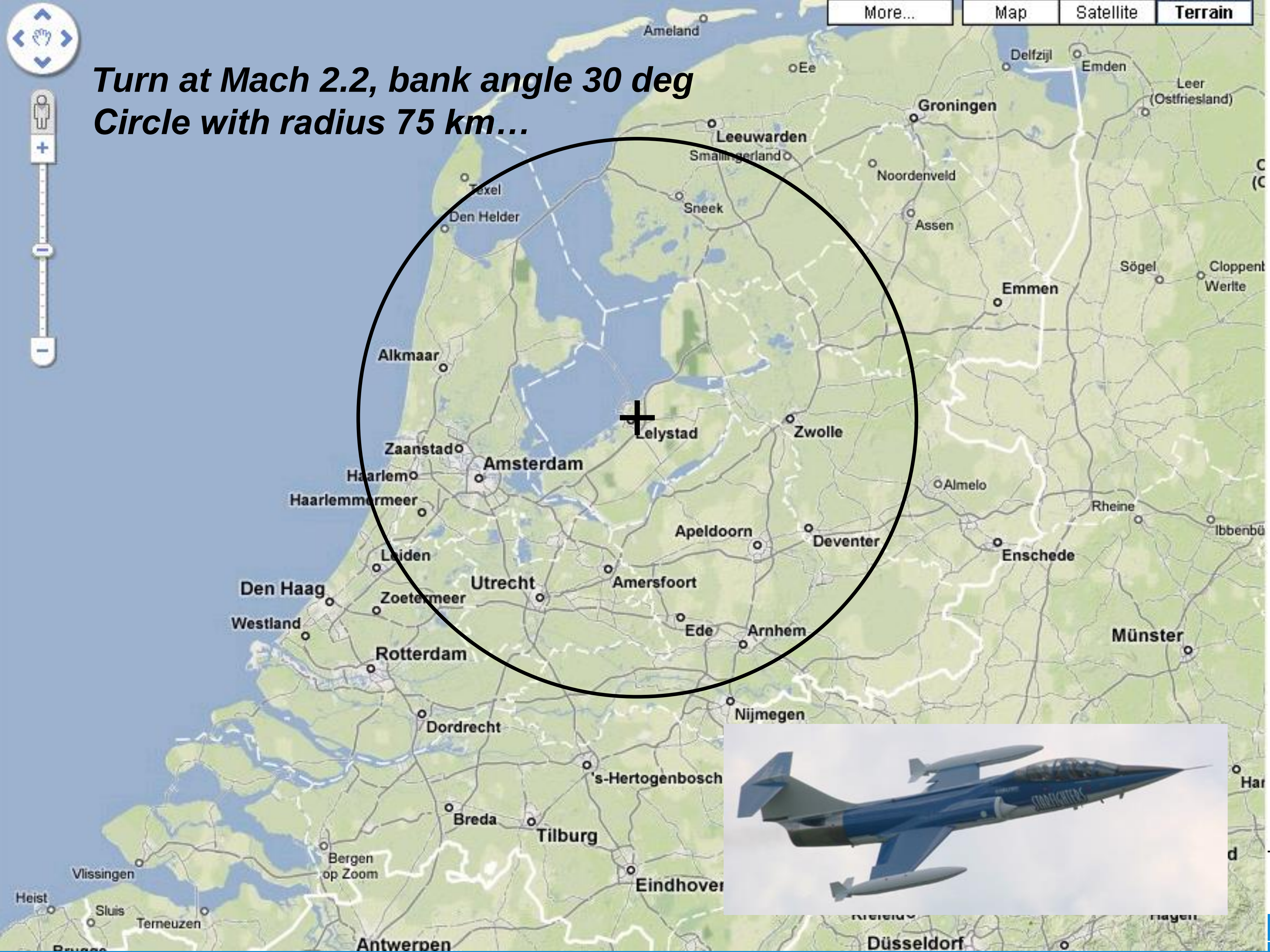
An increase in n for a given V results in a decrease in R

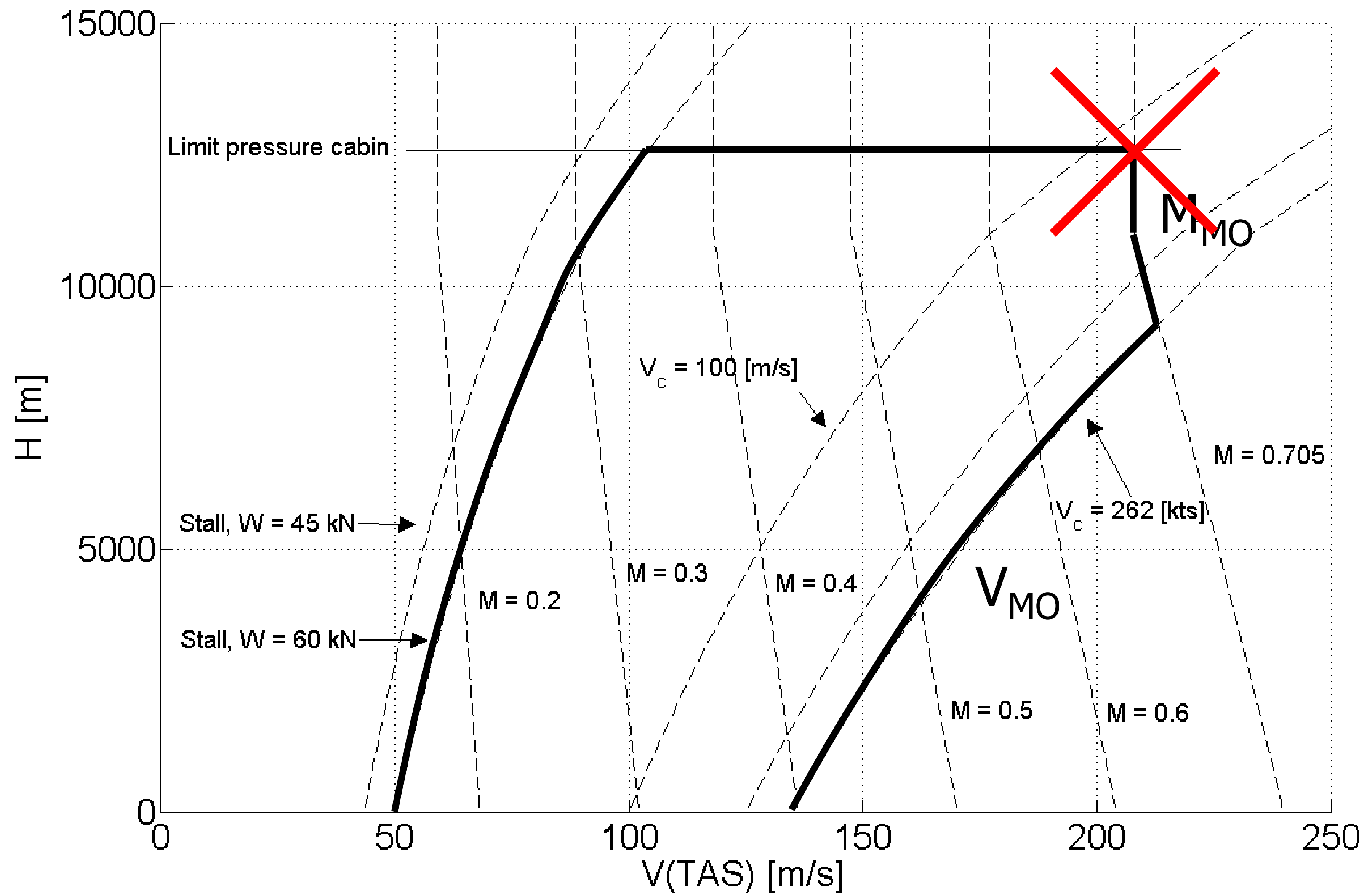
(remember the example of the aircraft turning at Mach 2)



$$R = \frac{V^2}{g\sqrt{n^2 - 1}}$$

Turn at Mach 2.2, bank angle 30 deg
Circle with radius 75 km…




15000
10000
5000
0
H [m]
Limit pressure cabin
M_{MO}
V_c = 100 [m/s]
M = 0.705
V_c = 262 [kts]
Stall, W = 45 kN
Stall, W = 60 kN
M = 0.2
M = 0.3
M = 0.4
V_{MO}
M = 0.5
M = 0.6
0
50
100
150
200
250
V(TAS) [m/s]

Turning performance

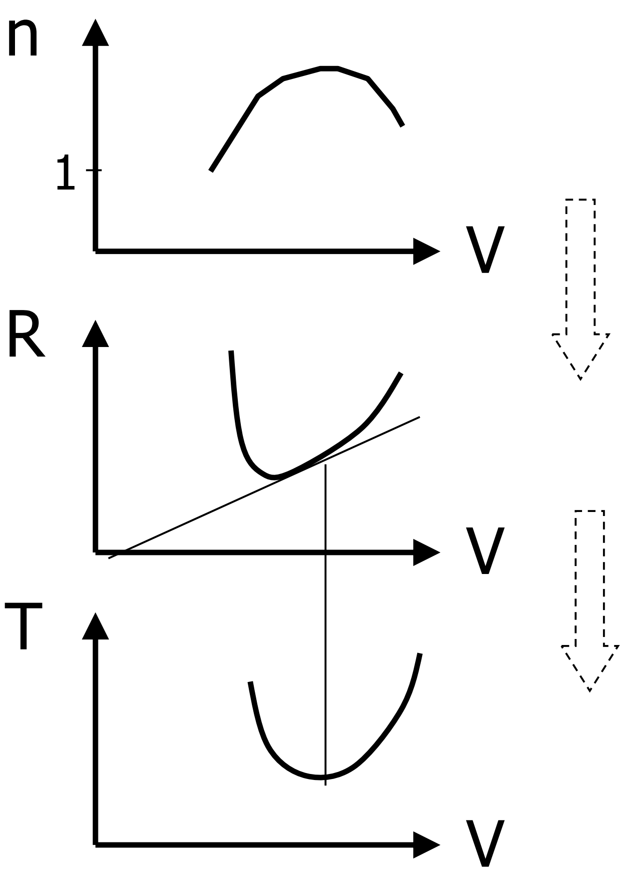
- Maximum load factor (steepest turn)
- Minimum turn radius (tightest turn)
- **Minimum time to turn (fastest turn)**
- Rate one/two/three turn (civil operations)
- Assumption: sustained turns

Fastest turn

Minimize time to achieve fastest turn
Time to complete a full circle:

$$T_{2\pi} = \frac{2\pi R}{V}$$

(Circumference of turn: $2\pi R$)



Turning performance

- Maximum load factor (steepest turn)
- Minimum turn radius (tightest turn)
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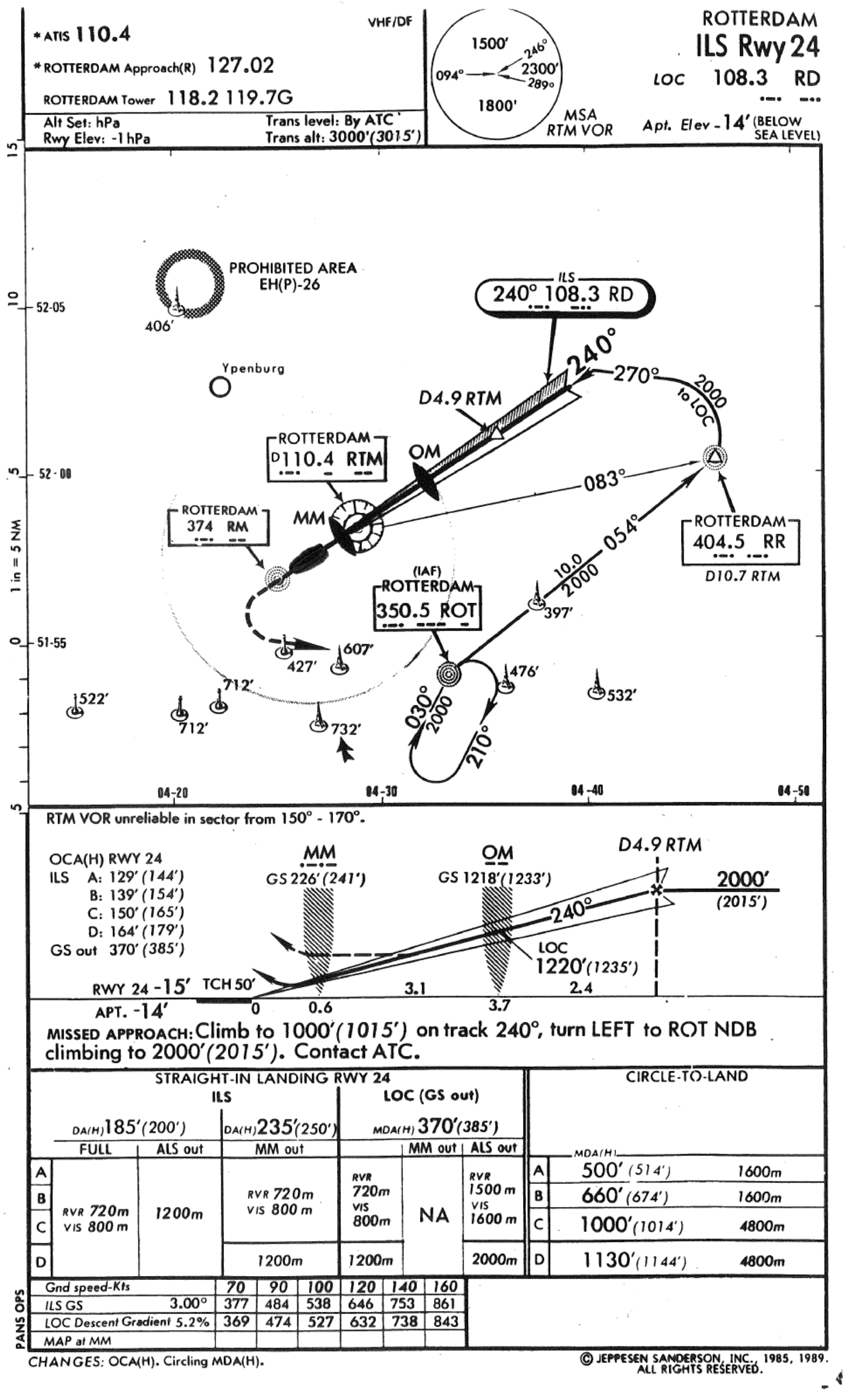
- **Rate one/two/three turn (civil operations)**
- Assumption: sustained turns

Rate one turn

- Rate one turn: 180 deg/min
- Rate two turn: 360 deg/min

Standardized turns are useful for air traffic control

Safety:
< 200’ no turns light a/c
< 1000’ no turns heavy a/c



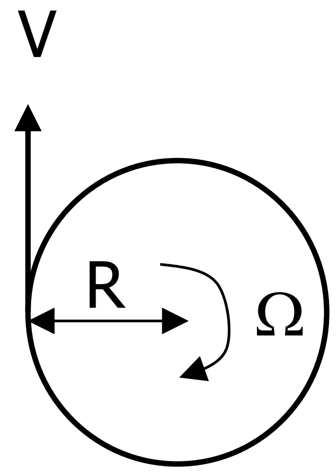
Rate one turn

Example:
Approach: V = 80 m/s, T_π = 60 [s]

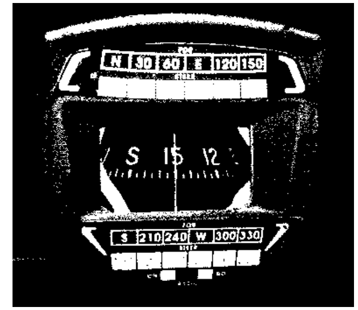
$$\Omega = \frac{V}{R} \Rightarrow R = \frac{V}{\Omega} = \frac{80}{\pi / 60} = 1502 \text{ [m]}$$

$$R = \frac{V^2}{g\sqrt{n^2 - 1}} \Rightarrow n = \sqrt{\left(\frac{V^2}{gR}\right)^2 + 1} = 1.09$$

$$\phi = \arccos\left(\frac{1}{n}\right) = 23°$$



Rate one turn



Rate one turn

Coordinated Turn

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Example calculations

Numerical example – turning performance at 7000 [m] altitude

Lift drag polar is known for this aircraft $C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$

Aircraft weight, maximum thrust and dimensions are known as well

Example calculations

Aircraft and atmospheric data at 7000 [m]

$$\rho = 0.59 \text{ [kg/m}^3\text{] (ISA)}$$
$$T_{\max} = 8.67 \text{ [kN]}$$
$$C_{D_0} = 0.021$$
$$Ae = 7 \text{ [-]}$$
$$W = 60000 \text{ [N]}$$
$$S = 30 \text{ [m}^2\text{]}$$

Calculate:

1. Maximum load factor and corresponding airspeed
2. Radius of tightest turn (R_{min}) and corresponding airspeed
3. Time for fastest turn ($T_{2\pi,min}$) and corresponding airspeed

Example calculations

Maximum load factor

$$n_{max} = \frac{T_{max}}{W}\left(\frac{C_L}{C_D}\right)_{max}$$

$$T_{max} = 8670 \text{ [N]}$$

$$\left(\frac{C_L}{C_D}\right)_{max} \Rightarrow C_L = \sqrt{C_{D_0}\pi A e}$$

$$C_L = \sqrt{0.021\cdot\pi\cdot 7} = 0.68 \text{ [-]}$$

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

$$C_D = 0.021 + \frac{0.68^2}{7\pi} = 0.042 \text{ [-]}$$

$$n_{max} = \frac{8670}{60000}\frac{0.68}{0.042} = 2.34$$

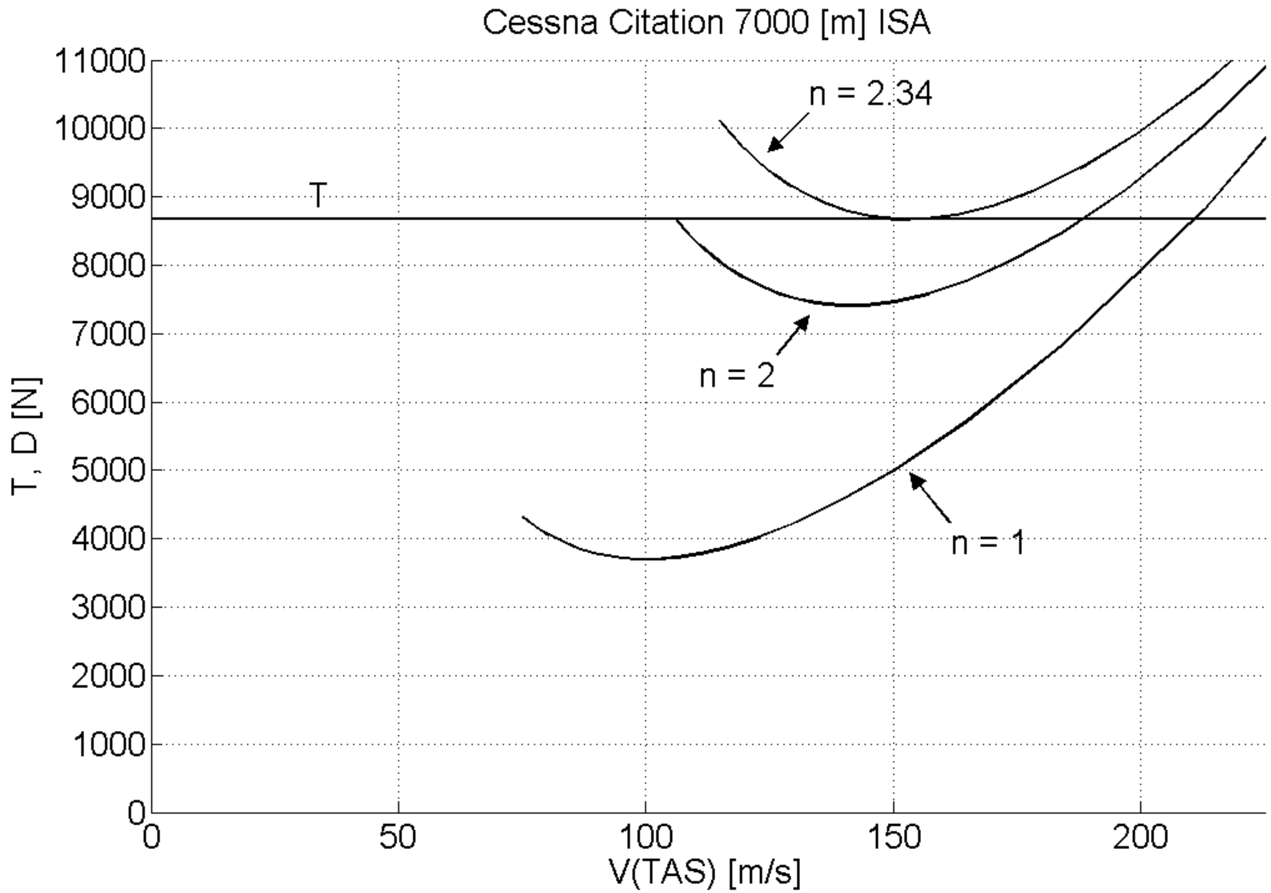
$$V_{n\,max} = \sqrt{\frac{n_{max}W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

$$= \sqrt{\frac{2.34\cdot 60000}{30}\frac{2}{0.59}\frac{1}{0.68}}$$

$$= 152.7 \text{ [m/s]}$$

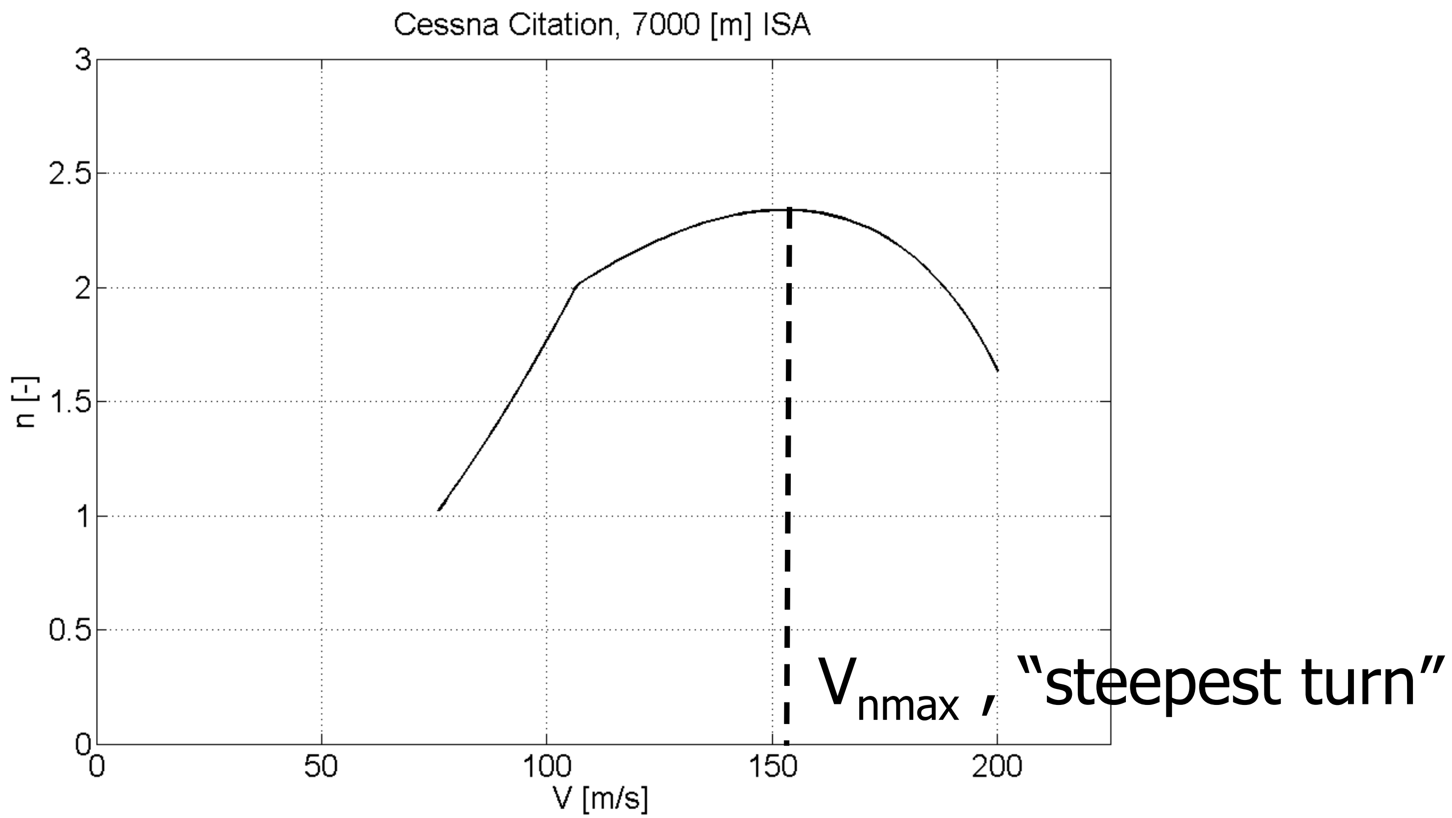
Example calculations

Maximum load factor



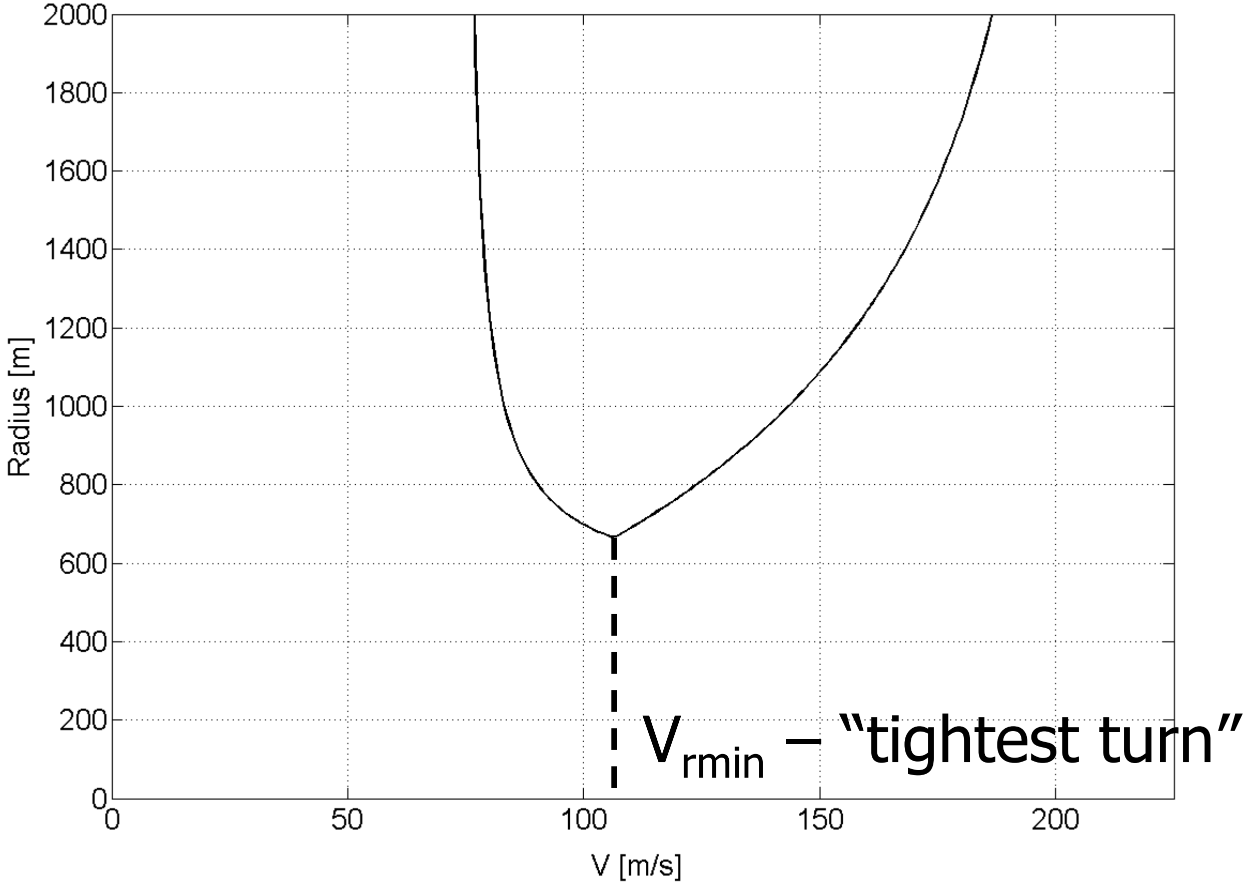
Example calculations

Maximum load factor



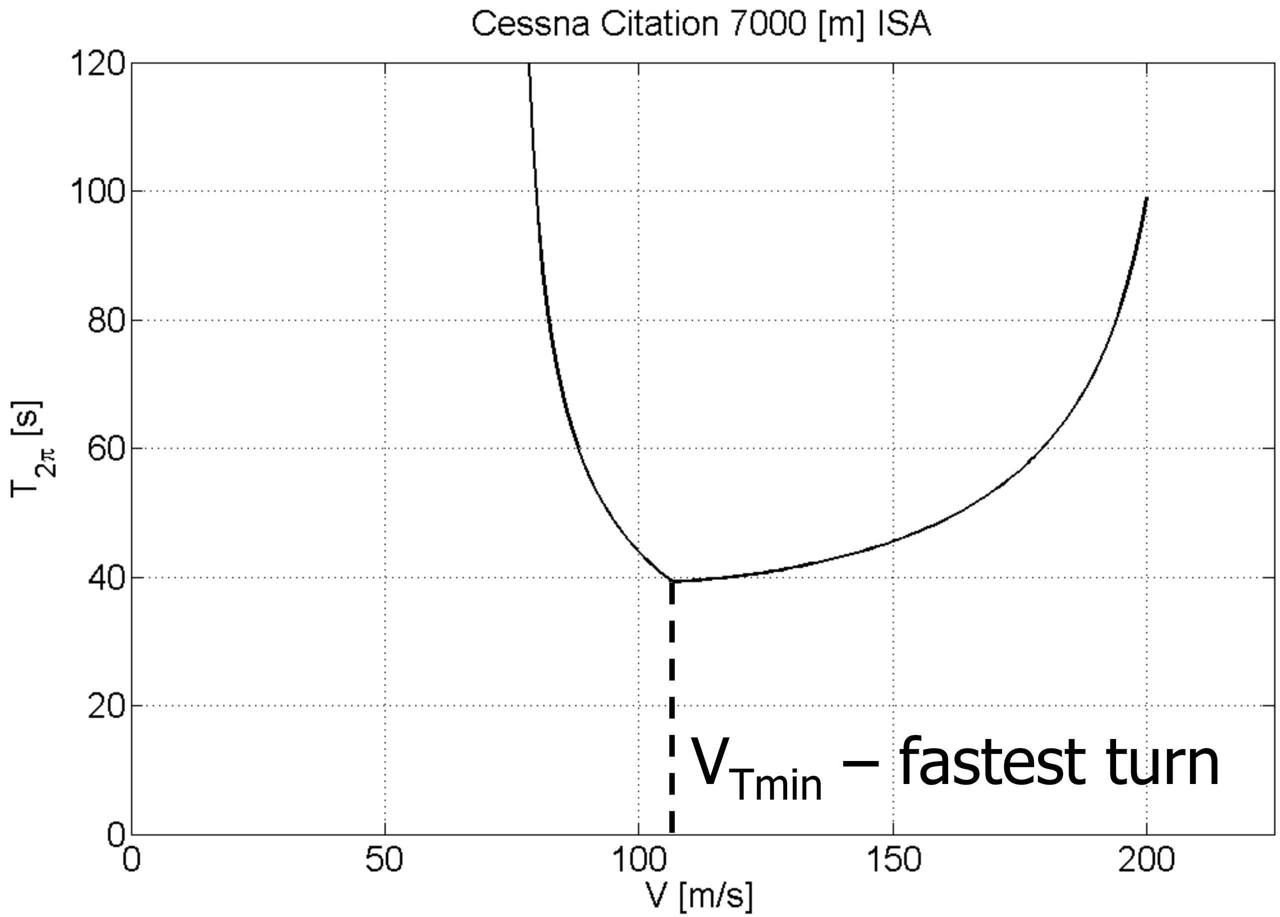
Example calculations

Minimum turn radius



Example calculations

Minimum time to turn (fastest turn)



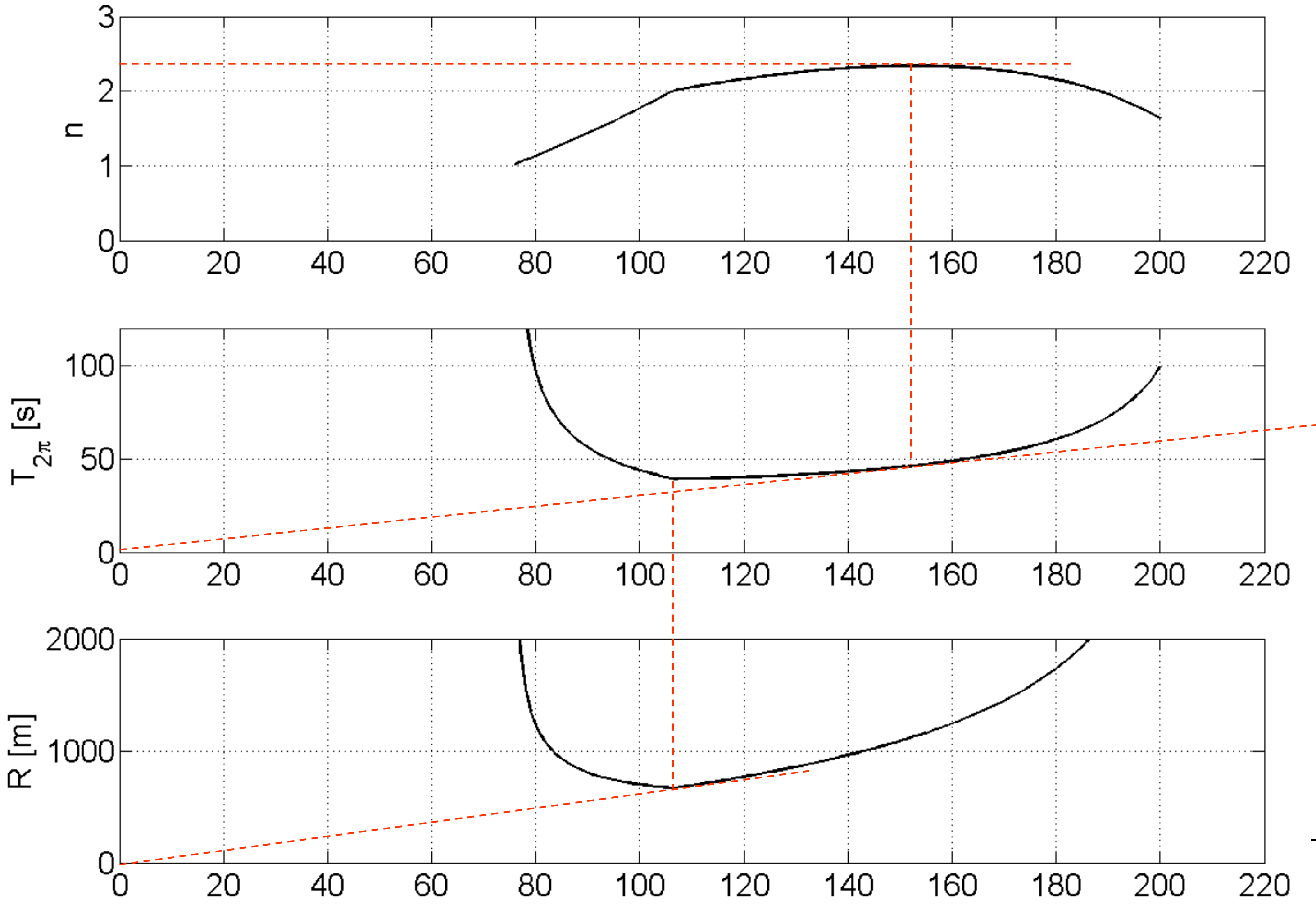
Example calculations

All results combined

$$n_{max} = \mu_{max}$$

$$\tan\mu = \frac{2\pi V}{g T_{2\pi}}$$

$$T_{2\pi} = \frac{2\pi R}{V}$$



TU Delft

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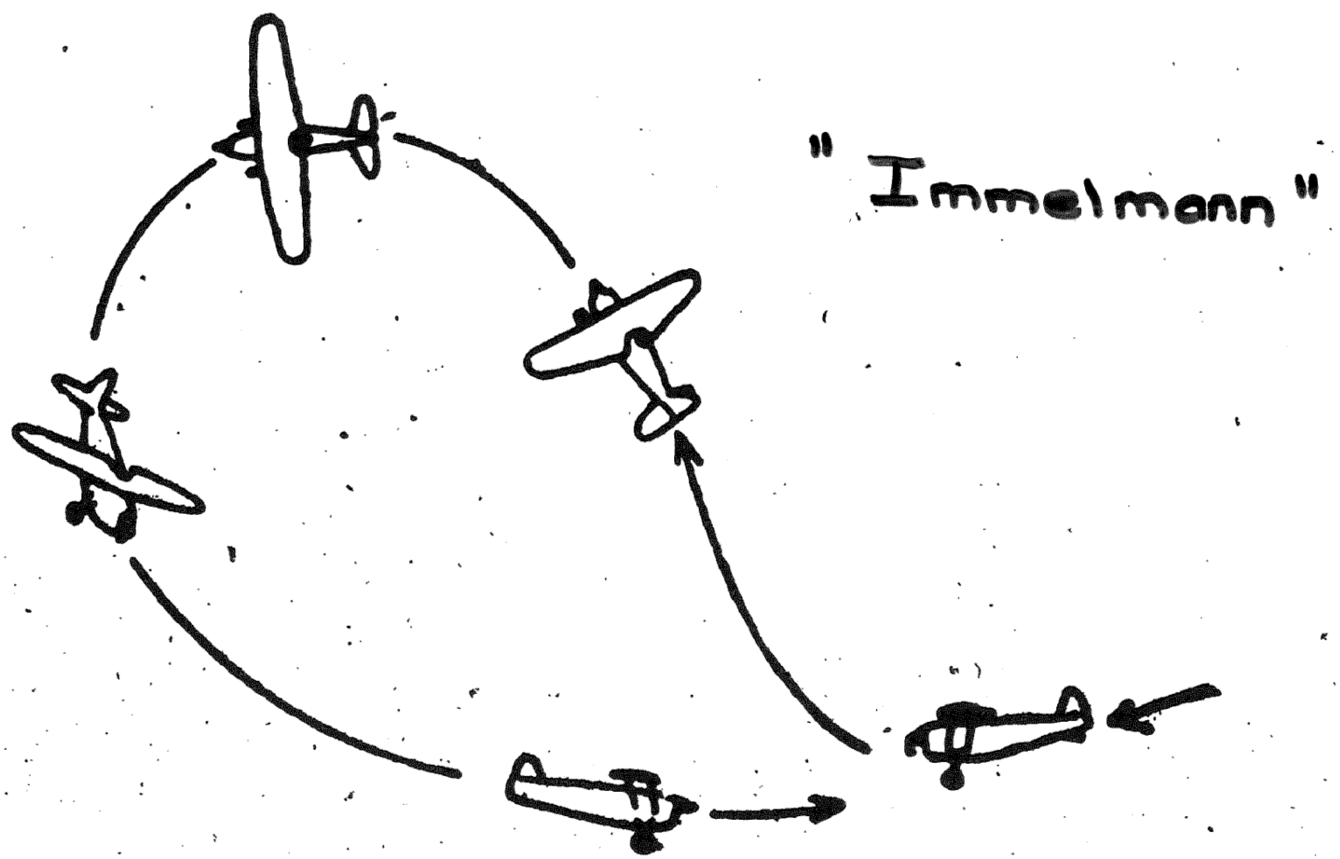
Advanced manoeuvres

Immelmann



http://www.youtube.com/watch?v=CGbOs0vgYOA&NR=1
(thrust vecotring turn at 2:50)

Advanced Manoeuvres

Thrust vectoring

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Altitude effects

Equations

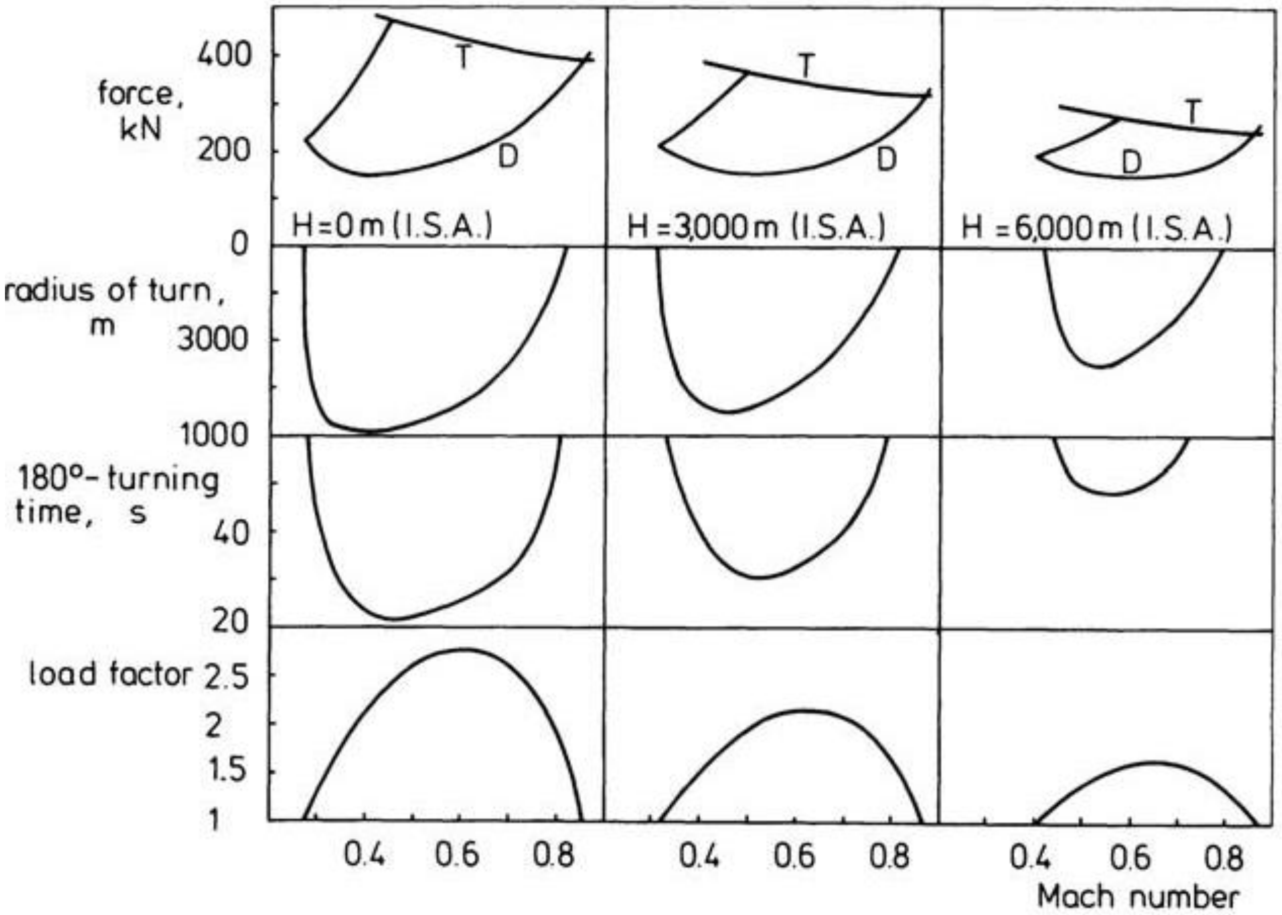
$$n_{\max} = \frac{T}{W}\left(\frac{C_L}{C_D}\right)_{\max} \propto \rho^{0.75}$$

$$V_{n_{\max}} = \sqrt{\frac{nW}{S}\frac{2}{\rho}\frac{1}{C_L}} \propto \rho$$

$$R_{n_{\max}} = \frac{V^2}{g\sqrt{n^2-1}} \uparrow \text{ if } \rho \downarrow$$

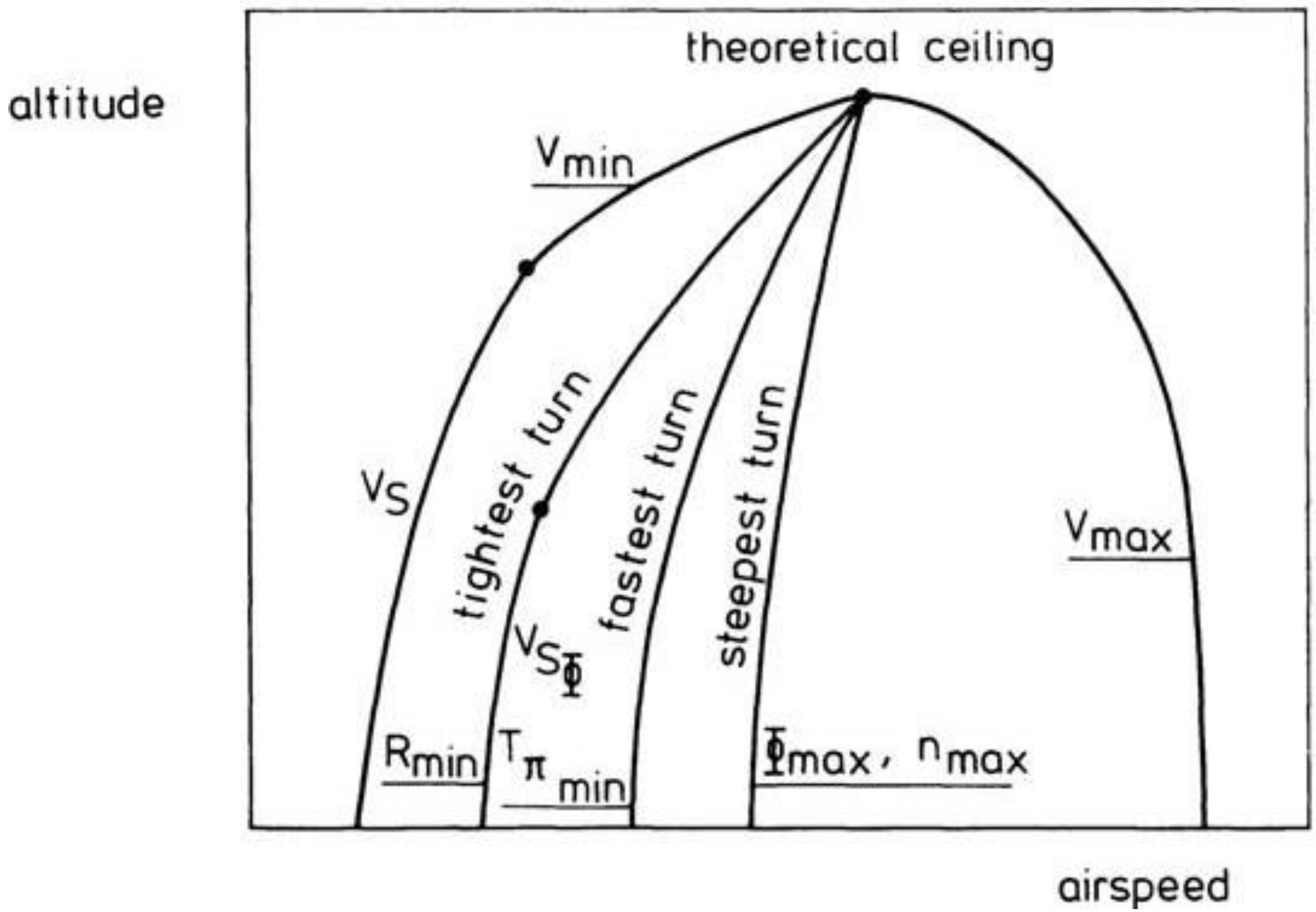
Altitude effects

Example



Altitude effects

Example



Content

- Aim
- How to fly a turn
- Equations of motion
- Load factor and performance diagrams
- Turning performance
- Example calculations
- Advanced manoeuvres
- Altitude effects
- **Summary**
- Example exam question

Summary

- Be able to derive the equations of motion for a horizontal sustained turn (what is the definition of horizontal and sustained?)

$$T = D$$

$$\frac{mV^2}{R} = L\sin\mu$$

$$L\cos\mu = W$$

- Load factor is defined as n = L/W

- In a horizontal sustained turn, n = 1/cosμ

Summary

- Maximum load factor for jet aircraft: $$n_{\max} = \frac{T_{\max}}{W}\left(\frac{C_L}{C_D}\right)_{\max}$$
- Turn radius: $$R = \frac{V^2}{g\sqrt{1-n^2}}$$
- You must be able to derive the equations mentioned above
- Time to turn 360 deg: $$T_{2\pi} = \frac{2\pi R}{V}$$
- Rate one turn is defined as 180 deg / min

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Example exam question

The following data is known for a small propeller aircraft (Cessna 172)

P_a = 100kW (independent of airspeed)

W = 10 kN

S = 15 m^2

The maximum rate of climb in steady symmetric flight of this aircraft equals 6 m/s when operating at sea level (0 m) in the International Standard Atmosphere.

a. Calculate the maximum load factor and corresponding bank angle of this aircraft in a horizontal steady turn when operating at the **same** altitude, power setting and angle of attack

b. The aircraft performs a rate one turn with an airspeed of 180 km/h at sea level in the international standard atmosphere. Calculate the required bank angle and corresponding turn radius.

Questions?