Flight and Orbital Mechanics

Lecture slides

Flight and Orbital Mechanics

Lecture hour 8, 9 – Take - off

Mark Voskuijl

Semester 1 - 2012

Content

- Introduction
- Equations of motion
- Accidents
- Take of distance analytical solution
  - Ground run
  - Airborne phase
- How to determine operational speeds
- Example exam question
- Summary
- *Additional topics*

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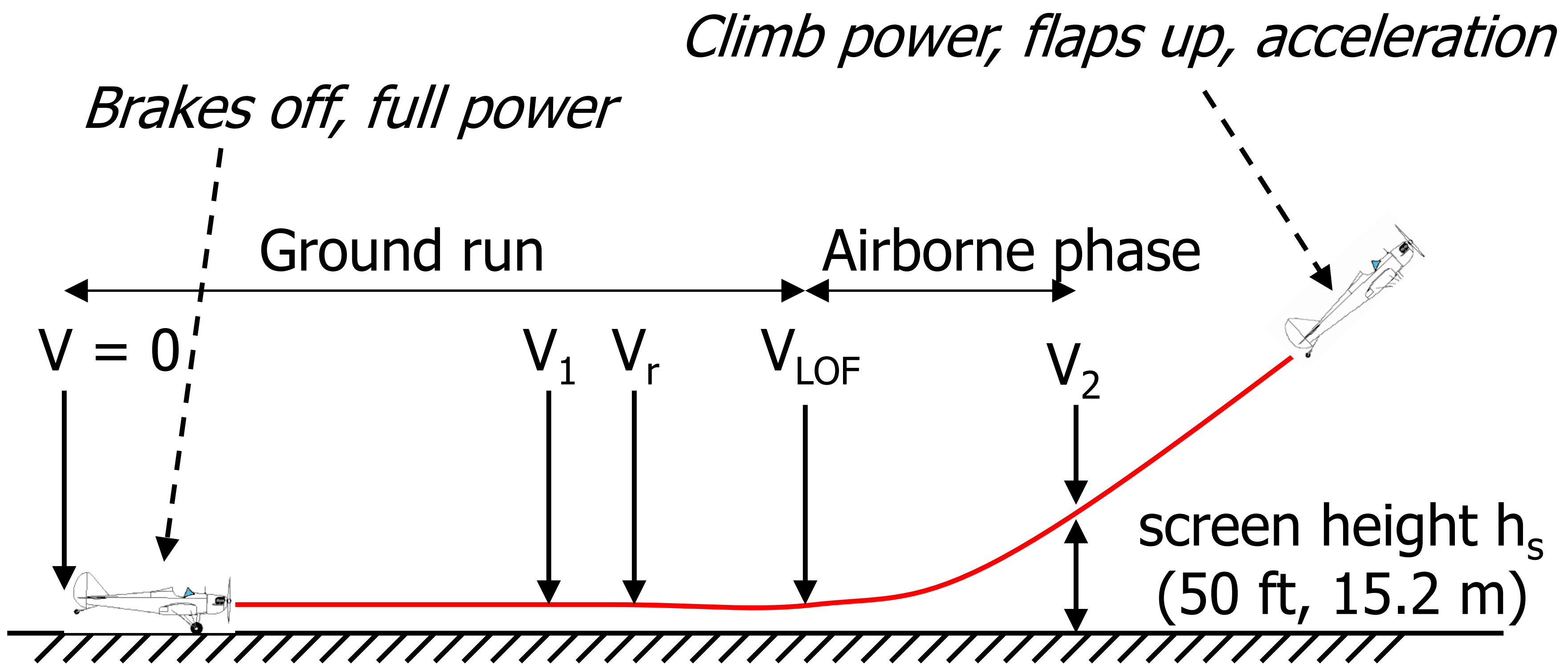
Introduction

Definition take-off manoeuvre

The take-off can be defined as the manoeuvre by which the airplane is accelerated from rest on the runway to the climb out speed V_c over a 10.7 m obstacle (screen height) for civil transports or a 15.2 m obstacle for light propeller-driven and military airplanes [Ruijgrok]

Introduction

Take-off manoeuvre



Introduction

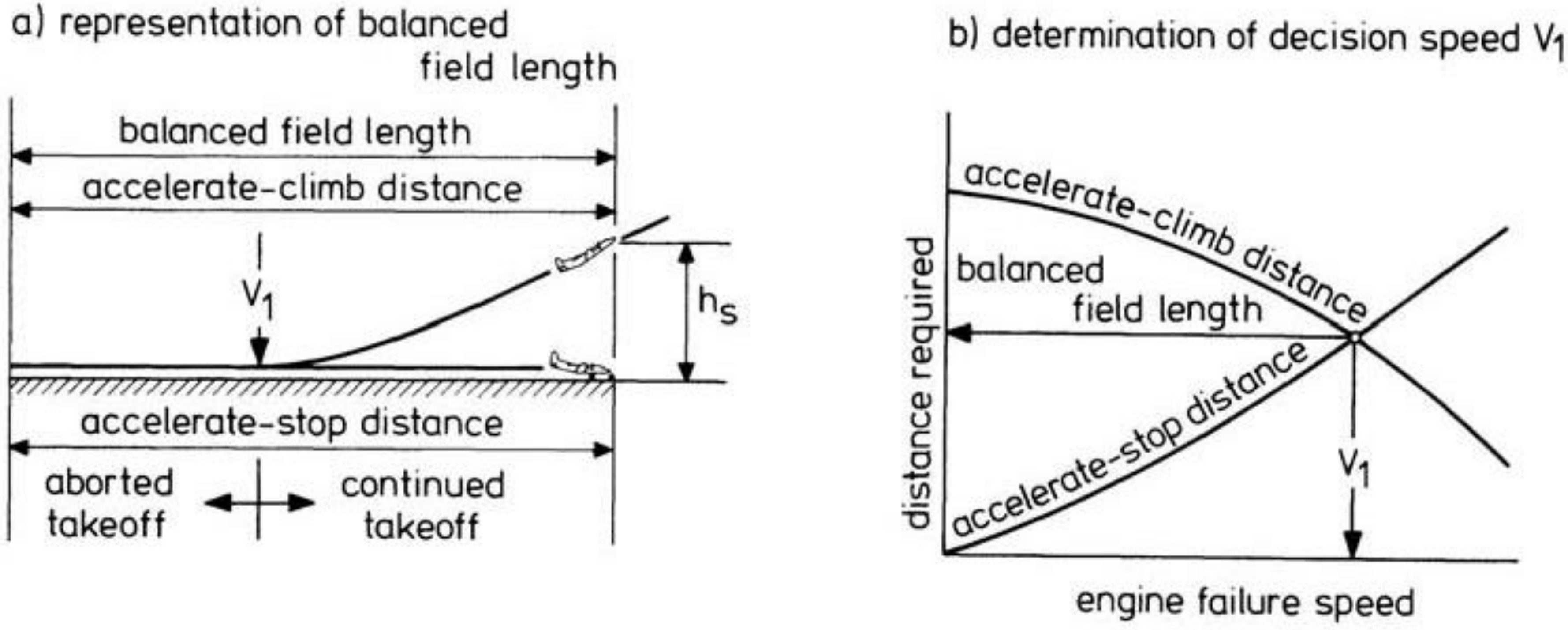
Objective

- Description take-off manoeuvre
- Understand balanced field length concept (engine failure during take off)
- Analytical calculation ground run distance
- Analytical calculation airborne distance

Introduction

Introduction

Balanced field length



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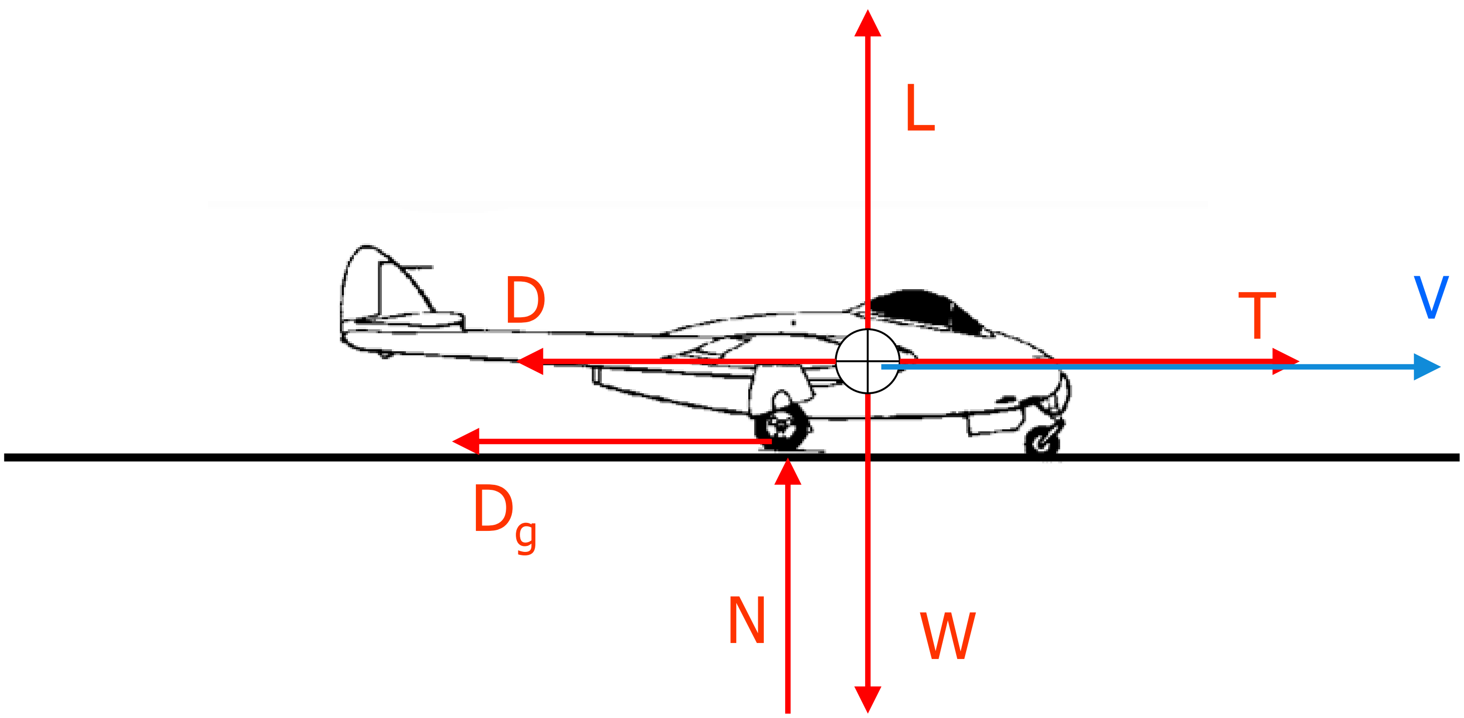
Equations of motion

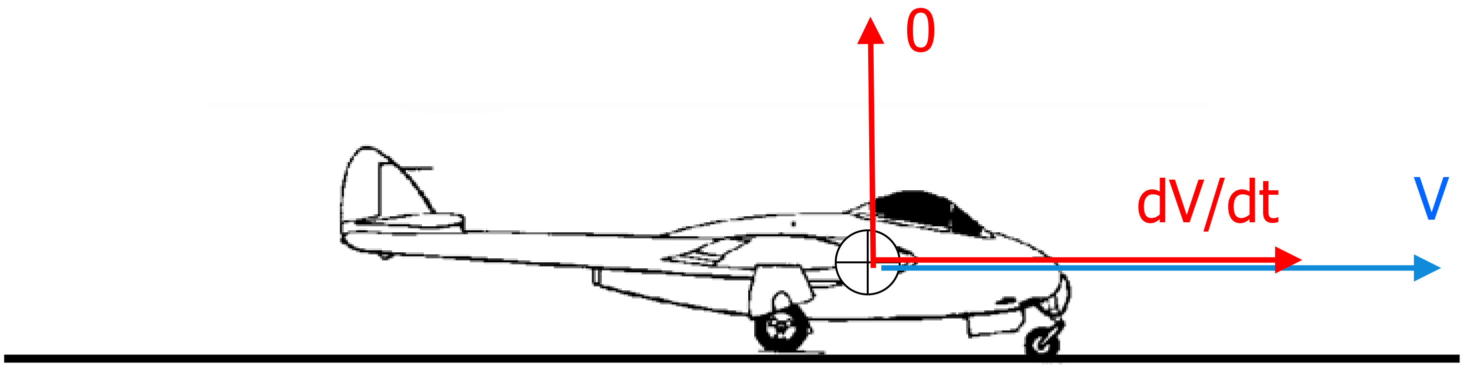
Ground run – Free body diagram (FBD)



Equations of motion

Ground run – Kinetic Diagram

There is no vertical acceleration during the ground phase (aircraft travels along a straight line)



Equations of motion

- Equations of motion: $\vec{F} = m\vec{a}$
- Parallel to the airspeed vector: $\frac{W}{g}\frac{dV}{dt} = T - D - D_g$
- Perpendicular to the airspeed $0 = N + L - W$
- Friction of wheels $D_g = \mu N$

$$\frac{W}{g}\frac{dV}{dt} = T - D - \mu\left(W - L\right)$$

$$N = W - L$$

Equations of motion

Assumptions:

- Horizontal runway
- No wind
- Constant weight
- Thrust vector parallel to airspeed

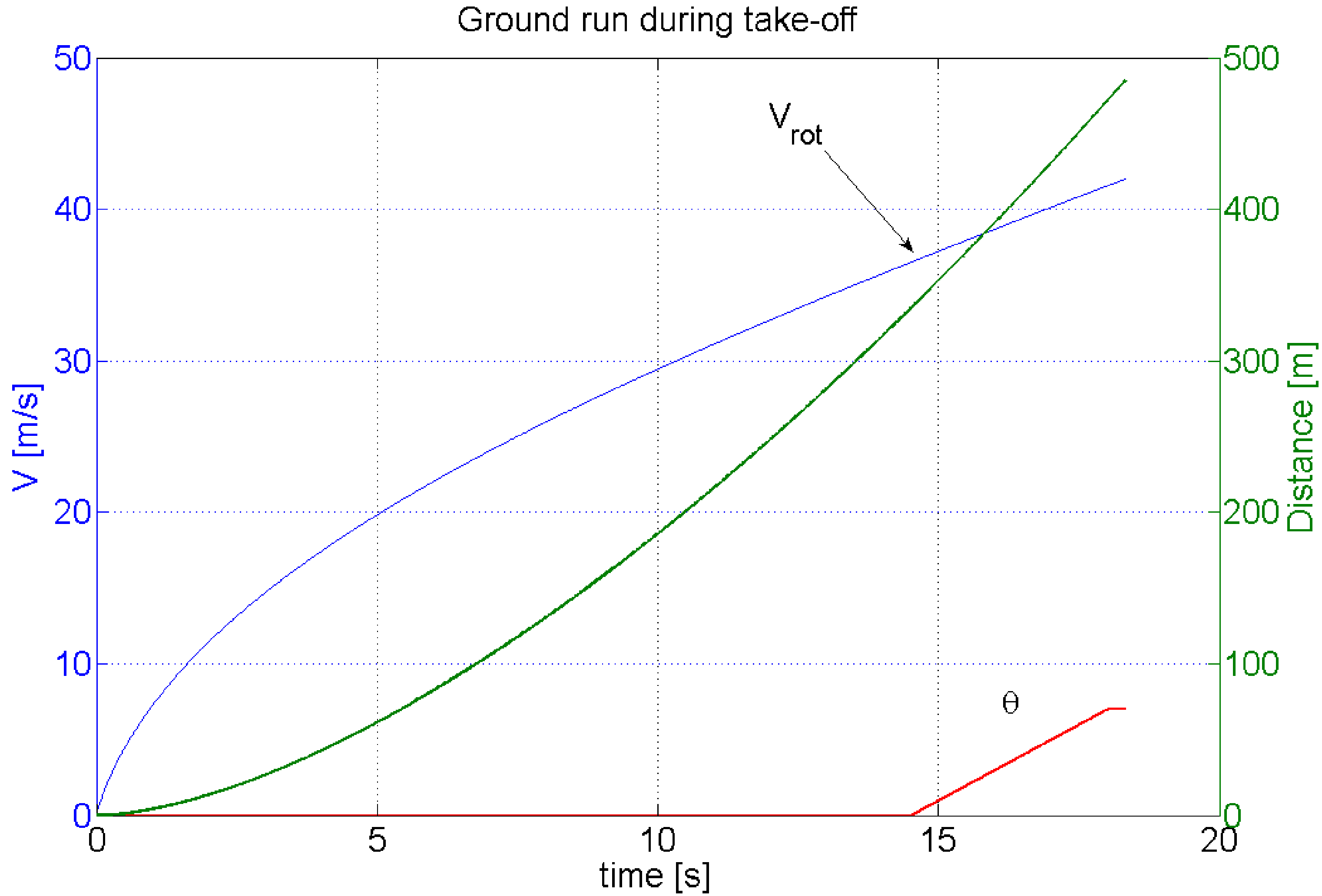
Kinematic equation:

$$\frac{ds}{dt} = V$$

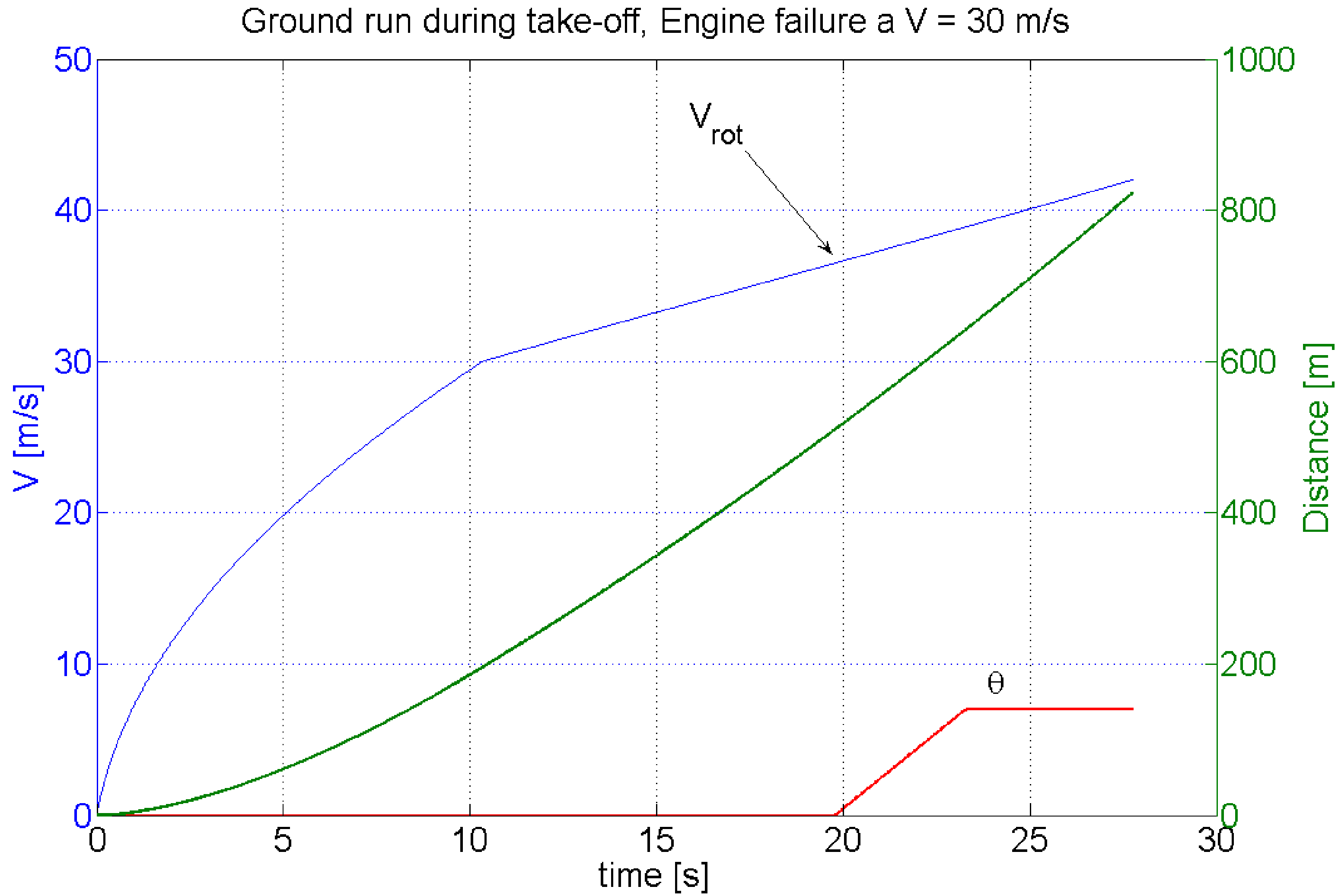
Equations of motion

Lift dumpers

Simulated take off



Simulated take off



Operational procedures

Decision speed V_1

- In case of an engine failure:

If $V < V_1$: Hit the brakes! (rejected take-off)
If $V > V_1$: Continue take-off!!

V_1 can be found in the flight manual as a function of W, H, temp

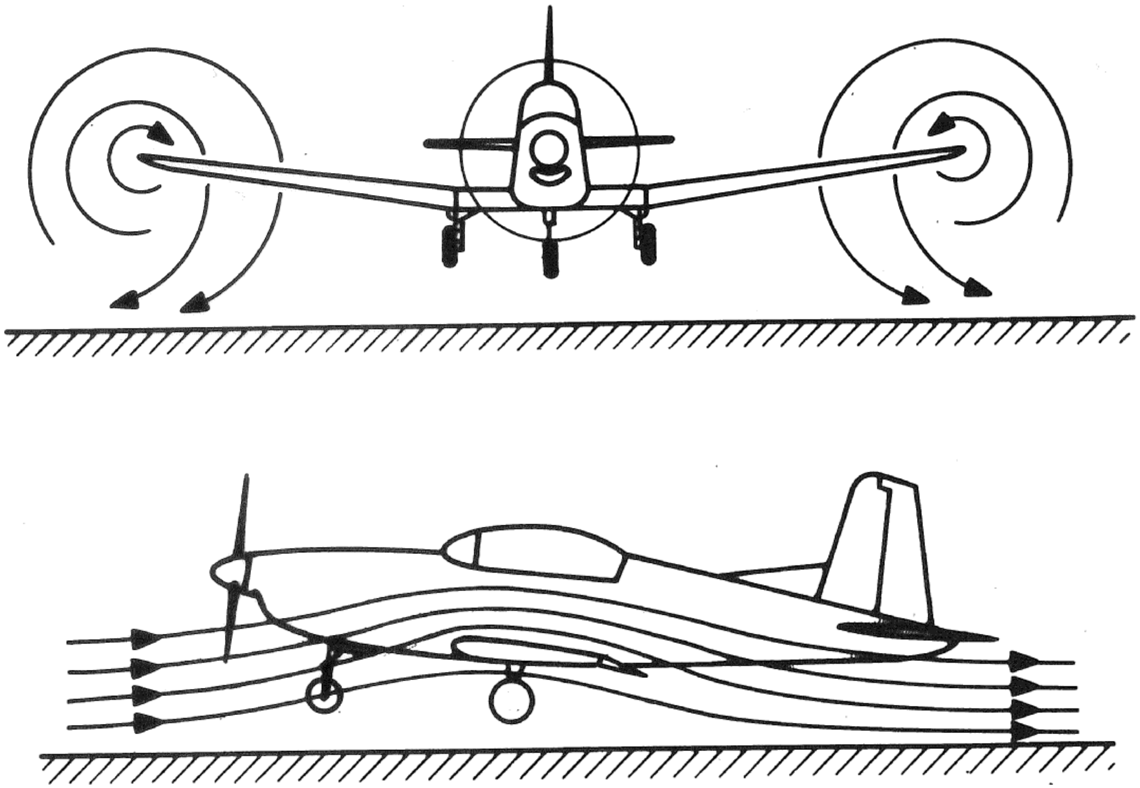
V_1 is generally only slightly smaller than V_R

Extra points of attention

- No wind
- No runway slope
- Ground effect is present
- Landing gear = down
- Flaps in take – off position

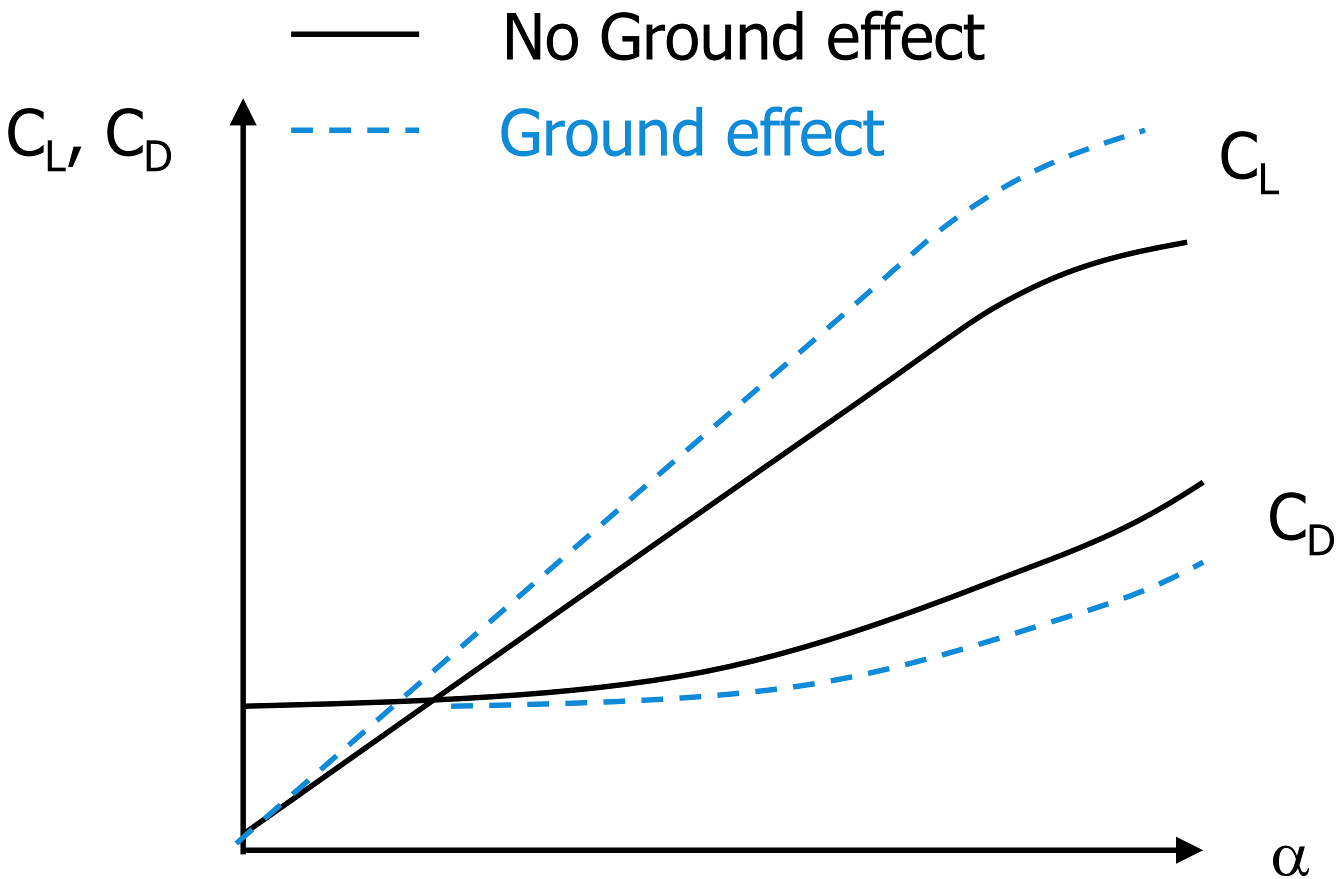
Extra points of attention

Ground effect



Extra points of attention

Ground effect



Extra points of attention

Ground effect

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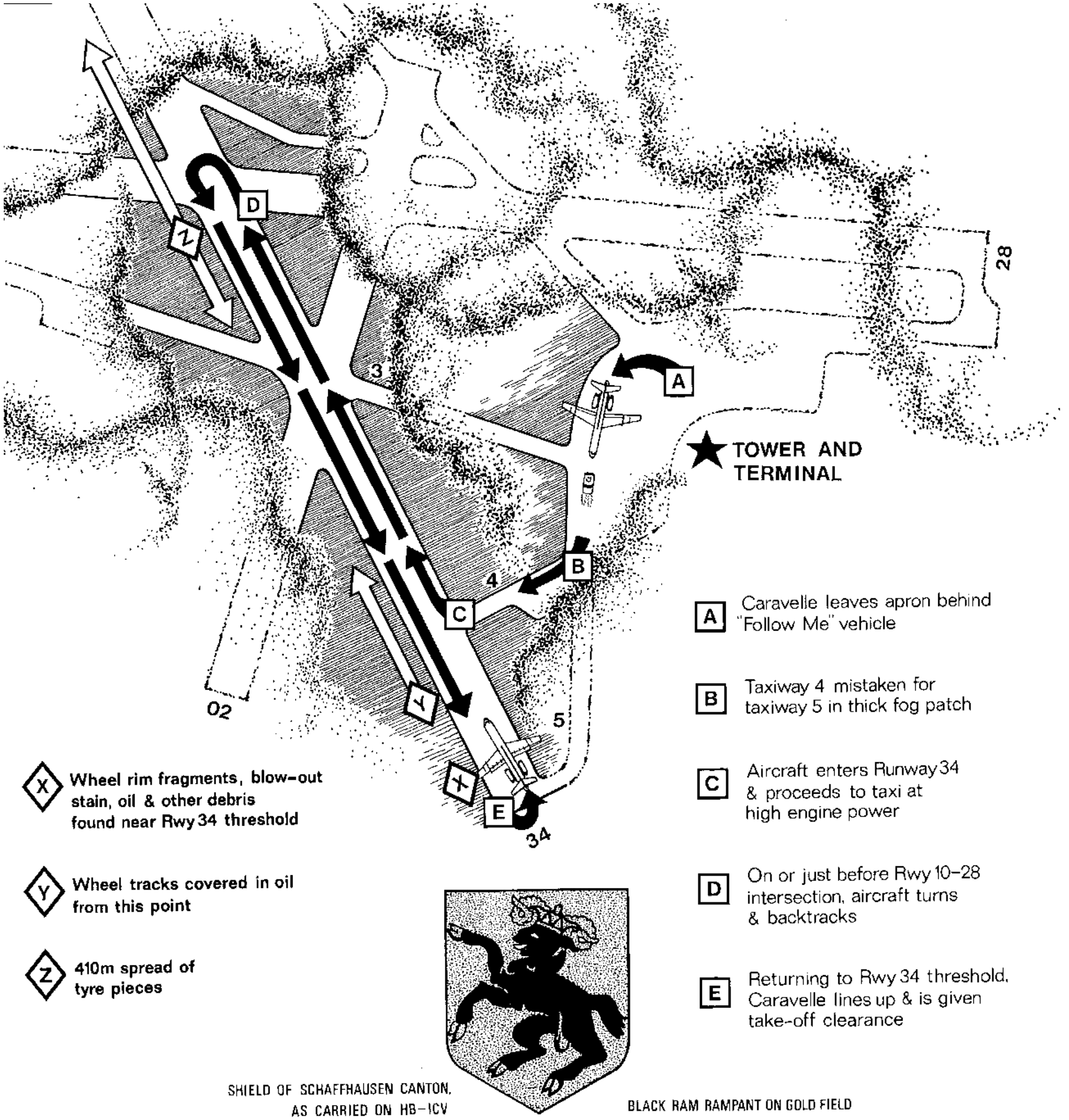
Accidents

- Caravelle III, September 1963
- Emergency stop El Al
- Aborted take-off KLM 1995

Accidents

Caravelle III (1963)



Accidents

El Al

Vrachtjumbo El Al maakt noodstop na storing aan motor

Van een onzer verslaggevers

SCHIPHOL – Een vrachtvliegtuig van het type Boeing 747 van de Israëlische luchtvaartmaatschappij El Al heeft gistermiddag tegen half vier de start vanaf Schiphol afgebroken.

De bemanning van het toestel maakte bij hoge snelheid een noodstop.

De Jumbo is identiek aan het El Al-toestel dat op 4 oktober 1992 op de Bijlmerflats Groeneveen en Klein Kruitberg stortte. Het toestel kon na de noodstop op eigen kracht naar het vrachtplatform taxiën. Daar werd de hulp ingeroepen van de brandweer om de remmen van het toestel te koelen. Die waren door krachtig remmen oververhit geraakt.

Volgens woordvoerder G. Knook van de verkeersleiding brak de gezagvoerder de start af toen hij merkte dat één van de vier motoren niet goed functioneerde.

Drie weken geleden brak een Jumbo 757 van El Al ook al de landing op Schiphol af. De bemanning slaagde er niet in de kleppen aan de voorzijde van de vleugels in de goede stand te krijgen. Volgens woordvoerder Knook vloog dit toestel een extra rondje in de buurt van Weesp om de zogenoemde slats alsnog in de juiste positie te brengen. Aan deze route die ook de ramp-jumbo in 1992 volgde en die over de Bijlmer voerde, werd door de bemanning de voorkeur gegeven.

Na enkele vergeefse pogingen slaagde de bemanning erin het defect te verhelpen en maakte het toestel een normale landing op de Buitenveldertbaan.

Accidents

KLM

KLM-toestel botst bijna op vliegveld Lissabon

LISSABON, AMSTERDAM (ANP, Reuter) – Een KLM-toestel heeft gisteren op het vliegveld van Lissabon op last van de verkeerstoren de start afgebroken, omdat gelijktijdig een Portugees militair vliegtuig op een kruisende baan startte. Een woordvoerder van de KLM op Schiphol heeft dat meegedeeld.

Het KLM-toestel, een Airbus A310 met 124 mensen aan boord, startte voor een vlucht van Lissabon naar Amsterdam. Het Portugese vliegtuig was een transporttoestel.

Een ooggetuige verklaarde dat de toestellen elkaar op een afstand van 50 meter passeerden. Volgens de woordvoerder van de KLM is dat moeilijk in te schatten. Volgens de zegsman hadden de vliegtuigen elkaar zonder de noodgreep niet geraakt. „Het Portugese vliegtuig was over het KLM-toestel heengevlogen. Maar de ingreep van de verkeerstoren was een goede." Het KLM-vliegtuig is een half uur na het incident alsnog vertrokken.

De KLM zei dat de bemanning toestemming had van de verkeerstoren te vertrekken. Door een misverstand dacht de bemanning van het Portugese vliegtuig dat zij ook mocht starten.

Toen de KLM-bemanning vol gas gaf en het toestel in beweging kwam, beval de verkeerstoren de start af te breken. Het Portugese toestel maakte toen al veel meer snelheid.

Trouw, 4-2-95

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Ground run distance

Analytical calculation

Equation of motion

$$\frac{dV}{dt} = \frac{g}{W}\left(T - D - D_g\right)$$

Introduce variable for the distance

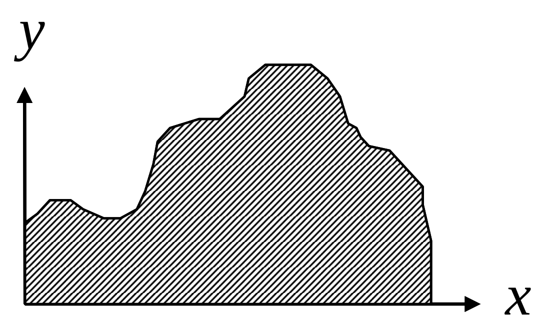
$$a = \frac{dV}{dt} = \frac{dV}{ds}\frac{ds}{dt} = \frac{dV}{ds}V$$

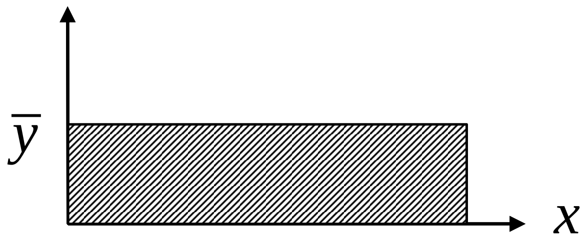
$$ds = \frac{VdV}{a}$$

$$s = \int_0^{V_{LOF}} \frac{V}{a}dV$$

Take the average acceleration

$$s = \frac{1}{\bar{a}}\int_0^{V_{LOF}} VdV = \frac{V_{LOF}^2}{2\bar{a}}$$





Ground run distance

Analytical calculation

- So what is the average acceleration?

$$\bar{a} = \frac{g}{W}\left(\bar{T} - \bar{D} - \bar{D}_g\right)$$

- Average acceleration typically occurs when:

$$V = \frac{V_{LOF}}{\sqrt{2}}$$

Ground run distance

Summary

$$s = \frac{V_{LOF}^2}{2\bar{a}} = \frac{WV_{LOF}^2}{2g\left(\bar{T} - \bar{D} - \bar{D}_g\right)}$$

The average thrust, aerodynamic drag and ground drag are the values that occur at:

$$V = \frac{V_{LOF}}{\sqrt{2}}$$

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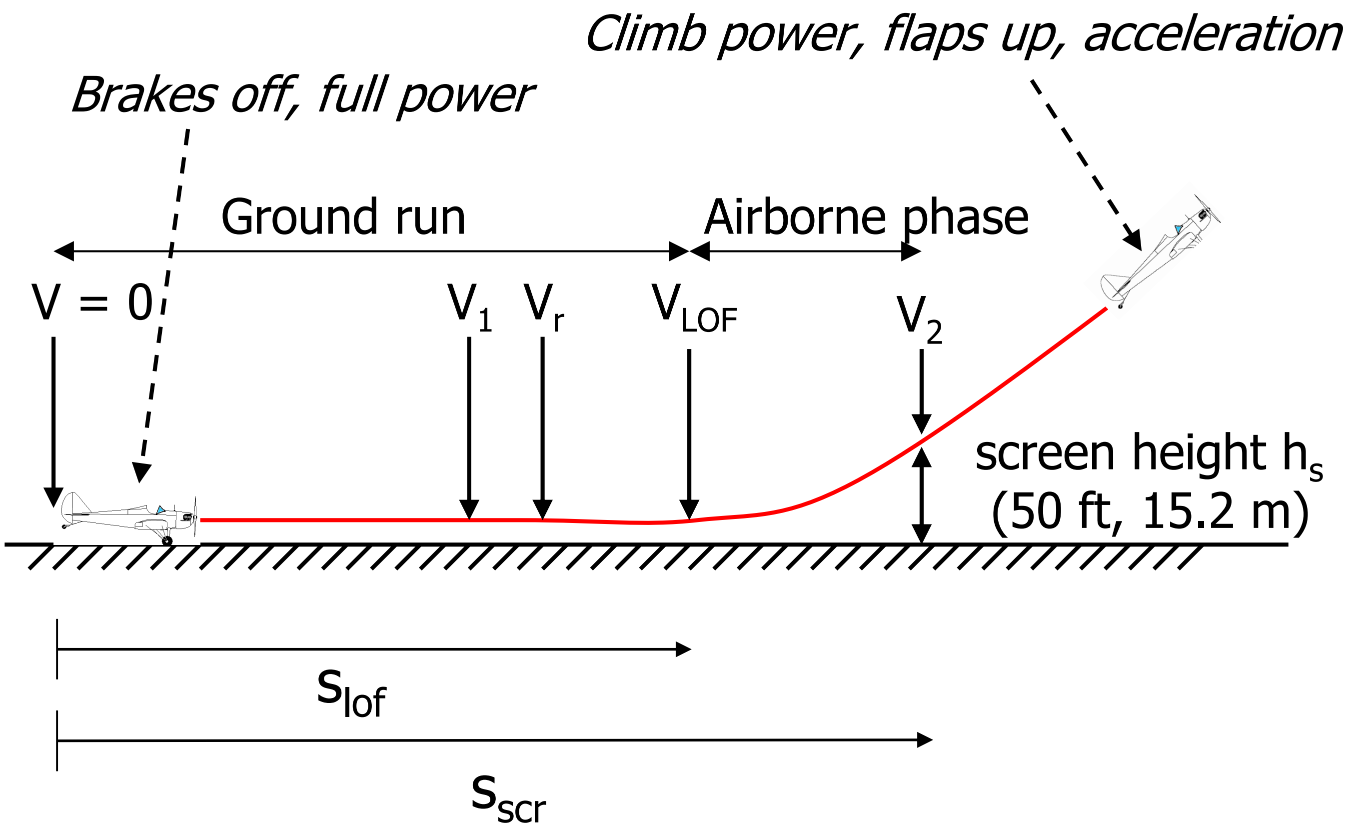
Airborne distance



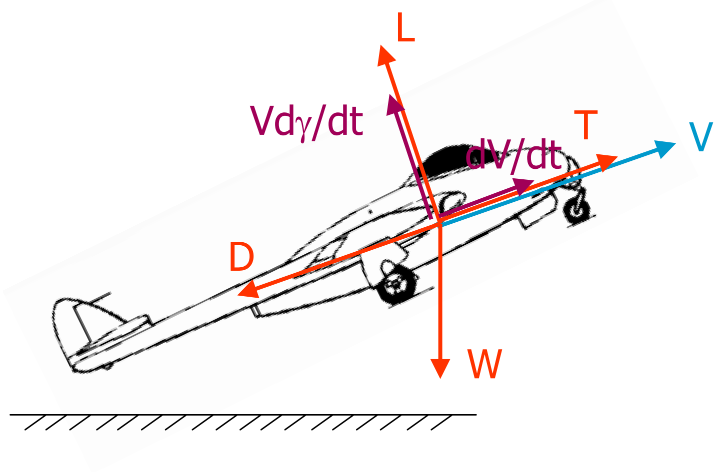
Airborne distance

Analytical solution

- Equations of motion

$$\frac{W}{g}\frac{dV}{dt} = T - D - W\sin\gamma$$

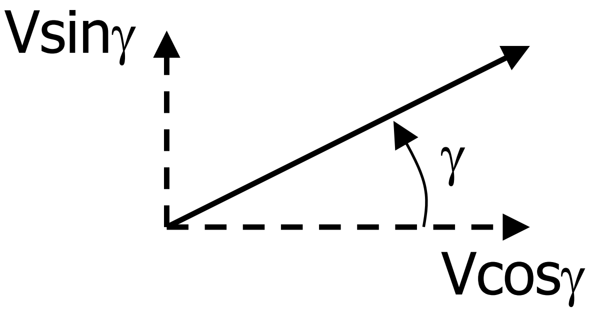
$$\frac{W}{g}V\frac{d\gamma}{dt} = L - W\cos\gamma$$



- Kinematic equations

$$\frac{dH}{dt} = V\sin\gamma$$

$$\frac{ds}{dt} = V\cos\gamma$$



Airborne distance

Analytical solution

Introduce variable s in equations of motion

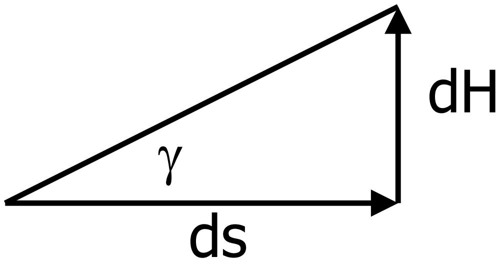
$$\frac{W}{g}\frac{dV}{dt}$$

$$\frac{W}{g}\frac{dV}{ds}V = T - D - W\sin\gamma$$

$$\frac{W}{g}VdV = (T-D)ds - W\sin\gamma ds$$

$$\frac{W}{g}VdV = (T-D)ds - Wdh$$

$$\frac{W}{g}\int_{V_{LOF}}^{V_{scr}} VdV = \int_{s_{LOF}}^{s_{scr}} (T-D)ds - \int_{0}^{h_{scr}} Wdh$$



$$\sin\gamma \approx \tan\gamma = \frac{dh}{ds}$$

$$\sin\gamma ds = dh$$

Airborne distance

Analytical solution

$$\frac{W}{g}\int_{V_{LOF}}^{V_{scr}} V dV = \int_{s_{LOF}}^{s_{scr}} (T-D)\,ds - \int_{0}^{h_{scr}} W dh$$

Assume constant (average) T-D

$$T-D=\bar{T}-\bar{D}$$

$$\frac{W}{2g}\left(V_{scr}^2-V_{LOF}^2\right)=\left(\bar{T}-\bar{D}\right)\left(s_{scr}-s_{LOF}\right)-Wh_{scr}$$

Assume steady climb at screen height

$$\frac{\bar{T}-\bar{D}}{W}=\sin\gamma_{scr}$$

$$\frac{V_{scr}^2-V_{LOF}^2}{2g}=\sin\gamma_{scr}\left(s_{scr}-s_{LOF}\right)-h_{scr}$$

Take-off distance

Summary

$$s_{airborne} = \frac{\frac{1}{2g}V_{scr}^2 - \frac{1}{2g}V_{LOF}^2 + h_{scr}}{\sin\gamma_{scr}}$$

Airborne distance

Summary

$$s_{ground} = \frac{W V_{LOF}^2}{2g\left(\bar{T} - \bar{D} - \bar{D}_g\right)}$$

$$s_{airborne} = \frac{\frac{1}{2g} V_{scr}^2 - \frac{1}{2g} V_{LOF}^2 + h_{scr}}{\sin \gamma_{scr}}$$

$$s_{total} = s_{ground} + s_{airborne}$$

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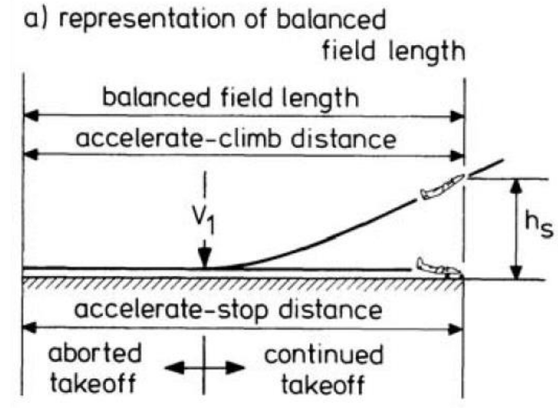
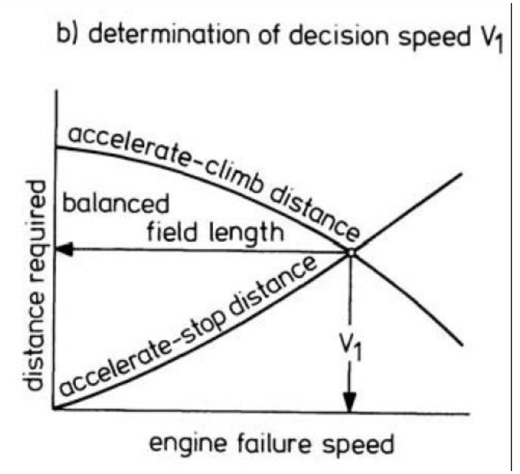
How to determine operational speeds

Take off speeds

- V_1 = Decision speed
- V_R = Rotation speed
- V_2 = Safety speed (speed to maintain during engine failure in the second climb segment)
- $V_2 \geq 1.2\ V_{min}$
- Without engine failure at screen height in general $V \cong V_2 + 10$ kts
- V_1, V_R, V_2 are given in the flight manual as a function of W, H, T





How to determine operational speeds

Flight manual

FAR PART 25 REQUIRED TAKEOFF FIELD LENGTH

Takeoff Weight = 11,500 lbs.

Field Elevation — Ft. / Ambient Temperature °C (°F)	SL	2000	4000	6000	8000
−20 (−4)	1950	2090	2230	2520	2970
−10 (14)	2030	2160	2360	2720	3200
0 (32)	2090	2240	2580	2990	3530
10 (50)	2200	2470	2860	3320	4030
20 (68)	2410	2780	3220	3870	4750
30 (86)	2760	3230	3870	4680	5870
40 (104)	3280	3890	4740	5860	

Takeoff Weight = 10,500 lbs.

Field Elevation — Ft. / Ambient Temperature °C (°F)	SL	2000	4000	6000	8000
−20 (−4)	1660	1760	1880	2120	2490
−10 (14)	1710	1820	2000	2290	2690
0 (32)	1770	1890	2170	2510	2950
10 (50)	1860	2080	2410	2780	3280
20 (68)	2030	2330	2690	3130	3790
30 (86)	2310	2660	3110	3720	4580
40 (104)	2660	3120	3750	4550	

DECISION, ROTATION AND TAKEOFF SAFETY SPEEDS

For FAR Part 25 Required Takeoff Field Length
15° Flaps
(STD Day At Sea Level)

Takeoff Wt. Lbs.	Decision Speed V_1 — KIAS	Rotation Speed V_R — KIAS	Safety Speed V_2 — KIAS
13,300	105	106	114
13,000	103	105	113
12,500	100	103	111
12,000	98	100	109
11,500	94	98	107
10,500	88	93	102
9,500	81	88	97

SECOND SEGMENT CLIMB TEMPERTURE LIMITS

Takeoff Weight = 13,300 Lbs.
Flaps Up, Zero Wind

Field Elevation — Ft.	S.L.	2000	4000	6000	8000
Maximum allowable takeoff temperature °C	54	50	46	40	30
Maximum allowable takeoff temperature °F	129	122	115	104	86

How to determine operational speeds

Choice by the manufacturer

- In general $V_2 = 1.2\ V_{min}$ and checking the compliance with the regulations with regarding the climb gradient at V_2
- V_R such that V_2 is reached at screen height in case of an engine failure (in general)
- V_1 by using the principle of balanced field length

How to determine operational speeds

Minimum climb gradient at V_2

No. of engines	2	3	4
1st segment (t.o. power, flaps, gear down)	>0%	>0.3%	0.5%
2nd segment (t.o. power, flaps, gear up)	2.4%	2.7%	3.0%

How to determine operational speeds

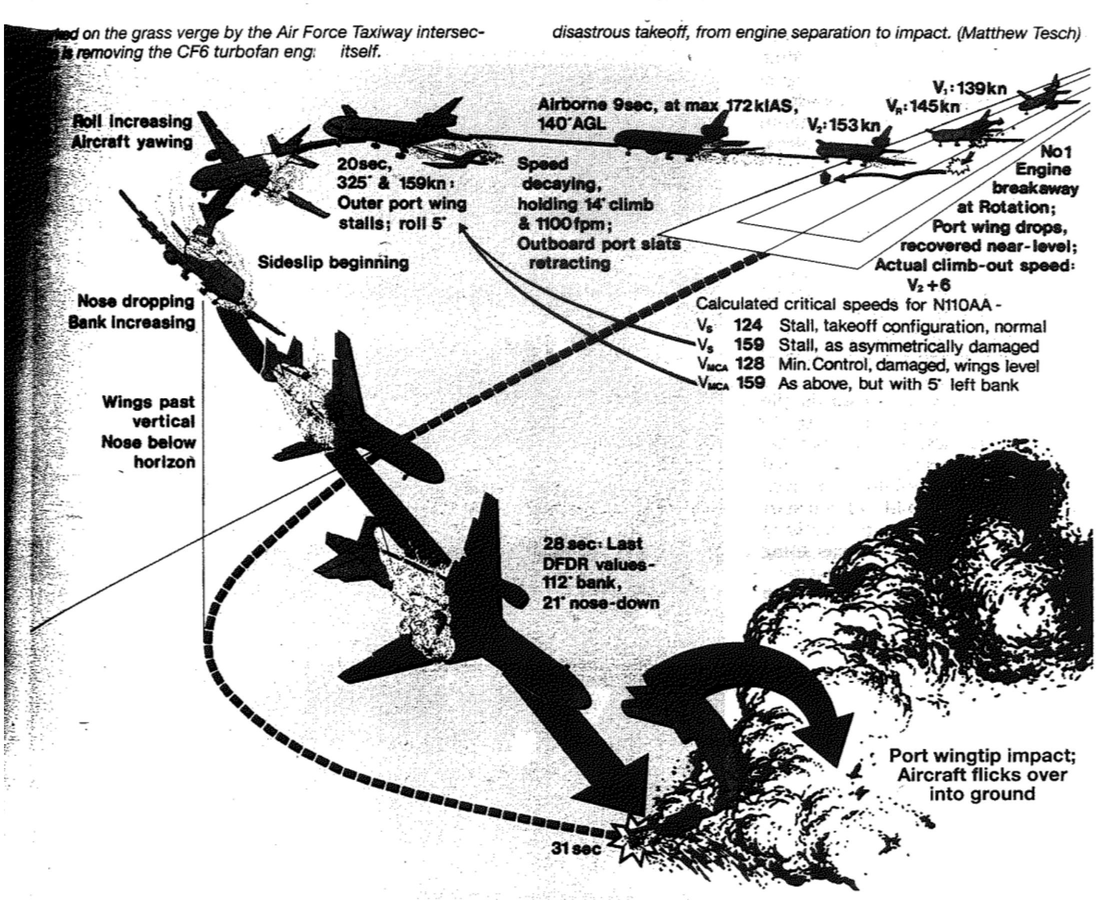
Airworthiness regulations

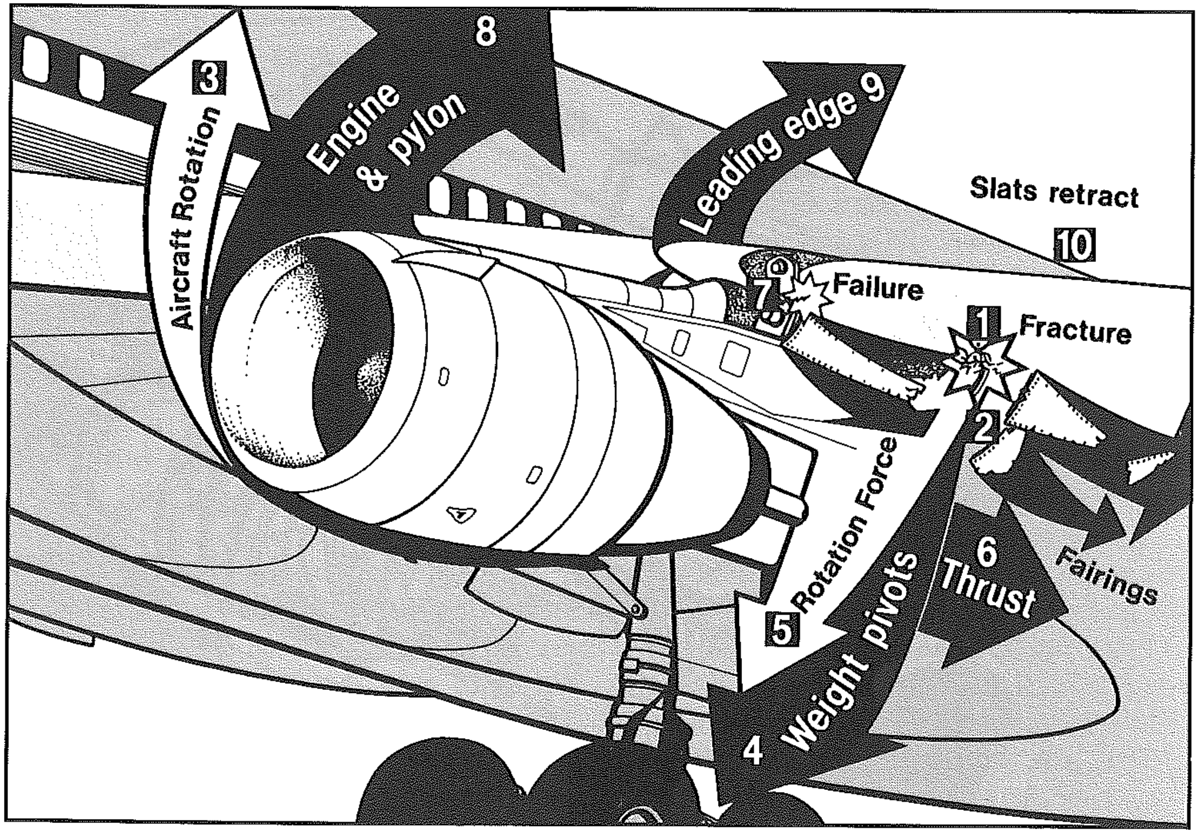
- Required field length is the largest of:
  - Balanced field length
  - 1.15 x take off distance without engine failure

- At V_2 a minimum climb gradient is required with an engine failure

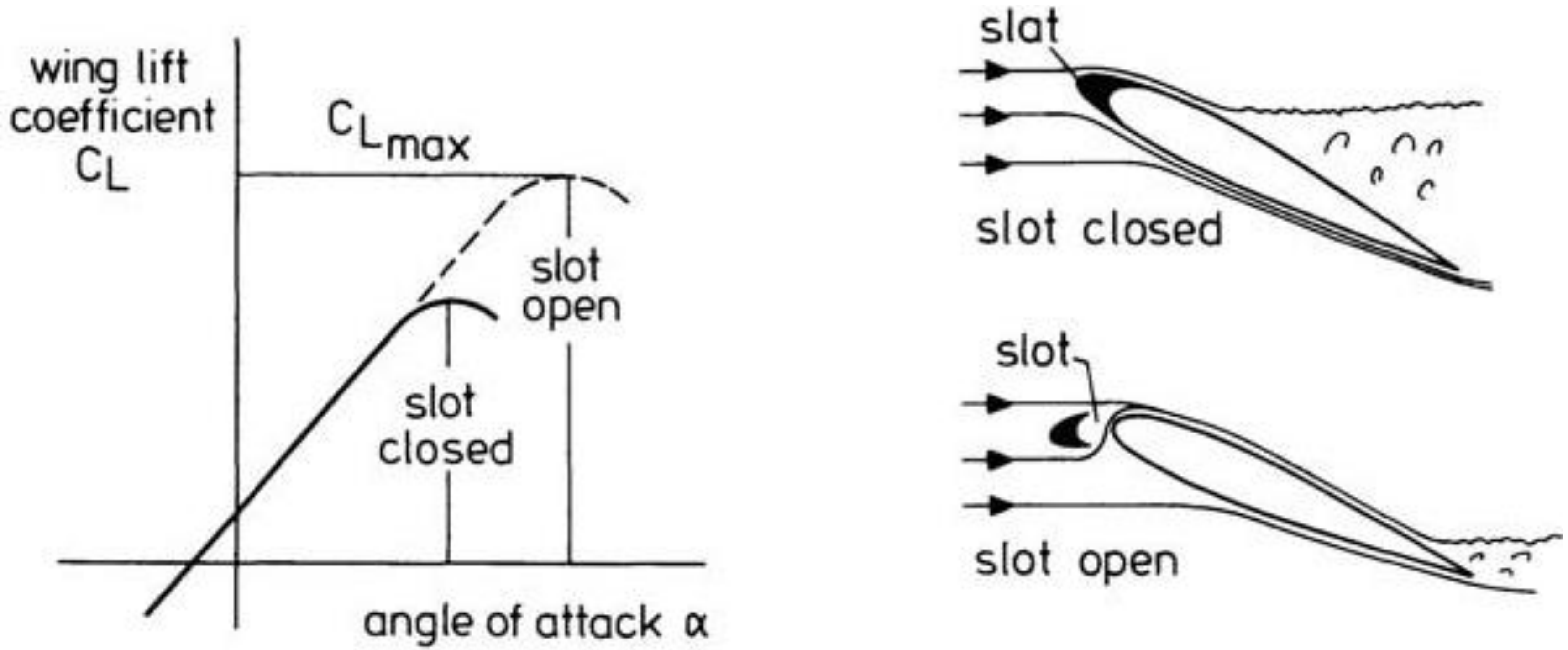
How to determine operational speeds

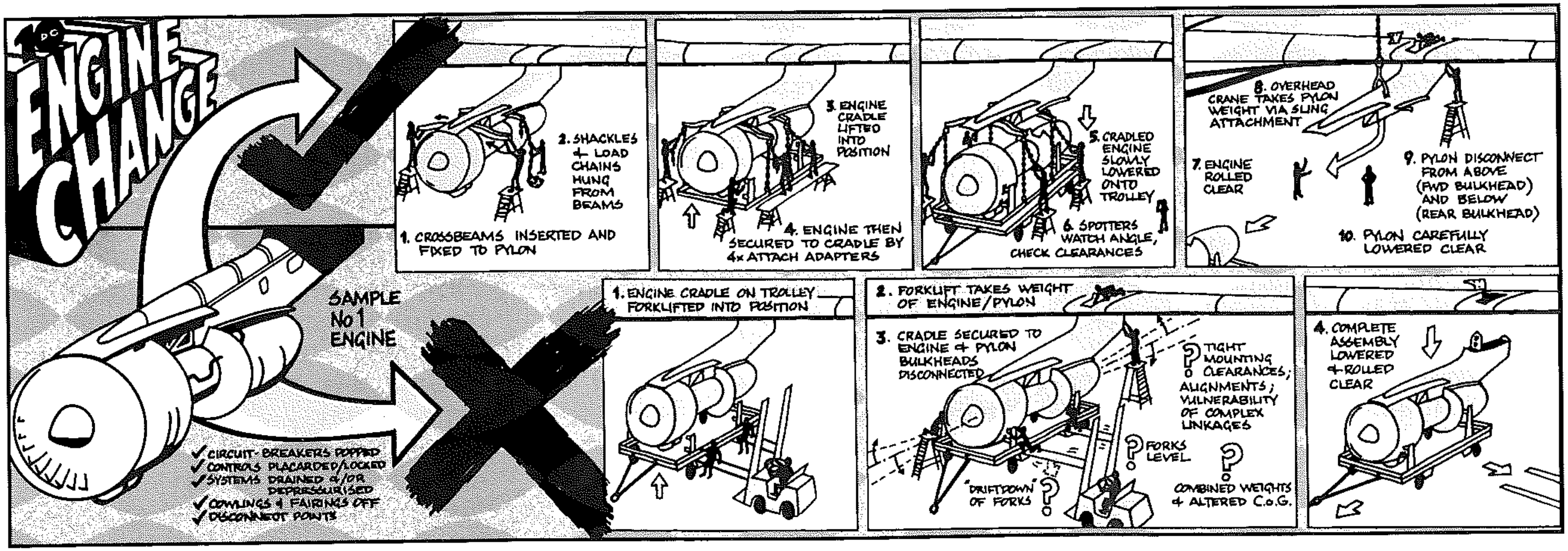
Story





Close-up of N110AA's port wing at the instant of engine separation. The drawing shows the deduced failure sequence of the pylon components and the engine's dramatic trajectory over the top of the wing. (Matthew Tesch)


wing lift coefficient C_L
$C_{L_{max}}$
slot open
slot closed
angle of attack α
slat
slot closed
slot
slot open


ENGINE CHANGE
SAMPLE No 1 ENGINE
✓CIRCUIT-BREAKERS POPPED
✓CONTROLS PLACARDED/LOCKED
✓SYSTEMS DRAINED &/OR DEPRESSURISED
✓COWLINGS & FAIRINGS OFF
✓DISCONNECT POINTS
1. CROSSBEAMS INSERTED AND FIXED TO PYLON
2. SHACKLES & LOAD CHAINS HUNG FROM BEAMS
3. ENGINE CRADLE LIFTED INTO POSITION
4. ENGINE THEN SECURED TO CRADLE BY 4x ATTACH ADAPTERS
5. CRADLED ENGINE SLOWLY LOWERED ONTO TROLLEY
6. SPOTTERS WATCH ANGLE, CHECK CLEARANCES
7. ENGINE ROLLED CLEAR
8. OVERHEAD CRANE TAKES PYLON WEIGHT VIA SLING ATTACHMENT
9. PYLON DISCONNECT FROM ABOVE (FWD BULKHEAD) AND BELOW (REAR BULKHEAD)
10. PYLON CAREFULLY LOWERED CLEAR
1. ENGINE CRADLE ON TROLLEY FORKLIFTED INTO POSITION
2. FORKLIFT TAKES WEIGHT OF ENGINE/PYLON
3. CRADLE SECURED TO ENGINE & PYLON BULKHEADS DISCONNECTED
? TIGHT MOUNTING CLEARANCES; ALIGNMENTS; VULNERABILITY OF COMPLEX LINKAGES
? FORKS LEVEL
"DRIFTDOWN" OF FORKS ?
? COMBINED WEIGHTS & ALTERED C.o.G.
4. COMPLETE ASSEMBLY LOWERED & ROLLED CLEAR

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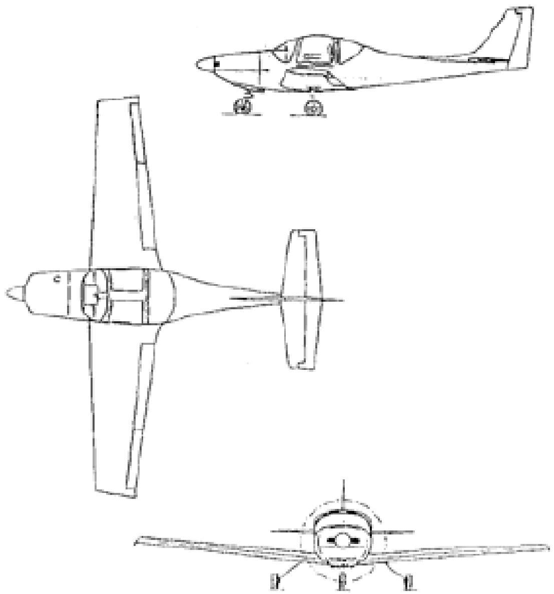
Example exam question

From the Euro-ENAER Eaglet, a small propeller airplane built in the Netherlands, the following data are available for the calculation of the takeoff performance:

takeoff weight	: W = 8500 N,
wing area	: S = 9.84 m^2.
lift-drag polar	: $C_D = C_{D_0} + C_L^2/\pi Ae$;
in the takeoff configuration	: C_{D_0} = 0.03, Ae = 5,
maximum lift-coefficient	: $C_{L_{max}}$ = 1.4,
lift-coefficient during takeoff roll	: C_{L_g} = 0.8,
maximum engine power at 0 m ISA	: P_{br} = 115 kW,
coefficient of friction during takeoff roll	: μ = 0.05
lift-off speed	: $V_{LOF} = 1.05\ V_{min}$,
propeller efficiency	: $\eta = \dfrac{P_a}{P_{br}}$
propeller efficiency during takeoff roll:	: η_a = 0.5,
propeller efficiency at lift-off and during the airborne phase	: η_{vl} = 0.8.



Example exam question

a. Draw a clear Free Body Diagram (FBD) and Kinetic Diagram (KD) visualizing all forces and accelerations that act on the aircraft **during the ground run**. Clearly indicate the direction of the velocity and all angles that are relevant for any further calculations.

b. Derive the equations of motion using the FBD and KD for the aircraft during the ground run. Clearly indicate all assumptions.

c. Derive an expression for the ground run distance s_{LOF} in terms of a mean acceleration $\overline{a}$ and the lift off speed V_{LOF}.

d. Calculate the lift off speed V_{LOF}.

e. Show that the mean acceleration $\overline{a}$ is equal to 2.2 m/s^2. Assume that the ground run is a uniformly accelerated motion. The mean acceleration must be calculated at the characteristic airspeed; $V = V_{LOF}/\sqrt{2}$.

f. Calculate the ground run distance s_{LOF}.

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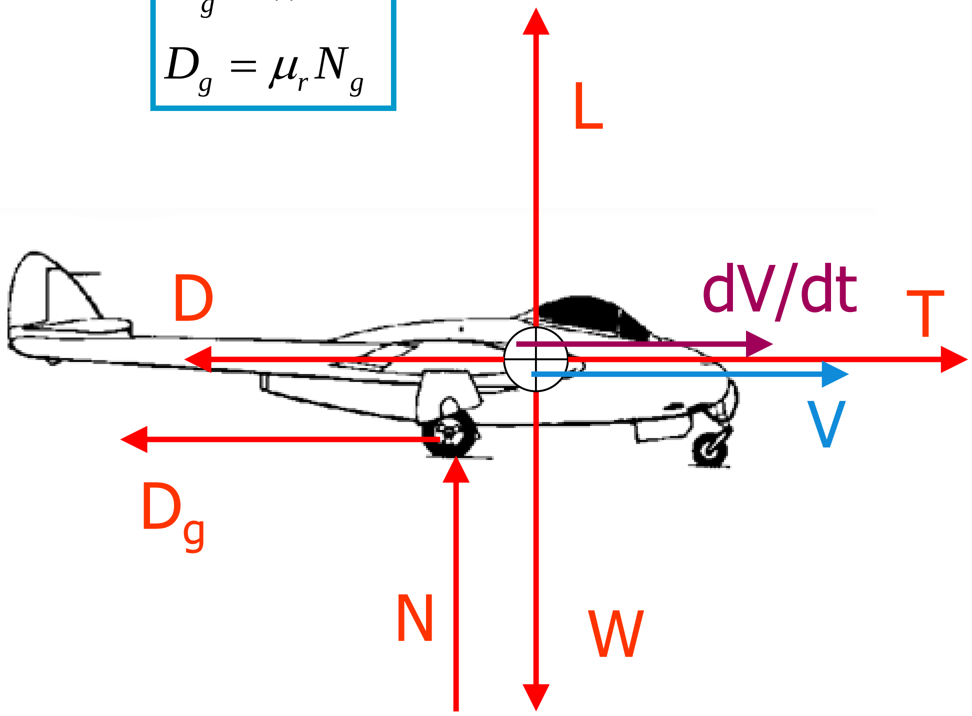
Summary

Ground run

$$N_g = W - L$$
$$D_g = \mu_r N_g$$



Equation of motion:

$$\frac{W}{g}\frac{dV}{dt} = T - D - \mu_r\left(W - L\right)$$

Kinematic equation:

$$\frac{ds}{dt} = V$$

Analytical approximation:

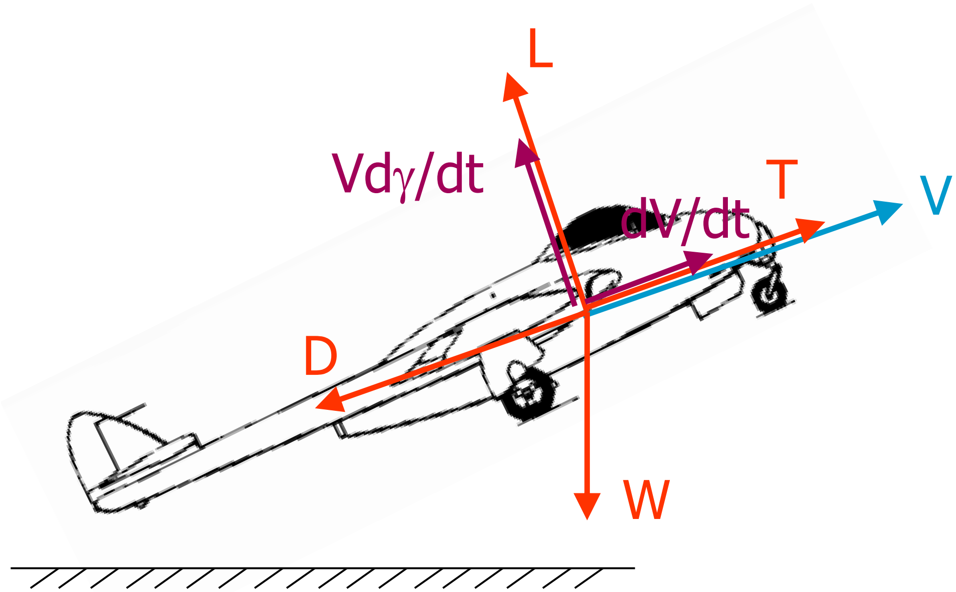
$$s = \int_0^{V_{LOF}} \frac{VdV}{a} = \frac{1}{2\bar{a}} V_{LOF}^2$$

Mean acceleration

$$\bar{a} = \frac{g}{W}\left[\bar{T} - \bar{D} - \bar{D}_g\right] \text{ at } \frac{V_{LOF}}{\sqrt{2}}$$

Summary

Airborne phase



Equations of motion:

$$m\dot{V} = T - D - W\sin\gamma$$

$$mV\dot{\gamma} = L - W\cos\gamma$$

Kinematic equations:

$$\dot{H} = V\sin\gamma$$

$$\dot{s} = V\cos\gamma$$

Analytical solution airborne phase (distance)

$$\frac{W}{2g}\int_{V_{LOF}}^{V_{scr}} dV^2 = \int_{s_{LOF}}^{s_{scr}} (T-D)\,ds - \int_{0}^{h_{scr}} W dh$$

⇒

$$\frac{V_{scr}^2 - V_{LOF}^2}{2g} = \sin\gamma_{scr}\cdot\left(s_{scr} - s_{LOF}\right) - h_{scr}$$

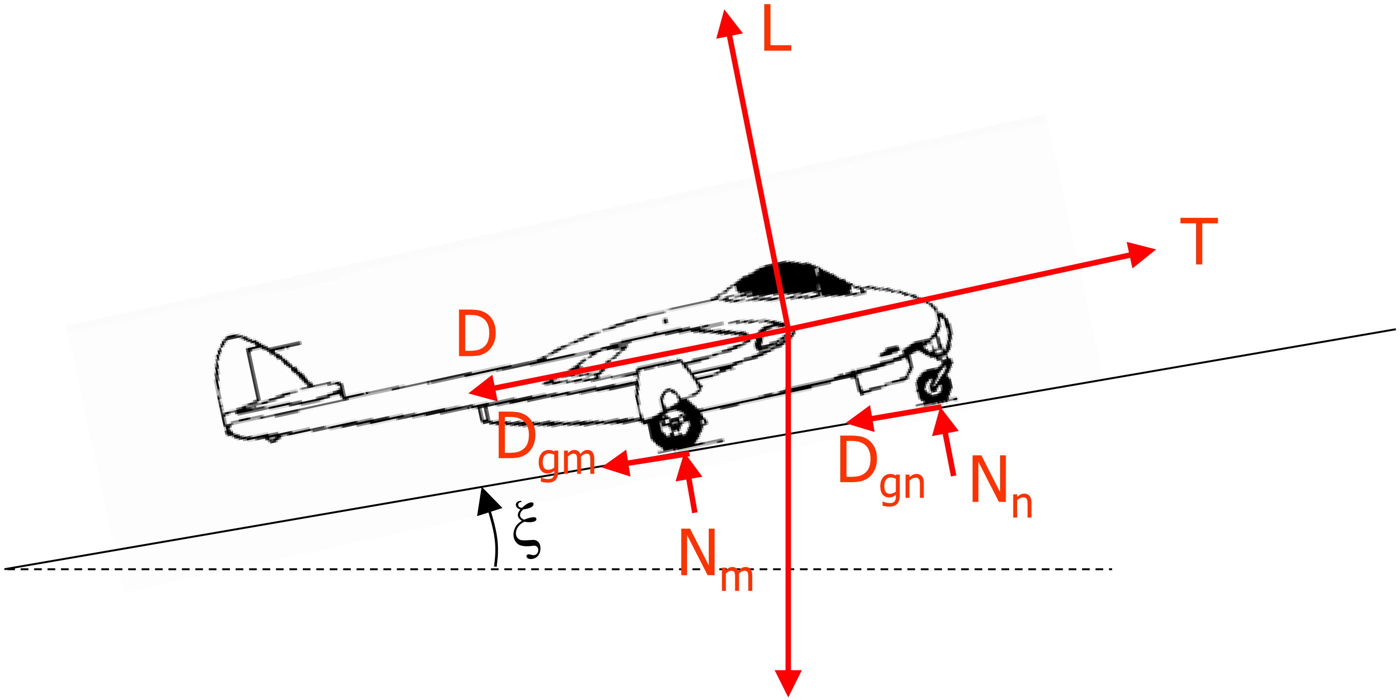
At given $V_{scr} = V_2 = 1.2\cdot V_{min} \Rightarrow s_{scr}$

Content

- Introduction
- Equations of motion
- Accidents
- Take of distance analytical solution
  - Ground run
  - Airborne phase
- How to determine operational speeds
- Example exam question
- Summary
- ***Additional topics***

Advanced topic

Effect of ground run slope



Additional topics

Effect of ground run slope

$$a = \frac{g}{W}\left[T - D - D_g - W\sin\xi\right]$$

$$a = \frac{g}{W}\left[T - D - \mu\left(W\cos\xi - L\right) - W\sin\xi\right]$$

$$\xi \text{ small} \quad \Rightarrow \quad \cos\xi = 1 \quad \sin\xi = \sin\xi$$

$$a = \frac{g}{W}\left[T - D - \mu\left(W - L\right) - W\sin\xi\right]$$

$$a = a_0 - g\sin\xi$$

$$s = \int_0^{V_{LOF}} \frac{dV^2}{2a} = \frac{V_{LOF}^2}{2\left(\bar{a}_0 - g\sin\xi\right)} = \frac{V_{LOF}^2 / 2\bar{a}_0}{1 - \frac{g}{\bar{a}_0}\sin\xi} = s_0 \frac{1}{1 - \frac{g}{\bar{a}_0}\sin\xi}$$