

Flight and Orbital Mechanics

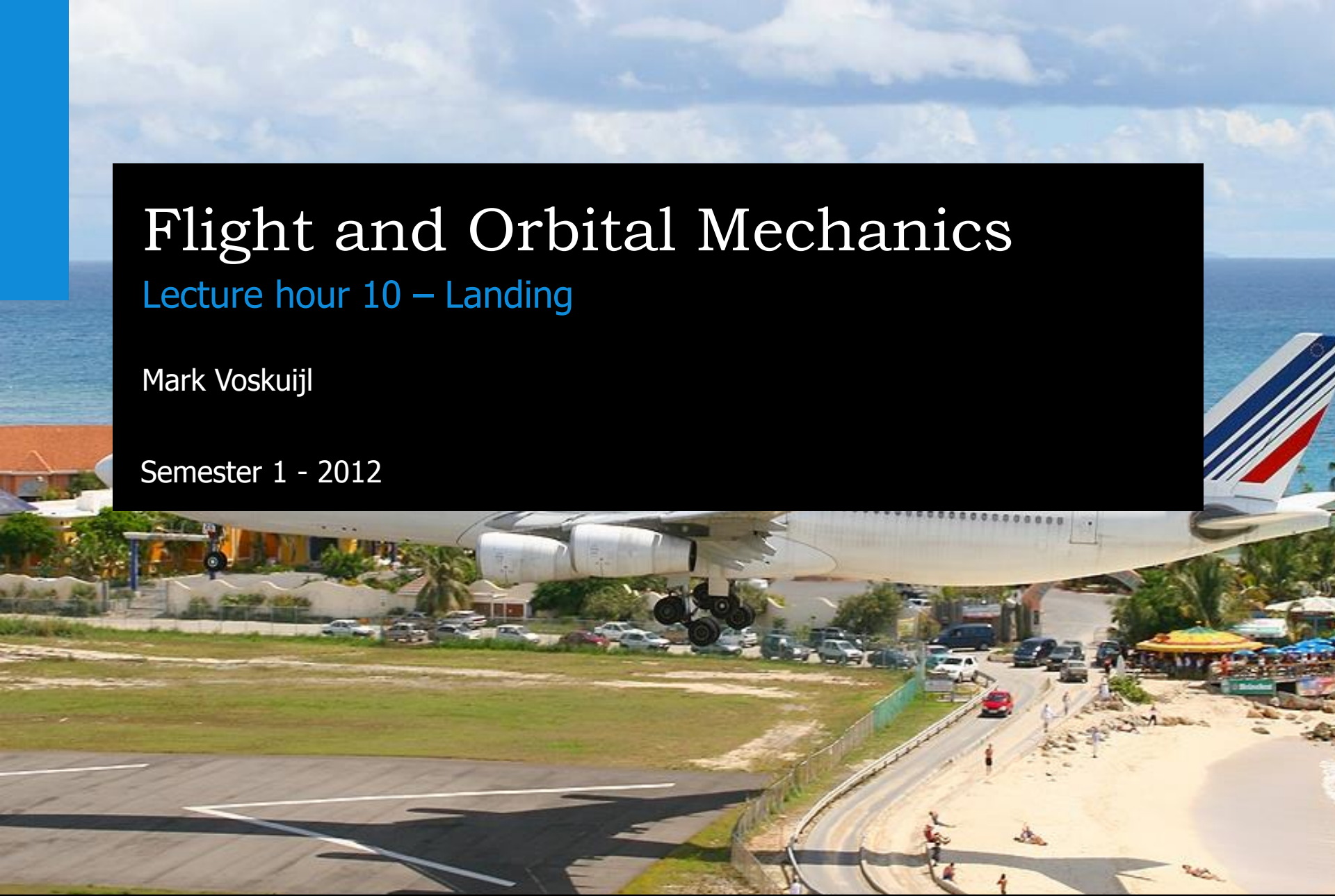
Lecture slides

Flight and Orbital Mechanics

Lecture hour 10 – Landing

Mark Voskuijl

Semester 1 - 2012



Content

- Introduction
- Equations of motion
- Analytical solution landing distance
- Spoilers, thrust reversers, etc.
- Tire, runway and brake properties
- Airworthiness Regulations
- Example exam question
- Summary

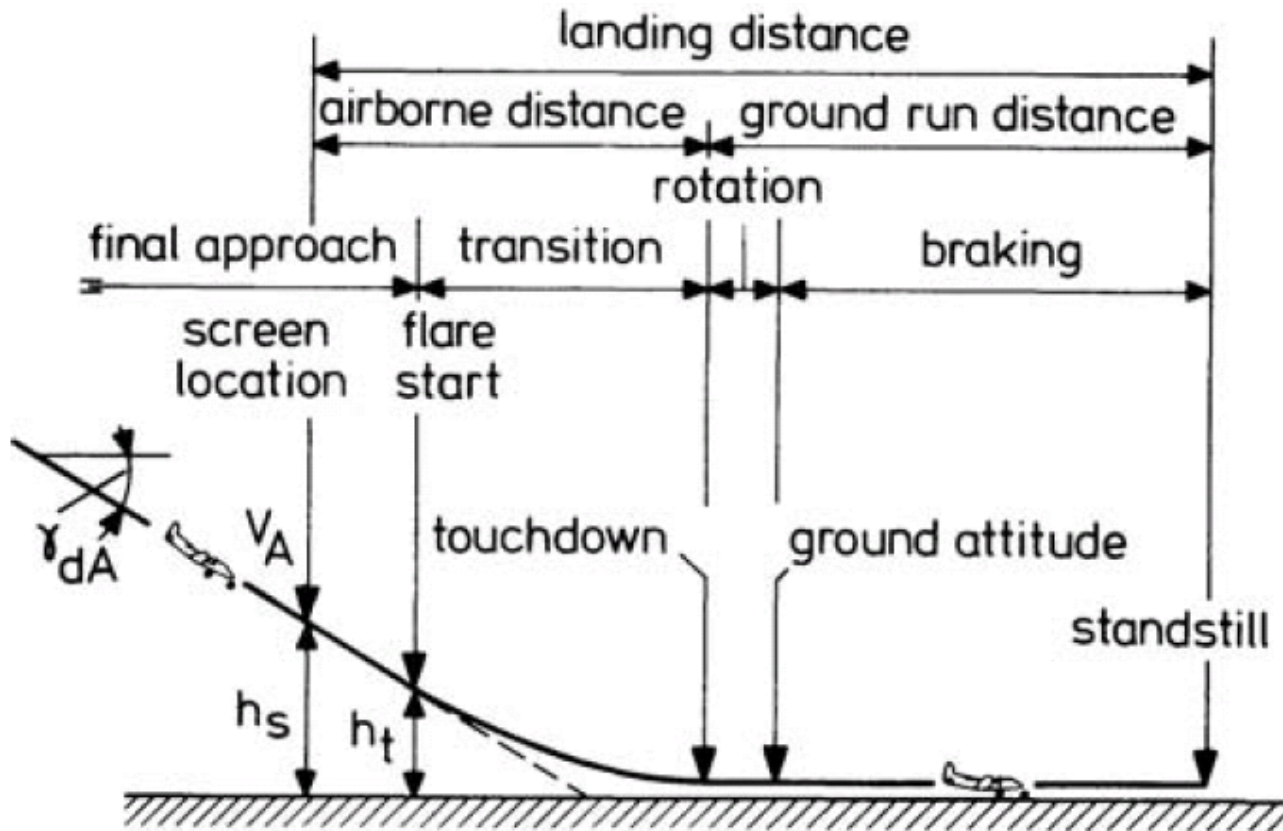


Content

- **Introduction**
- Equations of motion
- Analytical solution landing distance
- Spoilers, thrust reversers, etc.
- Tire, runway and brake properties
- Airworthiness Regulations
- Example exam question
- Summary



Introduction



Typical numbers
 $\gamma_{dA} = 3^\circ$
 $V_a \geq 1.3 V_{\min}$
 $V_T \approx 1.15 V_{\min}$
 $h_s = 50 \text{ ft}$

Introduction

The landing is the maneuver by which the airplane is brought from a steady approach speed V_A over a 15 m obstacle at the runway threshold to standstill at the runway.

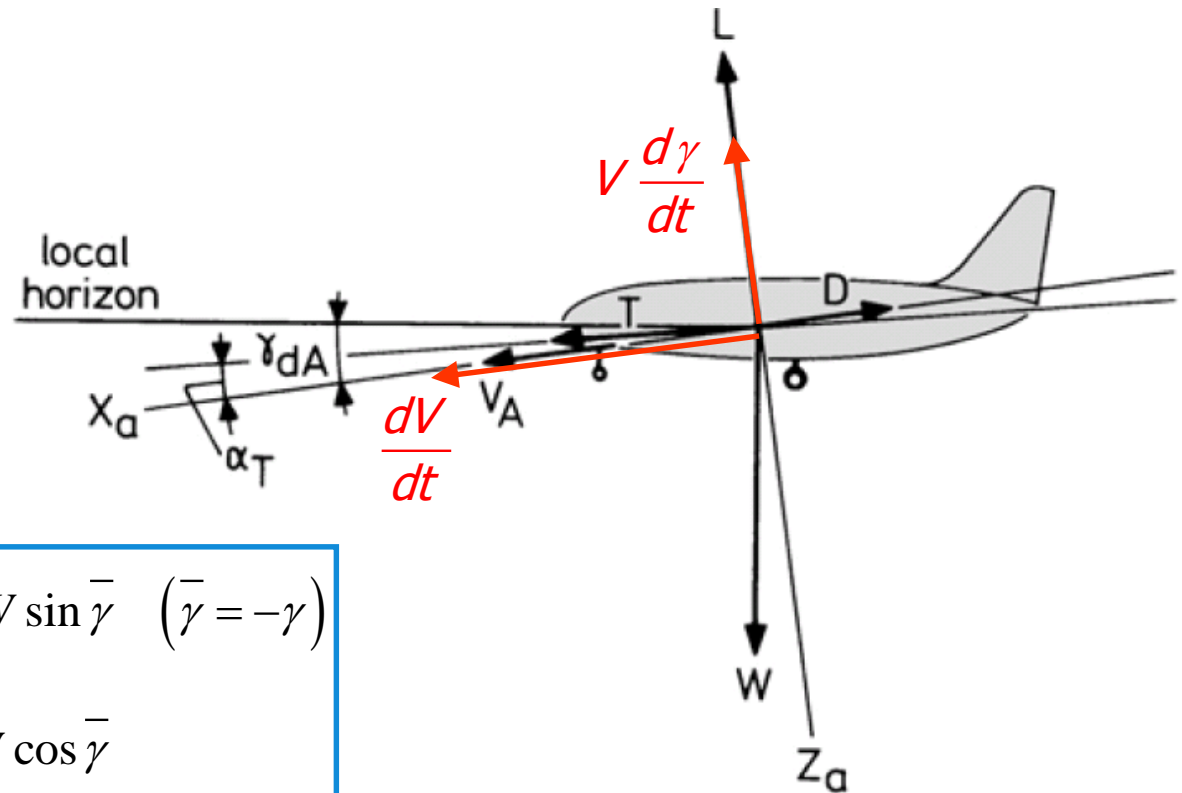
Content

- Introduction
- **Equations of motion**
- Analytical solution landing distance
- Spoilers, thrust reversers, etc.
- Tire, runway and brake properties
- Airworthiness Regulations
- Example exam question
- Summary



Equations of motion

Airborne phase



$$\sum F_{\parallel V} : \frac{W}{g} \frac{dV}{dt} = T - D + W \sin \bar{\gamma} \quad (\bar{\gamma} = -\gamma)$$

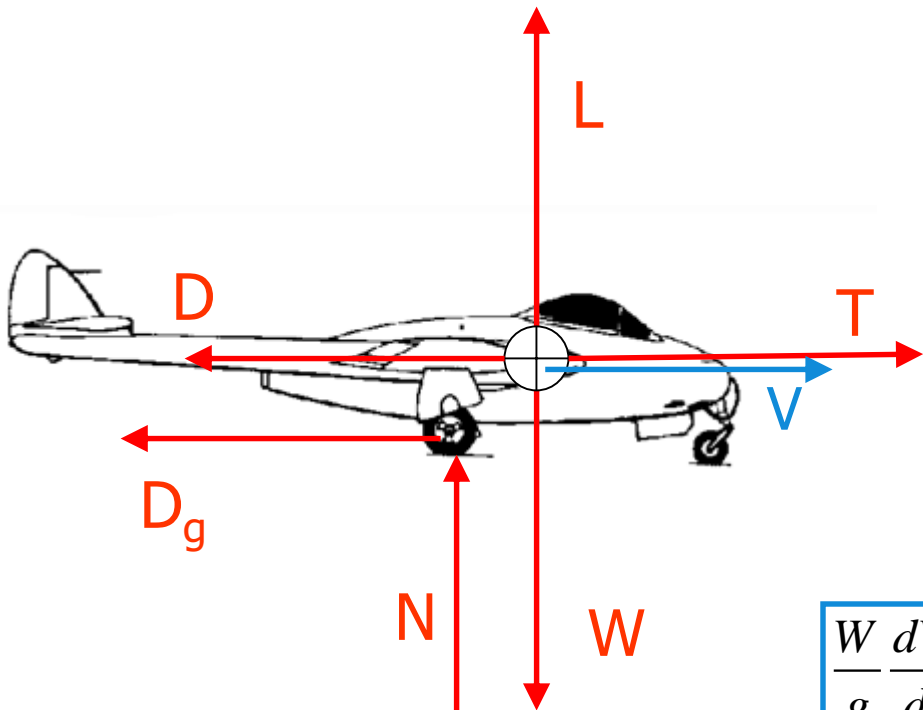
$$\sum F_{\perp V} : -\frac{W}{g} V \frac{d\bar{\gamma}}{dt} = L - W \cos \bar{\gamma}$$

Assumption: $\alpha_T = 0$

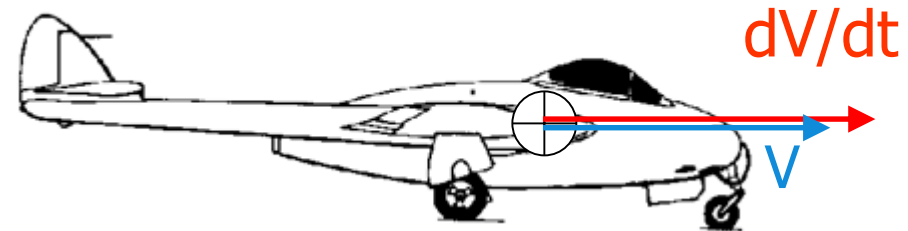
Equations of motion

Ground phase

Free body diagram



Kinetic diagram



$$\frac{W}{g} \frac{dV}{dt} = T - D - \mu_r (W - L)$$

Content

- Introduction
- Equations of motion
- **Analytical solution landing distance**
- Spoilers, thrust reversers, etc.
- Tire, runway and brake properties
- Airworthiness Regulations
- Example exam question
- Summary



Analytical solution

Airborne phase

$$\sum F_{//V} : \frac{W}{g} \frac{dV}{dt} = T - D + W \sin \bar{\gamma} \quad (\bar{\gamma} = -\gamma)$$

$$\sum F_{\perp V} : -\frac{W}{g} V \frac{d\bar{\gamma}}{dt} = L - W \cos \bar{\gamma}$$

(Equations of motion)

$$\frac{W}{g} \frac{dV}{ds} \frac{ds}{dt} = T - D + W \sin \bar{\gamma}$$

(Introduce variable s)

$$\frac{W}{g} V dV = (T - D) ds + W \sin \bar{\gamma} ds$$

(Rewrite)

$$\frac{W}{g} V dV = (T - D) ds - W dh$$

(dh \cong sin γ ds)

$$\frac{W}{2g} \int_{V_A}^{V_T} dV^2 = \int_0^s (T - D) ds - \int_{h_{scr}}^0 W dh$$

(Integrate)

$$\frac{W}{2g} (V_T^2 - V_A^2) = (\bar{T} - \bar{D}) s + W h_{scr}$$

(Result)

Analytical Solution

Airborne phase

$$\frac{W}{2g}(V_T^2 - V_A^2) = (\bar{T} - \bar{D})s + Wh_{scr}$$

What is $\bar{T} - \bar{D}$???

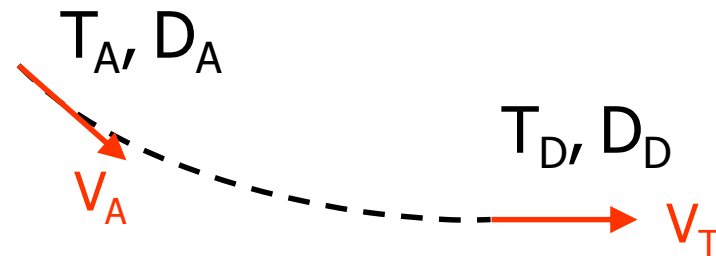
$$\bar{T} - \bar{D} = \frac{(T-D)_A + (T-D)_T}{2}$$

Approach

Assume steady flight

$$0 = T - D + W \sin \bar{\gamma}_A$$

$$(T - D)_A = -W \sin \bar{\gamma}_A$$



Touch down

$$\frac{W}{g} \frac{dV}{dt} = T - D, \quad L = W$$

$$T_T = 0$$

$$D_T = \left(\frac{C_D}{C_L} \right)_T W$$

$$(T - D)_T = - \left(\frac{C_D}{C_L} \right)_T W$$

Analytical solution

Airborne phase

$$\frac{W}{2g}(V_T^2 - V_A^2) = (\bar{T} - \bar{D})s + Wh_{scr}$$

$$\bar{T} - \bar{D} = \frac{(T-D)_A + (T-D)_T}{2} = -\frac{W}{2} \sin \bar{\gamma}_A - \left(\frac{C_D}{C_L} \right)_T \frac{W}{2}$$

$$\frac{V_A^2 - V_T^2}{2g} = \frac{s}{2} \left[\sin \bar{\gamma}_A + \left(\frac{C_D}{C_L} \right)_T \right] - h_{scr}$$

$$s = \frac{\frac{V_A^2}{2g} - \frac{V_T^2}{2g} + h_{scr}}{\frac{1}{2} \left[\sin \bar{\gamma}_A + \left(\frac{C_D}{C_L} \right)_T \right]}$$

Analytical solution

Ground run

$$a = \frac{dV}{dt} = V \frac{dV}{ds}$$

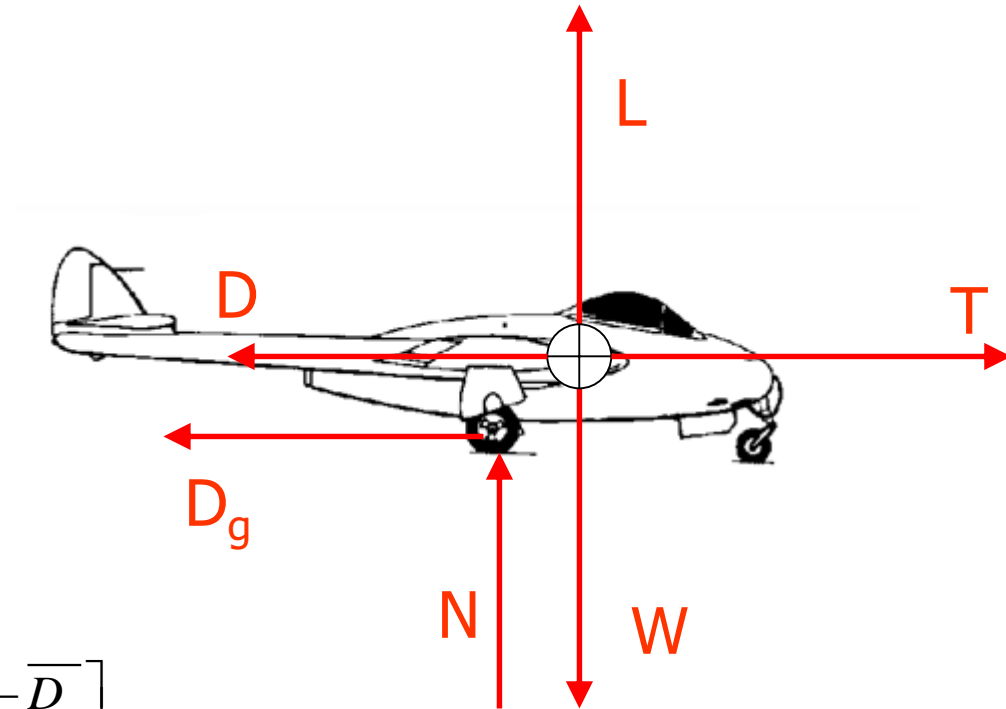
$$s = \int_{V_T}^0 \frac{V dV}{a} = -\frac{V_T^2}{2a}$$

$$\frac{W}{g} \frac{dV}{dt} = T - D - \mu_r (W - L)$$

$$\bar{a} = \frac{g}{W} [\bar{T} - \bar{D} - \bar{D}_g] = \frac{g}{W} [-\bar{T}_{rev} - \bar{D} - \bar{D}_g]$$

Reverse thrust

At $V_T / \sqrt{2}$



$$\frac{W}{g} \frac{dV}{dt} = T - D - \mu_r (W - L)$$

Analytical solution

Ground run

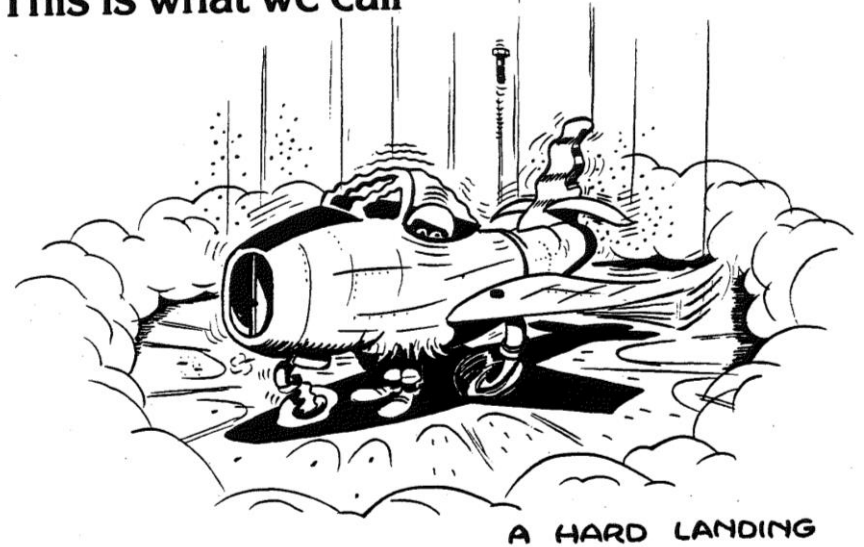
Final result:

$$s_g = \frac{W}{2g} \frac{V_T^2}{\left[\bar{T}_{rev} + \bar{D} + \bar{D}_g \right]}$$

$$V_T \approx 1.15V_{\min}$$

$$s_g \approx \frac{W}{2g} \frac{W}{S} \frac{2 \cdot 1.15^2}{\rho C_{L_{\max}}} \frac{1}{\left(\bar{T}_{rev} + \bar{D} + \bar{D}_g \right)}$$

This is what we call





DID WE LAND OR WERE WE SHOT DOWN ?

Content

- Introduction
- Equations of motion
- Analytical solution landing distance
- **Spoilers, thrust reversers, etc.**
- Tire, runway and brake properties
- Airworthiness Regulations
- Example exam question
- Summary



Spoilers, thrust reversers, etc.

Overview

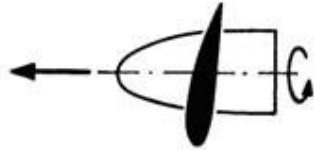
$$s_g \approx \frac{W}{2g} \frac{W}{S} \frac{2 \cdot 1.15^2}{\rho C_{L_{\max}}} \frac{1}{(T_{rev} + \bar{D} + \bar{D}_g)}$$

- W^2 !!
- Flaps and small W/S
- T_{rev} (but: asymmetry)
- D : Spoilers, chute
- D_g : ABS, Lift dumpers, clean runway



Propeller

forward thrust



fine (low) pitch,
takeoff,
small blade angle

forward thrust



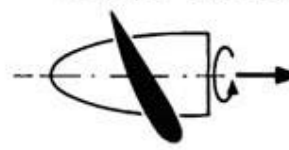
coarse (high) pitch,
cruise flight,
normal blade angle

zero thrust



full feathering,
propeller stopped,
large blade angle

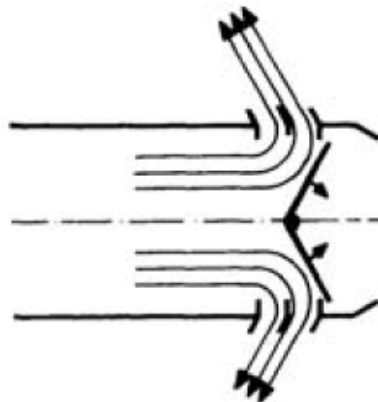
reverse thrust



landing brake,
large negative
blade angle

Jet

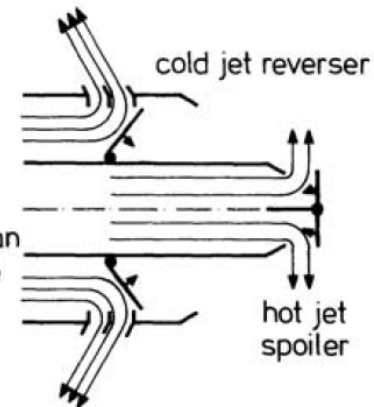
turbojet
engine



hot jet
reverser

Turbofan

turbofan
engine



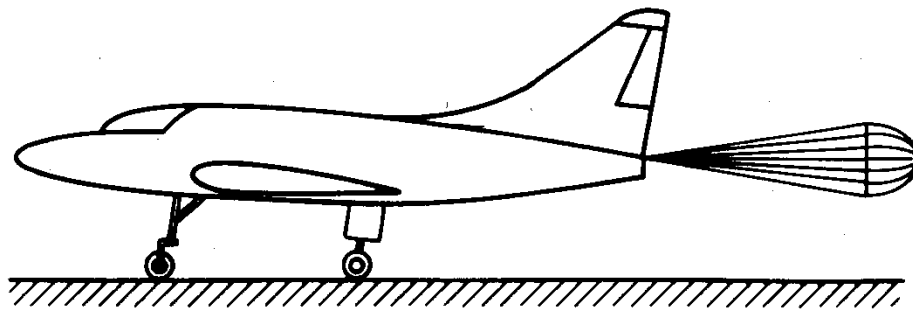
cold jet reverser

hot jet
spoiler

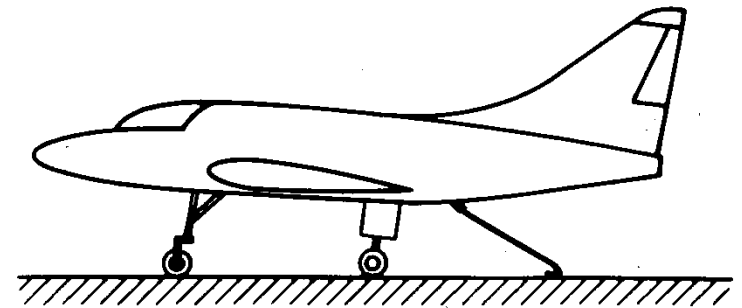
Spoilers, thrust reversers, etc.

Special devices

a) drag parachute

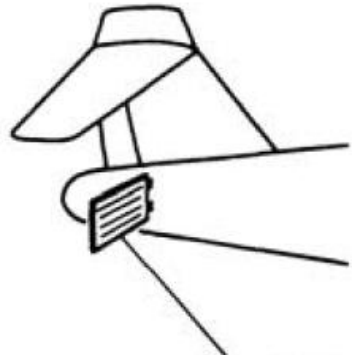


b) arrester hook

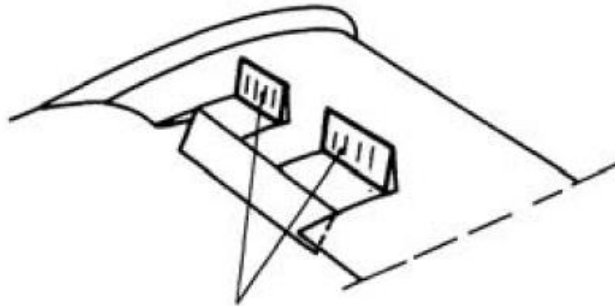


Spoilers, thrust reversers, etc.

Speed brakes and spoilers



b. speedbrake



c. spoilers



Spoilers, thrust reversers, etc.

Summary

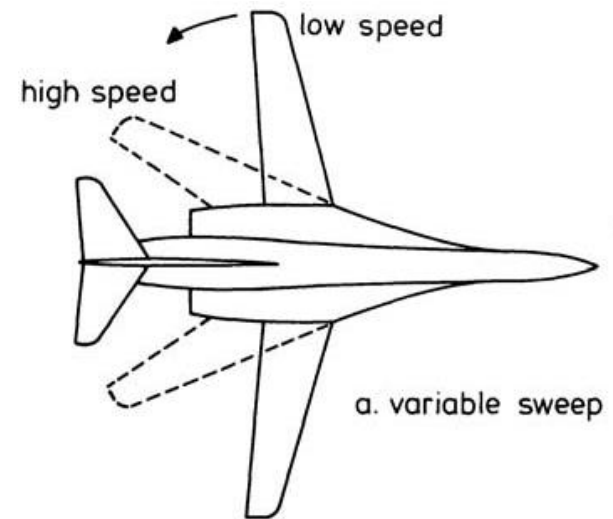
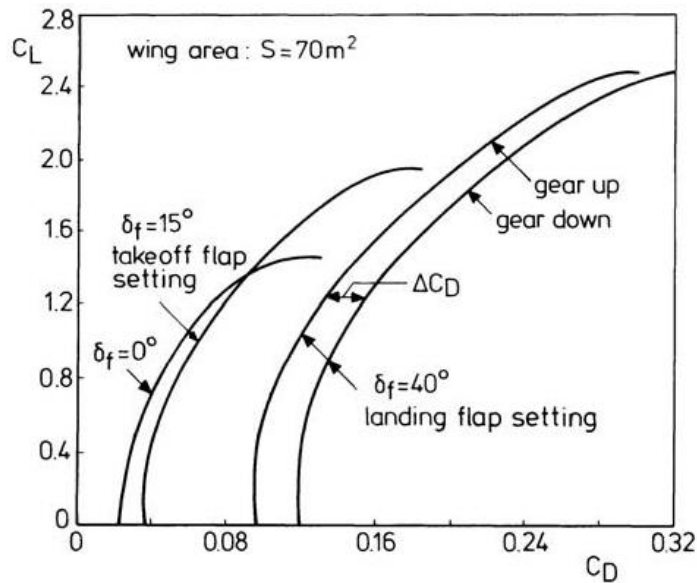
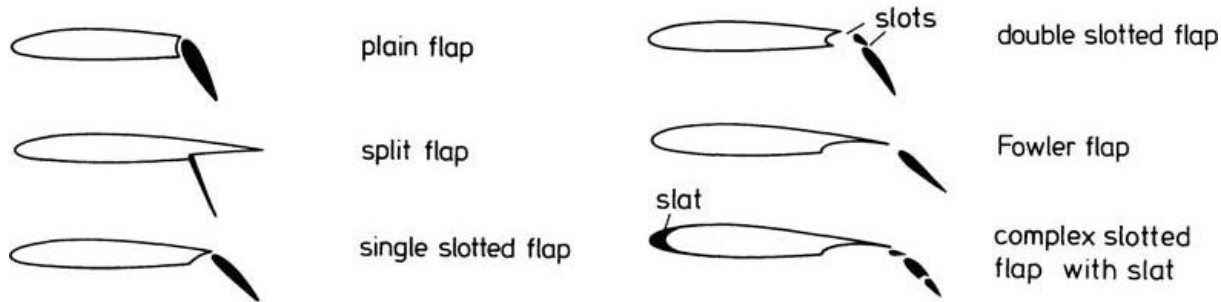
- Speed brakes
- Used to increase aerodynamic drag (D)

- Lift Dumpers
- Reduce lift and thereby increase normal force on wheels, which increases D_g

- Spoilers
- Used both to reduce lift and to increase aerodynamic drag. Furthermore, used for (high-speed) roll control

Spoilers, thrust reversers, etc.

Airspeed - Flaps



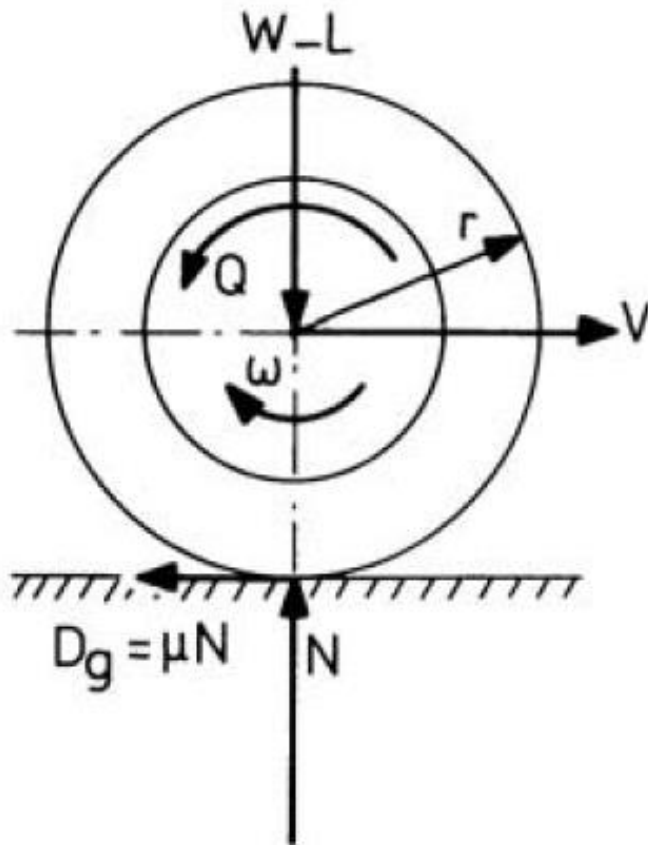
Content

- Introduction
- Equations of motion
- Analytical solution landing distance
- Spoilers, thrust reverser, etc.
- **Tire, runway and brake properties**
- Airworthiness Regulations
- Example exam question
- Summary



Tire, runway and brake properties

Equation of motion



Moments about centre of wheel:

$$D_g r - Q$$

Accelerations about centre of wheel:

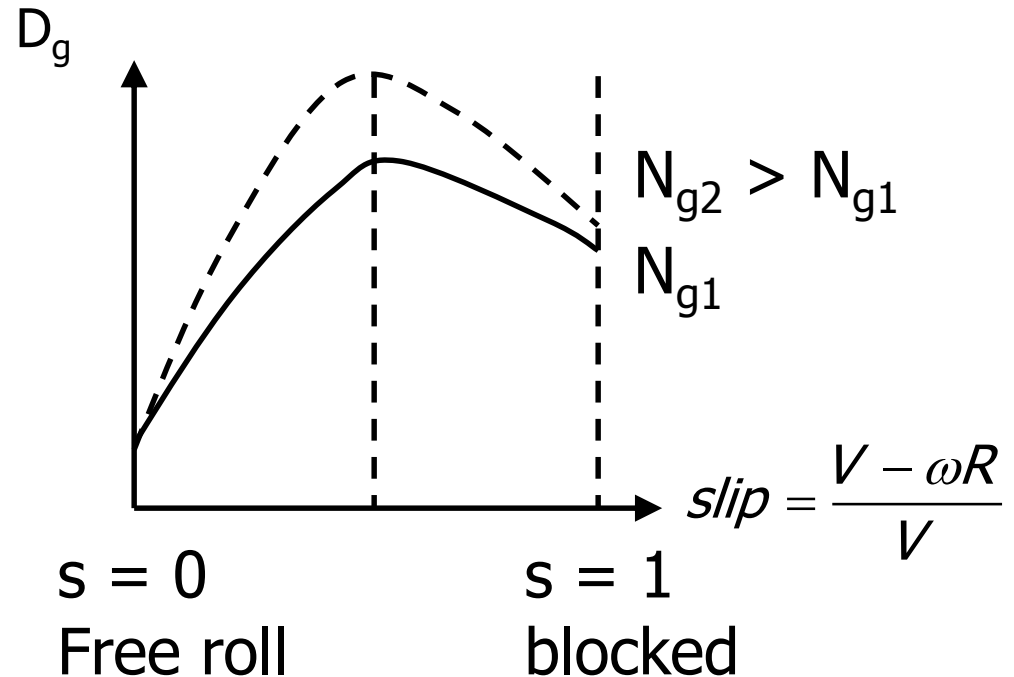
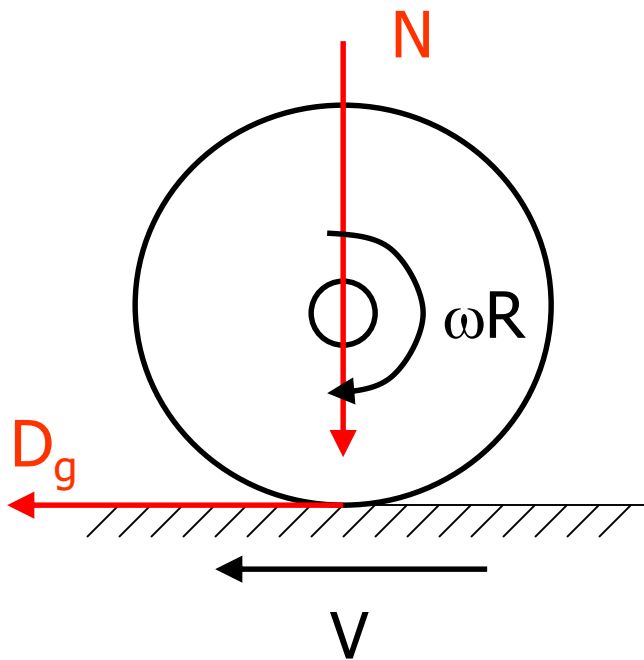
$$\frac{d\omega}{dt}$$

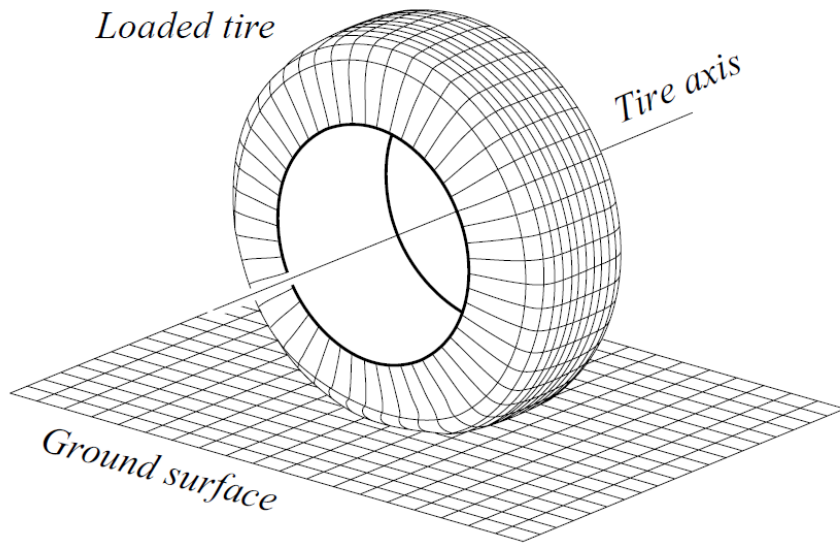
Combined

$$I \frac{d\omega}{dt} = D_g r - Q$$

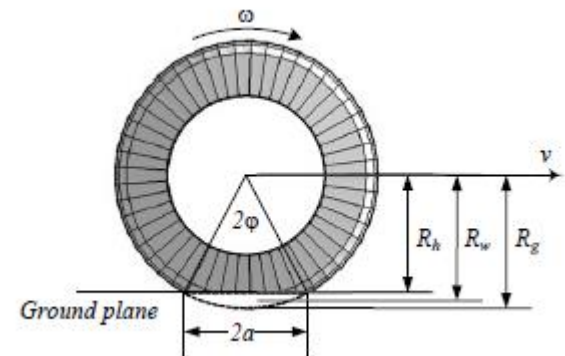
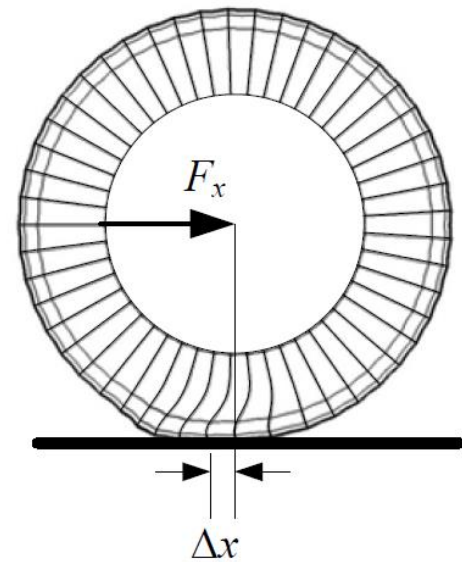
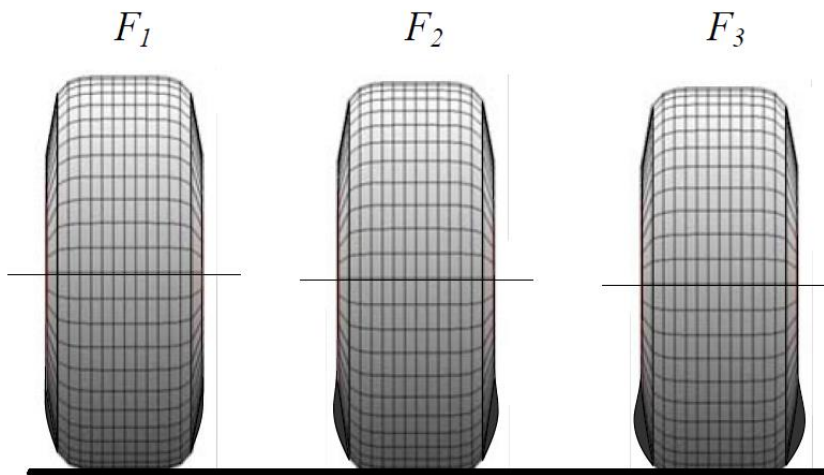
Tire, runway and brake properties

Experiment





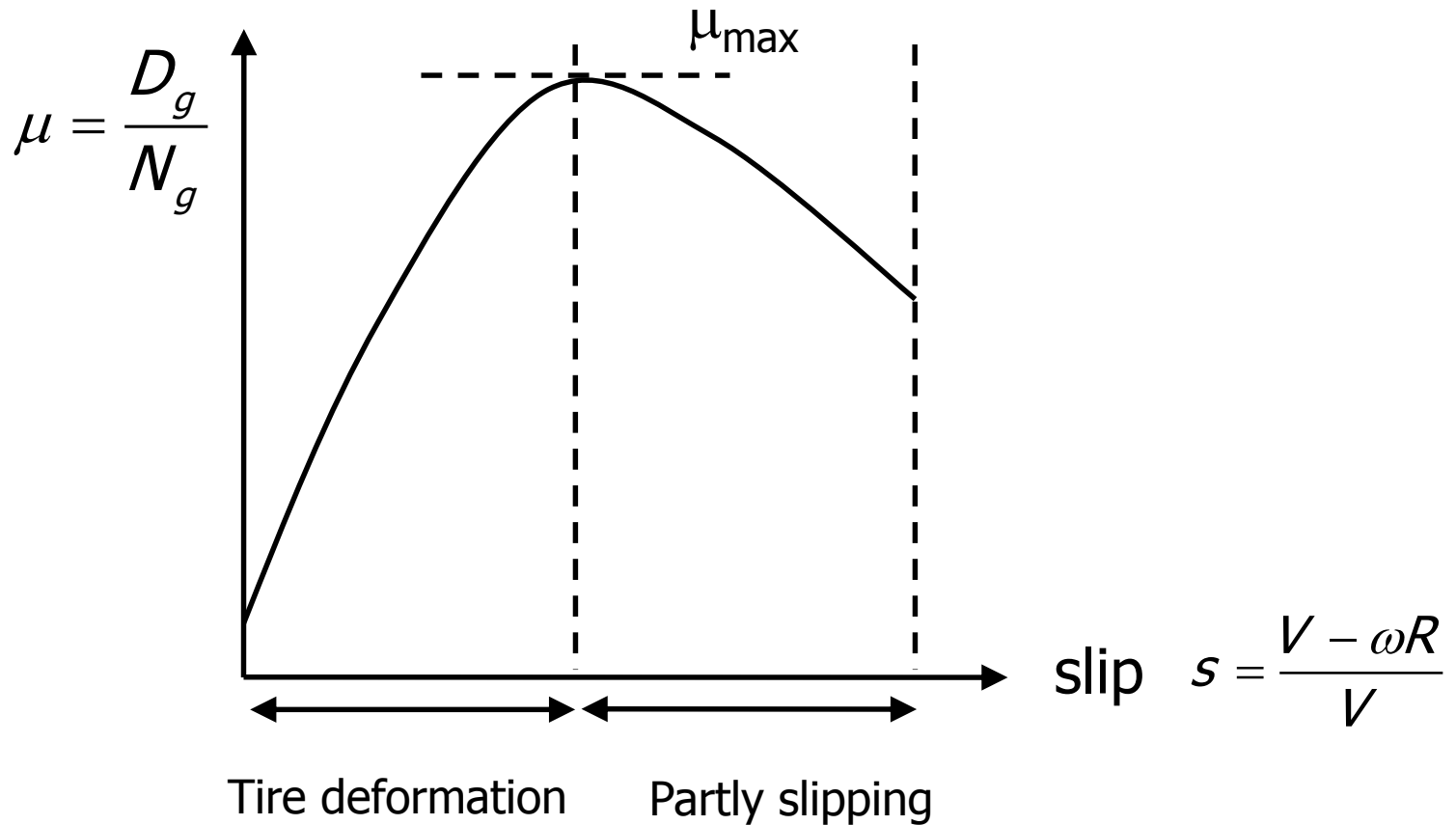
$$F_1 < F_2 < F_3$$



Images from: Jazar, R. "Vehicle Dynamics: Theory and Application"

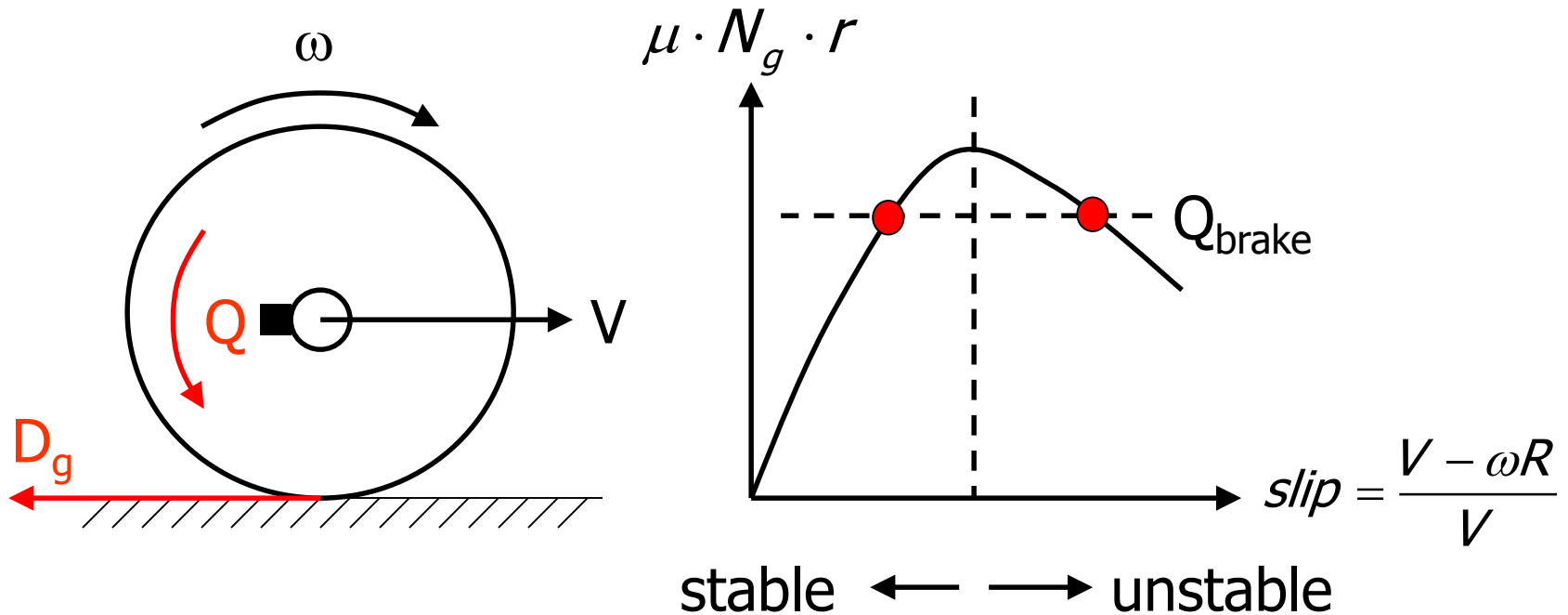
Tire, runway and brake properties

Dimensionless



Tire, runway and brake properties

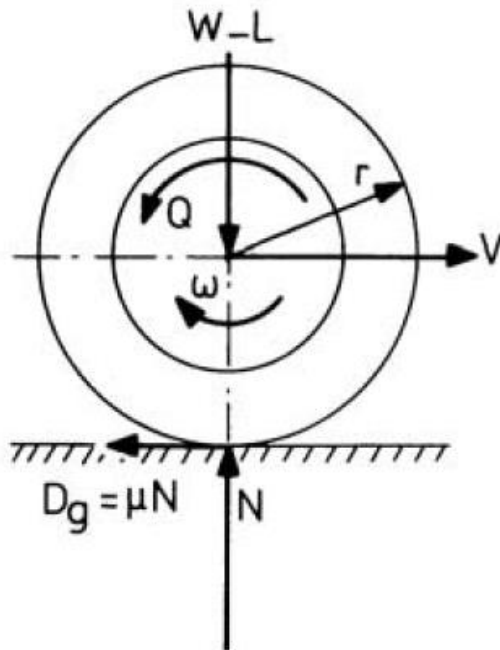
Brake stability



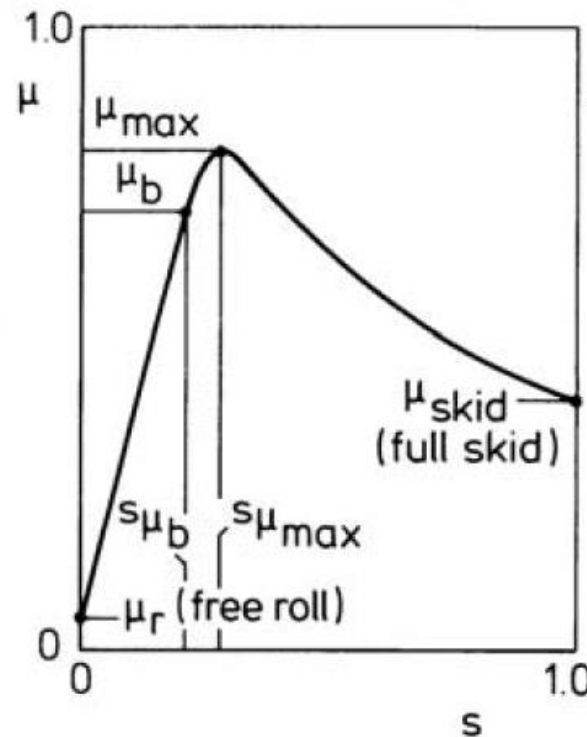
Tire, runway and brake properties

Summarized

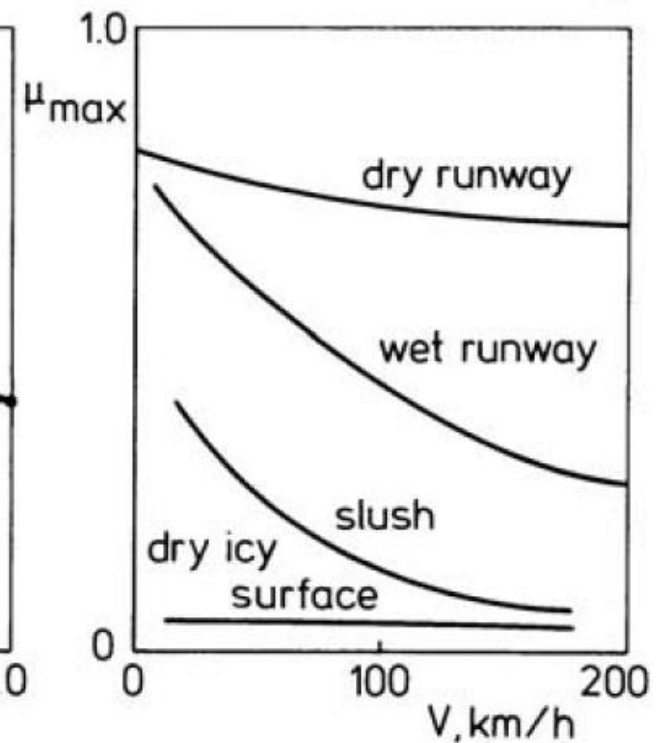
a) forces and moments



b) typical variation of friction coefficient with slip ratio



c) typical friction coefficients





<http://www.youtube.com/watch?v=UPnaBN95Npw>

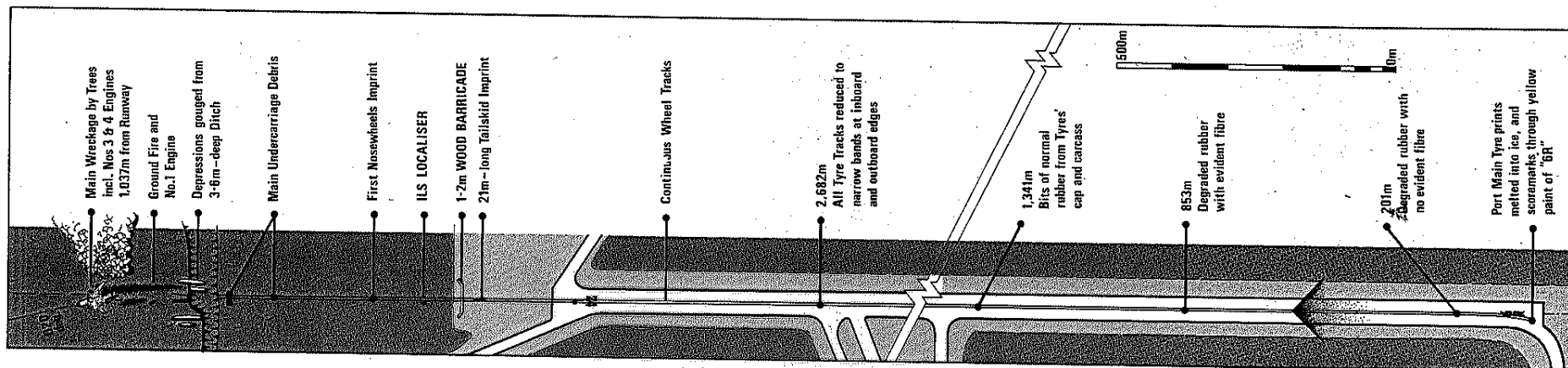
Tire, runway and brake properties

DC-8 Anchorage 1970

- Take-off with blocked brakes on icy runway
- $\mu_{\text{glide}} = 0.025$ (Compare to $\mu_{\text{roll}} = 0.02$)
- Normal ground run acceleration
- Tire blow out at V_1
- Increasing wear: no acceleration at $V > V_1$

Lessons:

1. Function indication: not position of controls
2. Take-off monitoring system



Content

- Introduction
- Equations of motion
- Analytical solution landing distance
- Spoilers, thrust reverser, etc.
- Tire, runway and brake properties
- **Airworthiness Regulations**
- Example exam question
- Summary



Airworthiness regulations

Summary

- $V_a \geq 1.3 V_{\min}$
- Required field length = $10/6$ x demonstrated landing distance
- Wet: x 1.15
- Landing climb potential: All engines and flaps in landing configuration

Content

- Introduction
- Equations of motion
- Analytical solution landing distance
- Spoilers, thrust reverser, etc.
- Tire, runway and brake properties
- Airworthiness Regulations
- **Example exam question**
- Summary



Example exam question

Question 1

The following data of the Cessna Citation in the approach shortly before the landing on an airfield at sea level are given:

Airplane weight	: $W = 60 \text{ kN}$
Wing area	: $S = 30 \text{ m}^2$
Lift-Drag polar	: $C_D = C_{D0} + C_L^2/\pi A e$
In landing configuration	: $C_{D0} = 0.07, A e = 5.8$
Maximum lift coefficient	: $C_{Lmax} = 2$
Airspeed during the approach	: $V_A = 1.3 V_{min}$
Flight path angle during approach	: $\gamma_A = 3^\circ$ (glide path)
Airspeed at touchdown	: $V_T = 1.2 V_{min}$
Lift coefficient in ground run attitude	: $C_{Lg} = 1.1$
Friction coefficient at maximum brake power	: $\mu_r = 0.4$ (constant)

- a. Draw a clear Free Body Diagram (FBD) and Kinetic Diagram (KD) of the aircraft visualizing all forces and accelerations that act on the aircraft during the **approach phase**. You can assume that the aircraft performs a steady descent during this approach phase along the glide slope ($\gamma = -3^\circ$). Clearly indicate the direction of the velocity and all angles that are relevant for any further calculations. Also explain what steady means, and what the consequences are of a steady descent for the forces in the FBD and/or the accelerations in the KD.

Example exam question

- b. Derive the equations of motion using the FBD and KD for the aircraft during the approach phase. Clearly indicate all assumptions.
- c. Calculate the approach speed V_A .
- d. Calculate the thrust during the approach.
- e. Draw a clear FBD and KD visualizing all forces and accelerations that act on the aircraft during the **airborne phase of the landing** (the phase between screen height and touch down). Clearly indicate the direction of the velocity and all angles that are relevant for any further calculations.
- f. Derive the equations of motion using the FBD and KD for the aircraft during the airborne phase of the landing. Clearly indicate all assumptions.
- g. Use the energy method to derive an expression for the distance covered during the airborne phase of the landing from screen height until touch down. This expression will be a function of the airspeed at screen height V_A , the touch down speed V_T , the screen height h_{scr} , the aircraft weight W and the mean excess thrust $\overline{T-D}$. The screen height is equal to 15 m.

Example exam question

- h. Calculate the mean excess thrust $\overline{T-D}$ during the airborne phase of the landing. The mean excess thrust is the average of the excess thrust values at the screen height and at touchdown. Assume a steady descent ($\gamma = -3^\circ$) at approach speed for the screen height, and note that at touch down the power setting is equal to zero thrust ($T = 0$). At touch down it can also be assumed that vertical equilibrium still exists ($L = W$) and that the flight path of the aircraft is tangent to the runway.
- i. Show that the distance covered during the airborne phase of the landing is equal to 400 m.
- j. Draw a clear FBD and KD visualizing all forces and accelerations that act on the aircraft during the **landing ground run**. Clearly indicate the direction of the velocity and all angles that are relevant for any further calculations.
- k. Derive the equations of motion using the FBD and KD for the aircraft during the landing ground run. Clearly indicate all assumptions.

Example exam question

- l. Calculate the ground run distance (s_g) assuming that maximum brake power is available immediately after touchdown. The landing ground run is assumed to be a uniformly decelerated motion. The characteristic airspeed is calculated at the average dynamic pressure occurring during this ground run (therefore: $V = V_T/\sqrt{2}$).
- m. Which auxiliary devices can be applied on modern (civil) airplanes to shorten the landing distance? Explain, in a few words, the main physical effect that is utilized by each of these devices in order to assist in shortening the landing distance.

Content

- Introduction
- Equations of motion
- Analytical solution landing distance
- Spoilers, thrust reverser, etc.
- Tire, runway and brake properties
- Airworthiness Regulations
- Example exam question
- **Summary**



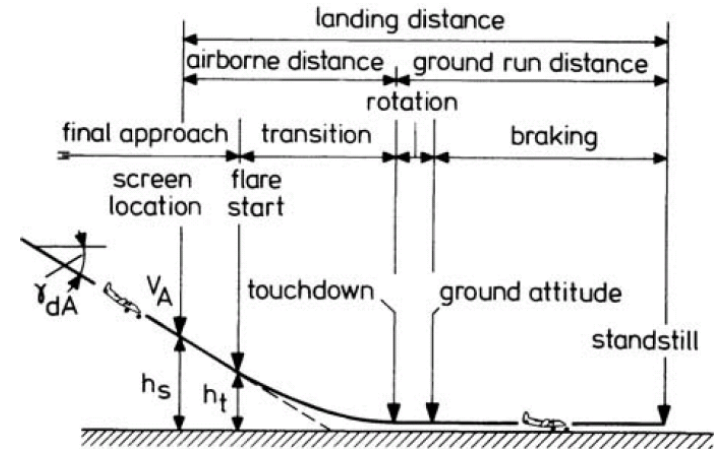
Summary

- Airborne distance required for landing:

$$s = \frac{\frac{V_A^2}{2g} - \frac{V_T^2}{2g} + h_{scr}}{\frac{1}{2} \left[\sin \bar{\gamma}_A + \left(\frac{C_D}{C_L} \right)_T \right]}$$

- Distance required on ground during landing

$$s_g \approx \frac{W}{2g} \frac{W}{S} \frac{2}{\rho} \frac{1.15^2}{C_{L_{max}}} \frac{1}{T_{rev} + \overline{D} + \overline{D}_g}$$



Landing procedure



You should be able to derive these equations

Summary

- Landing performance is influenced by
 - Aircraft weight
 - Airspeed (C_{Lmax} , flaps)
 - Drag devices (speed brakes, spoilers)
 - Thrust reversers
 - Lift dumpers
 - Tire and brake properties
- If tires block due to brake instability, then the friction reduces; need for ABS

Questions?

