Flight and Orbital Mechanics

Lecture slides

Flight and Orbital Mechanics

Lecture hours 11, 12 – Cruise

Mark Voskuijl

Semester 1 - 2012

Content

- Introduction
- Optimum cruise profile
  - Optimal airspeed for given H, W
  - Effect of altitude
  - Effect of weight
  - Best flying strategy
- Analytic Range equations
- Story
- Weight breakdown
- Economics
- Summary

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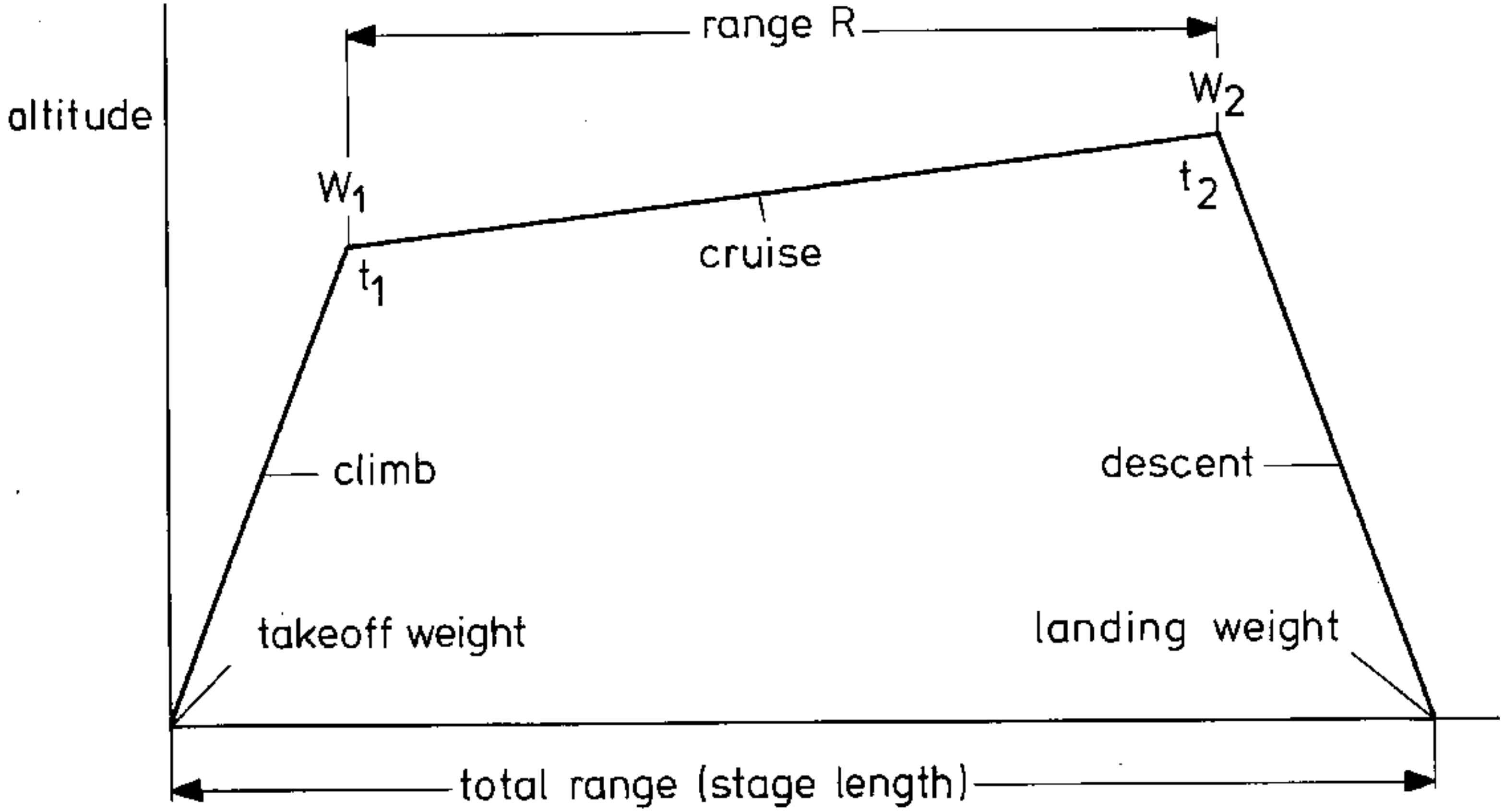
Introduction

Typical cruise flight



Introduction

Objective

- Range (distance)
- Endurance (Maximum time)

Introduction

Equations of motion

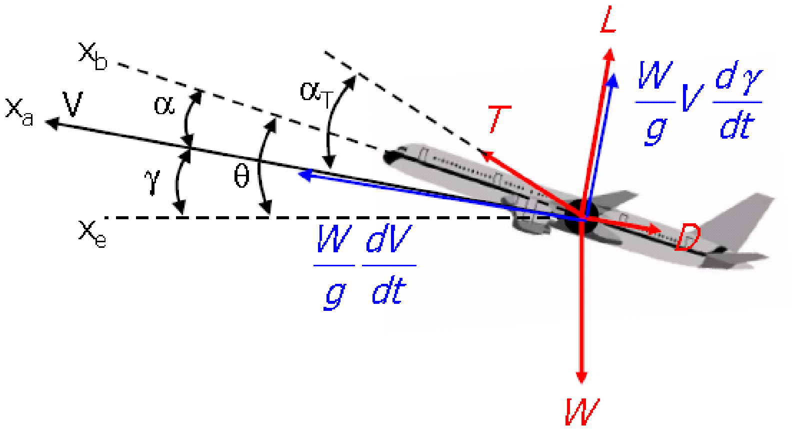
General 2D equations of motion and power equation

Unsteady curved symmetric flight

$$T\cos\alpha_T - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

$$L - W\cos\gamma + T\sin\alpha_T = \frac{W}{g}V\frac{d\gamma}{dt}$$

$$\frac{P_a - P_r}{W} = RC + \frac{V}{g}\frac{dV}{dt}$$



Cruise flight

Quasi steady ($dV/dt \cong 0$), quasi-rectilinear, ($d\gamma/dt \cong 0$)

Weight of the aircraft is **not constant**

Small flight path angle → $\cos\gamma = 1$, $\sin\gamma \neq 0$

Assume that the thrust vector acts in the direction of flight ($\alpha_T \cong 0$)

Introduction

Equations of motion

Equations of motion cruise flight

$$0 = \frac{g}{W}\left(T - D - W\sin\gamma\right)$$

$$L = W \qquad (\text{ quasi rectilinear })$$

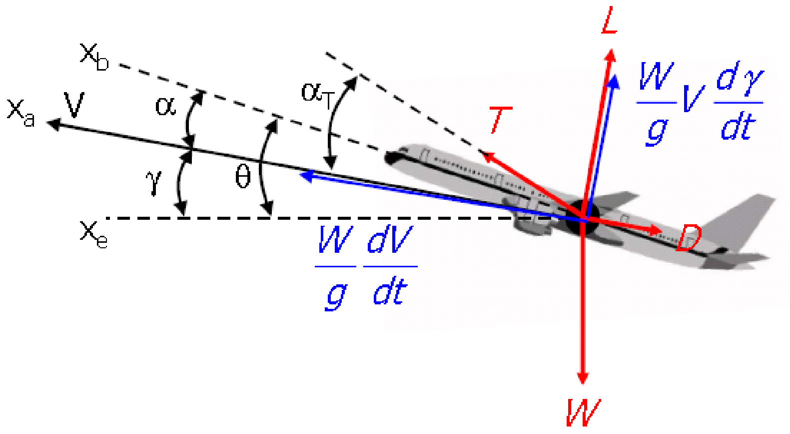
Additional equation

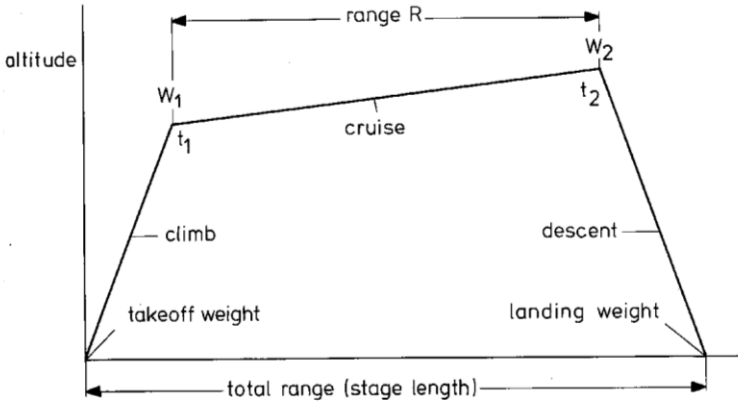
$$\frac{dW}{dt} = -F\left(\Gamma, V, H\right)$$

Kinematic equations

$$\frac{ds}{dt} = V\cos\gamma$$

$$\frac{dH}{dt} = V\sin\gamma$$





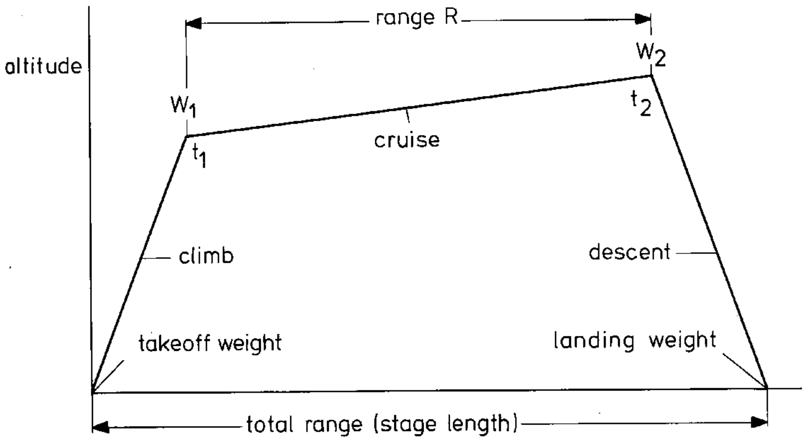
Introduction

Problem definition

Pilot can choose a certain airspeed and altitude: H(t), V(t)

What is the best flight condition?

1. Optimal initial conditions (V, H at initial weight)
2. Optimal flying strategy (V, H at decreasing weight)



Introduction

Criteria for optimal flight

1. Maximum endurance E:
Fuel flow F_{min} at every point in time

2. Maximum range R:
Specific range $(V/F)_{max}$ at every point in time

3. Given range, minimum fuel:
Specific range $(V/F)_{max}$ at every point in time

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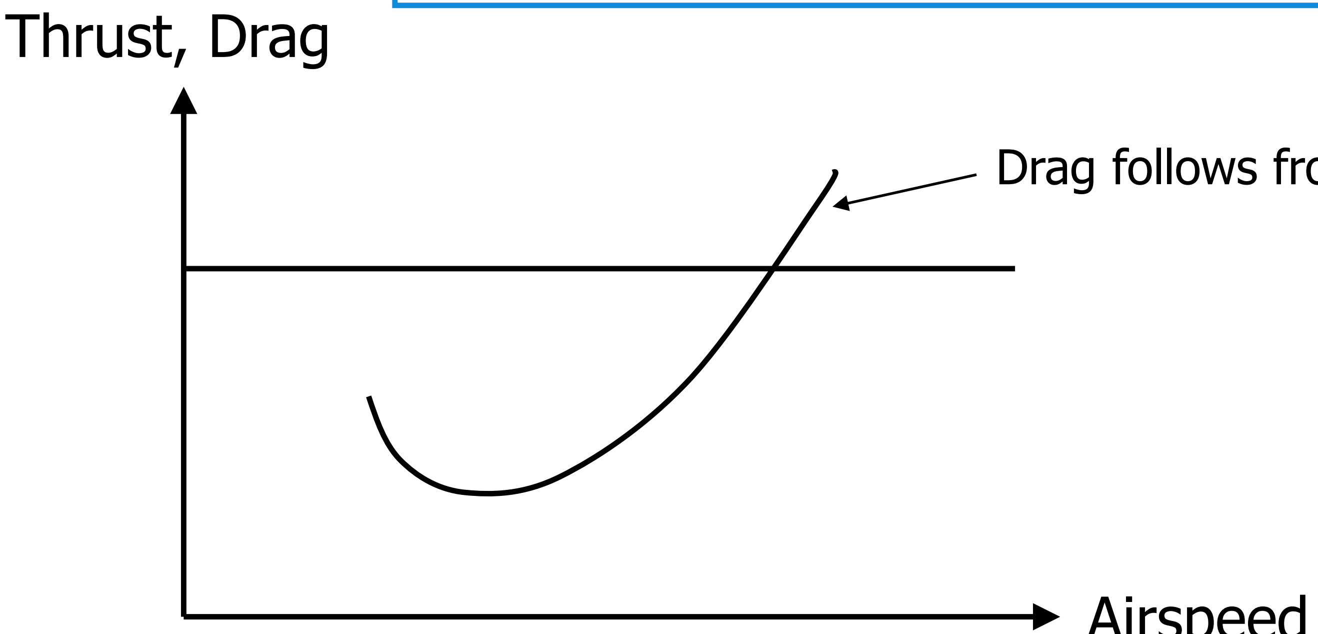
Optimum cruise profile (Jet)

Performance diagram - Jet

*For basic flight mechanics applications, **thrust** of a **turbojet** can be assumed to be **constant with airspeed** for a given flight altitude*

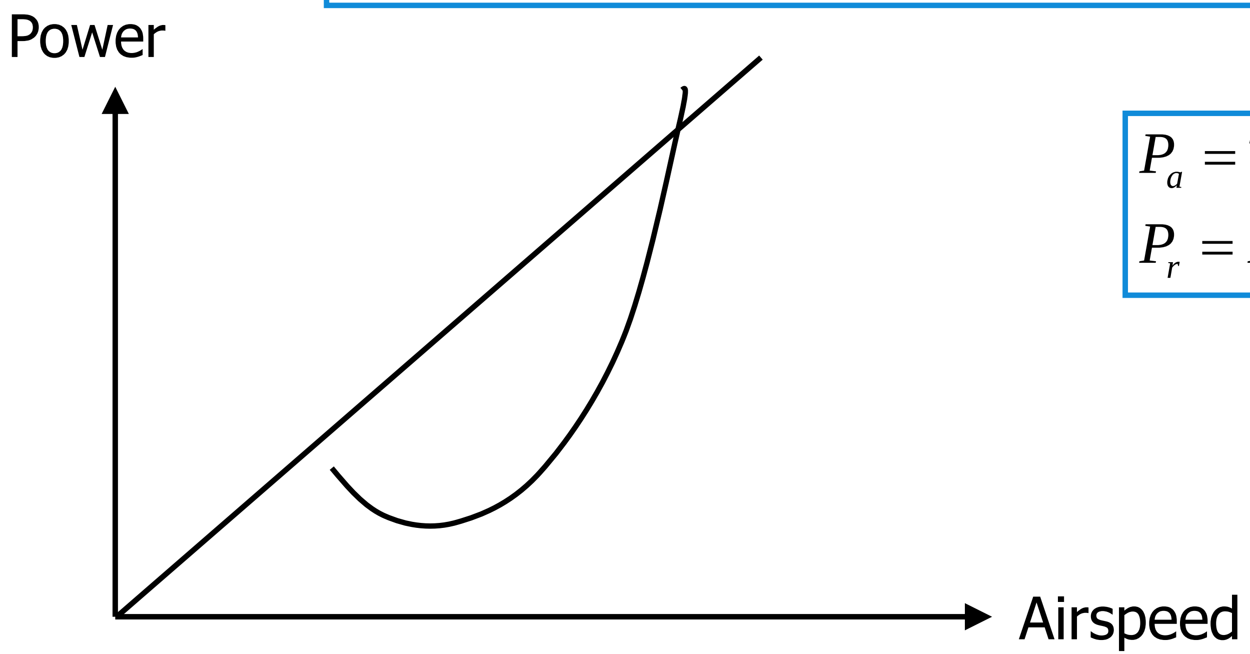


$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

Optimum cruise profile (Jet)

Performance diagram - Jet

*For basic flight mechanics applications, **thrust** of a **turbojet** can be assumed to be **constant with airspeed** for a given flight altitude*



$$P_a = TV$$
$$P_r = DV$$

Optimum cruise profile (Jet)

Thrust specific fuel consumption

$$F = c_T T$$

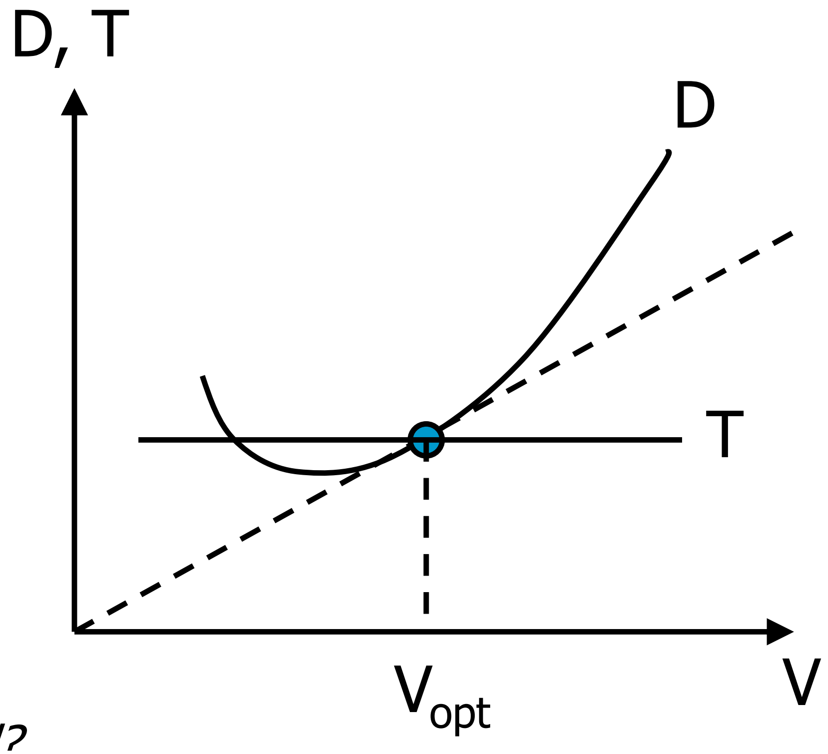
Additional assumption:
c_T constant

Specific range

$$\frac{V}{F} = \frac{V}{c_T T} = \frac{V}{c_T D}$$

$$\left(\frac{V}{F}\right)_{\max} \quad \text{if} \quad \left(\frac{D}{V}\right)_{\min}$$

What is the corresponding airspeed?



Optimum cruise profile (Jet)

Optimal airspeed for given altitude and weight

<u>Method to calculate the best airspeed</u>

Optimum criterion	Lift over drag ratio	Angle of attack	Airspeed (for given H,W)
$(D/V)_{min}$ →	C_L^X / C_D^Y →	C_L →	V

Optimum cruise profile (Jet)

Airspeed

1. Optimum criterion

$$\left(\frac{D}{V}\right)_{\min} \Rightarrow \left(\frac{V}{D}\right)_{\max}$$

2. Airspeed

$$L = W$$

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

3. Drag

$$D = \frac{L}{L}D = \frac{C_D}{C_L}W$$

4. Ratio

$$\frac{V}{D} = \frac{\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}}{\left(C_D/C_L\right)W} = \sqrt{\frac{1}{W \cdot S}\frac{2}{\rho}\frac{C_L}{C_D^2}}$$

5. For a given weight

$$\frac{V}{D} \propto \sqrt{\frac{C_L}{C_D^2} \cdot \frac{1}{\rho}}$$

6. Angle of attack for given altitude

$$\left(\frac{V}{D}\right)_{\max} \Rightarrow \left(\frac{C_L}{C_D^2}\right)_{\max}$$

Optimum cruise profile (Jet)

Airspeed

$$\left(\frac{V}{F}\right)_{\max} \Rightarrow \left(\frac{V}{D}\right)_{\max} \Rightarrow \left(\frac{C_L}{C_D^2}\right)_{\max}$$

$$\overset{lift-drag-polar}{\Rightarrow} \quad C_{L_{opt}} = \sqrt{\tfrac{1}{3} C_{D_0} \pi A e}$$

First year:

$$\left(\frac{C_L}{C_D^2}\right)_{\max} \Rightarrow \frac{d}{dC_L}\left(\frac{C_L}{C_D^2}\right) = 0$$

$$\frac{C_L \cdot 2C_D \dfrac{dC_D}{dC_L} - C_D^2 \cdot 1}{C_D^4} = 0$$

$$\frac{dC_D}{dC_L} = \frac{2C_L}{\pi A e}$$

$$C_D^4 \neq 0$$

$$\frac{2C_L}{\pi A e} = \tfrac{1}{2}\frac{C_D}{C_L} = \tfrac{1}{2}\frac{C_{D_0} + \dfrac{C_L^2}{\pi A e}}{C_L}$$

$$C_L = \sqrt{\tfrac{1}{3} C_{D_0} \pi A e}$$

<u>Airspeed for given altitude and weight</u>

$$L = W$$

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L,opt}}}$$

Optimum cruise profile (Jet)

Example question

Question 1 – Cruise flight

The Gulfstream IV, indicated in Figure 1 is a twin-turbofan executive transport aircraft. Data for this aircraft are given below:

$S = 88.3$ [m^2]

$b = 23.7$ [m]

$A = 6.36$ [-]

$e = 0.67$ [-]

$C_{D_0} = 0.015$ [-]

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

Figure 1 : Gulfstream IV

For jet aircraft, fuel consumption can be represented with the following equation:

$$F = c_T T$$

Aircraft Weight W = 300.000 [N] (start of cruise)

What is the best airspeed to fly (for max range) at 9000 [m] altitude?
(ρ = 0.4663 [kg/m^3], T = 229.65 [K])

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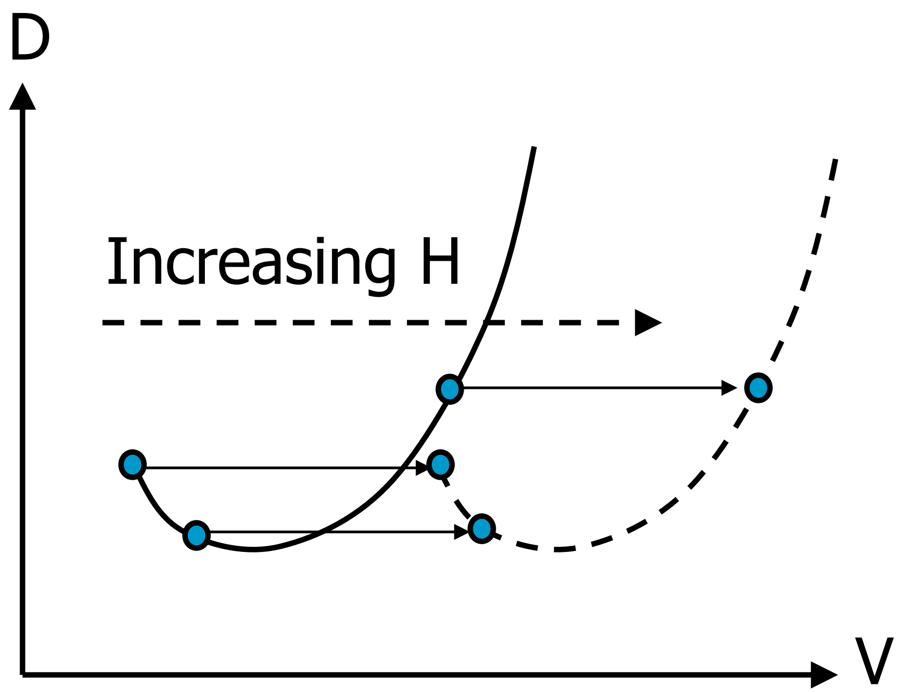
Effect of altitude

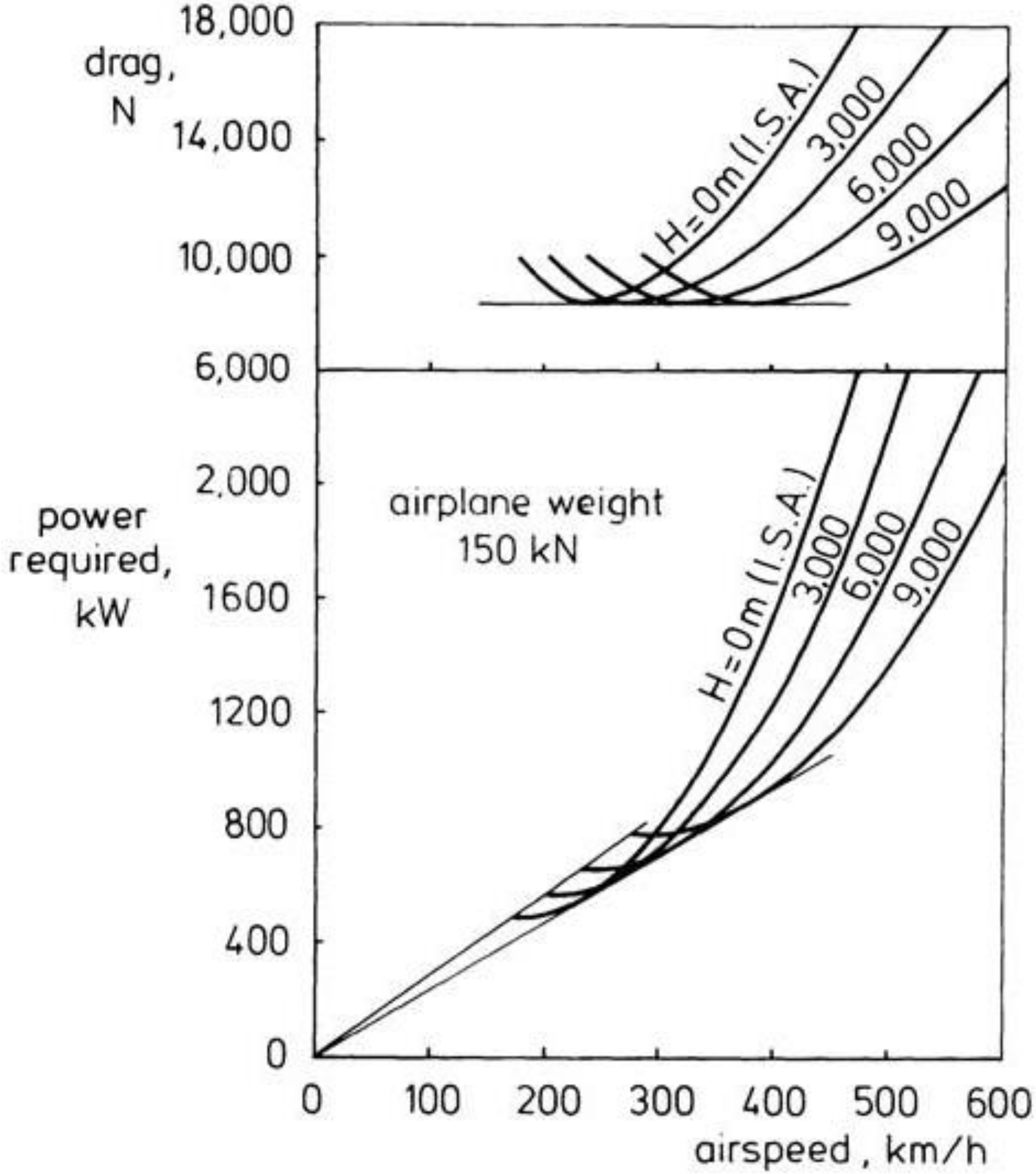
Performance diagram



Angle of attack is constant for a given point on the drag curve

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} \propto \frac{1}{\sqrt{\rho}}$$

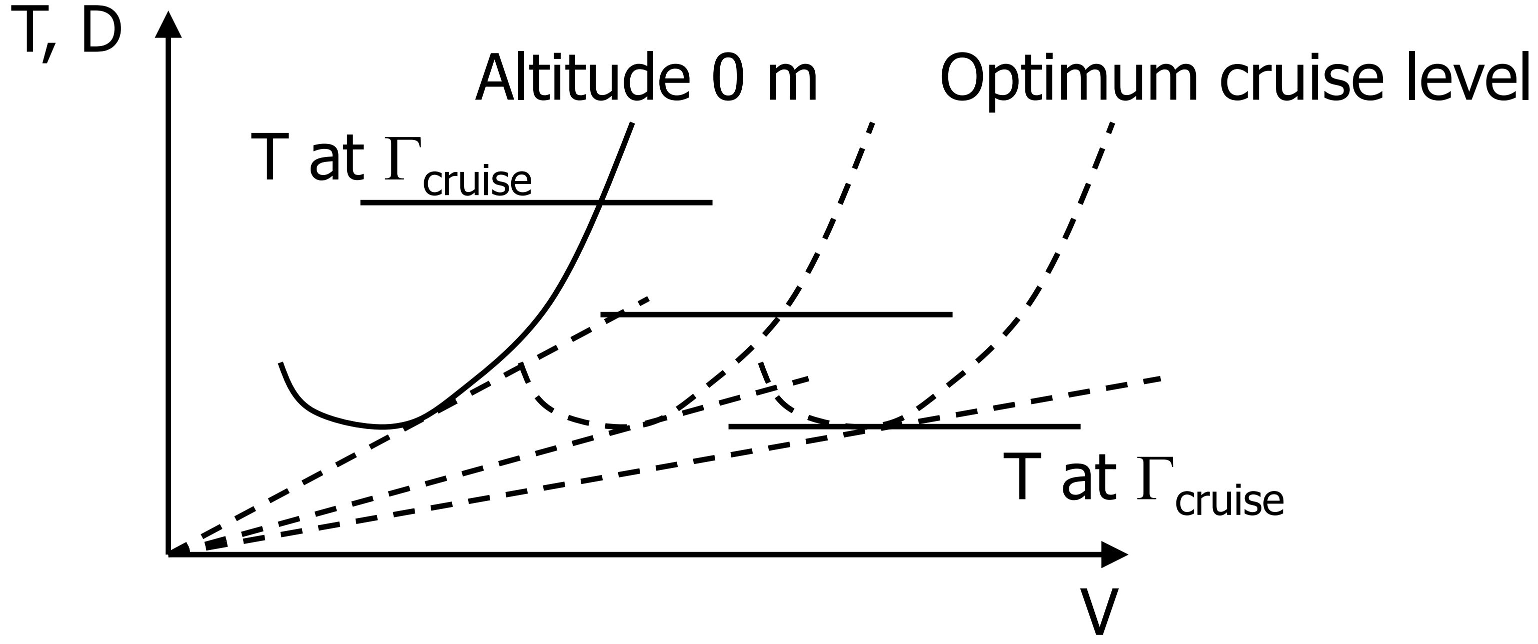
$$D = \frac{C_D}{C_L} W \propto \rho^0$$



drag, N
18,000
14,000
10,000
6,000
H=0m (I.S.A.)
3,000
6,000
9,000
power required, kW
2,000
1600
1200
800
400
0
airplane weight 150 kN
H=0m (I.S.A.)
3,000
6,000
9,000
0
100
200
300
400
500
600
airspeed, km/h

Effect of altitude

Specific range



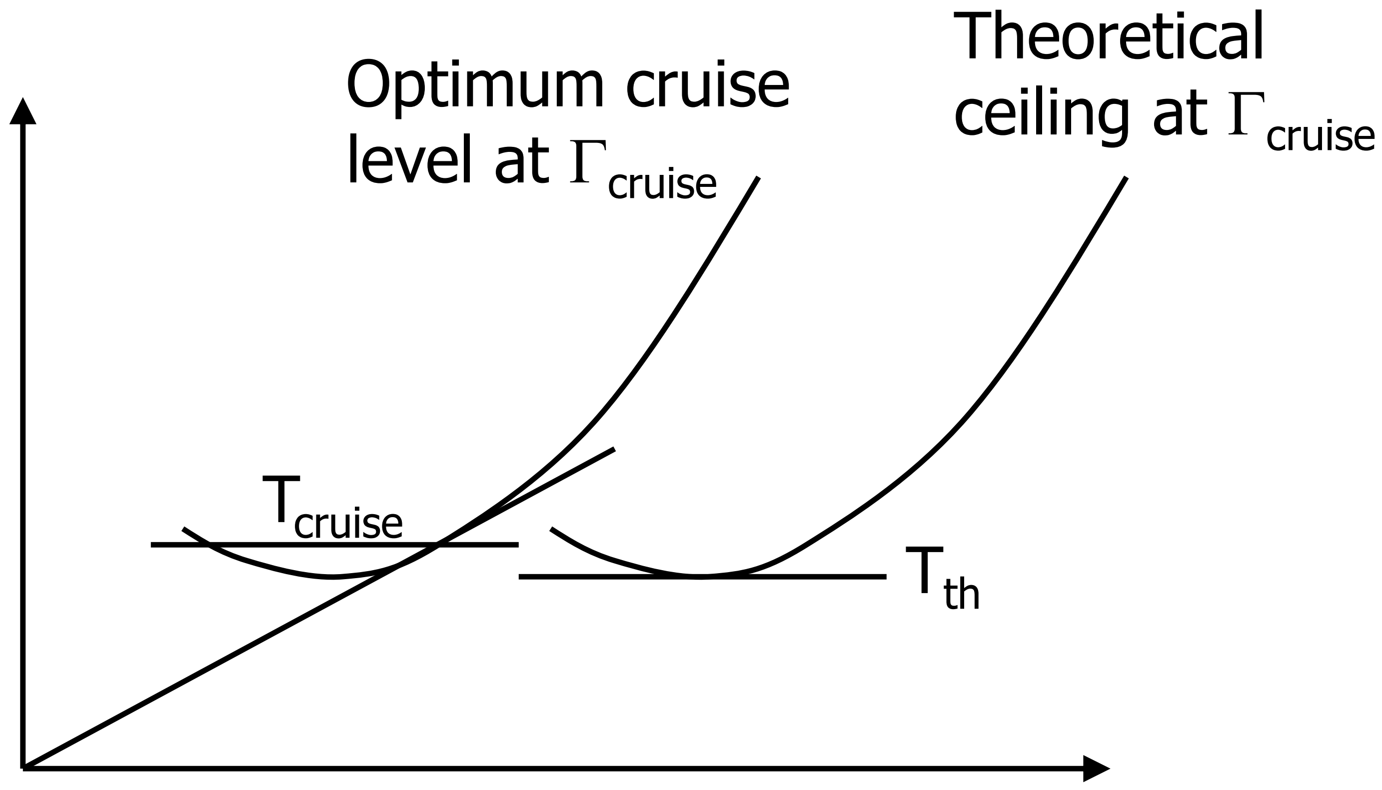
Effect of altitude

Conclusion

At increasing altitude:
- V/F increases
- V increases
- Engine more efficient

Thus: **fly as high as possible !** (up to the limits of the engine)

Effect of altitude

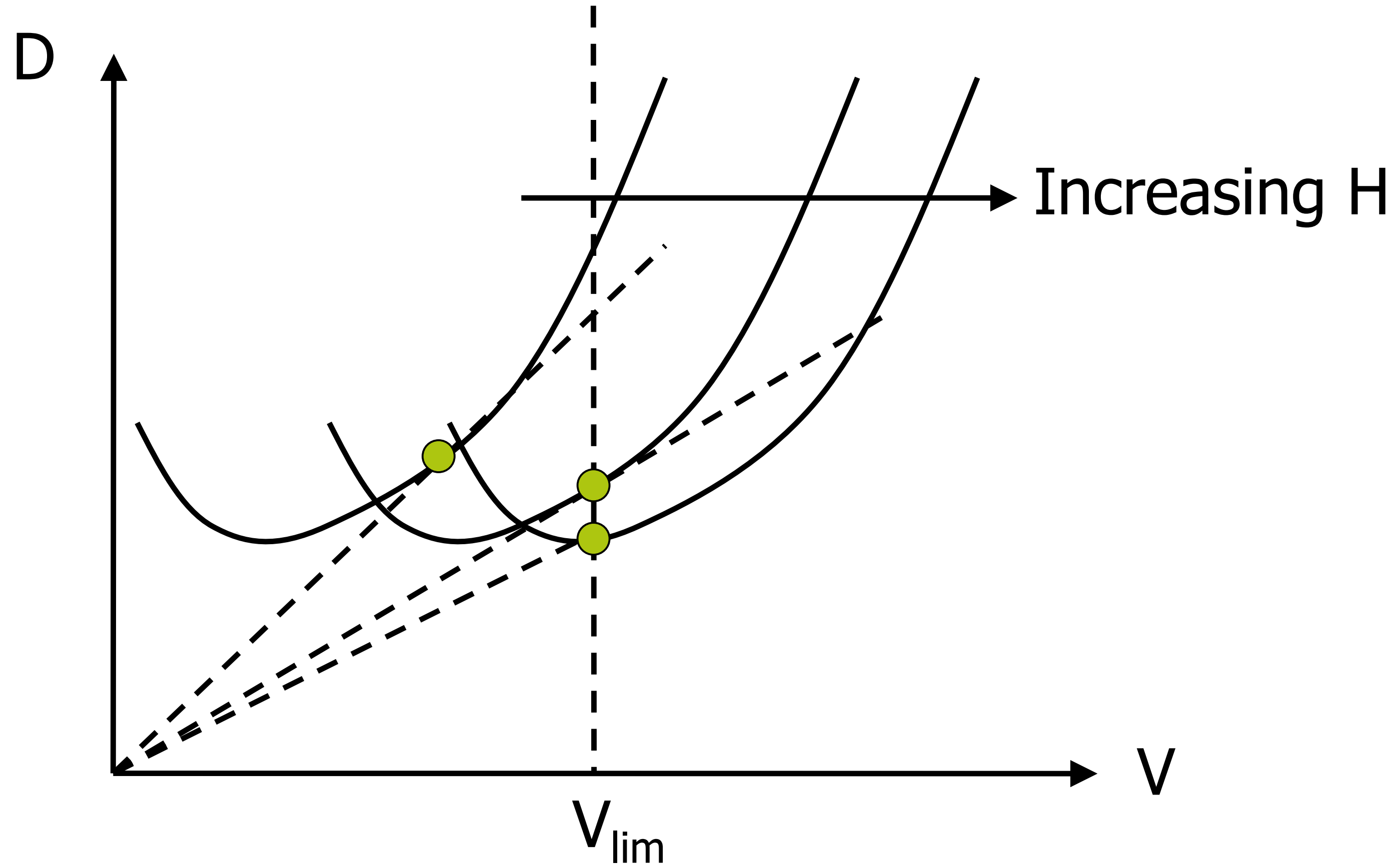


$H_{cr} < H_{th}$ (at Γ_{cruise})

Optimum $H_{cr} \approx H_s$ (service ceiling)

Effect of altitude

In the presence of speed limits (e.g. M_{MO})



Effect of altitude

In the presence of speed limits (e.g. M_{MO})

- Optimum V/F at V = V_{lim}

$$D_{min} \Rightarrow \left(\frac{C_L}{C_D}\right)_{max} \Rightarrow C_L = \sqrt{C_{D_0} \pi A e}$$

First year:

$$\left(\frac{C_L}{C_D}\right)_{max} \Rightarrow \frac{d}{dC_L}\left(\frac{C_L}{C_D}\right) = 0$$

$$\frac{C_L \cdot \frac{dC_D}{dC_L} - C_D \cdot 1}{C_D^2} = 0$$

$$\frac{dC_D}{dC_L} = \frac{2C_L}{\pi A e}$$

$$C_D^2 \neq 0$$

$$\frac{2C_L}{\pi A e} = \frac{C_D}{C_L} = \frac{C_{D_0} + \frac{C_L^2}{\pi A e}}{C_L}$$

$$C_L = \sqrt{C_{D_0} \pi A e}$$

Summary – Jet aircraft

- Choose V such that $(V/F)_{max} \rightarrow (C_L / C_D^2)_{max}$
- H as high as possible (limited by the engine)
- If the speed limit is reached at lower altitude:
  - $V = V_{lim}$
  - H is such that C_L / C_D is max

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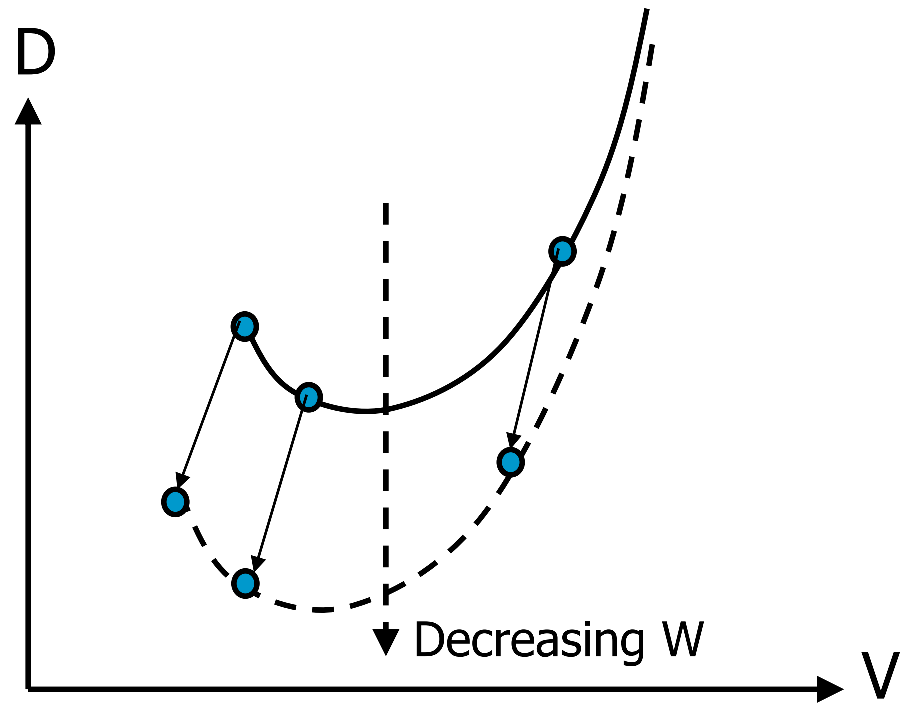
Effect of weight

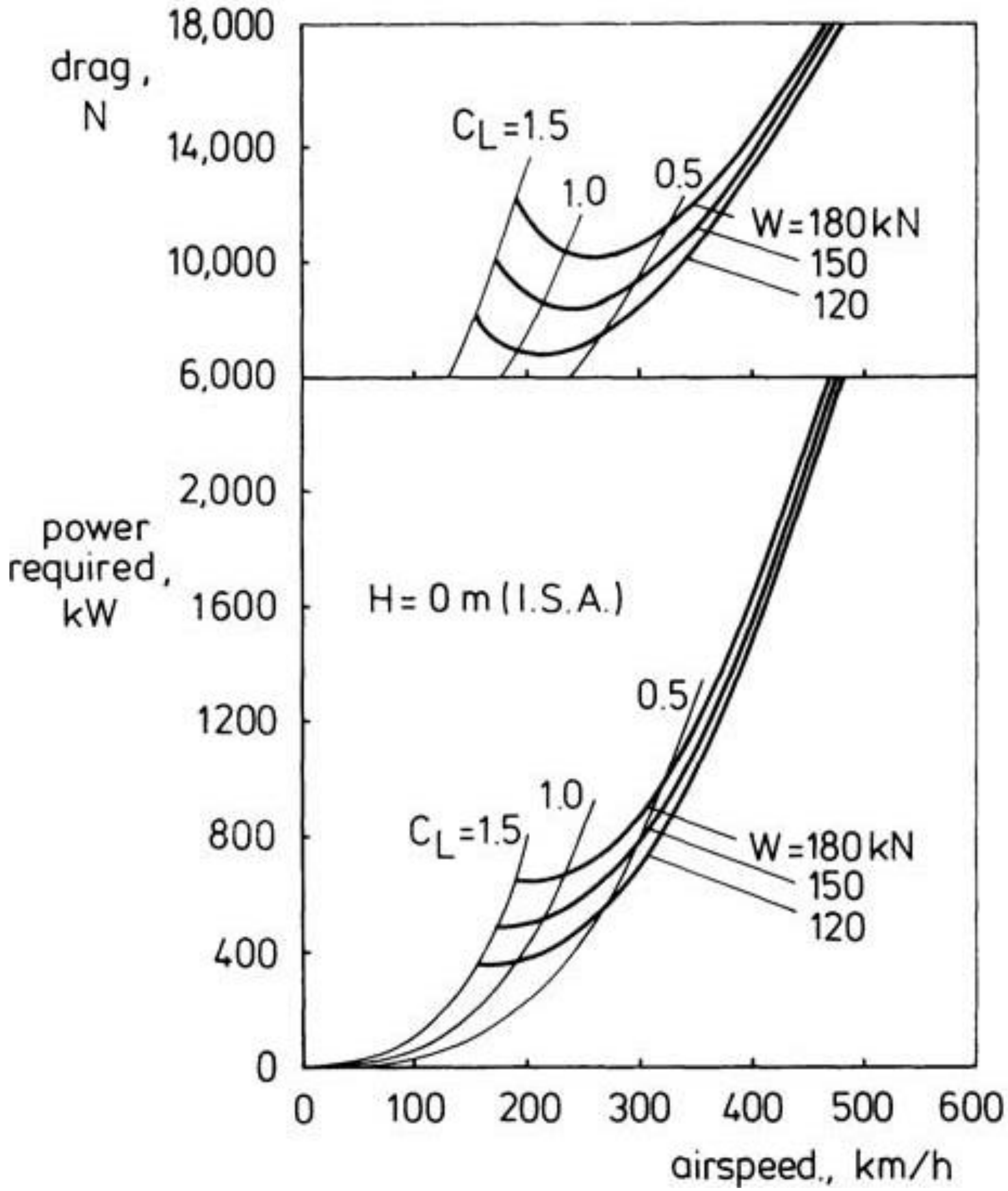
$$\begin{cases} V = \sqrt{\dfrac{W}{S}\dfrac{2}{\rho}\dfrac{1}{C_L}} \propto \sqrt{W} \\ D = \dfrac{C_D}{C_L} W \propto W \\ P_r = DV \propto W\sqrt{W} \end{cases}$$

$\Rightarrow$

$D \propto V^2$ at constant α

$P_r \propto V^3$ at constant α





drag, N
18,000
14,000
10,000
6,000
C_L=1.5
1.0
0.5
W=180 kN
150
120
power required, kW
2,000
1600
1200
800
400
0
H= 0 m (I.S.A.)
0.5
1.0
C_L=1.5
W=180 kN
150
120
0
100
200
300
400
500
600
airspeed., km/h

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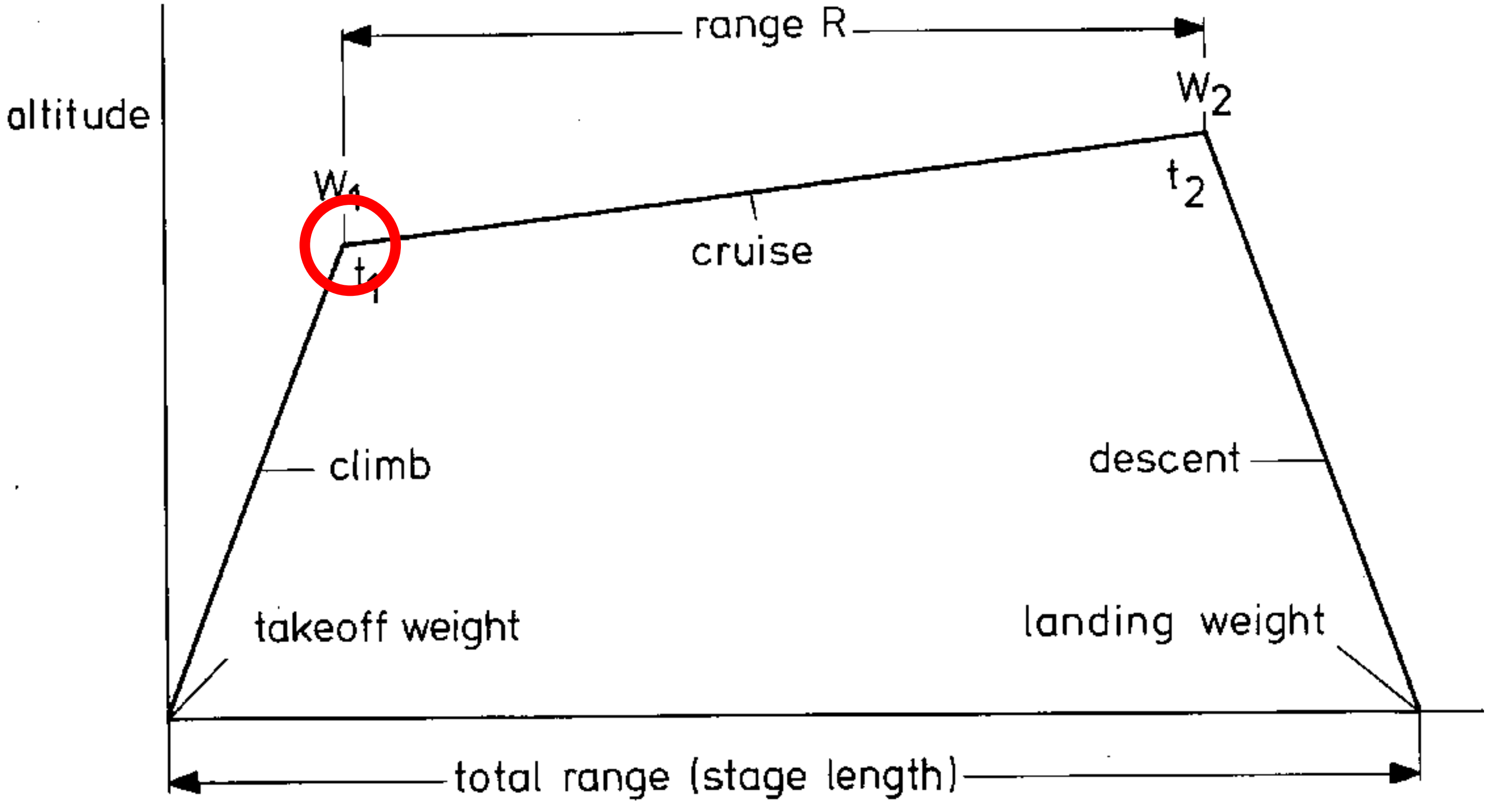
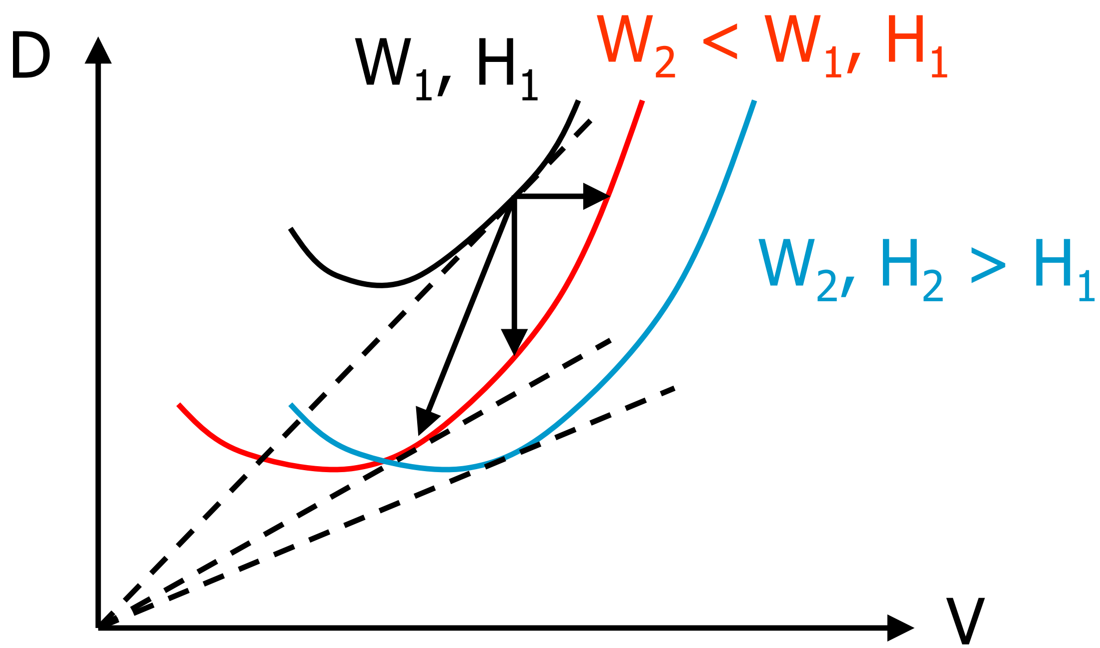

range R
altitude
W_2
W_1
t_2
t_1
cruise
climb
descent
takeoff weight
landing weight
total range (stage length)

Best flying strategy

Strategies:
I: constant altitude and engine setting
II: constant altitude and airspeed
III: constant angle of attack



Best flying strategy

<u>Constant altitude</u>

I. Γ = constant: $(V/F) << (V/F)_{opt}$ but V ↑

II. V = constant $(V/F) < (V/F)_{opt}$

III. α = constant, V↓ but $(V/F) = (V/F)_{opt}$

<u>Climb</u>

IV. α = constant, V = constant, $(V/F) = (V/F)_{opt}$, even $> (V/F)_0$

V. α = constant, changing V?

Best flying strategy

Optimum cruise climb possible?

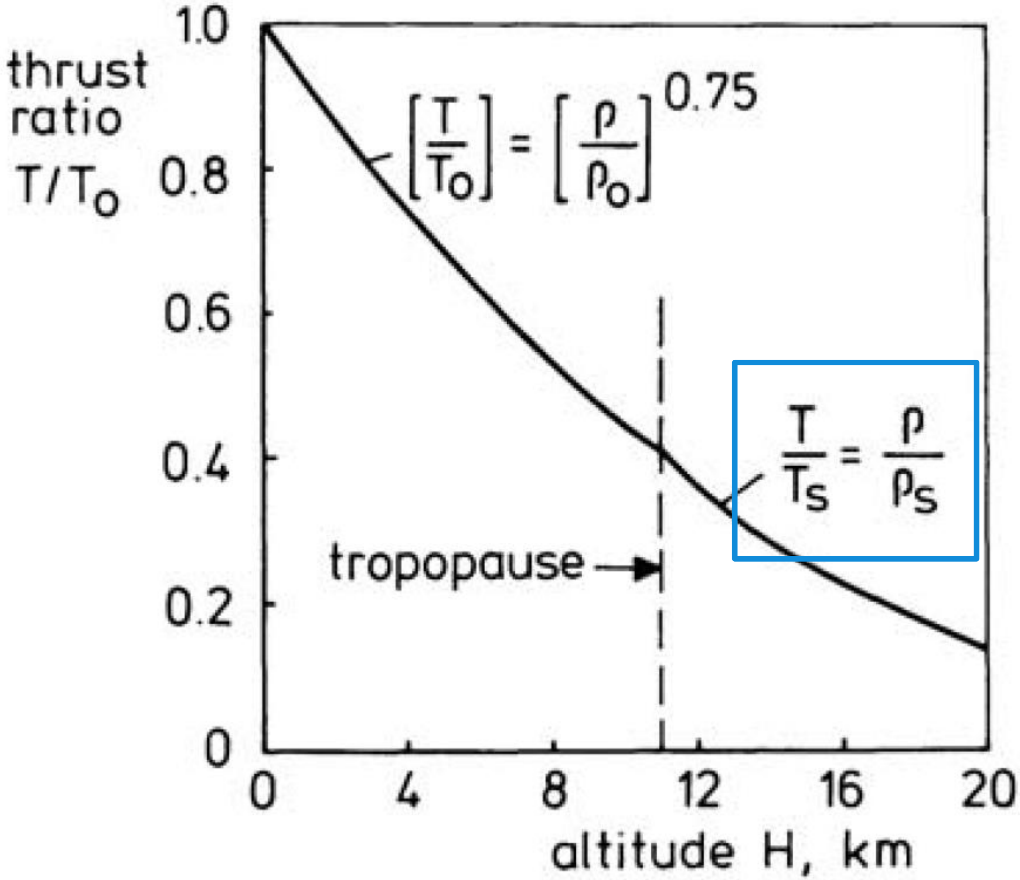
Strategy IV $\rightarrow \alpha_2 = \alpha_1$ (C_L is constant) and $V_2 = V_1$

$$V_1 = \sqrt{\frac{W_1}{S}\frac{2}{\rho_1}\frac{1}{C_L}}$$

Is this possible? Are the engines capable of providing enough thrust at higher altitude and lower weight?

Best flying strategy

Typical turbojet performance



Best flying strategy

Optimum cruise climb possible?

Strategy IV $\rightarrow \alpha_2 = \alpha_1$ (C_L is constant) and $V_2 = V_1$

$$\frac{W}{\rho} = \text{constant}$$

- This is exactly how a typical turbojet behaves above 11km. So there will be enough thrust.
- Strategy V is not feasible
- Below 11km there will be enough thrust as well

Best flying strategy

Optimum cruise climb possible?

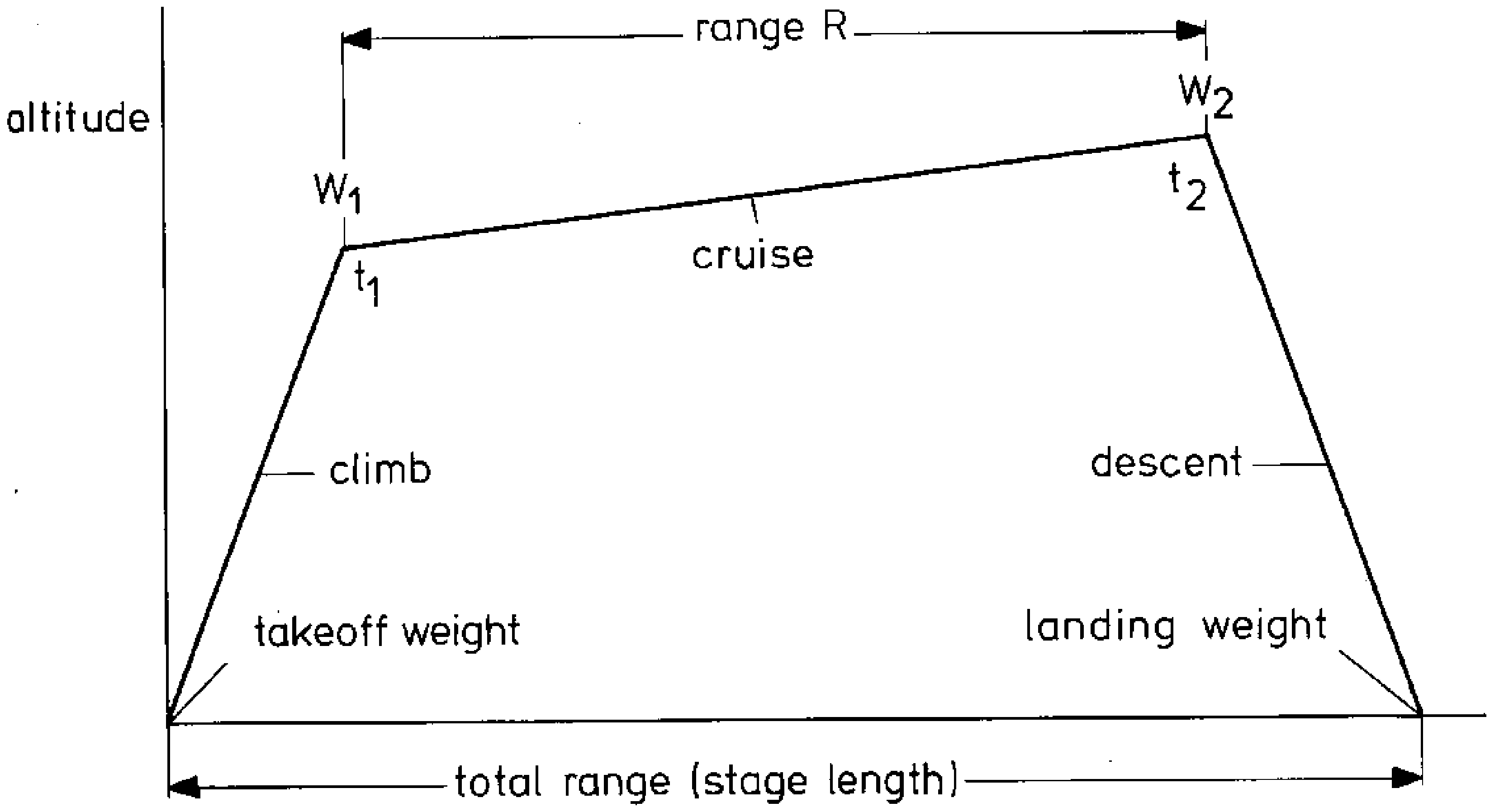
Strategy IV → $\alpha_2 = \alpha_1$ (C_L is constant) and $V_2 = V_1$

The engines can provide just enough thrust
(strategy V not possible)

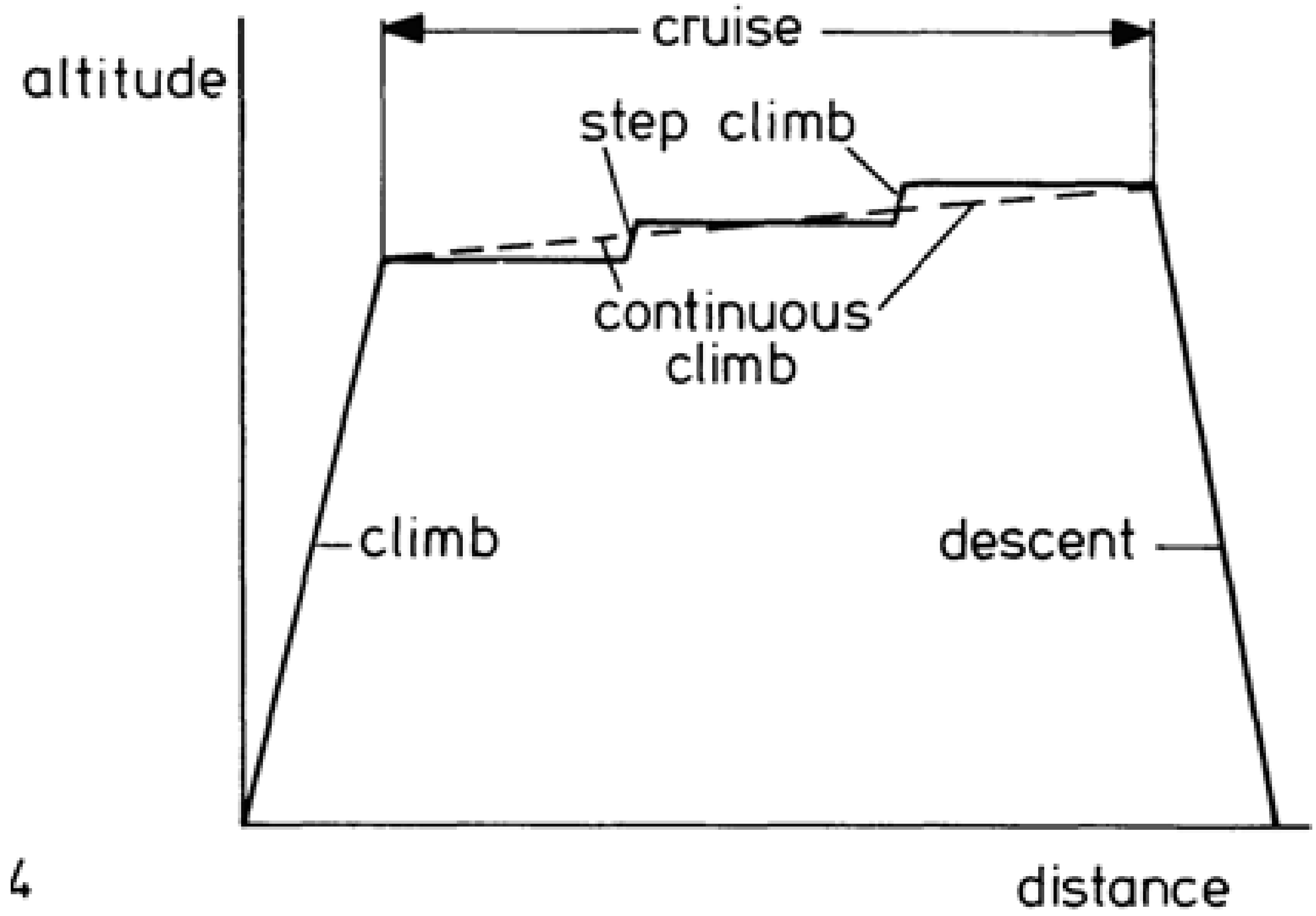
What happens in case of M_{lim}?

Mach number is constant at constant airspeed above 11km
→ No problem

Best flying strategy



Best flying strategy



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Analytic range equations

Breguet range equation

- Range

$$\frac{dW}{dt} = \frac{dW}{ds} V = -F$$

$$R = \int_{s_0}^{s_1} ds = \int_{W_1}^{W_0} \frac{V}{F} dW$$

Analytic range equations

Breguet range equation for jet aircraft

- Jet aircraft optimum climb cruise (α, V and c_T are constant during variation of W

$$R = \int_{W_1}^{W_0} \frac{V}{F} dW$$

$$R = \int_{W_1}^{W_0} \frac{V}{c_T D} dW$$

$$R = \int_{W_1}^{W_0} \frac{V}{c_T} \frac{C_L}{C_D} \frac{dW}{W}$$

$$R = \frac{V}{c_T} \frac{C_L}{C_D} \int_{W_1}^{W_0} \frac{dW}{W}$$

$$\boxed{R = \frac{V}{c_T} \frac{C_L}{C_D} \ln\left(\frac{W_0}{W_1}\right)}$$

Analytic range equations

Breguet range equation for jet aircraft

$$R = \frac{V}{c_T} \frac{C_L}{C_D} \ln\left(\frac{W_0}{W_1}\right)$$

- If V is not limited: R_{max} at $(V\,C_L / C_D)_{max} \rightarrow (C_L / C_D^2)_{max}$ and ρ_{min}

- If V is limited:
  R_{max} at $V = V_{lim}$ and ρ such that $(C_L / C_D)_{max}$

Analytic range equations

Breguet range equation for propeller aircraft

$$F = c_p P_{br}$$

$$\frac{V}{F} = \frac{\eta_j}{c_p}\frac{1}{T}$$

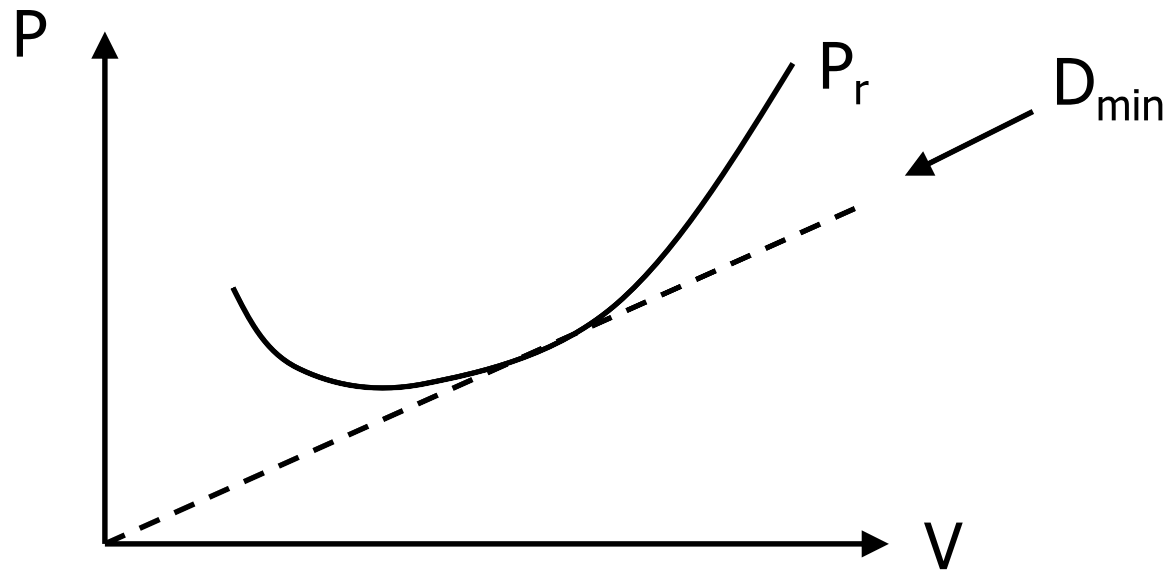
Cruise flight with constant α, c_p and η_j:

$$R = \int_{W_1}^{W_0} \frac{V}{F} dW$$

$$R = \frac{\eta_j}{c_p}\frac{C_L}{C_D}\ln\left(\frac{W_0}{W_1}\right)$$

Analytic range equations

Breguet range equation for propeller aircraft



Conclusion: Altitude is not important w.r.t V/F
But V is larger at high altitude

Analytic range equations

Unified Breguet range equation

- Jet aircraft

$$\eta_{tot} = \frac{TV}{H\dfrac{F}{g}}$$

- Propeller aircraft

$$\eta_{tot} = \frac{TV}{H\dfrac{F}{g}}$$

Analytic equations

Unified Breguet range equation

Time	1920	Lindbergh	Present
H	43000 kJ/kg	43000 kJ/kg	43000 kJ/kg
η_{tot}	0.20	0.20 – 0.30	>0.40
L/D	10	11	16-18
W_1/W_0	0.6 – 0.7	0.5	0.5

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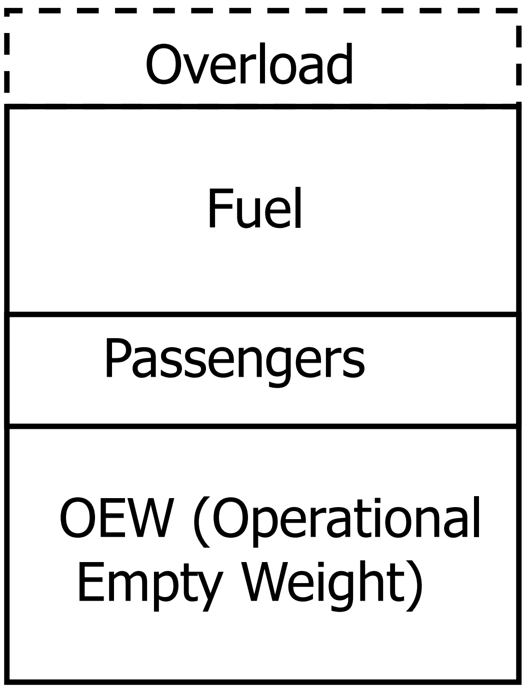
Story

History

- 1919: Alcock / Brown: Newfoundland → Ireland
- Fonck, Nungesser/Coli, Lindbergh: New York → Paris

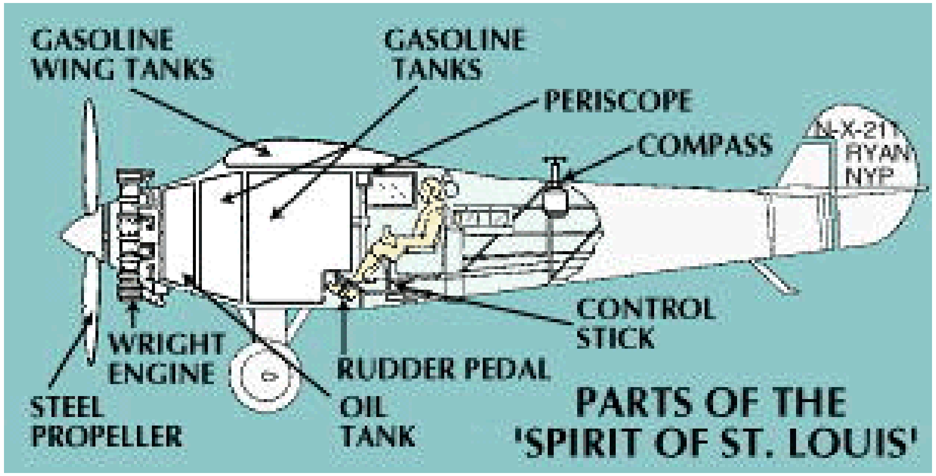


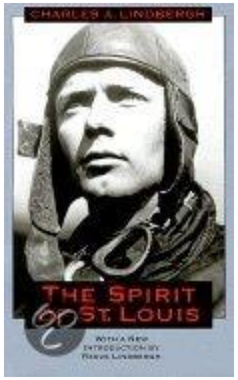
- Structural safety factor
- Take – off length (W^2)
- Climb gradient after take off
- Tailwind west → east

Story

Spirit of St. Louis

- Charles Lindbergh, 1927
- First solo, nonstop flight across the Atlantic Ocean

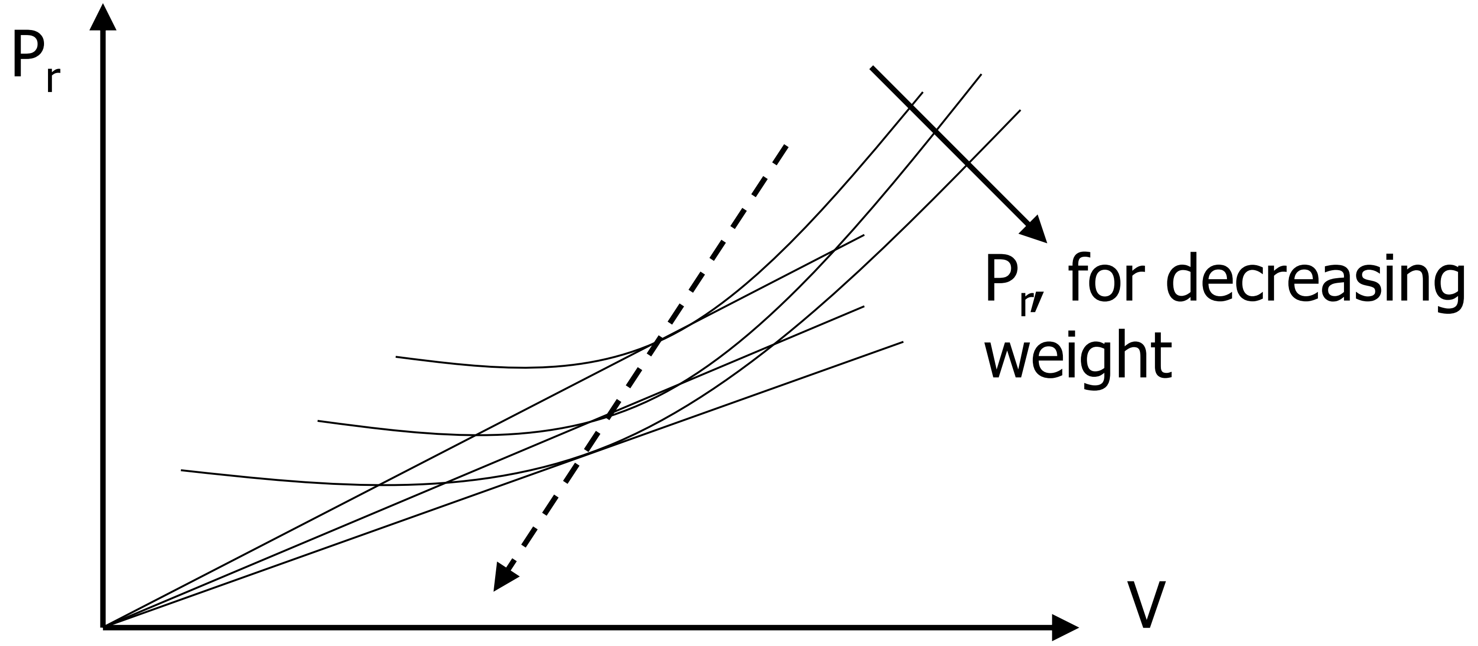




Story

Spirit of St. Louis

- Charles Lindbergh had to decrease airspeed to achieve maximum range



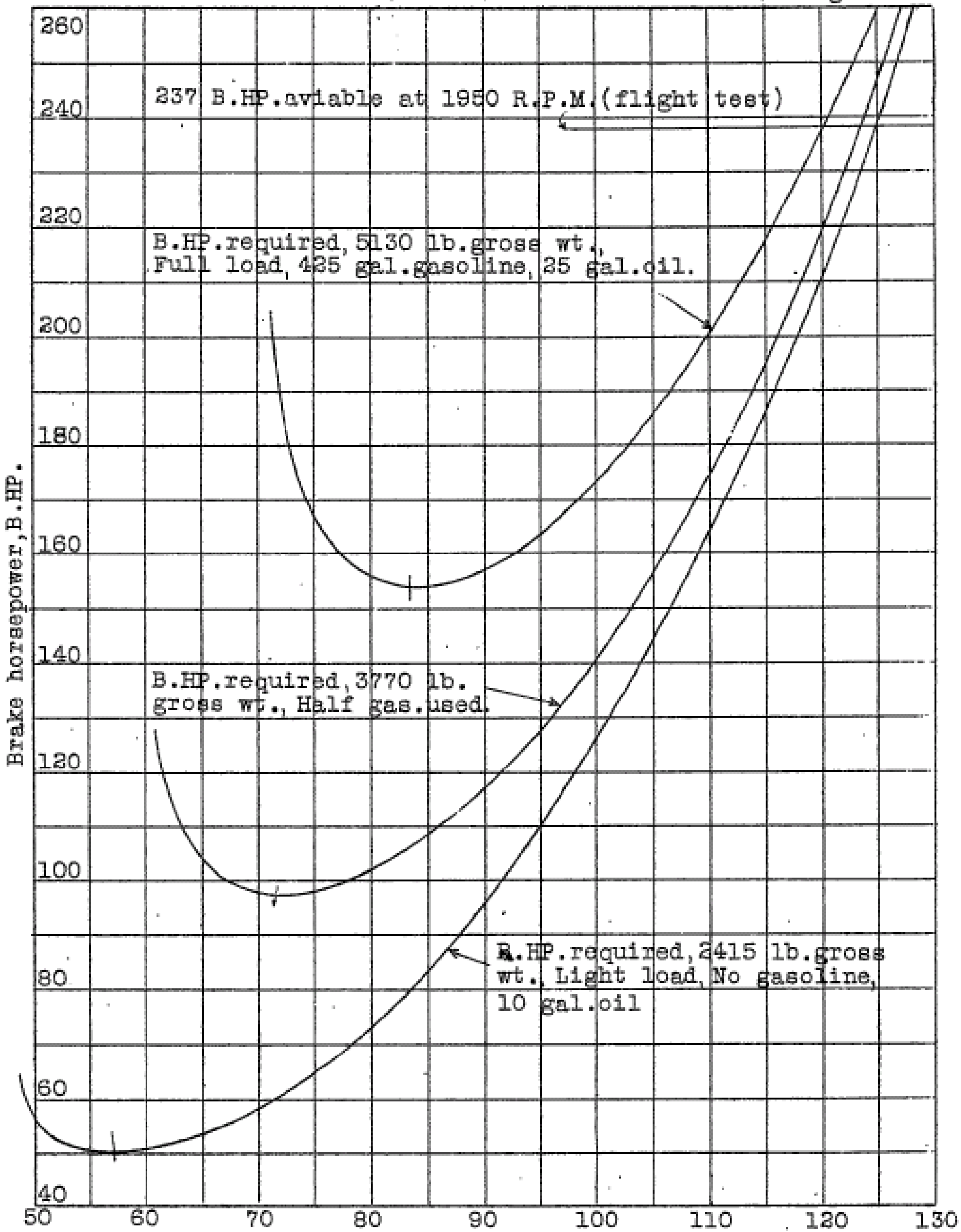


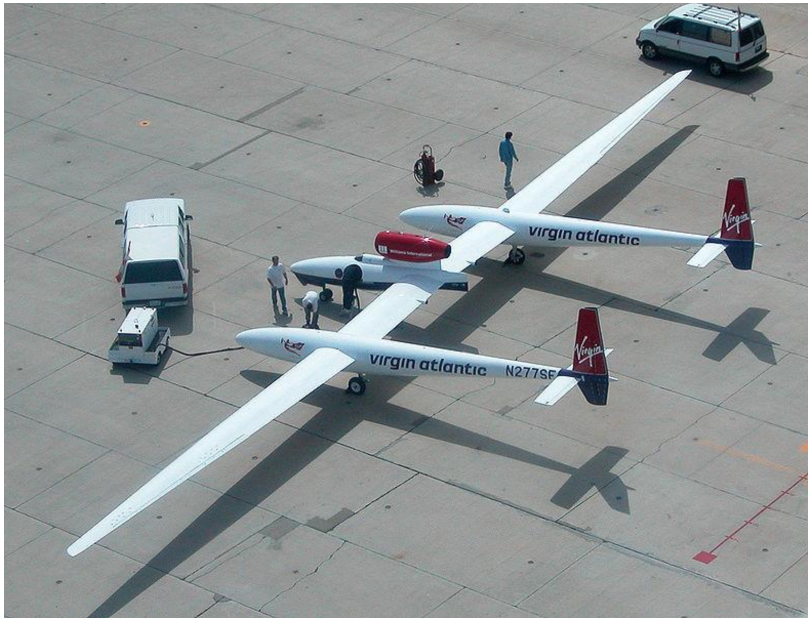
Fig.9 Ryan NYP airplane.

Story

Global flyer

Burt Rutan

Steve Fossett



Story

Global flyer

- First part of flight: insufficient strength to withstand gusts
- Best glide ratio: 1:37
  - $(C_L / C_D)_{max} = 37$
  - $C_{D0} = 0.018$ $e = 0.85$
- H_{cr} = 45000 ft = 13.716 m $\rightarrow$ $\rho = 0.2377$
- Distance flown 38000 km
- Time 66 hrs
- Fuel lost 2600 lbs $\rightarrow$ actual fuel fraction 71%

Story

Global flyer

- $(V/F)_{max}$: $C_L = \sqrt{\tfrac{1}{3} C_{D_0} \pi A e} = 0.72$

$$C_D = 0.024 \Rightarrow \frac{C_L}{C_D} = 30$$

- V for $(V/F)_{max}$, 45000 ft, W_{gross}: V = 175 m/s
- Time for 40.000 km at constant V: 64 hrs
- Guesstimate of η_{tot} :
  - High bypass fans at 1000 km/h: η_{tot} = 40%
  - Medium bypass fans η_{tot} = 35%, η_{th} = 50%, η_j = 70% → V_j / V = 1.86
- Correction for lower flight speed:
- Vj / V = 3 → η_j = 0.5 → η_{tot} = 25%

- Range in ideal climbing cruise: R = 53000 km

Story

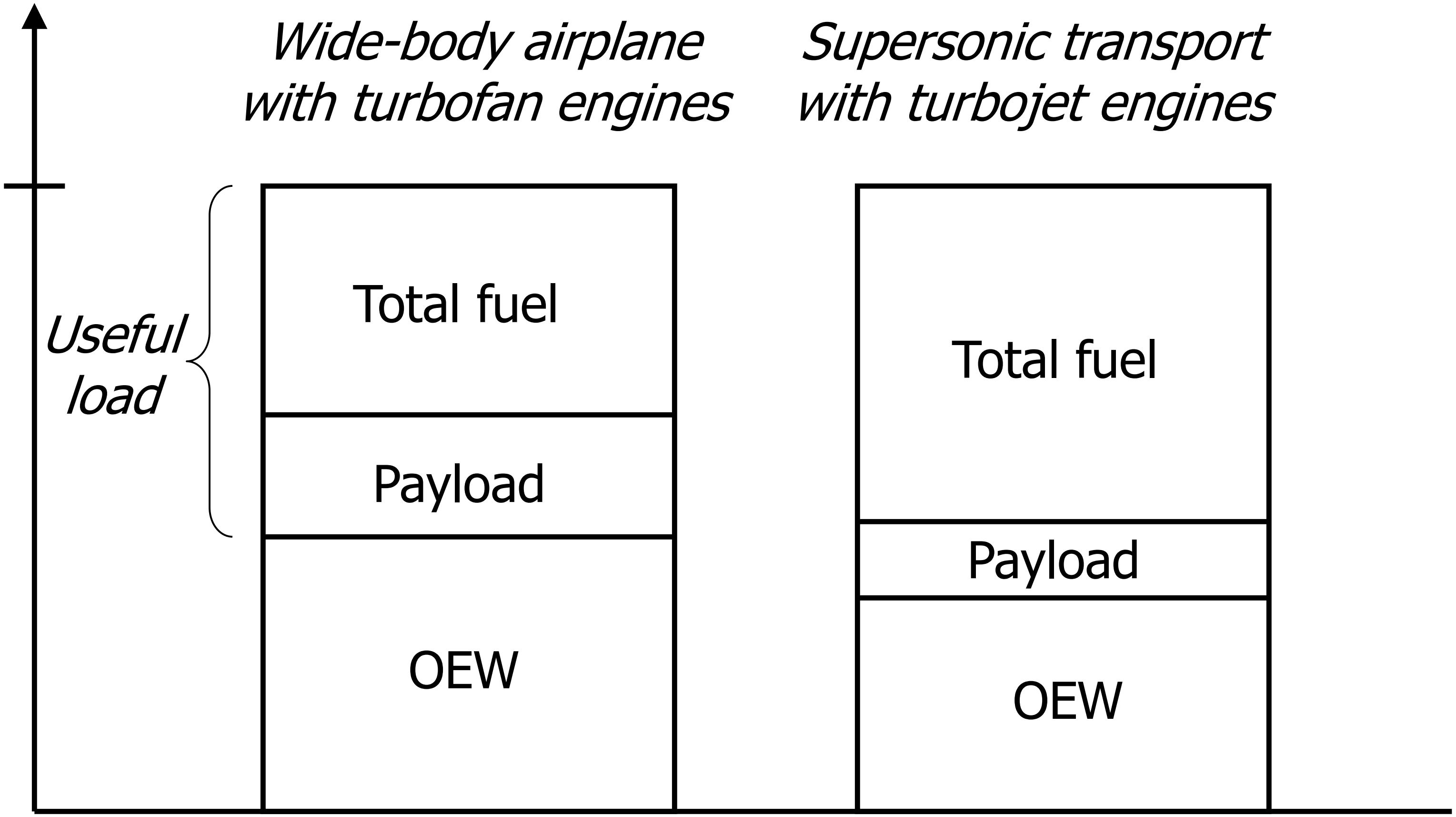
Global flyer

- Cruise at V = constant and H_{cr} = constant: R = 37.500 km
- At fuel fraction 70%: R = 28000 km
- Influence wind ?

Content

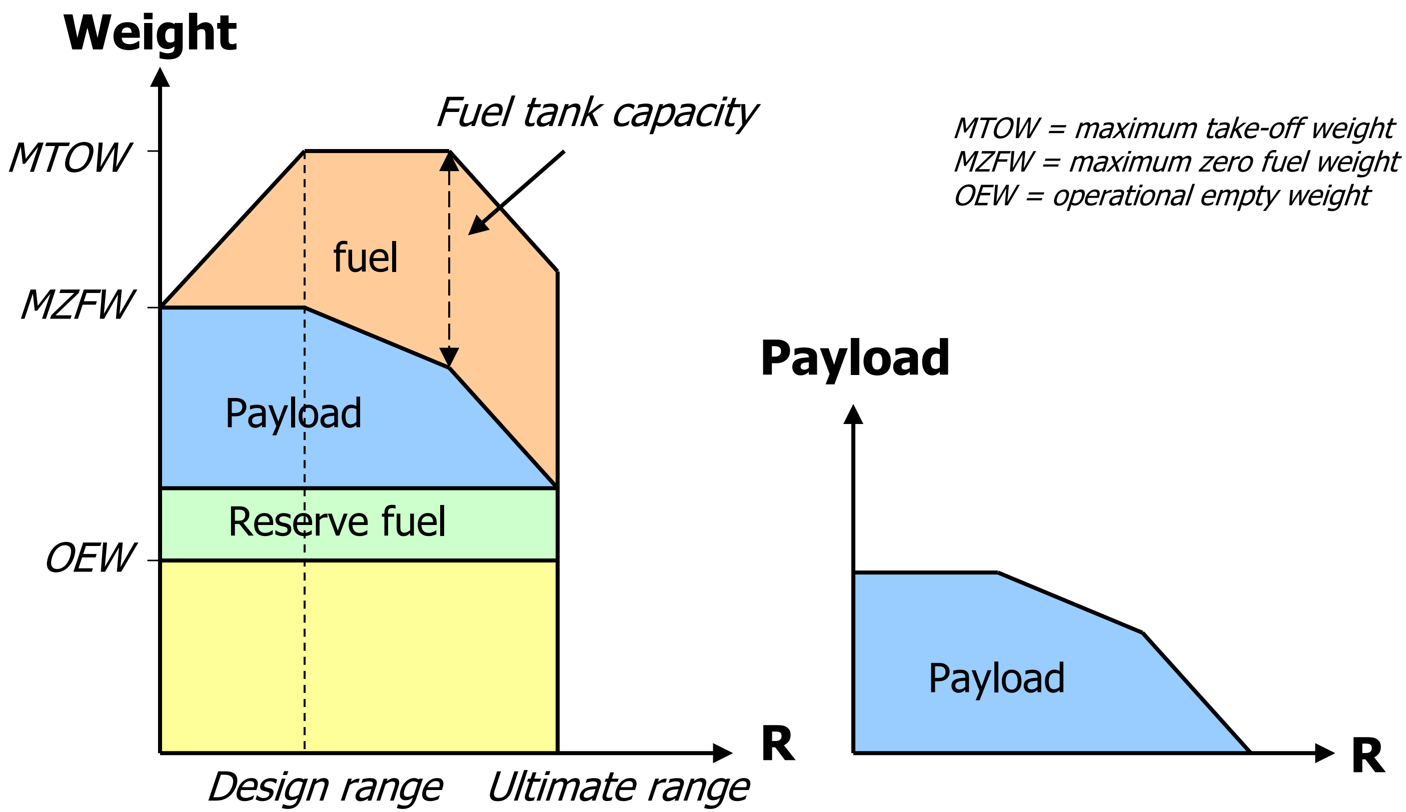
- Introduction
- Optimum cruise profile
  - Optimal airspeed for given H, W
  - Effect of altitude
  - Effect of weight
  - Best flying strategy
- Analytic Range equations
- Story
- **Weight breakdown**
- Economics
- Summary

Weight breakdown



Weight breakdown

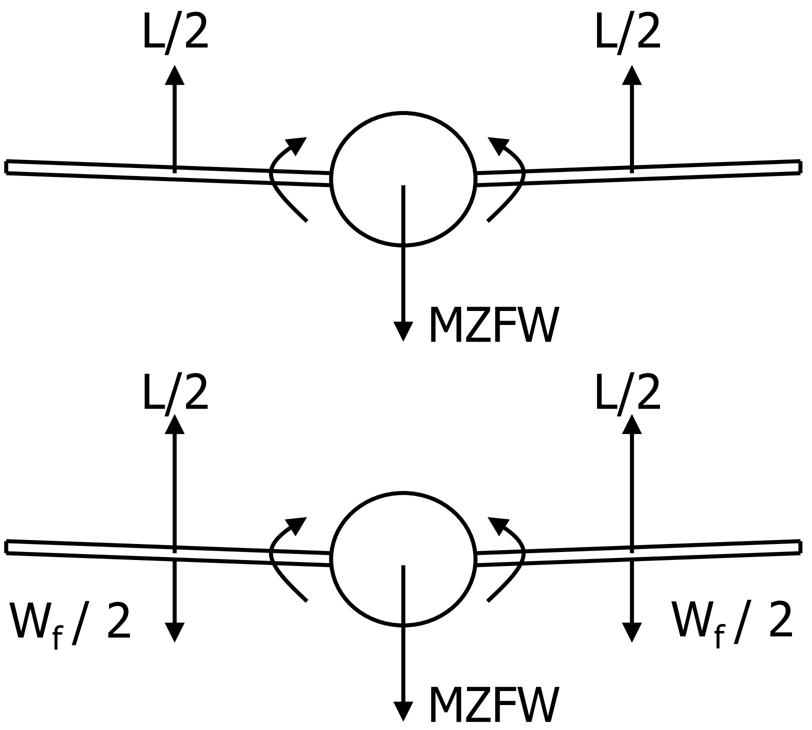
Payload range diagram



MTOW = maximum take-off weight
MZFW = maximum zero fuel weight
OEW = operational empty weight

Weight breakdown

Maximum zero fuel weight



MZFW limited, amongst others by bending moment of the wing

MTOW > MZFW at same bending moment. MTOW limited e.g. by landing gear

Weight breakdown

Reserve fuel

- Reserve fuel
  - In general:
    - Fuel to alternate
    - 45 minutes holding at altitude
- Fuel shortage:
  - In general:
    - Management problem
    - CRM Cockpit resource management

Content

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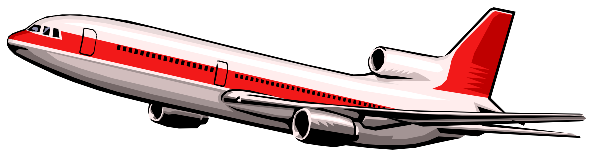
Economics

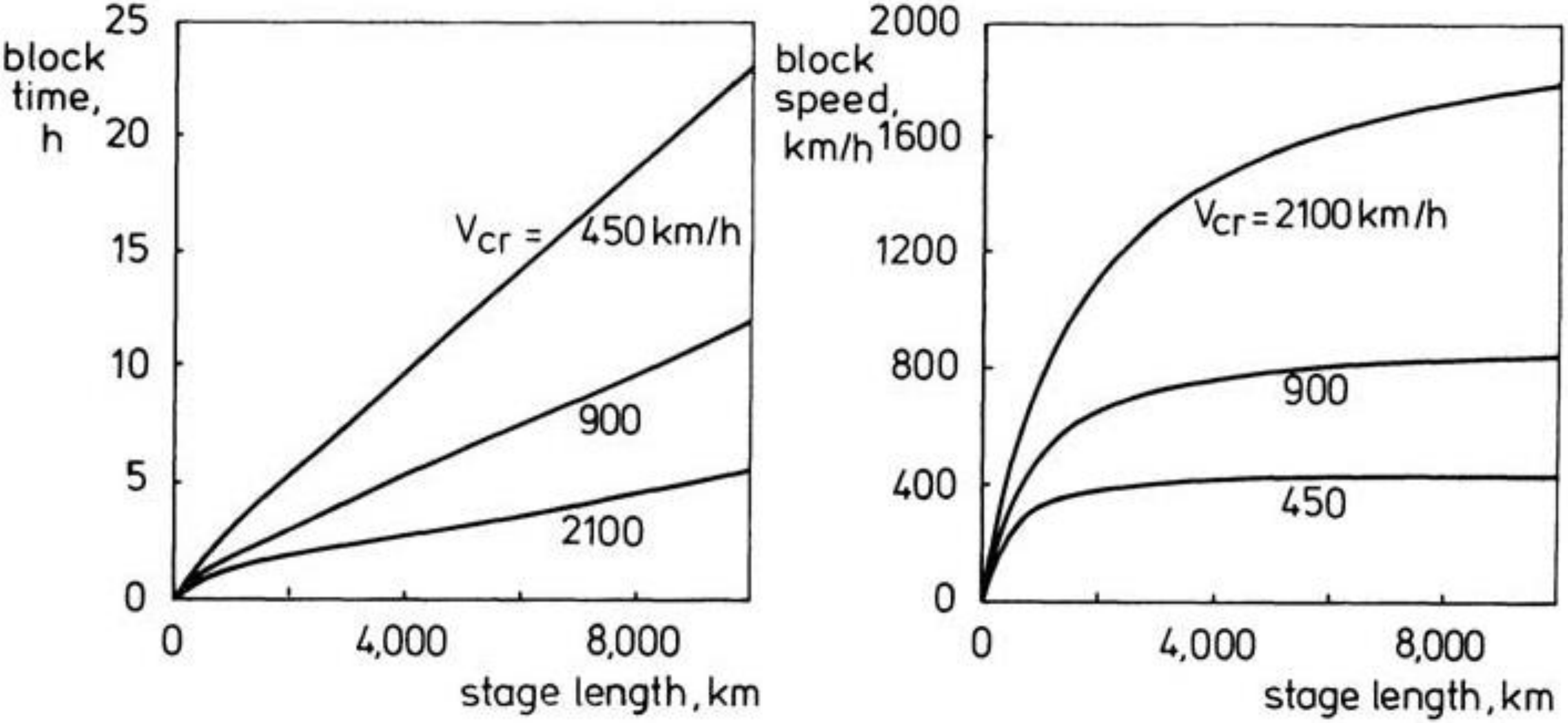
Block time and block speed

<u>Key Parameters</u>

Range R
Payload P
Block time E_B
Block speed V_B
Transport product P_R
Transport productivity P_h
Revenue earning capacity P_y

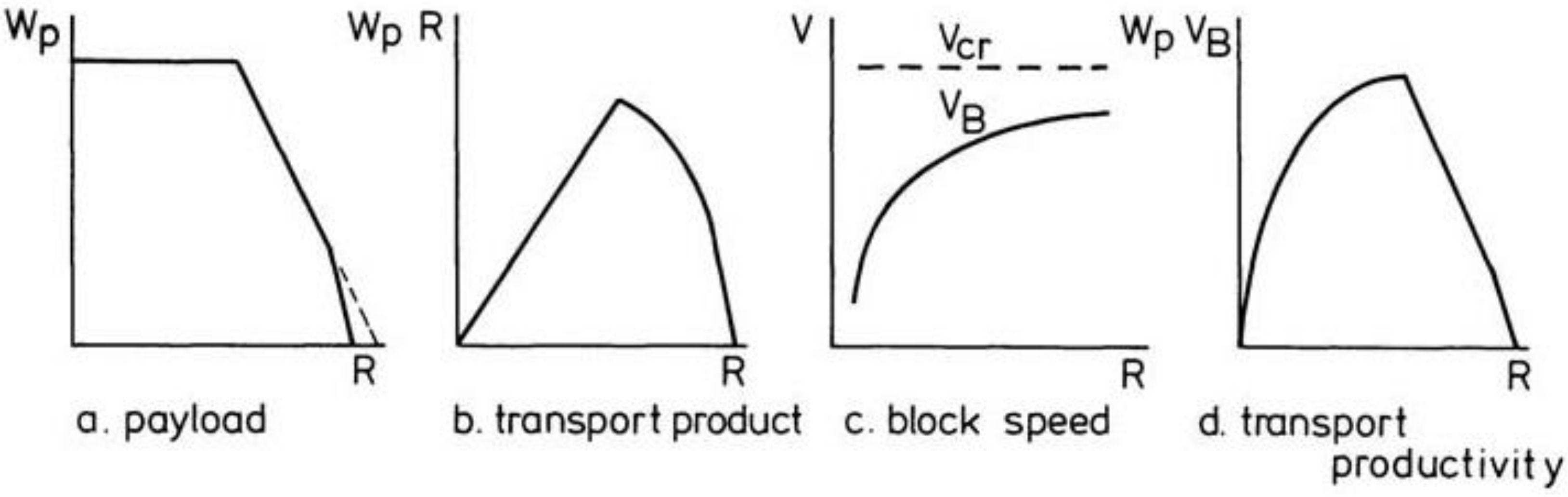


Economics


block time, h
25
20
15
10
5
0
0
4,000
8,000
stage length, km
Vcr = 450 km/h
900
2100
block speed, km/h
2000
1600
1200
800
400
0
0
4,000
8,000
stage length, km
Vcr = 2100 km/h
900
450

Economics

Transport productivity



Conclusion:

Maximum transport productivity is achieved at the design range

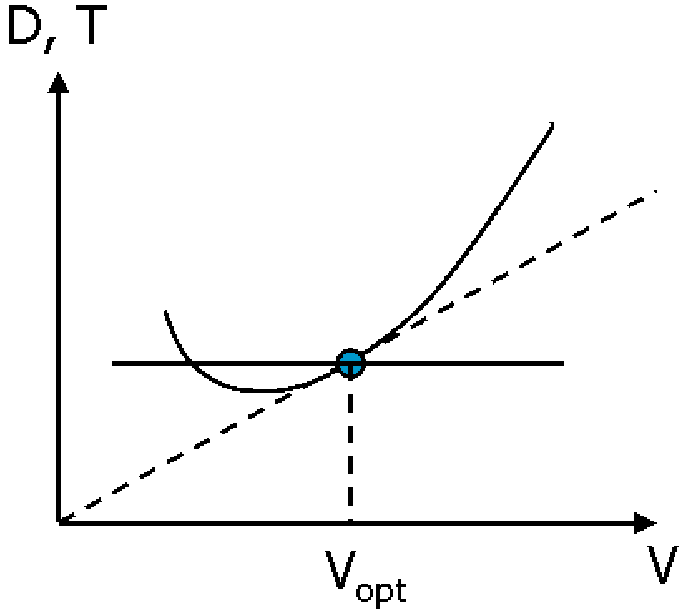
! Cost (direct operating cost) must be considered as well of course

Content

Summary

*The **objective** is to minimize fuel for a given range*

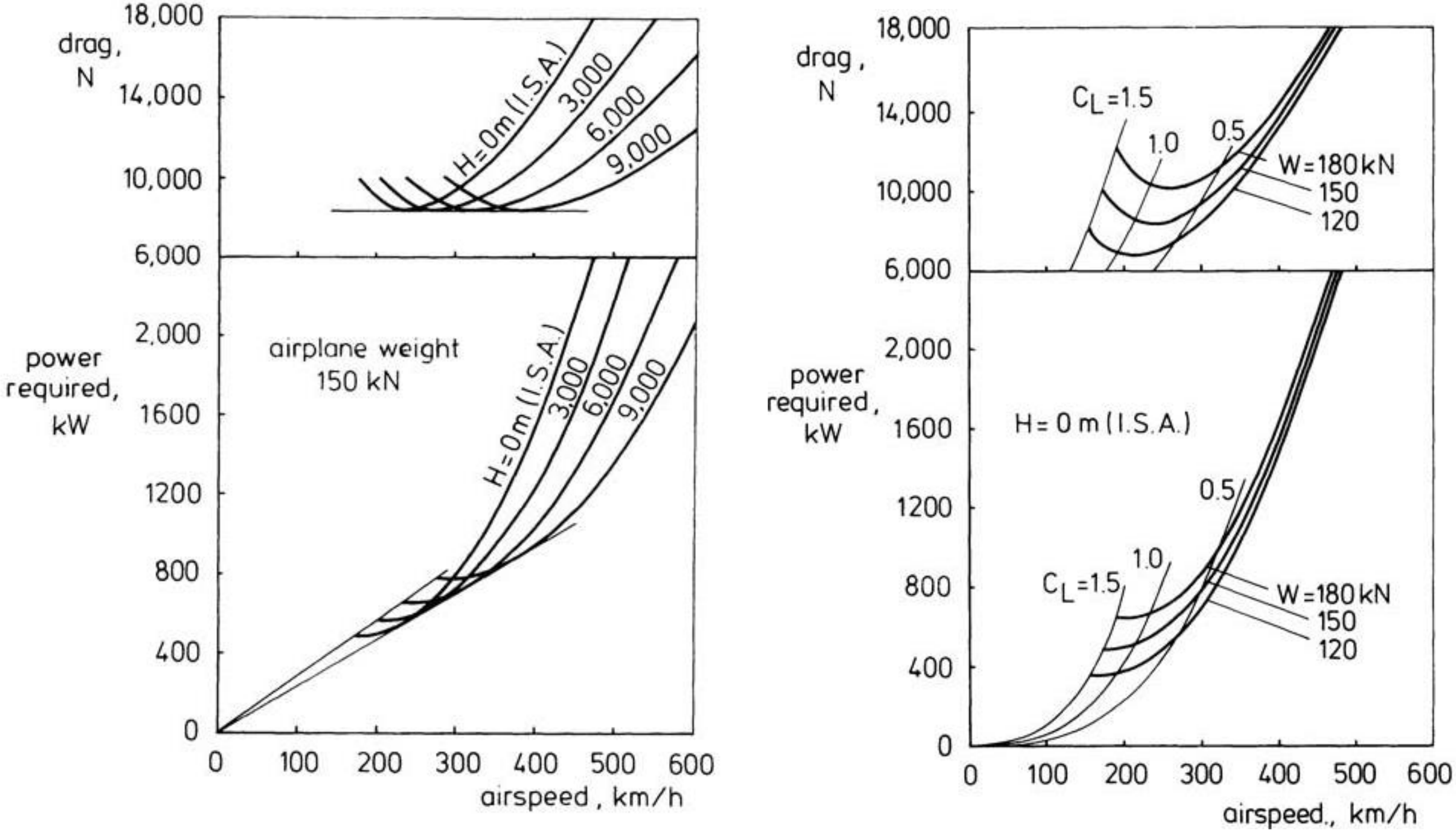
<u>Key parameter:</u>
Specific range V/F
[V]/[F] = [m/s]/[kg/s] = [m/kg]
So it is the distance travelled per unit of fuel



Performance diagram for jet aircraft

Summary

Effect of weight and altitude



Summary

Key conclusions

Jet aircraft (analytical approximation)

1. Choose V such that $(V/F)_{max} \rightarrow (C_L / C_D^2)max$
2. H as high as possible (limited by the engine)
3. If the speed limit is reached at lower altitude: $V = V_{lim}$, H is such that C_L / C_D is max

Propeller aircraft (analytical approximation)

1. Conclusion: Altitude is not important w.r.t V/F
2. But V is larger at high altitude

Summary

Breguet range equation

$$R = \int_{W_0}^{W_1} ds = \int_{W_1}^{W_0} \frac{V}{F} dW$$

Jet aircraft
(analytical approximation)

$$R = \int_{W_1}^{W_0} \frac{V}{c_T} \frac{C_L}{C_D} \frac{dW}{W}$$

Optimum cruise climb

$$R = \frac{V}{c_T} \frac{C_L}{C_D} \ln\left(\frac{W_0}{W_1}\right)$$

Propeller aircraft
(analytical approximation)

$$R = \int_{W_1}^{W_0} \frac{\eta_j}{c_p} \frac{C_L}{C_D} \frac{dW}{W}$$

Cruise flight with constant α, c_p and η_j:

$$R = \frac{\eta_j}{c_p} \frac{C_L}{C_D} \ln\left(\frac{W_0}{W_1}\right)$$

Summary

Unified Breguet range equation

- Jet aircraft

$$\eta_{tot} = \frac{TV}{H\frac{F}{g}} = \frac{V}{c_T}\frac{g}{H}$$

- Propeller aircraft

$$\eta_{tot} = \frac{TV}{H\frac{F}{g}} = \frac{\eta_j}{c_p}\frac{g}{H}$$

Fuel quality — *Propulsion efficiency* — *aerodynamic quality*

- Both:

$$R = \frac{H}{g}\eta_{tot}\frac{C_L}{C_D}\ln\left(\frac{W_0}{W_1}\right)$$

← *Structural characteristics*

Questions?