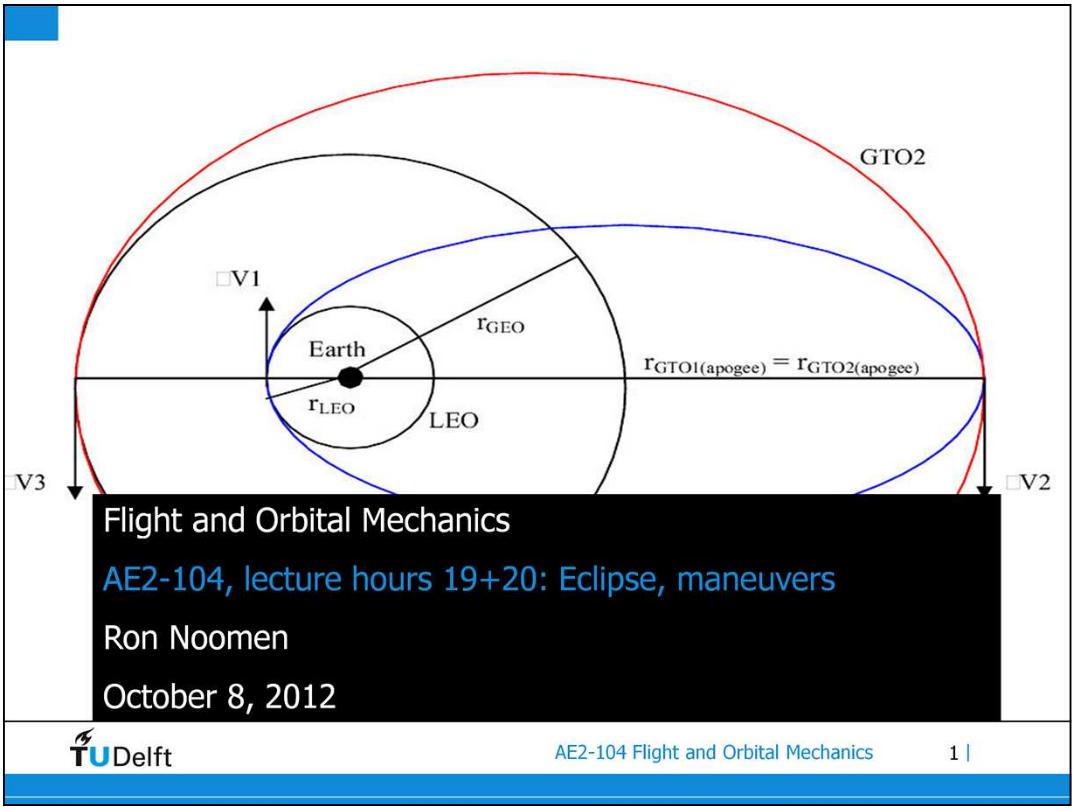
Flight and Orbital Mechanics

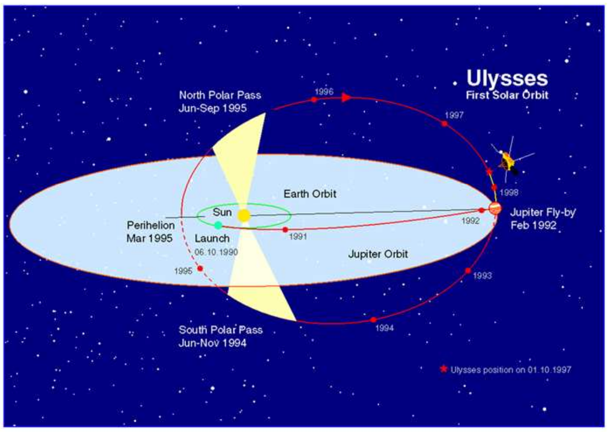
Lecture slides



Material for exam: this presentation (*i.e.*, no material from text book).

Illustration shows a 3-burn transfer. This can be the most efficient transfer from circular Low Earth Orbit (LEO) to Geostationary (GEO) orbit.

Example: Ulysses



[ESA, 2010]

Questions:

- what is the purpose of this mission?
- what is the operational orbit of the vehicle?
- what maneuver has been performed and why?
- ....

Introduction picture.

Overview

- Eclipse
  - 2D
  - 3D
- Maneuvers
  - In-plane
  - Out-of-plane
  - Combined
  - Disposal

Learning goals

The student should be able to:

- Describe the geometry Earth-Sun-satellite, and the relevant considerations for an eclipse to occur.
- Derive conditions (both qualitatively and quantitatively) to assess the occurrence of an eclipse.
- Make a first-order estimate of the duration of an eclipse period.
- Describe the possibilities for a satellite to be in permanent sunlight.
- Describe the various options for orbital transfers (both in-plane and out-of-plane).
- Compute the required ΔV for a Hohmann transfer.
- Quantify the benefits when applying different transfer scenarios.
- Describe options to reduce the required amount of ΔV for a specific transfer
- Describe and quantify maneuvers needed for end-of-life disposal (both LEO and GEO).

Lecture material:

- these slides (incl. footnotes)

Eclipse

What?

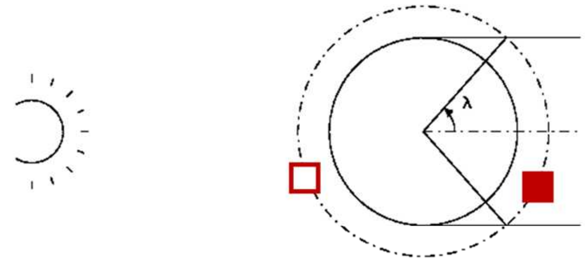
Obstruction of Sunlight (typically by Earth)

Consequences for

- power
- thermal control
- attitude control

For LEO satellites, an eclipse may take about one-third of an orbital revolution. During this period there is no inflow of energy for the power subsystem, nor for the thermal control of the vehicle. Also, the Sun is not available as a point of reference for the attitude control system (if only for the fail-safe mode). The design of the satellite has to be such that this situation can be accommodated for.

Eclipse (cnt'd)

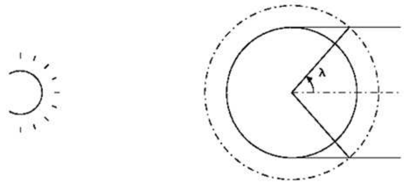


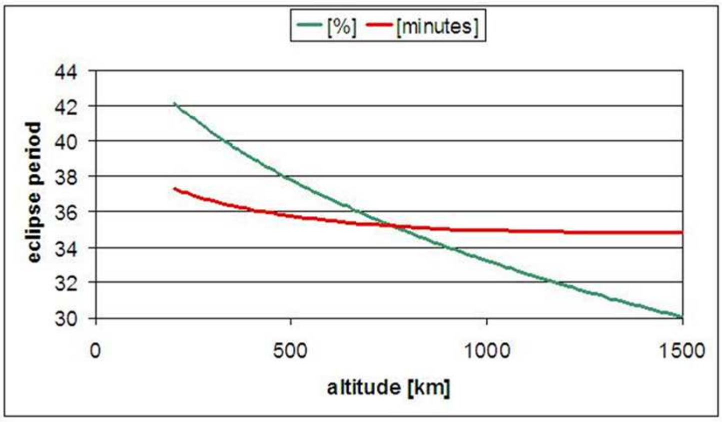
1st-order assessment:

- 2-dimensional (*i.e.*, Sun in orbital plane)
- Sun at infinite distance
- satellite in circular orbit
- length of eclipse: $\sin(\lambda) = R_e/a$ ➔ $T_{eclipse} = (2\lambda/360) \times T_{orbit}$

The far majority of satellites orbit Earth in (near-)circular orbits. Parameters "Re" and "a" represent the Earth radius and the semi-major axis of the satellite orbit (for circular orbits: the orbit radius r), respectively. $T_{orbit} = 2\pi\sqrt{(a^3/\mu)}$.

Eclipse (cnt'd)





For LEO satellites, eclipse length expressed in minutes is more-or-less constant (V_{circ} at 500 km altitude is 7.61 km/s, V_{circ} at 1500 km is 7.11 km/s). Consequences for the design of the (secondary) batteries.

Eclipse (cnt'd)

Question 1:

a) Compute the orbital period for an Earth-observing satellite at 800 km altitude

b) Compute the maximum eclipse length (in minutes) for this satellite.

c) Compute the semi-major axis of a geostationary satellite.

d) Compute the maximum eclipse length (in minutes) for this satellite.

In all cases, a 2-dimensional situation is to be considered.

Data: R_e = 6378.137 km, μ_{earth} = 398600.4415 km^3/s^2, T_{GEO} = $23^h56^m4^s$.

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

Answers **(TRY YOURSELF FIRST!!):**

a) T_{orbit} = 6052.4 s, or 100.87 min.

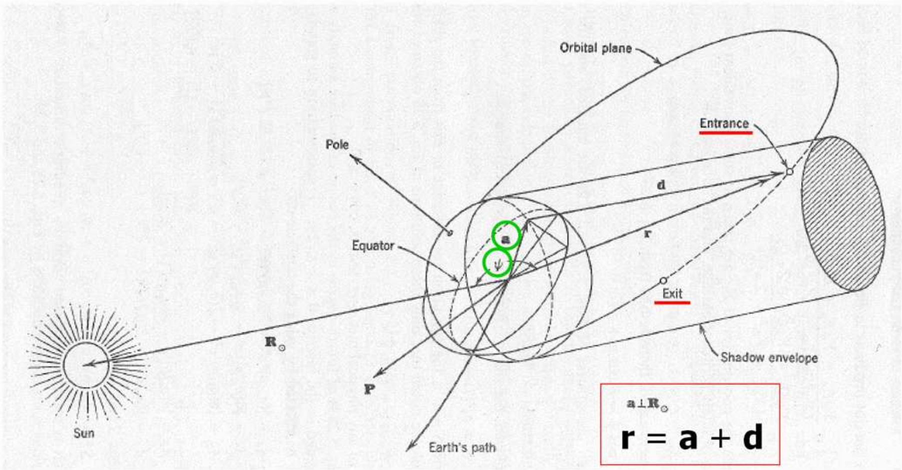
b) $T_{eclipse}$ = 35.1 min

c) a = 42164.14 km

d) $T_{eclipse}$ = 69.4 min

Eclipse (cnt'd)

3-dimensional situation [Escobal, 1976]:



The vector $\mathbf{R}_{\odot}$ gives the position of the Sun w.r.t. the center of the Earth. The vector $\mathbf{r}$ gives the position of the satellite w.r.t. the center of the Earth. The latter vector can be decomposed in components "d" (parallel to the direction towards the Sun) and "a" (perpendicular to the direction towards the Sun; not to be confused with the semi-major axis!). The angle Ψ is the angle between the direction to the Sun and the direction to the satellite.

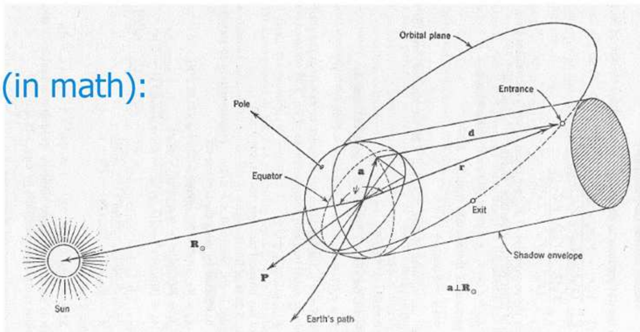
Eclipse (cnt'd)

Eclipse conditions (conceptual):

1. satellite on night-side of Earth
2. satellite "hides" behind Earth

Eclipse conditions (in math):

1. $\Psi > 90°$
2. $a < R_e$



Both conditions have to be satisfied!

Question: can we come up with expressions for the angle Ψ and the distance a?

Eclipse (cnt'd)

in - plane position satellite:

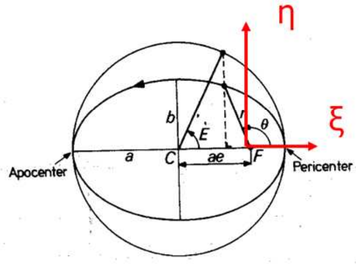
(r_{sat}, θ)

where

$$r_{sat} = \frac{a(1-e^2)}{1+e\cos(\theta)} = \frac{p}{1+e\cos(\theta)}$$

also

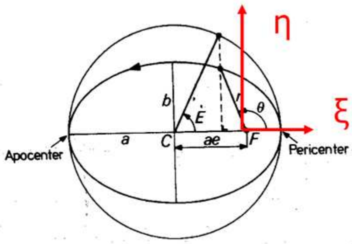
$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} r_{sat}\cos(\theta) \\ r_{sat}\sin(\theta) \end{pmatrix}$$



Parameters "ξ" and "η" represent two position components of the satellite within its orbital plane: the first parameter is measured from the focal center along the major axis, in the direction of the pericenter (for which the true anomaly θ is equal to 0°), whereas the 2nd parameter is measured in the direction where θ is equal to 90°.

Parameter p = a(1-e^2), "semi-latus rectum".

Eclipse (cnt'd)



2D in-plane ⇒ 3D :

transformation: $\mathbf{r}_{cart} = R_3(\Omega)R_1(i)R_3(\omega)\ \mathbf{r}_{in\text{-}plane}$

3D position satellite :

$$\begin{pmatrix} x_{sat} \\ y_{sat} \\ z_{sat} \end{pmatrix} = \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \\ n_1 & n_2 \end{pmatrix} \begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \\ n_1 & n_2 \end{pmatrix} \begin{pmatrix} r_{sat}\cos(\theta) \\ r_{sat}\sin(\theta) \end{pmatrix}$$

3D position Sun :

$X_{sun}, Y_{sun}, Z_{sun} \quad (\rightarrow \text{distance } R_{sun})$

The parameters "l_1" through "n_2" together represent the coordinates transformation from this in-plane situation to a full 3-dimensional situation, and are dependent on the <u>orientation</u> of the satellite orbit only (and not so much its position within the orbital plane):

$l_1 = \cos(\Omega)\cos(\omega) - \sin(\Omega)\sin(\omega)\cos(i)$

$l_2 = -\cos(\Omega)\sin(\omega) - \sin(\Omega)\cos(\omega)\cos(i)$

$m_1 = \sin(\Omega)\cos(\omega) + \cos(\Omega)\sin(\omega)\cos(i)$

$m_2 = -\sin(\Omega)\sin(\omega) + \cos(\Omega)\cos(\omega)\cos(i)$

$n_1 = \sin(\omega)\sin(i)$

$n_2 = \cos(\omega)\sin(i)$

So, the 3D position of a satellite in a Kepler orbit can be fully represented with a single time-dependent variable (*i.e.*, θ).

The position of the Sun can be obtained from an almanac (or a simple analytical model: a(nother) Kepler orbit if needed!).

Eclipse (cnt'd)

angle Ψ:

$$\mathbf{R}_{sun} \bullet \mathbf{r}_{sat} = R_{sun}\, r_{sat} \cos\Psi$$

or

$$\cos\Psi = \bar{\alpha}\cos\theta + \bar{\beta}\sin\theta$$

where

$$\bar{\alpha} = (l_1 X_{sun} + m_1 Y_{sun} + n_1 Z_{sun}) / R_{sun}$$

$$\bar{\beta} = (l_2 X_{sun} + m_2 Y_{sun} + n_2 Z_{sun}) / R_{sun}$$

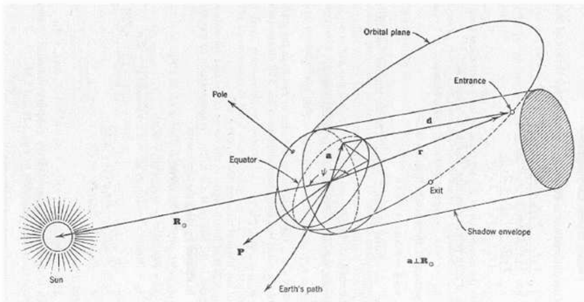
→ conditions for eclipse:

1) $\cos\Psi = \bar{\alpha}\cos\theta + \bar{\beta}\sin\theta < 0$
2) $a = r_{sat}\sin\Psi < R_e$ (a ≠ s.m.a.!!)

The (big) "dot" in the first equation represents the inner product between two vectors. This inner product can be developed in two ways: (1) multiplication of vector elements (as shown on the previous sheet) and summation, or (2) multiplication of vector lengths and cosine of angle-in-between. Parameters "αbar" and "βbar" can be considered as constants for a given epoch (or, 1st-order approximation, during one complete revolution – see subsequent sheets).

The conditions for eclipse are directly developed form the 2 elementary conditions mentioned on sheet 10.

Eclipse (cnt'd)



entering/leaving eclipse:

$$a = R_e$$

in general:

$$\sin^2\Psi + \cos^2\Psi = 1$$

$$r_{sat} = p / (1 + e\cos\theta)$$

so:

$$R_e^2 (1+e\cos\theta)^2 + p^2 (\overline{\alpha}\cos\theta + \overline{\beta}\sin\theta)^2 - p^2 = 0$$

shadow function S:

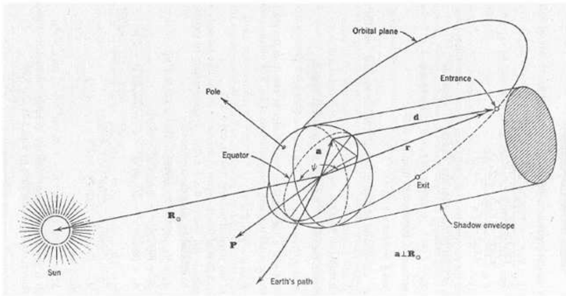
$$S(\theta) = R_e^2 (1+e\cos\theta)^2 + p^2 (\overline{\alpha}\cos\theta + \overline{\beta}\sin\theta)^2 - p^2$$

Parameter "p" represents the so-called semi-latus rectum: $p = a(1-e^2)$.

Equation after "so": combine expressions for $\sin\Psi$ and $\cos\Psi$ of previous sheet and re-arrange. Holds for condition of entering/leaving eclipse!

For a given epoch and satellite Kepler orbit, the shadow function S varies with true anomaly θ only. Solution?

Eclipse (cnt'd)



angle Ψ :

$$\cos\Psi = \bar{\alpha}\cos\theta + \bar{\beta}\sin\theta$$

shadow function S :

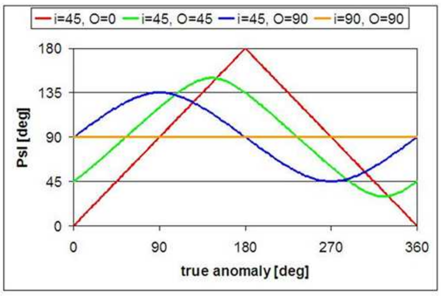
$$S(\theta) = R_e^2(1+e\cos\theta)^2 + p^2(\bar{\alpha}\cos\theta+\bar{\beta}\sin\theta)^2 - p^2$$

	S < 0	S = 0	S > 0
Ψ < 90°	in sunlight	in sunlight	in sunlight
Ψ > 90°	in sunlight	entering/leaving shadow cone	in shadow cone

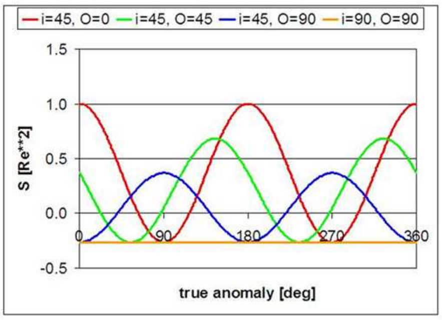
Whether the satellite is in eclipse (Earth shadow) or not is now fully determined by 2 parameters: Ψ and S. The (requirement on the) shadow function S replaces the (requirement on) the distance "a".

To interpret the consequences for *e.g.* S<0 and S>0, consider the situation where Ψ=90° (*i.e.* we know for sure that the satellite must be in Sunlight): it can easily be derived (do!) that for that case S<0, so this apparently corresponds with the situation "in sunlight".

Eclipse (cnt'd)



Example: 4 situations



All cases:

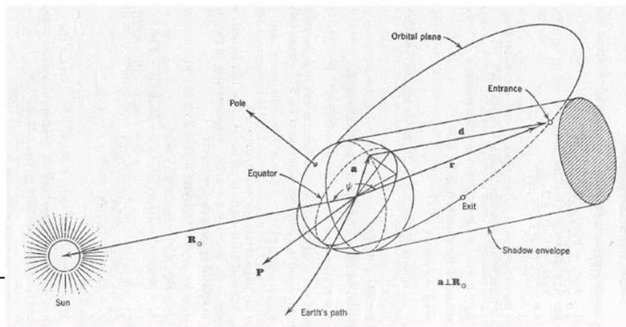
- circular orbit
- altitude 800 km
- Sun in equatorial plane

Sun is in direction (1,0,0). Legend: "i" is inclination, "O" is Ω (w.r.t. direction to Sun).

Eclipse (cnt'd)

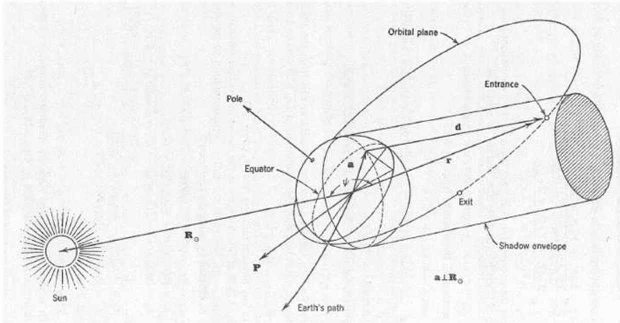
Question 2:

Derive the equations for the 2 conditions for a satellite to be in eclipse, using the 3-dimensional situation as sketched below.



Answer: see previous sheets.

Eclipse (cnt'd)



Question 3:

Given the 3-D situation shown above and the equations

$$\begin{pmatrix} x_{sat} \\ y_{sat} \\ z_{sat} \end{pmatrix} = \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \\ n_1 & n_2 \end{pmatrix} \begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \\ n_1 & n_2 \end{pmatrix} \begin{pmatrix} r_{sat}\cos(\theta) \\ r_{sat}\sin(\theta) \end{pmatrix}$$

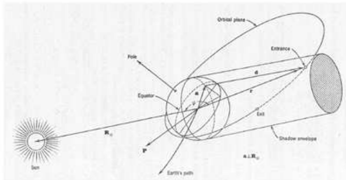
where

$$\begin{aligned}
l_1 &= \cos(\Omega)\cos(\omega) - \sin(\Omega)\sin(\omega)\cos(i) \\
l_2 &= -\cos(\Omega)\sin(\omega) - \sin(\Omega)\cos(\omega)\cos(i) \\
m_1 &= \sin(\Omega)\cos(\omega) + \cos(\Omega)\sin(\omega)\cos(i) \\
m_2 &= -\sin(\Omega)\sin(\omega) + \cos(\Omega)\cos(\omega)\cos(i) \\
n_1 &= \sin(\omega)\sin(i) \\
n_2 &= \cos(\omega)\sin(i)
\end{aligned}$$

a) Derive equations for the angle Ψ and the shadow function S, both as function of θ. Plot Ψ and S as function of θ. *(continued on next page)*

Eclipse (cnt'd)

Question 3 (cnt'd)



b) What are the 2 conditions for the satellite to be in eclipse?

c) Given the situation where the Sun is at (1,0,0) and the satellite parameters are Ω=45°, e=0, i=45°, make a plot of Ψ and S, both as a function of θ. Indicate when the satellite is in eclipse.

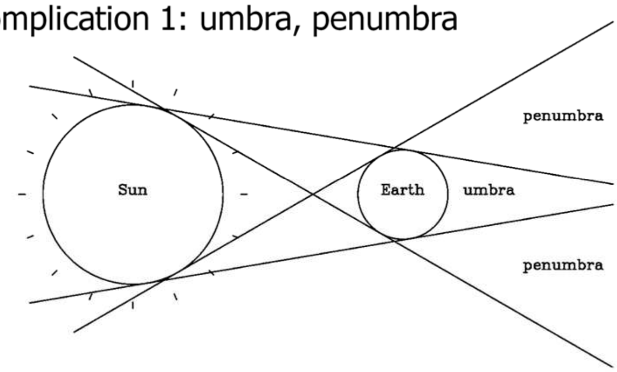
Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

Answers **(DID YOU TRY?):**

See treatment and derivation on previous pages.

Eclipse (cnt'd)

Complication 1: umbra, penumbra



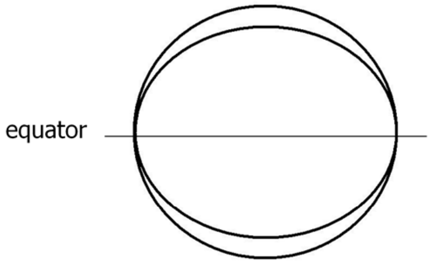
- top angle: ≈0.5°
- maximum effect on eclipse length: 8 s (LEO), 120 s (GEO)

Scales and effects are exaggerated. Umbra is the situation when the satellite is in full shadow; in penumbra part of the sunlight can still hit the vehicle.

GEO satellite: the vehicle lacks an inflow of 1371 W/m^2 during 120 sec….

Eclipse (cnt'd)

Complication 2: flattening Earth



- R_p = 6356.753 km
- R_e = 6378.137 km
- R_{avg} = 6371.0 km
- maximum effect (LEO) ≈ 22 s
- maximum effect (GEO) ≈ 0 s

Can you verify these numbers?

Eclipse (cnt'd)

Question 4:

a) Compute the orbital period for an Earth-observing satellite with a semi-major axis of 7178.137 km.

b) Compute the maximum eclipse length (in minutes) for this satellite, assuming that the Earth radius is equal to 6356.753 km.

c) Compute the maximum eclipse length (in minutes) for this satellite, assuming that the Earth radius is equal to 6378.137 km.

d) Compute the difference in eclipse length between the two cases.

In all cases, a 2-dimensional situation is to be considered.

Data: μ_{earth} = 398600.4415 km^3/s^2.

Answers **(TRY YOURSELF FIRST!!):**

a) T_{orbit} = 6052.4 s, or 100.87 min.

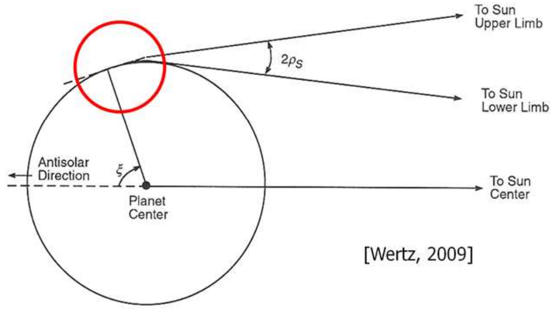
b) $T_{eclipse}$ = 34.93 min

c) $T_{eclipse}$ = 35.13 min

d) Difference $T_{eclipse}$ = 0.20 min

Eclipse (cnt'd)

Complication 3: atmosphere Earth



[Wertz, 2009]

- [Wertz, 2009]: maximum value around Earth 0.524°
- effect on eclipse length: 8 s (LEO), 120 s (GEO)

Magnitude of effect on eclipse length is similar to that of penumbra.

Eclipse (cnt'd)

Complication 4: motion Sun

Long-term: during year, North-South motion, ±23.5°

- See next sheets

Short-term: during orbit of satellite

- Sun moves 1°/day
- T_{orbit} ≈ 100 minutes (LEO)
- Sun motion ≈ 0.07°
- ignore

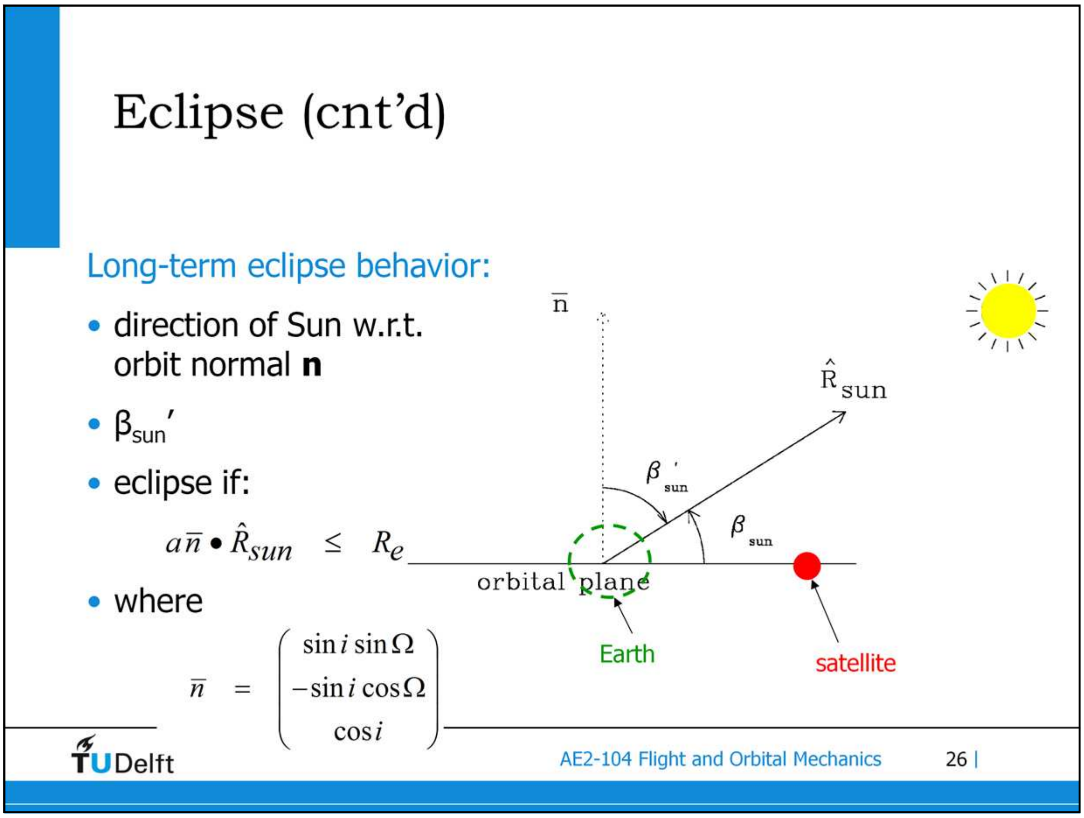
Eclipse (cnt'd)

Long-term eclipse behavior:

- lifetime mission
- mission design aspect
- orientation of orbit rather than instantaneous position satellite

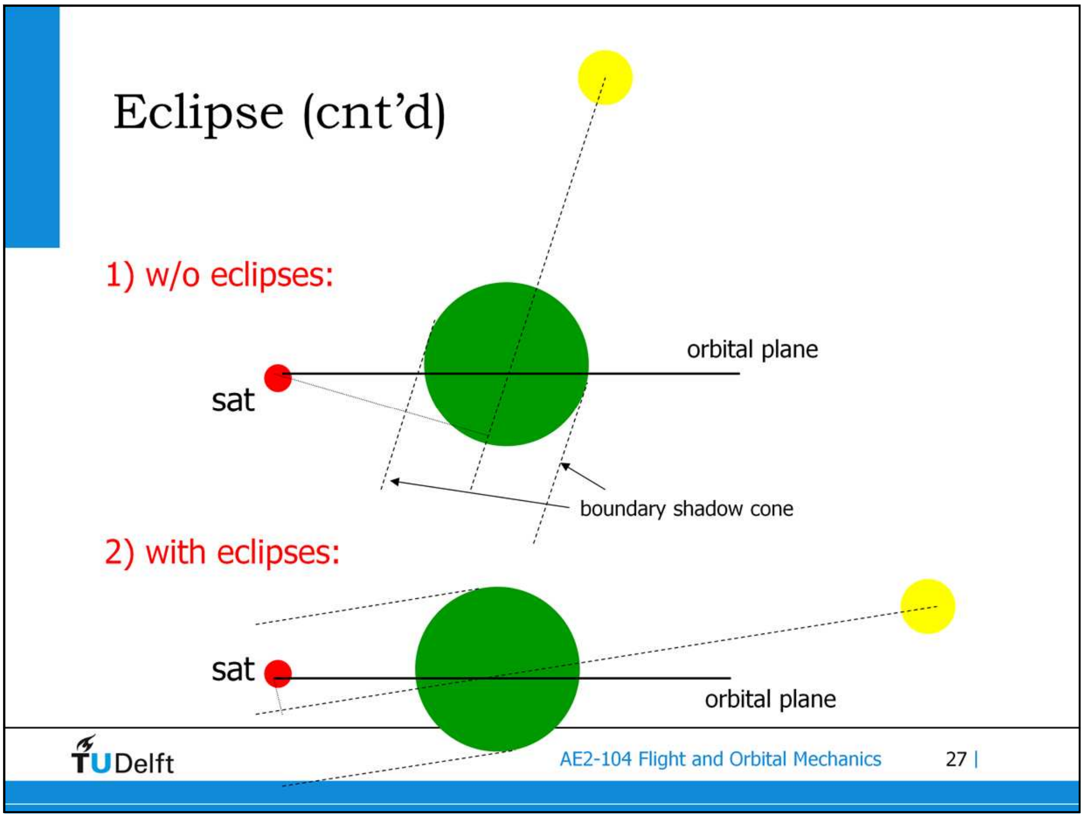
So far, the occurrence of eclipse during the course of a single revolution of the satellite around Earth was investigated. How about the behaviour during its overall lifetime?

Mission design aspect: power supply, thermal control, attitude control, …… What can we expect during say 10 years?



Fundamental criterion: hiding behind Earth, yes/no?

Eclipse condition: projection of orbit perpendicular to direction to Sun is smaller than Earth radius. Being in front of Earth or behind it does not matter since the satellite will be in both positions during one revolution. In the illustration, the size of the orbit is too large to be in eclipse anywhere, but when the orbit size reduces it becomes a possibility. Assumption: circular orbit. Note that in this and subsequent sheets, parameter “a” represents the semi-major axis again. The components of the vector **n** are given in an equatorial reference frame (*i.e.*, z-axis points in the direction of the Earth’s North pole). Check: $\cos(\beta_{sun}') = \sin(\beta_{sun})$.



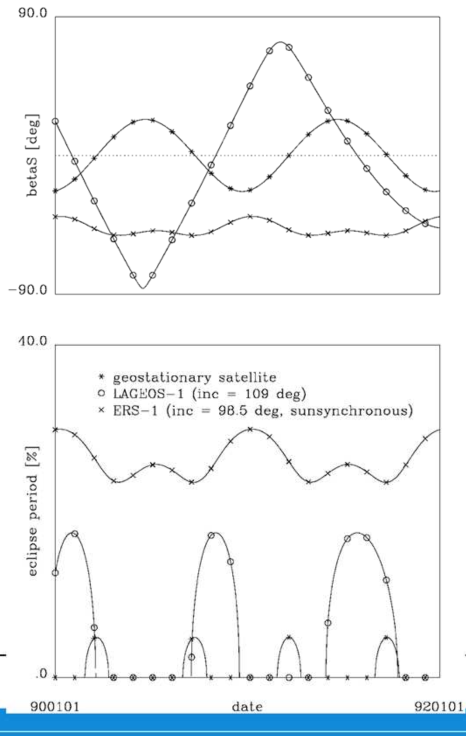
Eclipse condition: projection of orbit perpendicular to direction to Sun is smaller than Earth radius. Consider these 2 situations for a GEO satellite (*i.e.* at an altitude of 6 Earth radii), and the Sun at a declination of +23.5° (top plot; exaggerated angle…) or at +2°.

In both plots, the orbital plane is perpendicular to the sheet, *i.e.* the satellite exits (or enters) the sheet at right angles.

Eclipse (cnt'd)

3 examples:

- 2 full years
- eclipse seasons
- ERS-1: sun-synchronous orbit (but: N-S motion Sun)
- GEO satellite: max. eclipse 70 minutes
- eclipses unavoidable



TUDelft

Note: Sun-synchronous only says something about the (average) orientation of the orbital plane w.r.t. the direction to the Sun. It does NOT mean that the satellite is in full sunlight continuously!!!!

Eclipse (cnt'd)

"Eclipses unavoidable" ??

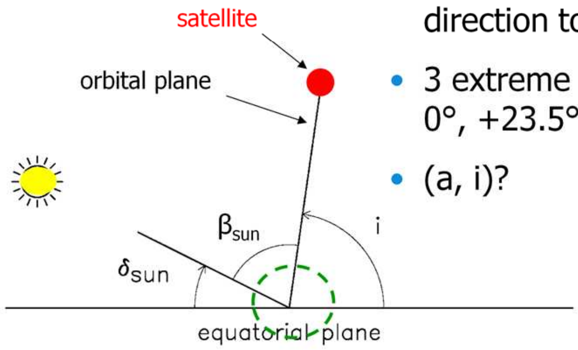
Eclipse (cnt'd)

Permanent sunlight (1)?

- illustrations previous sheet suggest "impossible"?
- mission lifetime 5-10 years
- arbitrary combinations (a, i, Ω)

For sake of simplicity, assume e=0.

Eclipse (cnt'd)



Permanent sunlight (2)?

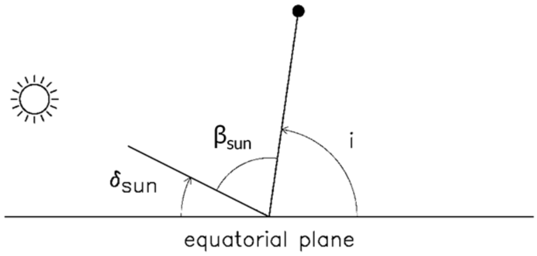
- Sun-synchronous orbit
- line of nodes perpendicular to direction to Sun (→ Ω)
- 3 extreme situations: δ_{sun} = -23.5°, 0°, +23.5°
- (a, i)?

In this picture, the satellite orbital plane is at right angles with the sheet (so the satellite is entering the sheet at right angles, in this sketch (or leaving it, for that matter….)). So, the vector normal to the orbital plane lays within the sheet.

If we do not have a sun-synchronous orbit, the relative orientation of the Sun w.r.t. the orbital plane will have all possible directions over the course of years. Unless it is "under control" by the precession of Ω induced by the J_2-effect.

The angle β_{sun} was already defined on sheet 26.

Eclipse (cnt'd)



Permanent sunlight (3)?

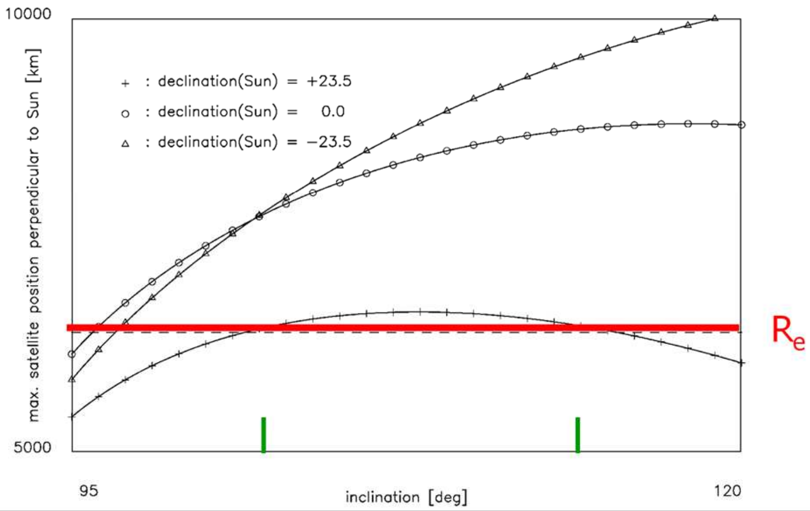
- Sun-synchronous orbit → a(i)
- $\delta_{sun} + \beta_{sun} + i = 180° \rightarrow \beta_{sun}(i)$
- $a(i) \sin \beta_{sun}(i) = f(i) \geq R_e$ (3 cases)

For sun-synchronous orbits, we have a strict 1-on-1 relation between semi-major axis and inclination (assuming e=0) (cf. lecture hours 15+16).

The 2nd equation holds for a given value of δ_{sun}, of course.

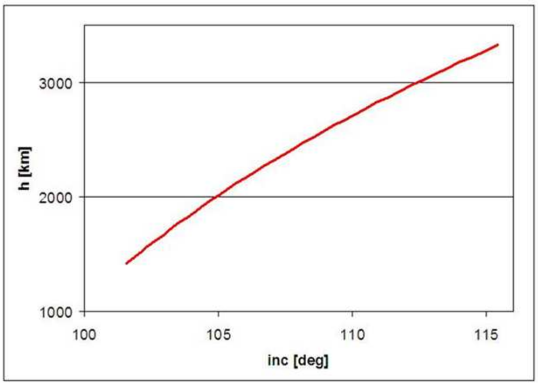
The last equation corresponds with the requirement on sheets 10 and 26.

Eclipse (cnt'd)



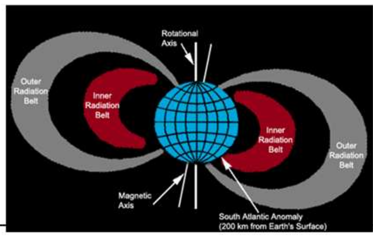
Requirement $a \times \sin\beta_{sun} > R_e$ satisfied for all 3 cases? Yes, for inclinations 101.5-115.4°.

Eclipse (cnt'd)



Permanent sunlight (4)?

- 101.5° < i < 115.4°
- h > 1400 km
- limited resolution ground observations
- Van Allen Belts



[Suntrek, 2010]

Inclination -> semi-major axis -> altitude.

Nice idea, but quite a number of drawbacks… → no practical application.

Van Allen Belts are regions in space with huge concentrations of (trapped) charged particles, causing fast degradation of instruments, solar cells, etcetera; they start at about 1000 km altitude.

Maneuvers

Definition
Change in velocity, to obtain change in orbit

Applications
- orbit transfers
- orbit maintenance
- rendezvous and docking
- end-of-life disposal

Orbit transfers: *e.g.* a transfer from Low Earth Orbit (LEO) to Geostationary Orbit (GEO). Orbit maintenance: *e.g.* to compensate for atmospheric drag losses.

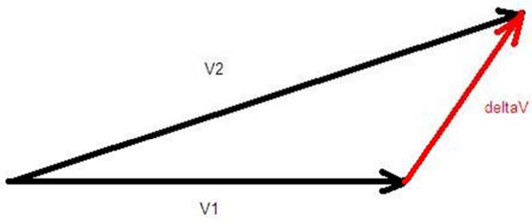
Maneuvers (cnt'd)

$$\Delta \mathbf{V} = \mathbf{V_2} - \mathbf{V_1}$$

$$\Delta V = \| \Delta \mathbf{V} \| = \| \mathbf{V_2} - \mathbf{V_1} \|$$

$$\Delta E = \frac{1}{2}(V_2^2 - V_1^2)$$

$$= \frac{1}{2}(\Delta V)^2 + \mathbf{V_1} \bullet \Delta \mathbf{V}$$



Observation 1:

ΔV most efficient when:

- parallel to original velocity
- original velocity largest

Note: ΔV is delivered instantaneously

Note the **bold** notation when refering to vectors, and the plain notation when refering to scalar values. The big "dot" in the last equation represents taking the inner product of two vectors.

The two observations hold for the case when doing in-plane maneuvers.

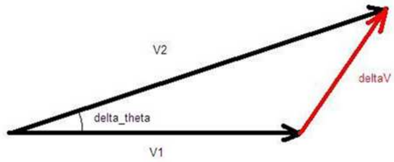
Maneuvers (cnt'd)

in general :

$$(\Delta V)^2 = V_1^2 + V_2^2 - 2V_1V_2\cos(\Delta\theta)$$

when $V_1 = V_2 = V$:

$$(\Delta V)^2 = V^2 + V^2 - 2VV\cos(\Delta\theta) = 2V^2(1-\cos(\Delta\theta))$$



Observation 2:

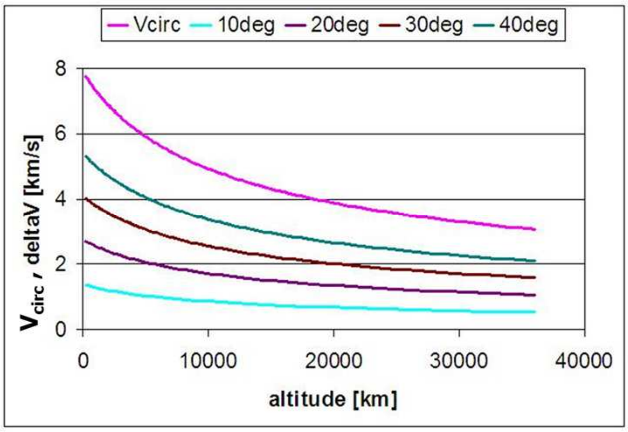
Dog-leg maneuvers: ΔV most efficient when:

- original velocity smallest

Note: ΔV is delivered instantaneously

So-called dog-leg maneuvers change the orientation of the velocity (and possibly also the magnitude).

Maneuvers (cnt'd)



Observation 2bis:

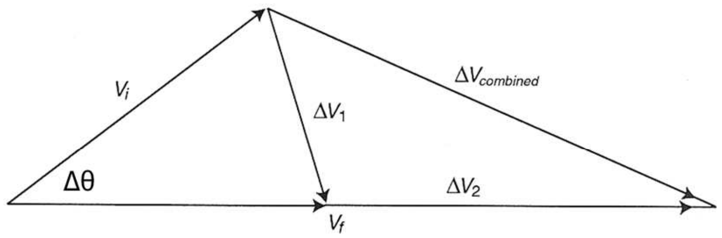
Dog-leg maneuvers:

- ΔV may approach circular velocity
- ΔV most efficient at highest altitude

$V_{circ} = \sqrt{(\mu/a)}$ (where "a" is semi-major axis, not altitude!). The most efficient launch from Kennedy Space Center results in an orbit with an inclination of 28.5°. When going for a GEO (i=0°), a large ΔV is required no matter what. Translate into propellant mass with Tsiolkovsky's equation….

Kourou (+Ariane, or Soyuz) is a better choice (since it is located at 5°)!

Maneuvers (cnt'd)

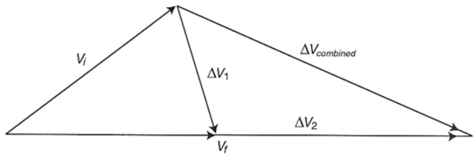


- $V_f > V_i$
- option 1: first ΔV_1, then ΔV_2
- option 2: $\Delta V_{combined}{}^2 = V_i^2 + V_f^2 - 2V_i\, V_f \cos(\Delta\theta)$
- $\Delta V_{combined} \leq \Delta V_1 + \Delta V_2$

Note: the target velocity V_f stretches all the way, from lower left to lower right!

General definitions; impulsive shots. $\Delta\theta$ is the angle between V_i and V_f. A combined maneuver is more efficient than 2 separate ones.

Maneuvers (cnt'd)

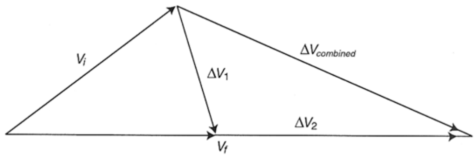


Example: transfer from circular parking orbit at 185 km and i =28.5° to pericenter of parabolic orbit with i=0°

- Approach?
- Answers?

General definitions; impulsive shots. The angle between V_i and V_f is equal to 28.5°. Additional parameter values: μ = 398600.4415 km^3/s^2; R_e = 6378.137 km.

Maneuvers (cnt'd)



Example (cnt'd): transfer from circular parking orbit at 185 km and i =28.5° to pericenter of parabolic orbit with i=0°

- $V_i = V_c = \sqrt{(\mu/r)} = 7.793$ km/s
- $V_f = V_{esc} = \sqrt{(2\mu/r)} = 11.021$ km/s
- $(\Delta V_1)^2 = 2\, V_1^2\, (1\text{-}\cos(28.5)) \rightarrow \Delta V_1 = 3.837$ km/s
- $\Delta V_2 = \text{abs}\,(V_f - V_i) = 3.228$ km/s
- option 1: $\Delta V_{total} = \Delta V_1 + \Delta V_2 \rightarrow \Delta V_{total} = 7.065$ km/s
- option 2: $(\Delta V_{comb})^2 = V_i^2 + V_f^2 - 2\, V_i\, V_f \cos(28.5) \rightarrow \Delta V_{comb} =$ 5.589 km/s
- Combined maneuver is better

General definitions; impulsive shots. The angle between V_i and V_f is equal to 28.5°. Additional parameter values: $\mu = 398600.4415$ km^3/s^2; $R_e = 6378.137$ km.

Maneuvers (cnt'd)

Question 5:

Consider a situation where a geostationary satellite has developed an inclination of 5°. The operators want to return it to its nominal orbit where i = 0°.

a) Knowing that the orbital period of a geostationary satellite is $23^h56^m4^s$, compute the semi-major axis of this orbit.

b) Compute the velocity in geostationary orbit.

c) Compute the ΔV required to make this inclination change.

Data: $\mu_{Earth} = 398600.4415$ km^3/s^2.

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

Answers **(DID YOU TRY??):**

a) a = 42164.125 km

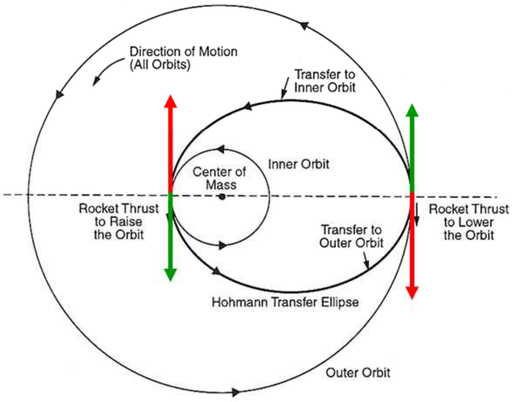
b) V_c = 3.075 km/s

c) ΔV = 0.268 km/s

Maneuvers (cnt'd)

Minimum energy transfers:

- Hohmann orbit
- coplanar, circular
- tangential to initial and target orbit
- low -> high
- high -> low



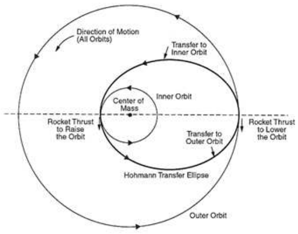
[Wertz, 2009]

Concept of Hohmann orbit. Essential: touches initial and final orbits in tangential direction. Impulsive shots.

This plot sketches Hohmann transfers, both for low→high (green velocity changes, 2x) and for high→low (red, also 2 maneuvres).

It holds for transfer of orbits around Earth (since the ΔV's directly translate into orbit changes). In the case of interplanetary missions, one would need to evaluate the effect of a ΔV in the pericenters of the departure and arrival hyperbola to the excess velocities.

Maneuvers (cnt'd)



Hohmann transfers:

$$a_T = \frac{1}{2}(r_1 + r_2)$$

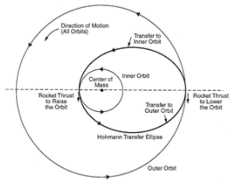
$$\Delta V_1 = V_{per,T} - V_{c,1} = \sqrt{\mu}\left[\sqrt{\frac{2}{r_1} - \frac{1}{a_T}} - \sqrt{\frac{1}{r_1}}\right]$$

$$\Delta V_2 = V_{c,2} - V_{apo,T} = \sqrt{\mu}\left[\sqrt{\frac{1}{r_2}} - \sqrt{\frac{2}{r_2} - \frac{1}{a_T}}\right]$$

$$T_{transfer} = \frac{1}{2} T_T = \pi\sqrt{\frac{a_T^3}{\mu}}$$

Equations for Hohmann transfers around one and the same planet. Equations hold for transfer from low orbit to high orbit, but similar expressions exist for transfer in other direction. Note: circular initial and target orbits, so $r_1=a_1$ and $r_2=a_2$. Use the vis-viva equation (*i.e.*, the energy equation, re-arranged) to compute the velocities in pericenter and apocenter.

Maneuvers (cnt'd)

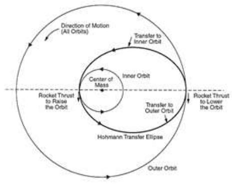


Example Hohmann transfers: initial parking orbit at 185 km altitude, target orbit at 1500 km altitude (both circular; coplanar)

- Approach?
- Answers?

Equations for Hohmann transfers around 1 and the same planet. Additional parameter values: μ = 398600.4415 km^3/s^2; R_e = 6378.137 km.

Maneuvers (cnt'd)

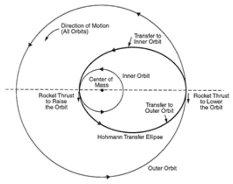


Example Hohmann transfers (cnt'd): initial parking orbit at 185 km altitude, target orbit at 1500 km altitude (both circular; coplanar)

- Initial orbit: $V_{c,1} = \sqrt{(\mu/r_1)} = 7.793$ km/s
- Target orbit: $V_{c,2} = \sqrt{(\mu/r_2)} = 7.113$ km/s
- Transfer orbit: $a_t = (r_1 + r_2) / 2 = 7220.637$ km
- Transfer orbit: $½ V^2 – \mu/r = -\mu/2a_t \rightarrow V_{per,t} = 8.140$ km/s; $V_{apo,t} = 6.781$ km/s
- $\Delta V_1 = abs(V_{per,t} – V_{c,1}) = 0.347$ km/s
- $\Delta V_2 = abs(V_{c,2} – V_{apo,t}) = 0.331$ km/s
- $\Delta V_{total} = 0.679$ km/s
- $T_{transfer} = \pi \sqrt{(a_t^3/\mu)} = 3053$ sec = 50.9 min

Equations for Hohmann transfers around 1 and the same planet. Additional parameter values: $\mu = 398600.4415$ km^3/s^2; $R_e = 6378.137$ km.

Maneuvers (cnt'd)



Question 6:

Consider a Hohmann transfer from an initial parking orbit at 185 km altitude, to a target orbit at 800 km altitude (both circular; coplanar)

a) Compute the velocities in the original and in the target obit.

b) Compute the semi-major axis and eccentricity of the transfer orbit.

c) Compute the required ΔV (1st maneuver, 2nd maneuver, total).

d) Compute the transfer time.

Data: μ_{Earth} = 398600.4415 km^3/s^2, R_{earth} = 6378.137 km.

. Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

Answers **(DID YOU TRY FIRST??):**

a) $V_{c,185}$ = 7.793 km/s, $V_{c,800}$ = 7.452 km/s

b) a = 6870.5 km, e = 0.045

c) ΔV_1 = 0.173 km/s, ΔV_2 = 0.169 km/s, ΔV_{tot} = 0.342 km/s

d) 2834 s = 47.2 min

Maneuvers (cnt'd)

LEO (at 250 km, i = 28.5°) to GEO: some options

approach	total ΔV [km/s]
Entire plane change at 250 km followed by separate Hohmann transfer to GEO	7.730
Hohmann transfer to GEO altitude followed by separate plane change	5.425
Combined maneuver, entire plane change at perigee	6.483
Combined maneuver, entire plane change at apogee	4.273
Combined maneuver, 4° plane change at perigee, 24.5° plane change at apogee	4.265

Real mission operations: a good understanding of what's ongoing is crucial for the lifetime of the mission (in terms of ΔV budget).

Translate these ΔV values to propellant mass (using Tsiolkovsky's equation) and you'll understand why you're either employee of the year or you find yourself fired…..

Maneuvers (cnt'd)

Question 7:

Consider a transfer from a circular parking orbit at 185 km and i =28.5° (*i.e.*, launch from Kennedy Space Center) to the International Space Station (h=400 km, e=0, i=55.6°)

a) Compute the velocities in the original orbit and in the target orbit.

b) Compute the total ΔV that would be required for the orbit raising, assuming that the two orbits are coplanar (Hohmann transfer).

c) Compute the ΔV that would be required to only change the inclination of the original parking orbit to that of the ISS orbit.

d) Compute the total ΔV if the sequence of maneuvers was (1) dog-leg maneuver (only) in initial orbit, and (2) Hohmann transfer.

e) Compute the total ΔV if the maneuver for inclination change is combined with the 2nd step of orbit raising (*i.e.*, a combined maneuver).

Data: μ_{Earth} = 398600.4415 km^3/s^2, R_{Earth} = 6367.137 km

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

Answers **(DID YOU TRY??):**

a) $V_{c,185}$ = 7.793 km/s, $V_{c,400}$ = 7.669 km/s

b) $\Delta V_{Hohmann}$ = 0.125 km/s

c) ΔV_{dogleg} = 3.652 km/s

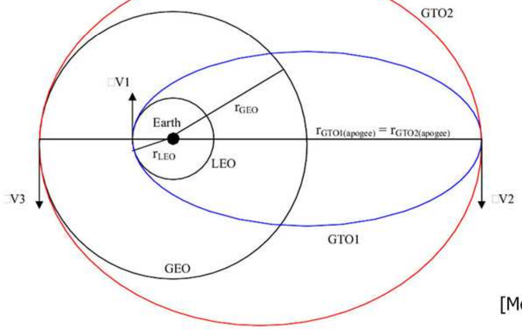
d) ΔV_{total} = 3.777 km/s

e) ΔV_{total} = 0.063 + 3.579 = 3.579 km/s

Maneuvers (cnt'd)

Can we do better?

1) Use super-synchronous apocenter



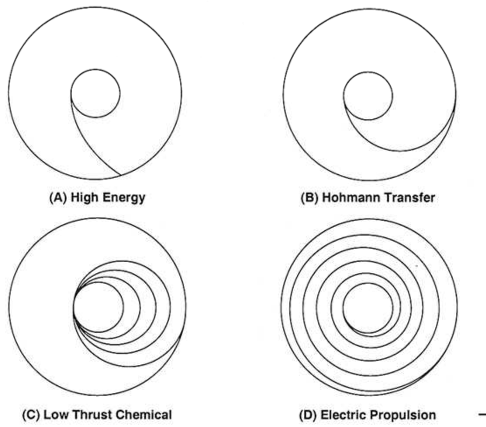
[Messidoro, 2010]

The plane change is being done at a very high altitude, where the in-plane velocity of the vehicle is very modest. Takes more (in-plane) energy to arrive at this point, though. Only interesting when $r_{target}/r_{initial} > 11$.

Maneuvers (cnt'd)

Can we do better (cnt'd)?

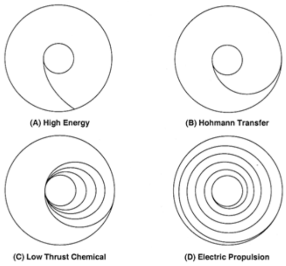
2) Use alternative techniques for in-plane transfer



[Wertz, 2009]

Consider the Hohmann trajectory as the reference; can we do faster? More efficient?

Maneuvers (cnt'd)



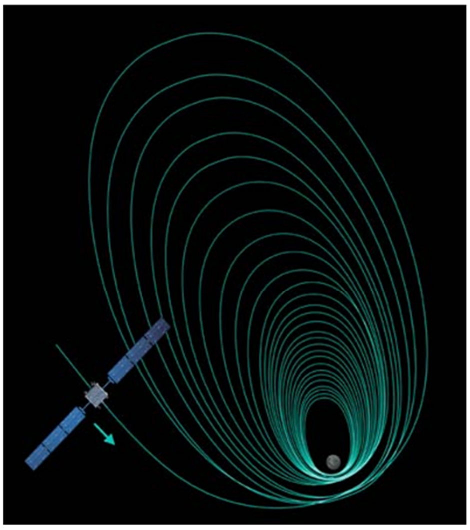
transfer type	orbit type	typical acceleration	ΔV	transfer time
high-energy	elliptic or hyperbolic	10 g	> Hohmann	< Hohmann
Hohmann	elliptic	1-5 g	Hohmann	Hohmann
low-thrust chemical	Hohmann segments	0.02-0.5 g	< Hohmann	6-8 * Hohmann
electric prop.	spiral transfer	0.0001-0.001 g	diff. between V_{circ}	60-120 * Hohmann

Design: trade time vs. propellant mass (or: payload mass).

Maneuvers (cnt'd)

Example in-plane maneuvers:

- SMART-1
- launched on September 27, 2003
- continuous low-thrust to reach Moon
- arrived in lunar orbit on November 15, 2004 (*i.e.*, transfer period 415 days)



[ESA, 2010]

The picture shows the phase around the Moon, where the orbit was lowered and made less eccentric.

Maneuvers (cnt'd)

Can we do better (cnt'd)?

3) Use alternative techniques for out-of-plane transfer

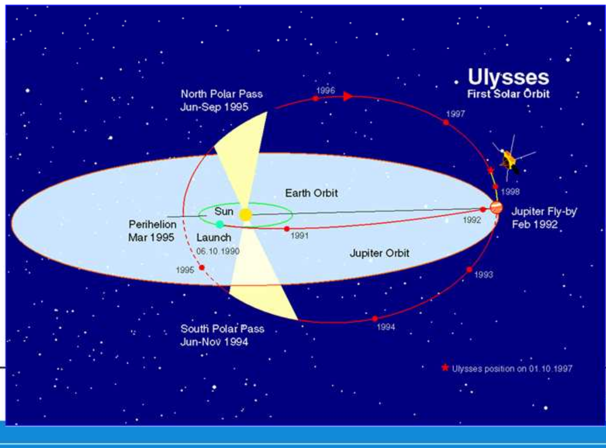
method	mechanism
ΔV at lowest velocity	small velocity is easier to change
combine ΔV with orbit raising	vector sum is smaller than sum of components
3-burn transfer	use intermediate high altitude for ΔV and then lower
differential node rotation	natural mechanism (J_2) for plane change: $f(\Delta a, i)$
aero-assist	aerodynamic forces cause plane change
planetary fly-by	gravity pull of planet causes plane change

Options in general, and (some) for interplanetary flight only.

Maneuvers (cnt'd)

Example out-of-plane maneuver:

- Ulysses
- 3-dimensional swing-by along Jupiter



[ESA, 2010]

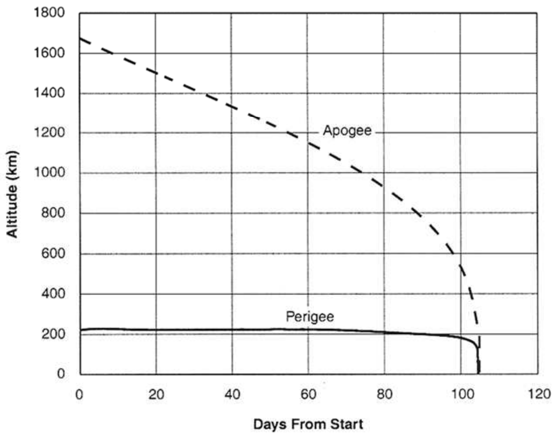
The interplanetary orbit went from i=0° to i=80.2° because of the 3-dimensional swingby (a.k.a. gravity-assist, slingshot, flyby) at Jupiter. An impulsive maneuver with traditional rocket engines would have cost us an impossible 42 km/s.

Maneuvers (cnt'd)

Disposal (1): LEO

- **lower orbit and burn up in atmosphere**
- **1 tangential maneuver**
- **directly? after 25 years?**

[Wertz, 2009]



De-orbit is costing energy. Here, you do only the first burn of a Hohmann transfer. How much do you want to spend, and how rapidly does your satellite burn up? Trade…

Maneuvers (cnt'd)

Disposal (2): GEO

- **raise circular GEO orbit to circular graveyard orbit (GEO + 300 km)**
- **2 tangential maneuvers → Hohmann transfer**
- **$\Delta V \approx 10$ m/s**
- **≈ 6.8 kg propellant ($m_{sat,dry}$ = 2000 kg, I_{sp} = 300 s)**

Use Tsiolkovsky's equation to get propellant mass.

Maneuvers (cnt'd)

Question 8:

Consider an end-of-life maneuver for a spacecraft originally in a circular orbit at 1500 km.

a) What would be the ΔV needed to lower the orbit to a pericenter at 800 km (apocenter remains at 1500 km)?

b) How much propellant would be required for such a maneuver? Assume an I_{sp} of 300 sec, and an initial spacecraft mass of 1000 kg.

c) What would be the expected lifetime with a pericenter at 800 km?

d) Same 3 questions, but now for the case that the pericenter is lowered to 500 km?

See next sheet for additional information

Answers: see footnotes below (BUT TRY YOURSELF FIRST!!)

ANSWERS: **(DID YOU TRY??)**

a) ΔV = 167.3 m/s

b) M_{prop} = 55.3 kg

c) Lifetime = -H/($\Delta a_{2\pi}$) (cf. topic "Perturbations") ~ 6,647,000 revs ~ 475,380 days ~ 1301 yrs. This estimate is based on drag conditions at an altitude of 900 km; with the pericenter of the orbit at 800 km and the apocenter at 1500 km, drag shows a big variation throughout a single revolution around Earth. Representative (constant) drag conditions which can be applied along the entire orbit (and which should be used in the computation of $\Delta a_{2\pi}$ and the selection of density scale height H) should hold at an altitude somewhere in between these 2 extremes. But: since atmospheric density behaves in an exponential way, it's not so much the direct average altitude ((1500+800)/2=1150km) where one finds the average drag, but somewhere below. Assume here that 900 km is reasonable.

d) ΔV = 245.2 m/s; M_{prop} = 79.9 kg; lifetime ~ 111,000 revs ~ 7462 days ~ 20.4 yrs (based on drag conditions at a representative altitude of 600 km)

Maneuvers (cnt'd)

Question 8 (cnt'd):

Effect of atmospheric drag on semi-major axis after 1 revolution, and on lifetime:

$$\Delta a_{2\pi} = -2\pi \, (C_D A/m) \, \rho \, a^2 \qquad [\text{m}]$$

$$L = - H / \Delta a_{2\pi} \qquad [\text{rev}]$$

$C_D = 2.2$

$A = 10$ m^2

m = 1000 kg

Altitude [km]	Atmospheric density [kg/m^3]	Density scale height [km]
500	4.9×10^{-13}	64.5
600	1.0×10^{-13}	74.8
700	2.7×10^{-14}	99.3
800	9.6×10^{-15}	151
900	4.7×10^{-15}	226
1000	2.8×10^{-15}	296
1250	1.1×10^{-15}	408
1500	5.2×10^{-16}	516

Maneuvers (cnt'd)

Question 9:

Consider an end-of-life maneuver for a GEO spacecraft.

a) What is the original semi-major axis? The orbital period in GEO is $23^h 56^m 4^s$.

b) What would be the ΔV needed to transfer to a graveyard orbit which is located 250 km higher?

c) How much propellant would be required to achieve this? Assume an I_{sp} of 300 sec, and an initial spacecraft mass of 1000 kg.

d) Same 2 questions, but now for the case that the graveyard orbit is 500 km higher?

Data: μ = 398600.4415 km³/s².

Answers: see footnotes below (BUT TRY YOURSELF FIRST!!)

ANSWERS: **(DID YOU TRY??)**

a) a_{GEO} = 42163 km

b) ΔV = 4.54 + 4.53 = 9.07 m/s

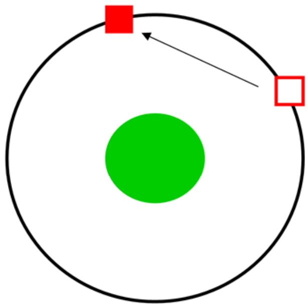
c) Tsiolkovsky: m_{prop} = 3.1 kg

d) ΔV = 9.04 + 9.02 = 18.06 m/s; m_{prop} = 6.2 kg

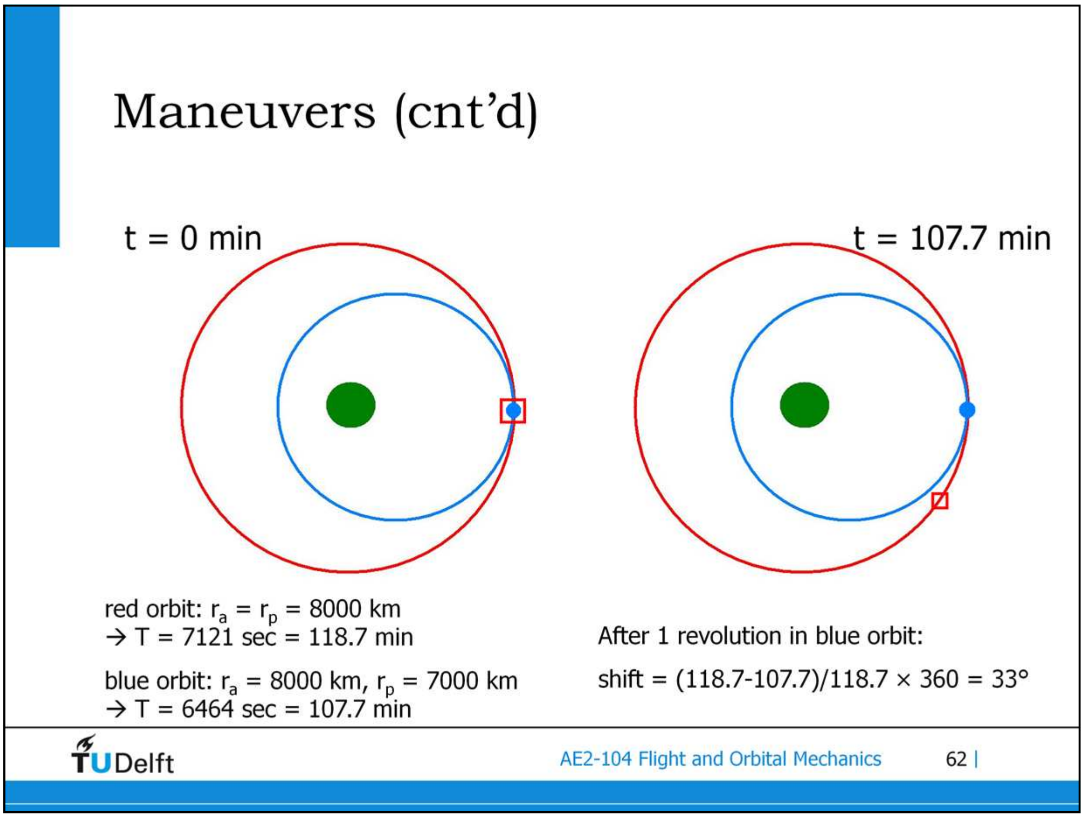
Maneuvers (cnt'd)

in-plane phase shift:

- **orbital plane remains the same**
- **reposition satellite at new slot**
- **GEO, GPS, Iridium**



Some space missions require guaranteed 100% continuity (*e.g.* telecommunication services, from GEO) -> a spare vehicle is available in the proper orbit already, but not at the location of the satellite that fails. Orbits with different semi-major axes will show a different orbital period -> can be used to drift in-plane w.r.t. original position.



The red box represents a satellite in the red orbit; the blue dot is a satellite in the blue orbit. The green circle represents Earth. Picture is not to scale. Because of the difference in orbital period, the blue satellite "overtakes" the red one.

So, if satellite is originally in red orbit, but temporarily switches to the blue orbit, it will alter its position w.r.t. the position it would have if it were to remain in the red orbit.

Maneuvers (cnt'd)

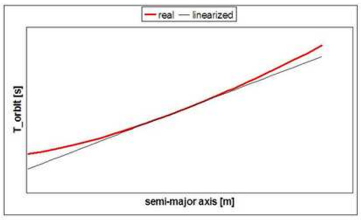
Option 1:

1. **Convert original circular orbit to elliptical orbit**
2. **Assume pericenter altitude (apocenter altitude = original altitude)**
3. **Use equations for Hohmann transfer (circ→ell) (first ΔV only!)**
4. **Compute orbital period in transfer orbit**
5. **Compute difference in orbital period ΔT**
6. **Translates into change in orbit position $n_{org} \times \Delta T$**
7. **Do N revolutions to accomplish total shift**
8. **Use equations for Hohmann transfer (ell→circ) (first ΔV only!)**
9. **Trial and error**

Parameter "n_{org}" is mean motion (in original orbit; $n_{org} = \sqrt{(\mu/a^3)}$ [rad/s]).

"Trial and error": try various altitudes for pericenter altitude of temporary orbit, and see what you end up with (transfer times, total ΔV).

Maneuvers (cnt'd)



Option 2: linearize

1. **$T = 2\pi\sqrt{(a^3/\mu)} \rightarrow \Delta T = (\partial T/\partial a)\Delta a = 3/2\ (T/a)\ \Delta a$**
2. **After 1 revolution:**

   $$\Delta pos_{rev} = n_{org}\Delta T = 3\pi\ (1/a)\ \Delta a$$
3. **After N revolutions (time T_{total}):**

   $$\Delta pos_{total} = N\ \Delta pos_{rev} = 3\pi\ (1/a)\ \Delta a\ N = 3\pi\ (1/a)\ \Delta a\ (T_{total}/T)$$
4. **Required change in semi-major axis:**

   $$\Delta a = (\Delta pos_{total}\ a)/(3\pi)\ (T/T_{total})$$
5. **Required change in velocity:**

   ....

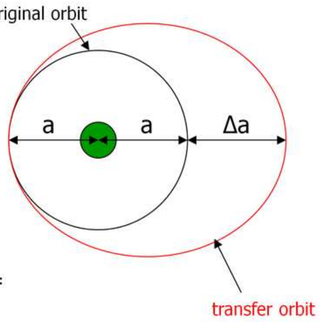
Parameters "a" and "T" refer to original orbit (*i.e.*, not the transfer orbit).

Note 1: the first equation linearizes the effect on orbital period of a small change in semi-major axis.

Note 2 : $n_{org}T = 2\pi$, so $n_{org} = 2\pi/T$ (easier here than the standard $n_{org}=\sqrt{(\mu/a^3)}$).

Note 3: $T_{total} = N \times T$ of course.

Maneuvers (cnt'd)



5. Required change in velocity:

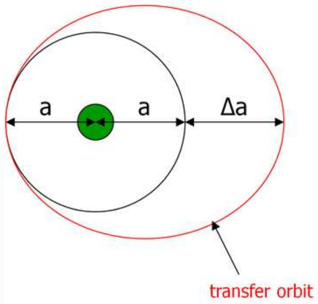
$$\Delta V_{tot} = 2 \times \Delta V_{1,H} = 2 \times (V_{per,T} - V_{circ,1}) =$$
$$= 2 \times \{\sqrt{(\mu(2/r_1 - 1/a_T))} - \sqrt{(\mu/r_1)}\}$$

Note:

- **a_T = semi-major axis of transfer orbit**
- **r_1 = radius of initial orbit**
- **2 times, since maneuver to enter transfer orbit and similar maneuver to exit transfer orbit**

Illustration: $2a_T = a + a + \Delta a$ → $a_T = a + \Delta a/2$. $r_1 = a$. Velocity change for entering transfer orbit is identical to first step in Hohmann transfer (cf. sheets 43,44). Return from transfer orbit to original circular orbit requires exactly the same ΔV. Velocity in pericenter follows from vis-viva equation.

Maneuvers (cnt'd)



pericenter velocity :

$$V_p = \sqrt{\mu}\sqrt{\frac{2}{r_1} - \frac{1}{a_T}} = \sqrt{\mu}\left(\frac{2}{a} - \frac{1}{a+\frac{\Delta a}{2}}\right)^{1/2} = \sqrt{\mu}\left(\frac{2\,(a+\frac{\Delta a}{2}) - a}{a\,(a+\frac{\Delta a}{2})}\right)^{1/2} =$$

$$= \sqrt{\mu}\left(\frac{a+\Delta a}{a\,(a+\frac{\Delta a}{2})}\right)^{1/2} = \sqrt{\frac{\mu}{a}}\left(\frac{a+\Delta a}{a+\frac{\Delta a}{2}}\right)^{1/2} = \sqrt{\frac{\mu}{a}}\left(\frac{1+\frac{\Delta a}{a}}{1+\frac{\Delta a}{2a}}\right)^{1/2} =$$

$$= \sqrt{\frac{\mu}{a}}\left(1+\frac{\Delta a}{a}\right)^{1/2}\left(1+\frac{\Delta a}{2a}\right)^{-1/2} \approx \sqrt{\frac{\mu}{a}}\left(1+\frac{1}{2}\frac{\Delta a}{a}+O((\frac{\Delta a}{a})^2)\right)\left(1-\frac{1}{2}\frac{\Delta a}{2a}+O((\frac{\Delta a}{2a})^2)\right) \approx$$

$$\approx \sqrt{\frac{\mu}{a}}\left(1-\frac{1}{2}\frac{\Delta a}{2a}+\frac{1}{2}\frac{\Delta a}{a}+O((\frac{\Delta a}{a})^2)\right) \approx \sqrt{\frac{\mu}{a}}\left(1+\frac{\Delta a}{4a}\right)$$

Not for exam.... !!!

circular velocity :

$$V_c = \sqrt{\frac{\mu}{a}}$$

so

$$\Delta V = V_p - V_c = \sqrt{\frac{\mu}{a}}\frac{\Delta a}{4a}$$

two maneuvers :

$$\Delta V_{tot} = 2\,\Delta V = \sqrt{\frac{\mu}{a}}\frac{\Delta a}{2a} = \sqrt{\frac{\mu}{a}}\frac{\Delta pos_{total}}{6\pi}\frac{T}{T_{total}}$$

Full linearization for small changes in orbits (to save propellant).

Parameter "a" refers to original orbit; parameter "a_T" refers to the transfer orbit. The terms between brackets are expanded with a Taylor series, with first 2 terms only $(1+\varepsilon)^k \sim 1+k\varepsilon + O(\varepsilon^2)$ for $\varepsilon<<1$.

The maneuver is given twice, so the value for ΔV here is the total amount required. The last equation follows from the equations of a Hohmann transfer.

Maneuvers (cnt'd)

Example option 2: reposition GEO satellite 60° further in orbit. First approach: within 3 days. 2nd approach: within 30 days.

	3 days	**30 days**
T [s]	86164	
a [km]	42163	
T_{total} [s]	259,200	2592,000
Δa [km]	1557.3	155.7
V_c [km/s]	3.075	
V_p [km/s]	3.102	3.078
ΔV_{tot} [km/s]	0.054	0.006
M_{prop} [kg]	37.0	4.1

Compare results with required ΔV to transfer to graveyard orbit, or ΔV to compensate for 3rd-body perturbations from Sun and Moon (total 51 m/s/yr).

Computation propellant mass: Tsiolkovsky, with dry mass satellite assumed to be 2000 kg and I_{sp} = 300 sec.

Maneuvers (cnt'd)

Question 10:

Re-locate a back-up GPS satellite at a position 60 degrees different in orbit (the orbital plane is assumed to be constant). Consider the options of doing this 2, 5, 10, 30 or 60 days. Compute, for each option, the Δa required and the total ΔV required. Also provide an estimate for the necessary propellant mass.

Data: μ = 398600.4415 km^3/s^2; T_{orbit} = $11^h58^m2^s$; I_{sp} = 300 s; $m_{sat,dry}$ = 500 kg

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

Answers: **(DID YOU TRY??)**

2 days: Δa = 735.8 km, ΔV = 53.8 m/s, m_{prop} = 9.2 kg

5 days: Δa = 294.3 km, ΔV = 21.5 m/s, m_{prop} = 3.7 kg

10 days: Δa = 147.2 km, ΔV = 10.8 m/s, m_{prop} = 1.8 kg

30 days: Δa = 49.1 km, ΔV = 3.6 m/s, m_{prop} = 0.6 kg

60 days: Δa = 24.5 km, ΔV = 1.8 m/s, m_{prop} = 0.3 kg