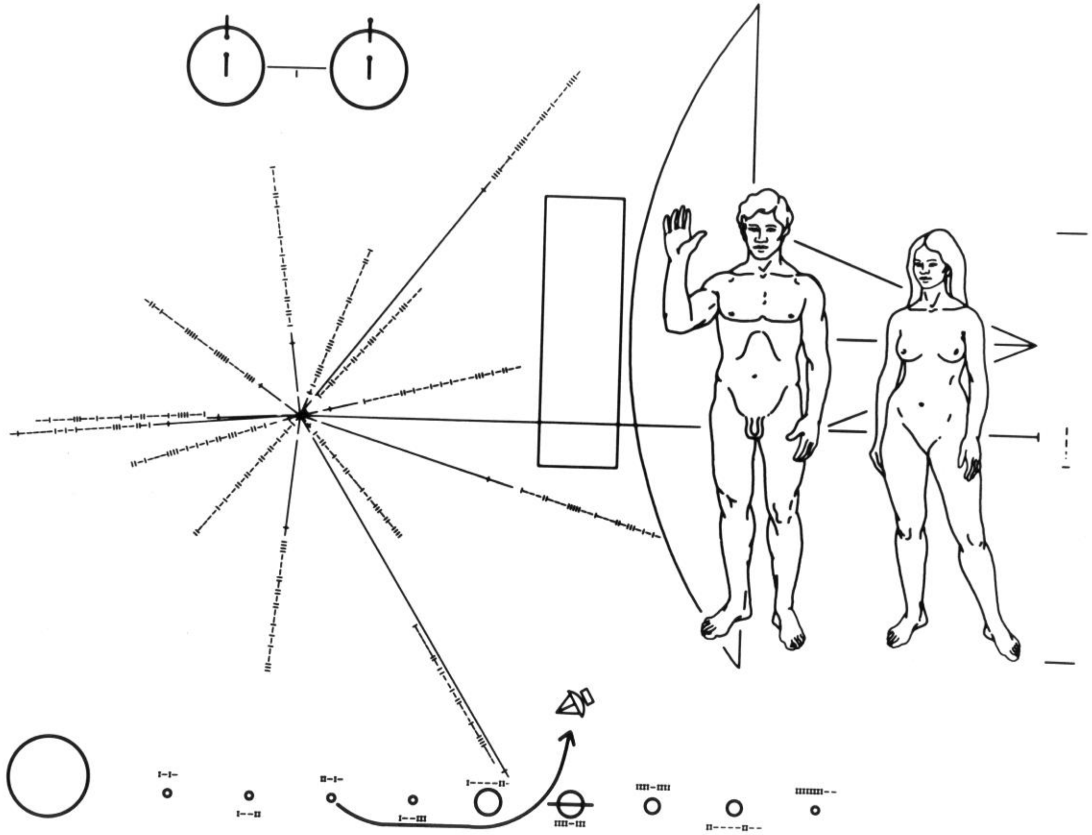
Flight and Orbital Mechanics

Lecture slides



Flight and Orbital Mechanics

AE2-104, lecture hours 21-24: Interplanetary flight

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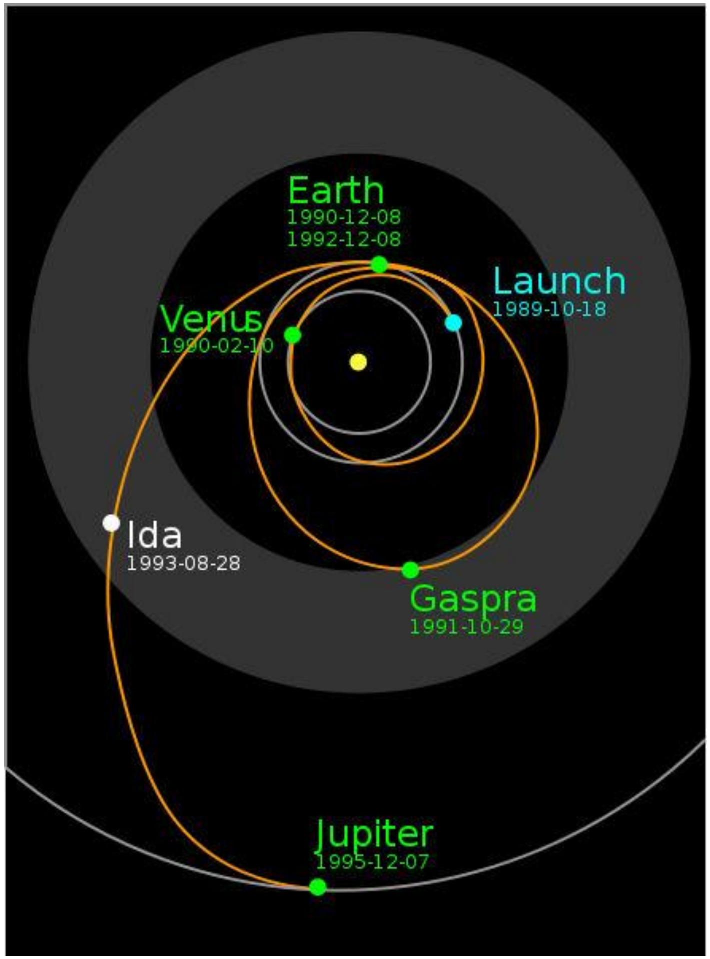
October 25, 2012

Example: Galileo VEEGA trajectory



[NASA, 2010]

Questions:

- what is the purpose of this mission?
- what propulsion technique(s) are used?
- why this Venus-Earth-Earth sequence?
- ….

Overview

- Solar System
- Hohmann transfer orbits
- Synodic period
- Launch, arrival dates
- Fast transfer orbits
- Round trip travel times
- Gravity Assists

Learning goals

The student should be able to:

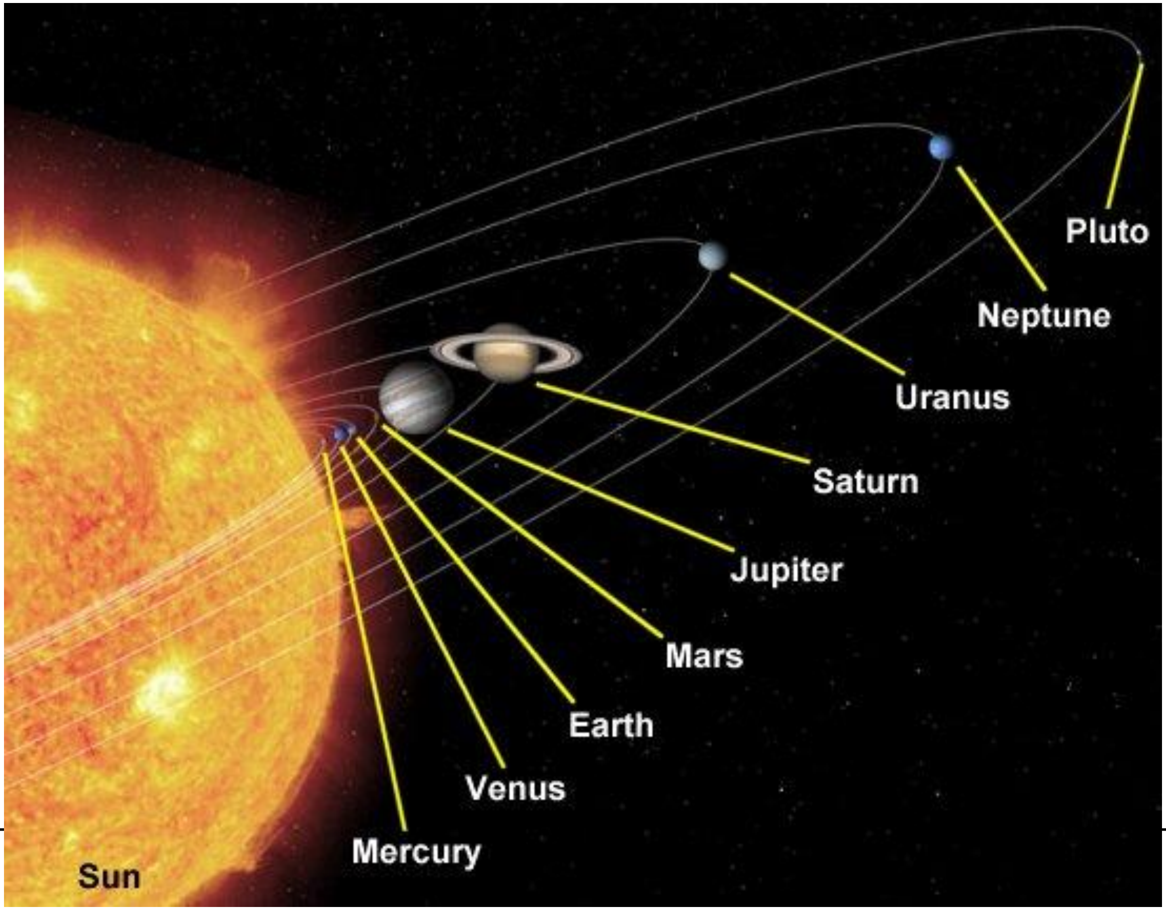
- describe and explain the concept of an interplanetary transfer, including that of patched conics;
- compute the main parameters of a Hohmann transfer between arbitrary planets (including the required ΔV);
- compute the main parameters of a fast transfer between arbitrary planets (including the required ΔV);
- derive the equation for the synodic period of an arbitrary pair of planets, and compute its numerical value;
- derive the equations for launch and arrival epochs, for a Hohmann transfer between arbitrary planets;
- derive the equations for the length of the main mission phases of a round trip mission, using Hohmann transfers; and
- describe the mechanics of a Gravity Assist, and compute the changes in velocity and energy.

Lecture material:

- these slides (incl. footnotes)

Introduction

The Solar System (not to scale):



[Aerospace web, 2010]

Introduction (cnt'd)

planet	mean distance [AU]	eccentricity [-]	inclination [°]
Mercury	0.387	0.206	7.0
Venus	0.723	0.007	3.4
Earth	1.000	0.017	0.0
Mars	1.524	0.093	1.9
Jupiter	5.203	0.048	1.3
Saturn	9.537	0.054	2.5
Uranus	19.191	0.047	0.8
Neptune	30.069	0.009	1.8
Pluto *	39.482	0.249	17.1

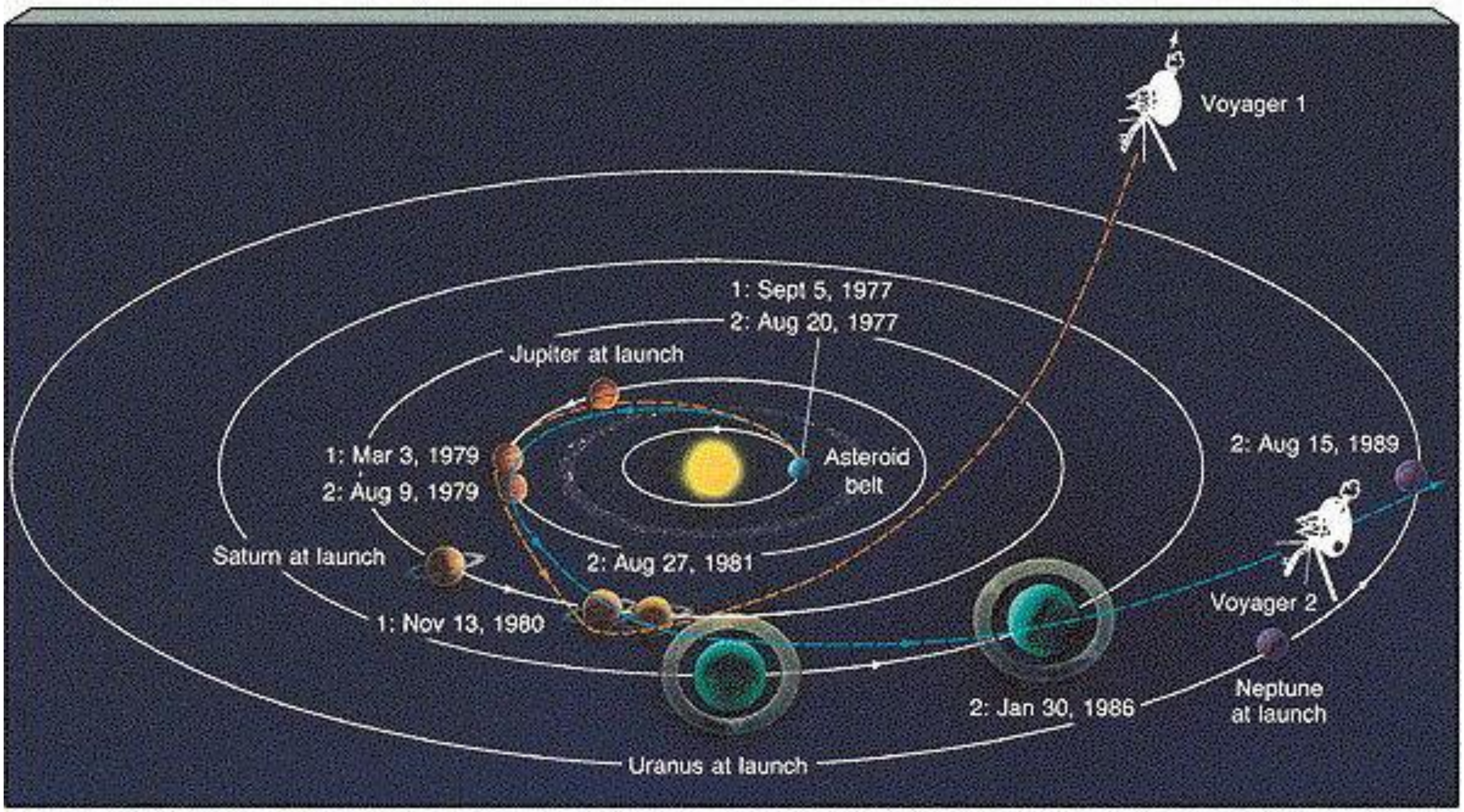
[Wertz, 2009]

Introduction (cnt'd)

Conclusions:

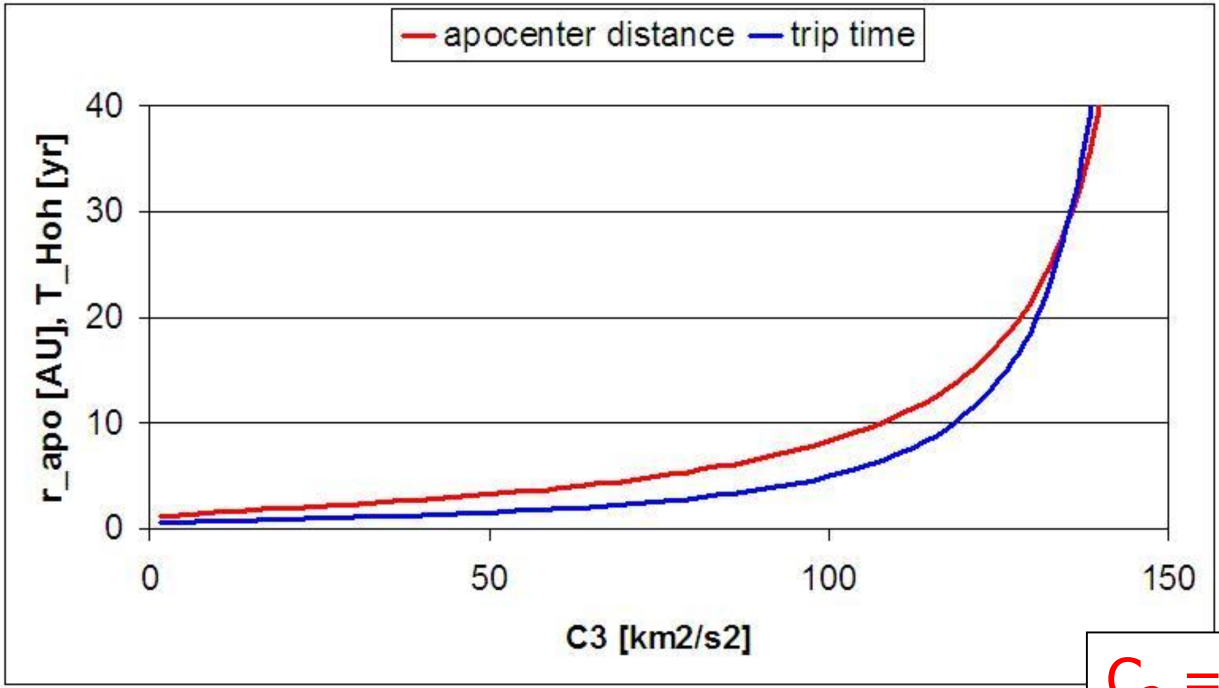
- scale of interplanetary travel >> scale of Earth-bound missions
- orbits of planets more-or-less circular (except Mercury and Pluto)
- orbits of planets more-or-less coplanar (except Pluto)
- 2-dimensional situation with circular orbits good 1st-order model
- (asteroids????)

Introduction (cnt'd)



[Virginia.edu, 2010]

Introduction (cnt'd)

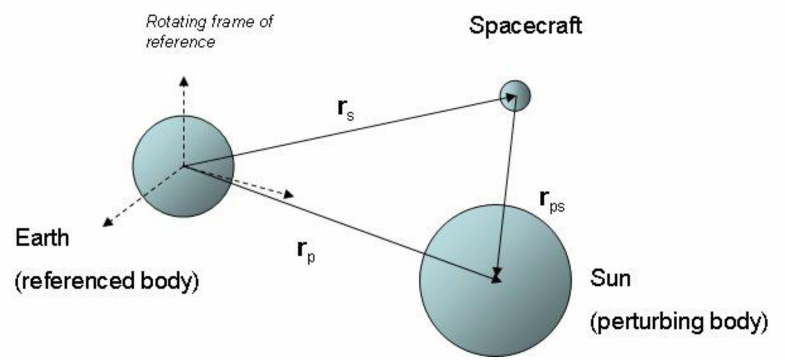


$C_3 = V_\infty^2$

- how can we escape from Earth gravity?
- how can we travel to other planets/asteroids in the most efficient way?
- how can we reach beyond 10 AU?

Basics

Interaction between 3 bodies:



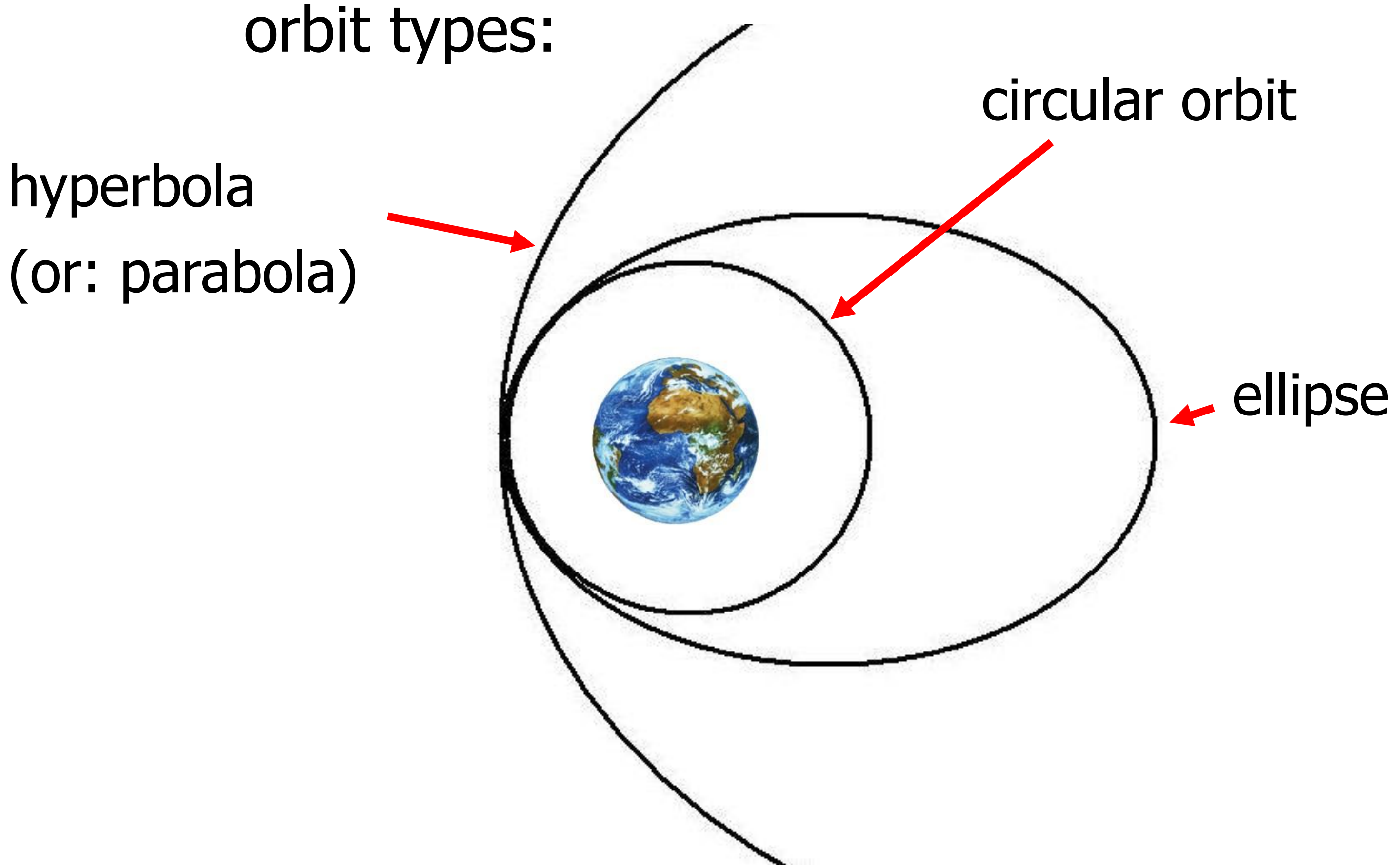
definition Sphere of Influence:

$$\frac{acc_{Sun,3rd}}{acc_{Earth,main}} = \frac{acc_{Earth,3rd}}{acc_{Sun,main}}$$

$$r_{SoI} = r_{3rd} \left(\frac{M_{main}}{M_{3rd}} \right)^{0.4}$$

- SoI Earth: ~930,000 km (0.006 AU; 0.6% distance Earth-Sun)
- at SoI Earth: $acc_{3rd}/acc_{main} = O(10^{-6})$
- 2-body approach very good 1st-order model

Basics (cnt'd)



Basics (cnt'd)

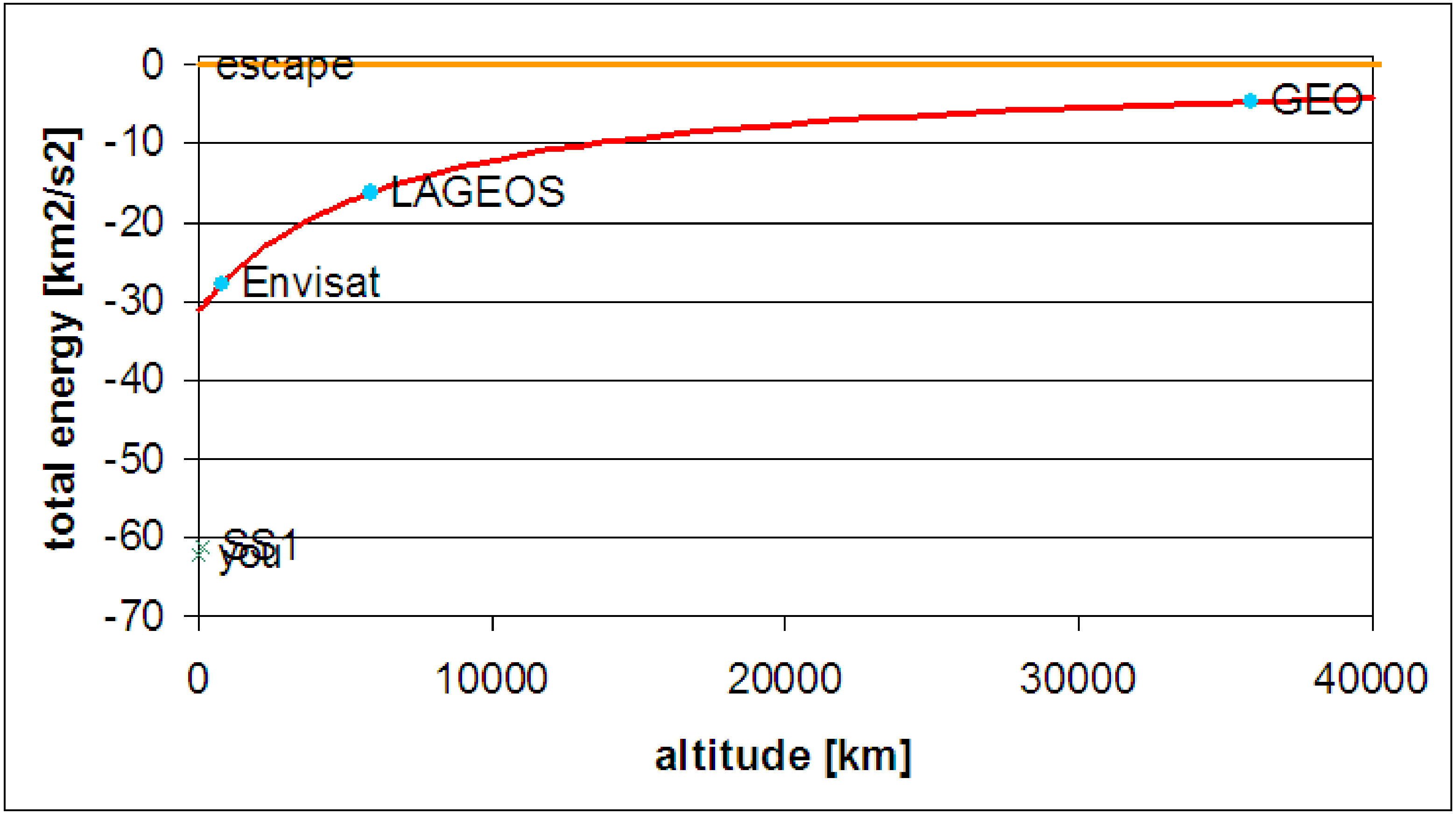
main characteristics:

symbol	meaning	ellipse	hyperbola
a	semi-major axis	> 0	< 0
e	eccentricity	< 1	> 1
E	(specific) energy	< 0	> 0
$r(\theta)$	radial distance	$a(1-e^2)/(1+e\cos(\theta))$	
r_{min}	minimum distance (pericenter)	$a(1-e)$	
r_{max}	maximum distance (apocenter)	$a(1+e)$	∞
V	velocity	$\sqrt{[\mu(2/r - 1/a)]}$	
V	velocity		$\sqrt{[V_{esc}^2 + V_\infty^2]}$

Basics (cnt'd)

satellite	**altitude [km]**	**specific energy [$km^2/s^2/kg$]**
SpaceShipOne	100+ (culmination)	-61.5
ENVISAT	800	-27.8
LAGEOS	5,900	-16.2
GEO	35,900	-4.7
Moon	384,000	-0.5
in parking orbit	185	-30.4
Hohmann orbit to Mars	185 (after 1st ΔV)	+4.3

Basics (cnt'd)

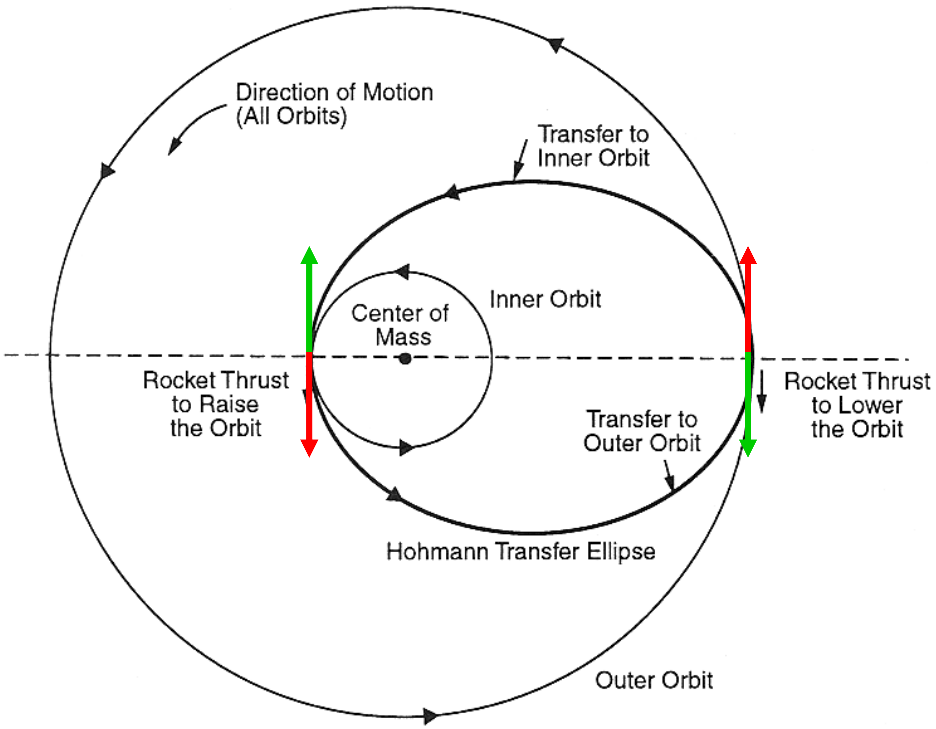


total energy [km2/s2]
0
-10
-20
-30
-40
-50
-60
-70
escape
GEO
LAGEOS
Envisat
SS1
you
0
10000
20000
30000
40000
altitude [km]

Hohmann transfer

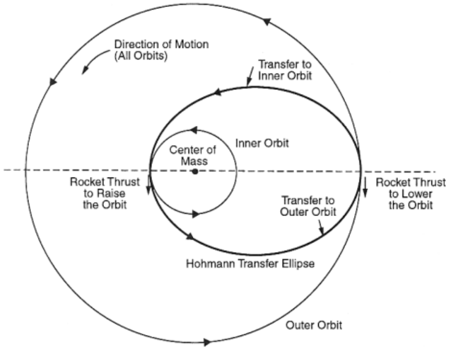
Hohmann transfer between orbits around Earth:

- coplanar orbits
- circular initial, target orbits
- impulsive shots
- transfer orbit touches tangentially
- minimum energy



[Wertz, 2009]

Hohmann transfer (cnt'd)



Hohmann transfer between orbits around Earth (cnt'd):

$$a_T = \frac{1}{2}(r_1 + r_2)$$

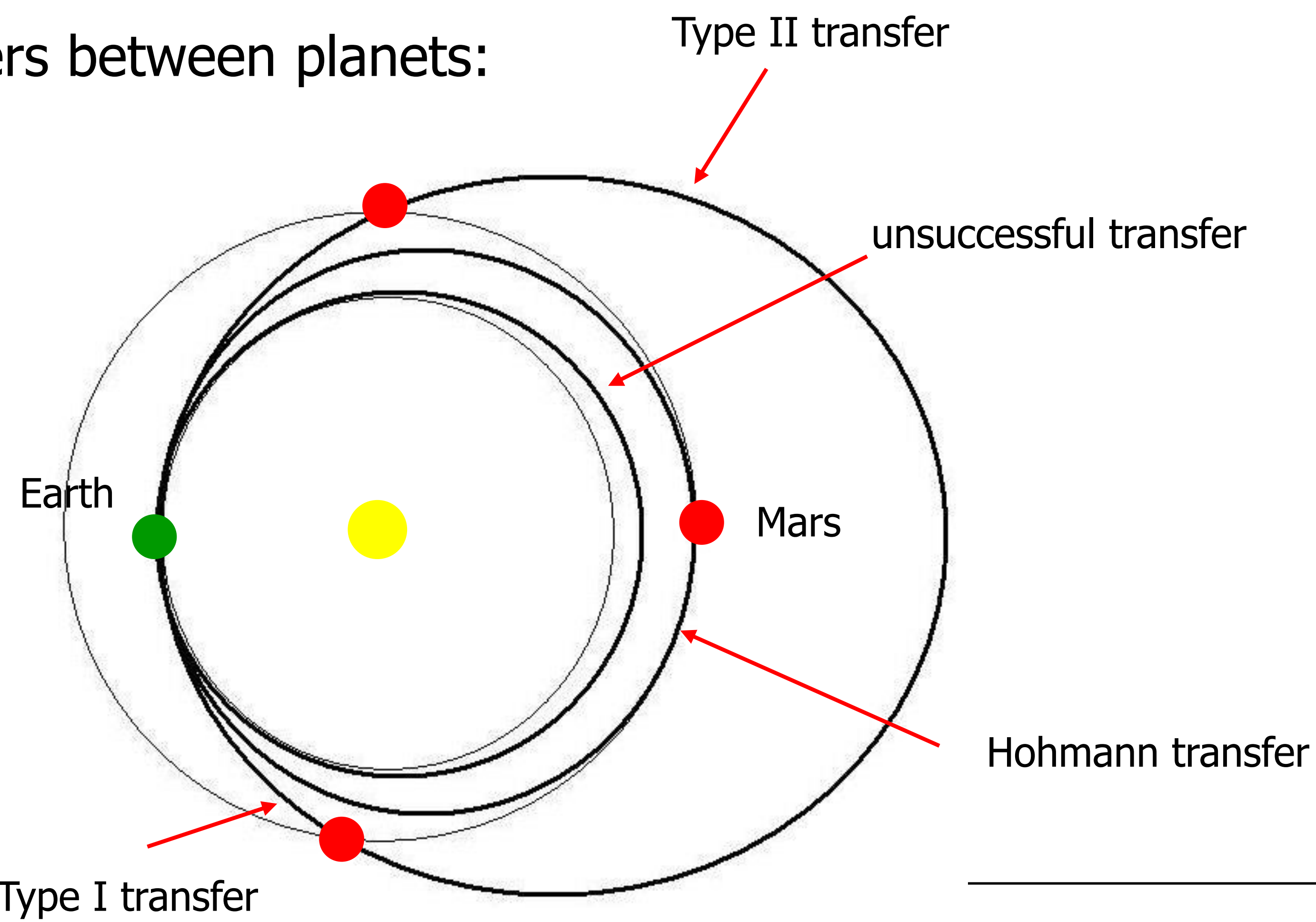
$$\Delta V_1 = V_{per,T} - V_{c,1} = \sqrt{\mu_{Earth}\left(\frac{2}{r_1} - \frac{1}{a_T}\right)} - \sqrt{\frac{\mu_{Earth}}{r_1}}$$

$$\Delta V_2 = V_{c,2} - V_{apo,T} = \sqrt{\frac{\mu_{Earth}}{r_2}} - \sqrt{\mu_{Earth}\left(\frac{2}{r_2} - \frac{1}{a_T}\right)}$$

$$T_{transfer} = \frac{1}{2}T_T = \pi\sqrt{\frac{a_T^3}{\mu_{Earth}}}$$

Hohmann transfer (cnt'd)

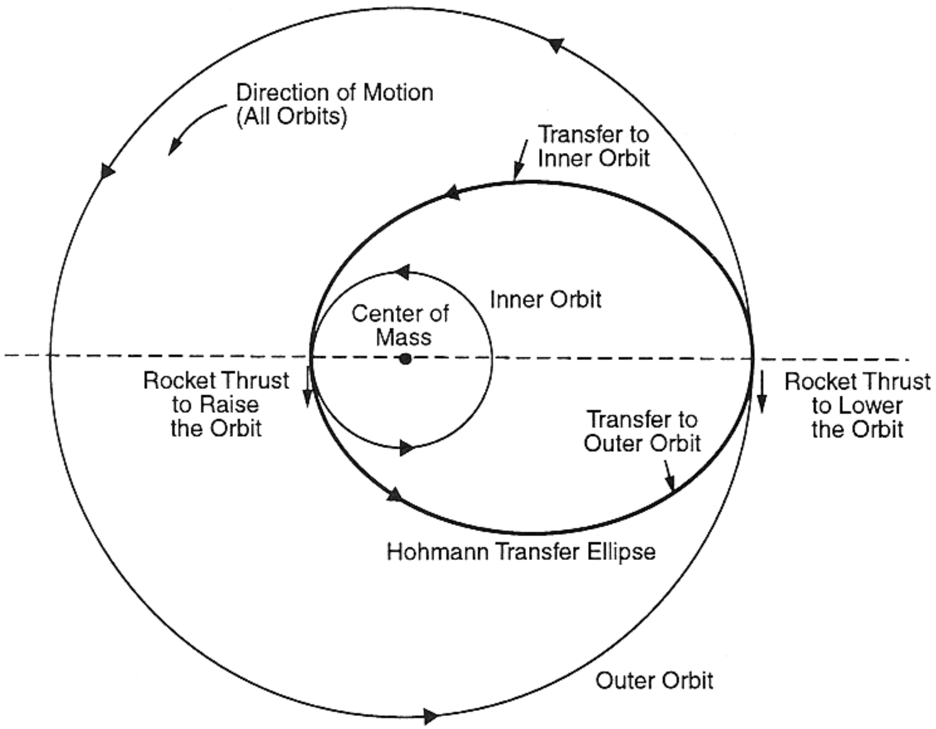
transfers between planets:



Hohmann transfer (cnt’d)

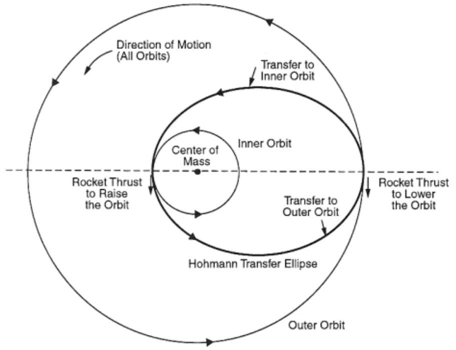
Hohmann transfer between planets around <u>Sun</u>:

- coplanar orbits
- circular orbits departure and target planet
- impulsive shots
- transfer orbit touches tangentially
- minimum energy



[Wertz, 2009]

Hohmann transfer (cnt'd)



Hohmann transfer between planets around Sun (cnt'd):

$$a_T = \frac{1}{2}(r_{dep} + r_{tar})$$

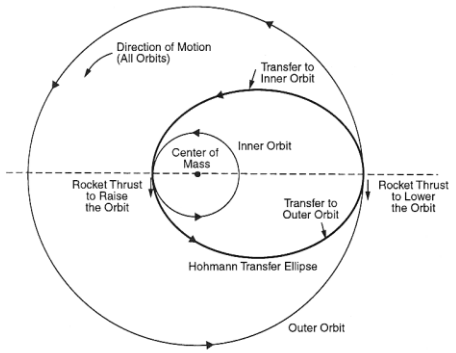
$$V_{\infty,1} = V_{per,T} - V_{dep,1} = \sqrt{\mu_{Sun}\left(\frac{2}{r_{dep}} - \frac{1}{a_T}\right)} - \sqrt{\frac{\mu_{Sun}}{r_{dep}}}$$

$$V_{\infty,2} = V_{tar,2} - V_{apo,T} = \sqrt{\frac{\mu_{Sun}}{r_{tar}}} - \sqrt{\mu_{Sun}\left(\frac{2}{r_{tar}} - \frac{1}{a_T}\right)}$$

$$T_{transfer} = \frac{1}{2} T_T = \pi \sqrt{\frac{a_T^3}{\mu_{Sun}}}$$

Question: identical to transfer around Earth?

Hohmann transfer (cnt'd)



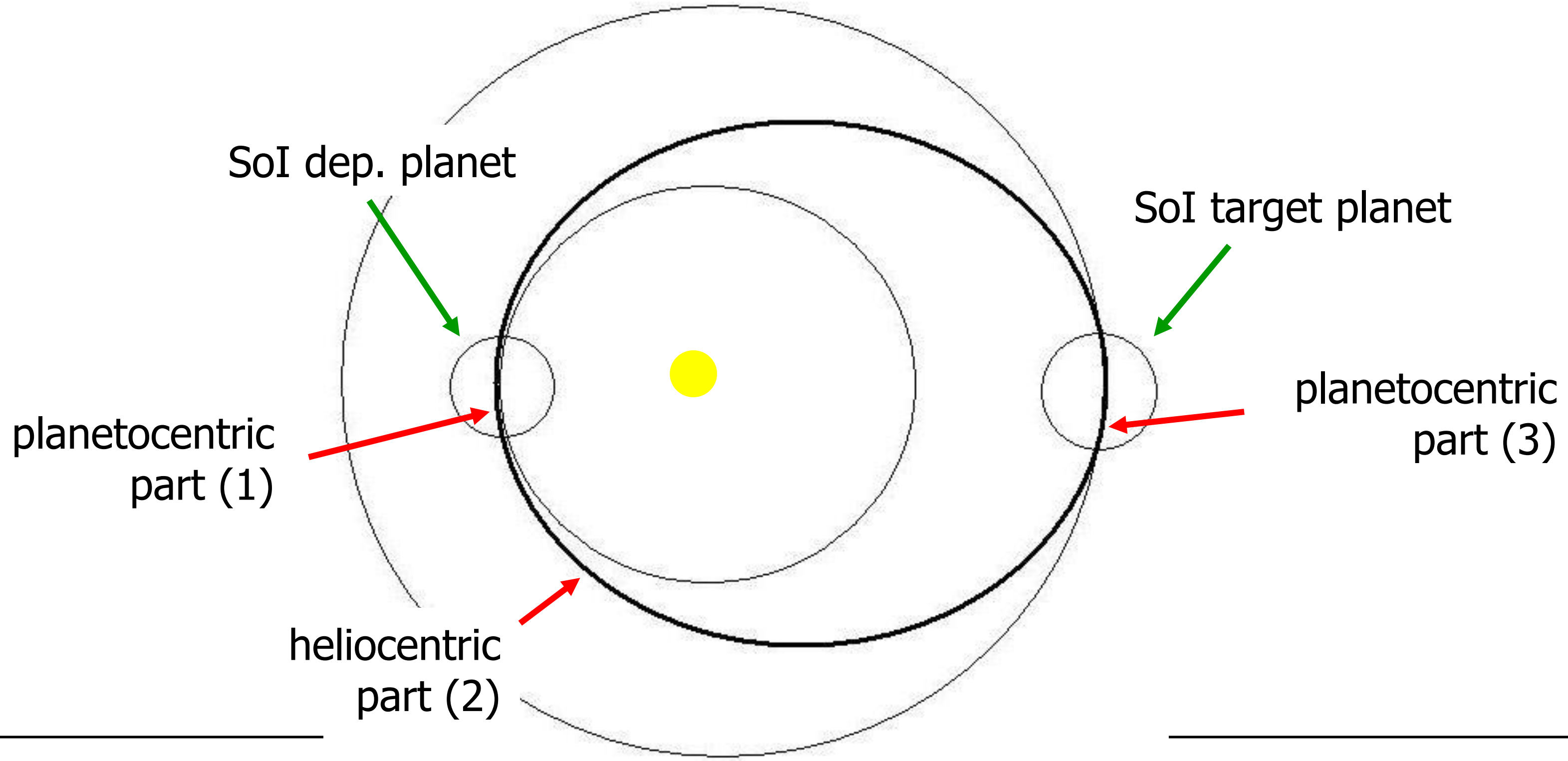
Hohmann transfer between planets around Sun (cnt'd):

Answer:

NO!!!

Hohmann transfer (cnt'd)

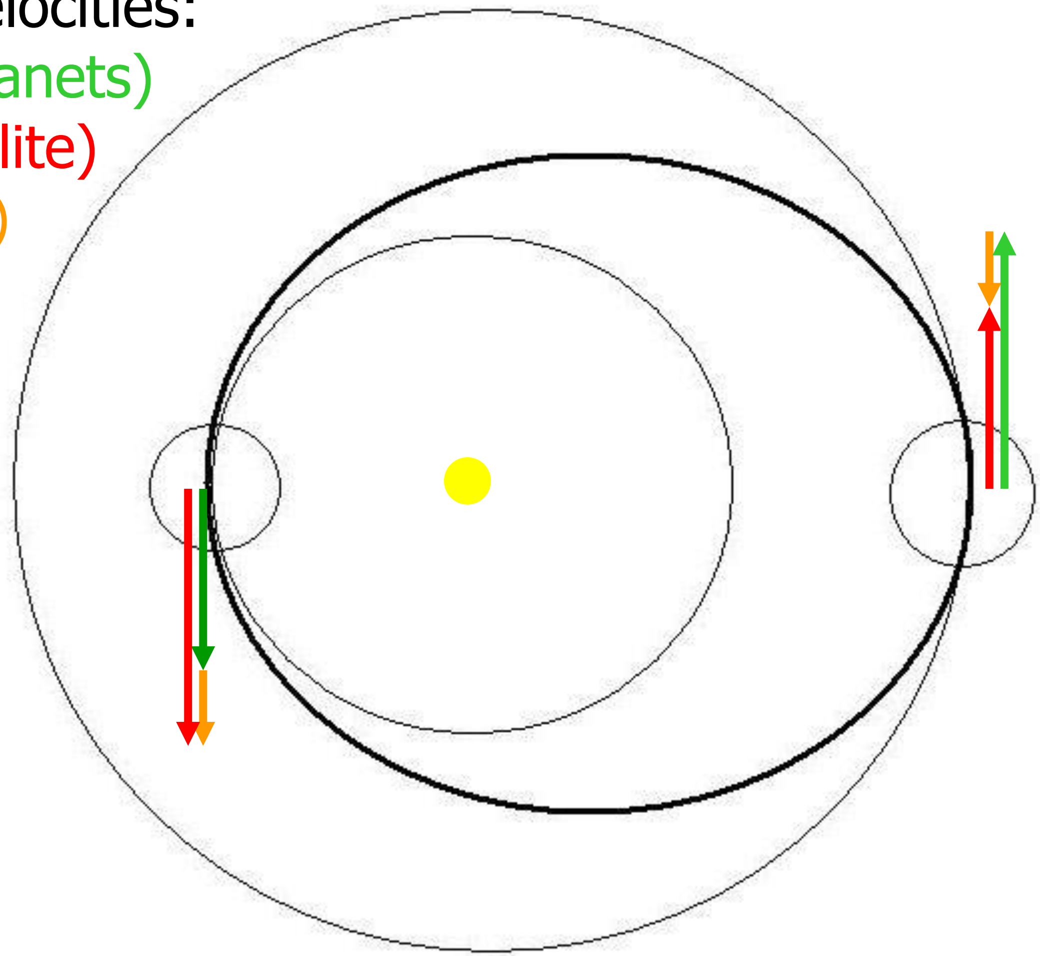
interplanetary trajectory: succession of 3 influence areas



Hohmann transfer (cnt'd)

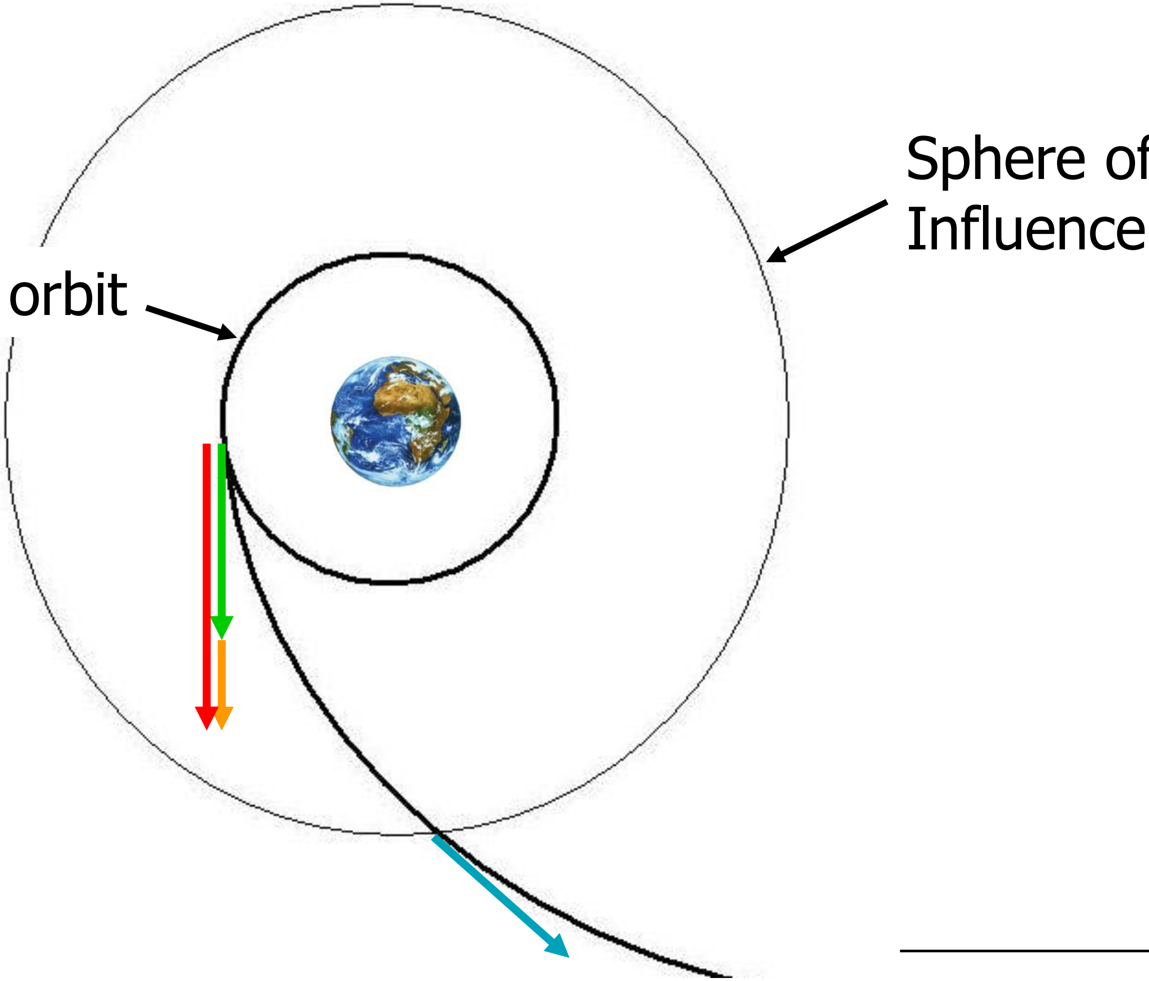
heliocentric velocities:

- V_{dep}, V_{tar} (planets)
- V_1, V_2 (satellite)
- V_∞ (relative)



Hohmann transfer (cnt'd)

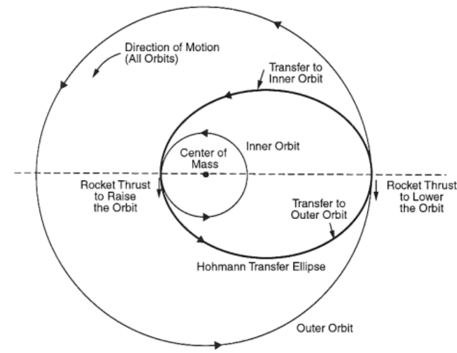
planetocentric scale:



planetocentric satellite velocities:

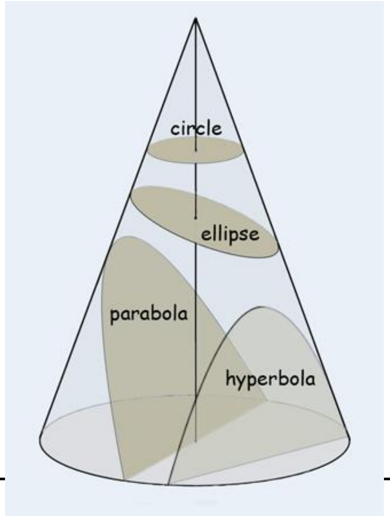
- V_c (parking orbit)
- ΔV (maneuver)
- V_0 (hyperbola)
- V_∞ (excess velocity)

Hohmann transfer (cnt'd)



Hohmann transfer between planets around <u>Sun</u> (cnt'd):

- transfer starts in parking orbit around departure planet
- planetocentric until leaving SoI
- relative velocity when crossing SoI: V_∞
- V_∞ achieved by maneuver ΔV in parking orbit
- similarly around target planet
- succession of 3 2-body problems
- "patched conics"



Hohmann transfer (cnt'd)

Essential difference between Hohmann transfer around Earth and around Sun:

- Earth missions: ΔV directly changes velocity from V_{circ} to V_{per} (or V_{apo}) of Hohmann transfer orbit
- interplanetary missions: ΔV changes velocity from V_{circ} to value (larger than) V_{esc}, which results in V_{∞}

Q: trips to the Moon?

Hohmann transfer (cnt'd)

Main elements of computation interplanetary Hohmann transfer:

- compute semi-major axis transfer orbit
- compute V_∞ at departure and target planet
- compute pericenter velocity of planetocentric hyperbolae
- compute ΔV's

Hohmann transfer (cnt'd)

Earth(185) → Mars(500)

Recipe (1-2):

step	parameter	expression	example
1	V_{dep} (heliocentric velocity of departure planet)	$V_{dep} = \sqrt{(\mu_{Sun}/r_{dep})}$	29.785 km/s
2	V_{tar} (heliocentric velocity of target planet)	$V_{tar} = \sqrt{(\mu_{Sun}/r_{tar})}$	24.130 km/s
3	V_{c0} (circular velocity around departure planet)	$V_{c0} = \sqrt{(\mu_{dep}/r_0)}$	7.793 km/s
4	V_{c3} (circular velocity around target planet)	$V_{c3} = \sqrt{(\mu_{tar}/r_3)}$	3.315 km/s
5	a_{tr} (semi-major axis of transfer orbit)	$a_{tr} = (r_{dep} + r_{tar}) / 2$	188.77×10^6 km
6	e_{tr} (eccentricity of transfer orbit)	$e_{tr} = \lvert r_{tar} - r_{dep} \rvert / (r_{tar} + r_{dep})$	0.208
7	V_1 (heliocentric velocity at departure position)	$V_1 = \sqrt{[\mu_{Sun}(2/r_{dep}-1/a_{tr})]}$	32.729 km/s
8	V_2 (heliocentric velocity at target position)	$V_2 = \sqrt{[\mu_{Sun}(2/r_{tar}-1/a_{tr})]}$	21.481 km/s

Hohmann transfer (cnt'd)

Recipe (2-2):

step	parameter	expression	example
9	$V_{\infty,1}$ (excess velocity at departure planet)	$V_{\infty,1} = \lvert V_1 - V_{dep} \rvert$	2.945 km/s
10	$V_{\infty,2}$ (excess velocity at target planet)	$V_{\infty,2} = \lvert V_2 - V_{tar} \rvert$	2.649 km/s
11	V_0 (velocity in pericenter of hyperbola around departure planet)	$V_0 = \sqrt{(2\mu_{dep}/r_0 + V_{\infty,1}^2)}$	11.408 km/s
12	V_3 (velocity in pericenter of hyperbola around target planet)	$V_3 = \sqrt{(2\mu_{tar}/r_3 + V_{\infty,2}^2)}$	5.385 km/s
13	ΔV_0 (maneuver in pericenter around departure planet)	$\Delta V_0 = \lvert V_0 - V_{c0} \rvert$	3.615 km/s
14	ΔV_3 (maneuver in pericenter around target planet)	$\Delta V_3 = \lvert V_3 - V_{c3} \rvert$	2.070 km/s
15	ΔV_{tot} (total velocity increase)	$\Delta V_{tot} = \Delta V_0 + \Delta V_3$	5.684 km/s
16	T_{tr} (transfer time)	$T_{tr} = \pi \sqrt{(a_{tr}^3/\mu_{Sun})}$	0.709 yr

Hohmann transfer (cnt'd)

target	ΔV_{dep} [km/s]	$V_{\infty,dep}$ [km/s]	C_3 [km²/s²]	ΔV_{tar} [km/s]	ΔV_{total} [km/s]
Mercury	5.556	7.533	56.7	7.565	13.122
Venus	3.507	2.495	6.2	3.258	6.765
Mars	3.615	2.945	8.7	2.086	5.701
Jupiter	6.306	8.793	77.3	16.905	23.211
Saturn	7.284	10.289	105.9	10.343	17.627
Uranus	7.978	11.281	127.3	6.475	14.452
Neptune	8.247	11.654	135.8	6.925	15.172
Pluto	8.363	11.814	139.6	3.048	11.412

What is possible? departure? arrival? total?

Hohmann transfer (cnt'd)

Question 1:

Consider a Hohmann transfer from Earth to Mercury. Begin and end of the transfer is in a parking orbit at 500 km altitude, for both cases.

a) What are the semi-major axis and the eccentricity of the transfer orbit?
b) What is the trip time?
c) What are the excess velocities at Earth and at Mercury (*i.e.*, heliocentric)?
d) What are the circular velocities in the parking orbit around Earth and Mercury (*i.e.*, planetocentric)?
e) What are the ΔV's of the maneuvers to be executed at Earth and Mercury? What is the total ΔV?

Data: μ_{Sun}=1.3271 × 10^{11} km^3/s^2; μ_{Earth}=398,600 km^3/s^2; $\mu_{Mercury}$=22,034 km^3/s^2; R_{Earth}=6378 km; $R_{Mercury}$=2440 km; distance Earth-Sun = 1 AU; distance Mercury-Sun = 0.387 AU; 1 AU = 149.6 × 10^6 km.

Answers: see footnote below **(BUT TRY YOURSELF FIRST!!)**

Hohmann transfer (cnt'd)

Question 2:

Consider a Hohmann transfer from Mars to Jupiter. Begin and end of the transfer is in a parking orbit at 500 km and 50,000 km altitude, respectively.

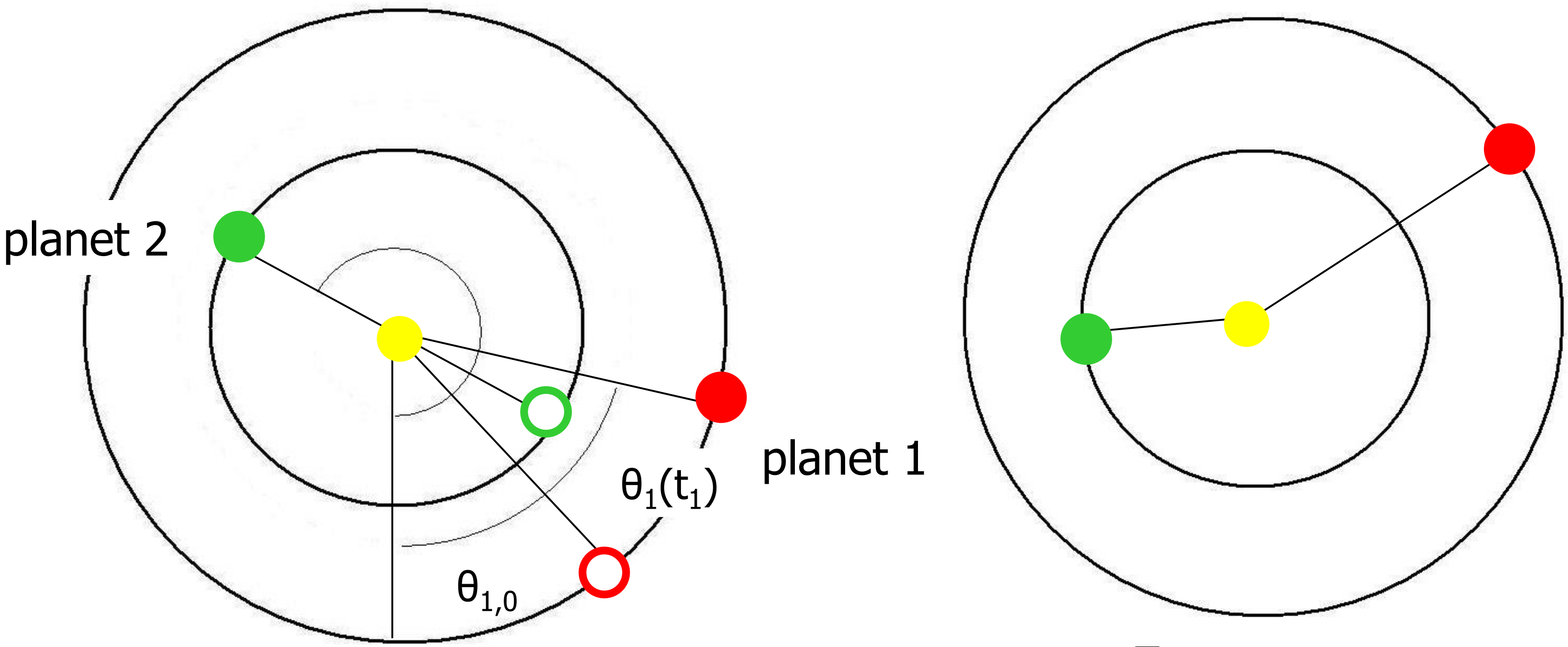
a) What are the semi-major axis and the eccentricity of the transfer orbit?
b) What is the trip time?
c) What are the excess velocities at Mars and at Jupiter (*i.e.*, heliocentric)?
d) What are the circular velocities in the parking orbit around Mars and Jupiter (*i.e.*, planetocentric)?
e) What are the ΔV's of the maneuvers to be executed at Mars and Jupiter? What is the total ΔV?

Data: μ_{Sun}=1.3271 × 10^{11} km^3/s^2; μ_{Mars}=42,832 km^3/s^2; $\mu_{Jupiter}$=1.267 × 10^8 km^3/s^2; R_{Mars}=3397 km; $R_{Jupiter}$=71,492 km; distance Mars-Sun = 1.52 AU; distance Jupiter-Sun = 5.20 AU; 1 AU = 149.6 × 10^6 km.

Answers: see footnote below **(BUT TRY YOURSELF FIRST!!)**

Timing

Synodic period (1):



$\Delta\theta(t_1) = \theta_2(t_1)-\theta_1(t_1)$

$t_2 = t_1 + T_{syn}$

$\Delta\theta(t_2) = \theta_2(t_2)-\theta_1(t_2) = \Delta\theta(t_1) + 2\pi$

Timing (cnt'd)

Synodic period (2):

positions of planet 1 and 2:

$$\begin{aligned}\theta_1(t) &= \theta_{1,0} + n_1\,(t - t_0)\\ \theta_2(t) &= \theta_{2,0} + n_2\,(t - t_0)\end{aligned}$$

difference:

$$\Delta\theta(t) = \theta_2(t) - \theta_1(t) = (\theta_{2,0} - \theta_{1,0}) + (n_2 - n_1)\,(t - t_0)$$

geometry repeats after T_{syn}:

$$\Delta\theta(t_2) - \Delta\theta(t_1) = 2\pi = (n_2 - n_1)\,(t_2 - t_1) = (n_2 - n_1)\,T_{syn}$$

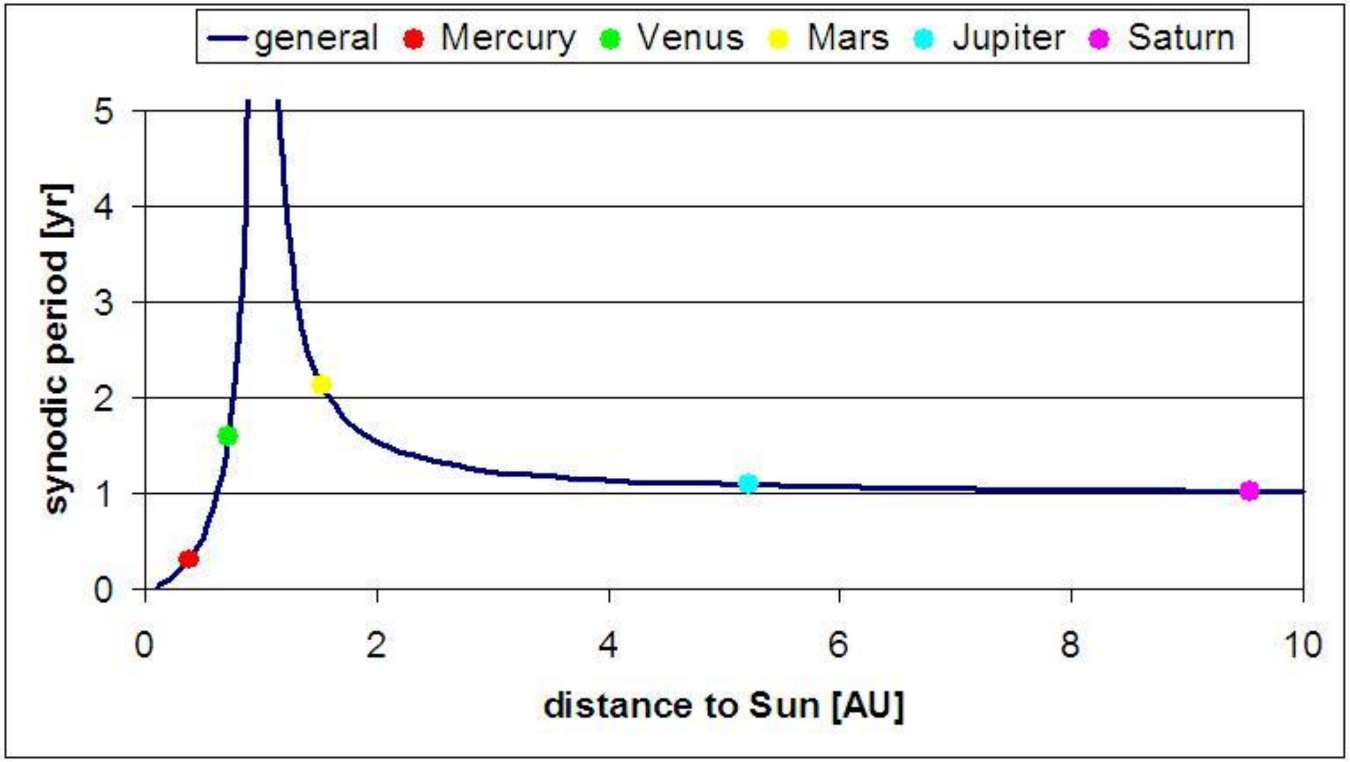
or

$$\frac{1}{T_{syn}} = \left|\frac{1}{T_2} - \frac{1}{T_1}\right|$$

Def: synodic period = time interval after which relative geometry repeats

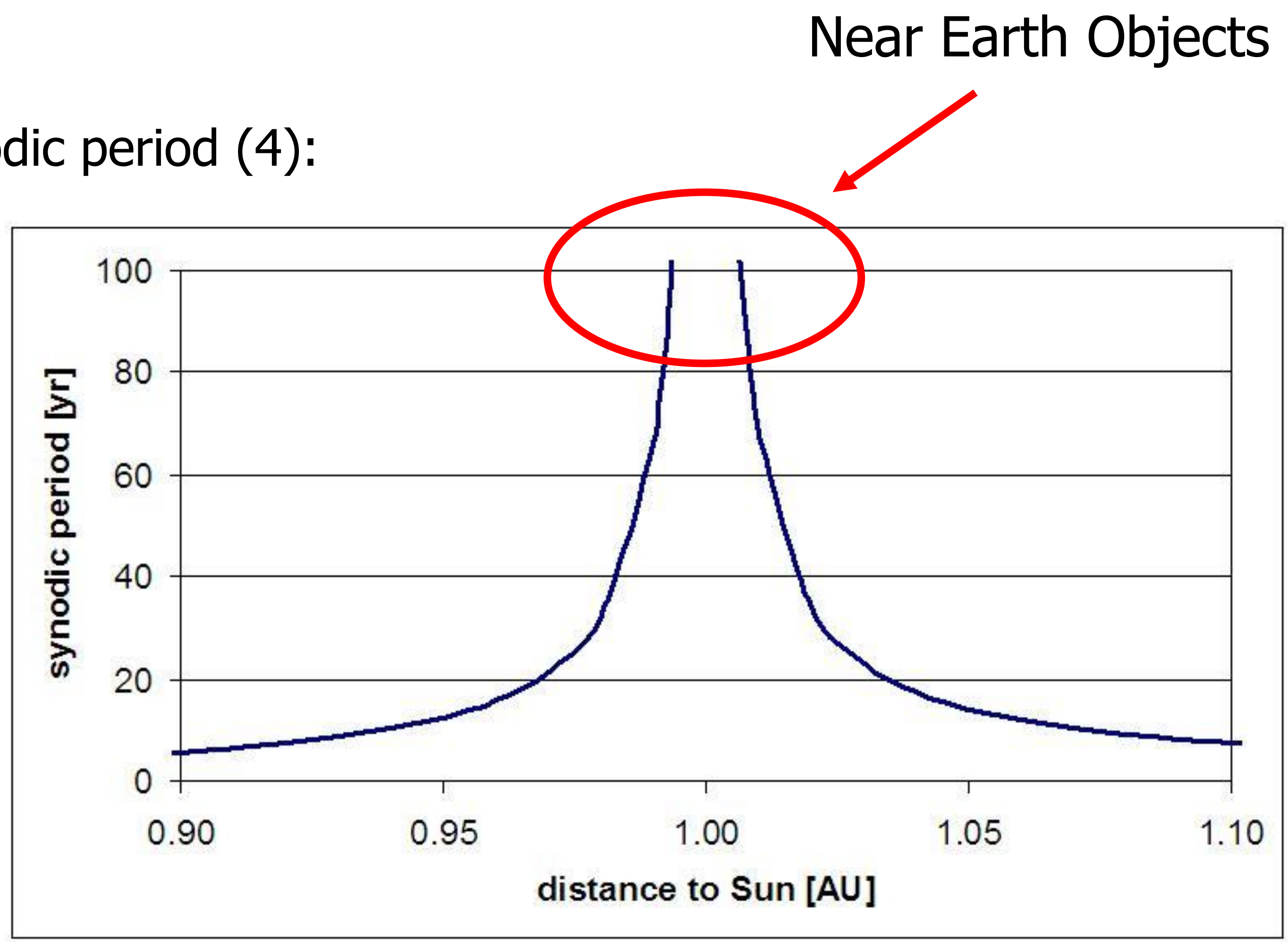
Timing (cnt'd)

Synodic period (3):



Timing (cnt'd)

Synodic period (4):



Timing (cnt'd)

Question 3:

Consider a trip from Earth to Saturn.

a) Compute the orbital period of the Earth around the Sun.
b) Compute the orbital period of Saturn around the Sun.
c) Derive a general equation for the synodic period, *i.e.*, the period after which the relative constellation of two planets repeats.
d) Compute the synodic period of the combination Earth-Saturn.
e) Now consider a Near-Earth Object (NEO) with a semi-major axis of 1.05 AU (circular orbit). Compute the orbital period of this object.
f) Compute the synodic period of the combination Earth-NEO.
g) Discuss the physical reason for the difference between the answers for questions (d) and (f).

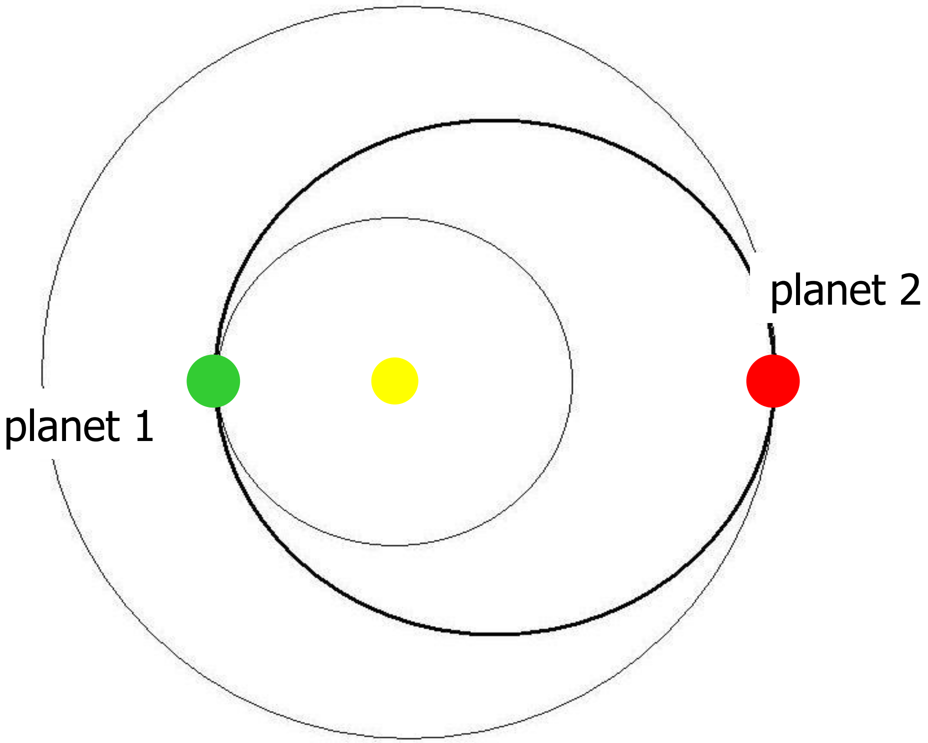
Data: $\mu_{Sun} = 1.3271 \times 10^{11}$ km^3/s^2; distance Earth-Sun = 1 AU; distance Saturn-Sun = 9.54 AU; 1 AU = 149.6×10^6 km.

Answers: see footnote below **(BUT TRY YOURSELF FIRST!!)**

Timing (cnt'd)

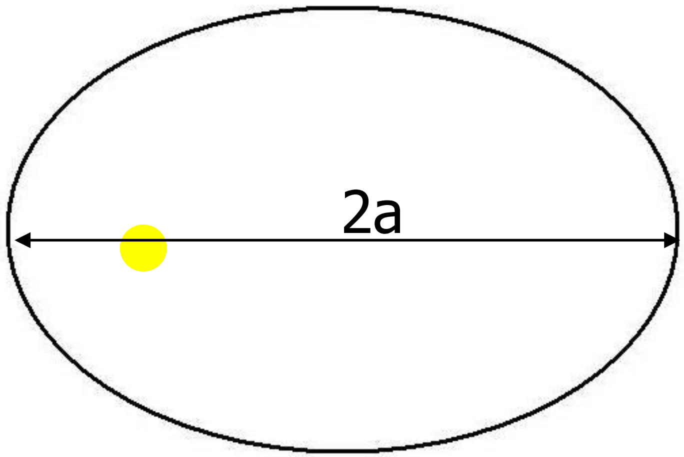
Interplanetary Hohmann transfer (1):

- when do we leave?
- when do we arrive?
- travel time?



Timing (cnt'd)

Interplanetary Hohmann transfer (2):



transfer time:

$$T_H = \frac{1}{2} T_{orbit} = \pi \sqrt{\frac{a^3}{\mu}}$$

Example:

Earth → Mars: a = (1+1.52)/2 = 1.26 AU → $T_H = 22.3 \times 10^6$ s = 258.3 days

Timing (cnt'd)

Interplanetary Hohmann transfer (3):

positions at epoch 1:

$$\theta_1(t_1) = \theta_1(t_0) + n_1\,(t_1 - t_0)$$
$$\theta_2(t_1) = \theta_2(t_0) + n_2\,(t_1 - t_0)$$
$$\theta_{sat}(t_1) = \theta_1(t_1) = \theta_1(t_0) + n_1\,(t_1 - t_0)$$

positions at epoch 2:

$$\theta_1(t_2) = \theta_1(t_1) + n_1\,T_H$$
$$\theta_2(t_2) = \theta_2(t_1) + n_2\,T_H$$
$$\theta_{sat}(t_2) = \theta_{sat}(t_1) + \pi = \theta_1(t_1) + \pi = \theta_2(t_2)$$

so

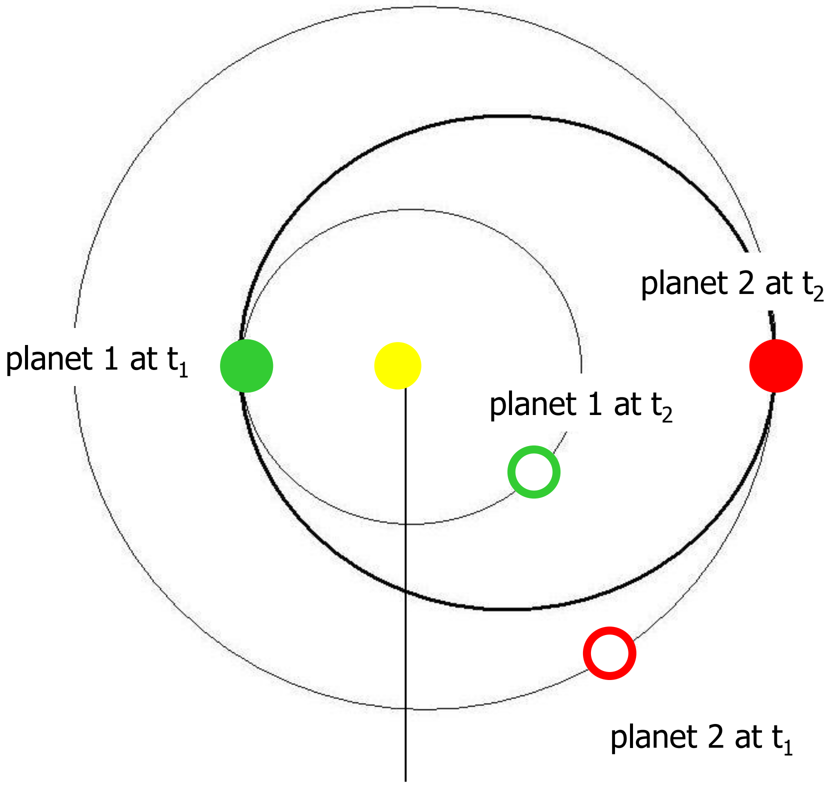
$$\theta_1(t_0) + n_1\,(t_1 - t_0) + \pi = \theta_2(t_0) + n_2\,(t_1 - t_0) + n_2\,T_H$$

or

$$t_1 = t_0 + \frac{\theta_2(t_0) - \theta_1(t_0) + n_2\,T_H - \pi}{n_1 - n_2}$$

and

$$t_2 = t_1 + T_H$$



Timing (cnt'd)

Interplanetary Hohmann transfer (4):

Example: Earth → Mars

Data:

- t_0 = January 1, 2000, 12^h
- $\theta_{Earth}(t_0) = 100.46°$; $\theta_{Mars}(t_0) = 355.45°$
- a_{Earth} = 1 AU; a_{Mars} = 1.52 AU
- $\mu_{Sun} = 1.3271 \times 10^{11}$ km^3/s^2

Results:

- a_H = 1.26 AU → T_H = 258 days = 0.709 yr
- n_{Earth} = 0.9856°/day; n_{Mars} = 0.5241°/day
  ➔ t_1 = 456 days after t_0 ≡ April 1, 2001
  ➔ t_2 = 715 days after t_0 ≡ Dec 16, 2001

Other options: modulo $T_{synodic}$!!

Timing (cnt'd)

Question 4:

Consider a Hohmann transfer from planet 1 to planet 2.

a) Derive the following general equations for the epoch of departure t_1 and the epoch of arrival t_2:

$$t_1 = t_0 + \frac{\theta_2(t_0) - \theta_1(t_0) + n_2\, T_H - \pi}{n_1 - n_2}$$

$$t_2 = t_1 + T_H$$

Here, t_0 is a common reference epoch, T_H is the transfer time in a Hohmann orbit, n_1 and n_2 are the mean motion of the two planets, and θ_1 and θ_2 are the true anomalies of the planetary positions, respectively. Assume circular orbits for both planets.

b) Consider a Hohmann transfer from Earth to Neptune. What is the transfer period?

c) Assuming that on January 1, 2010, θ_{Earth}=70° and $\theta_{Neptune}$=120°, what would be the epoch of departure (expressed in days w.r.t. this January 1)?

d) What would be the arrival epoch?

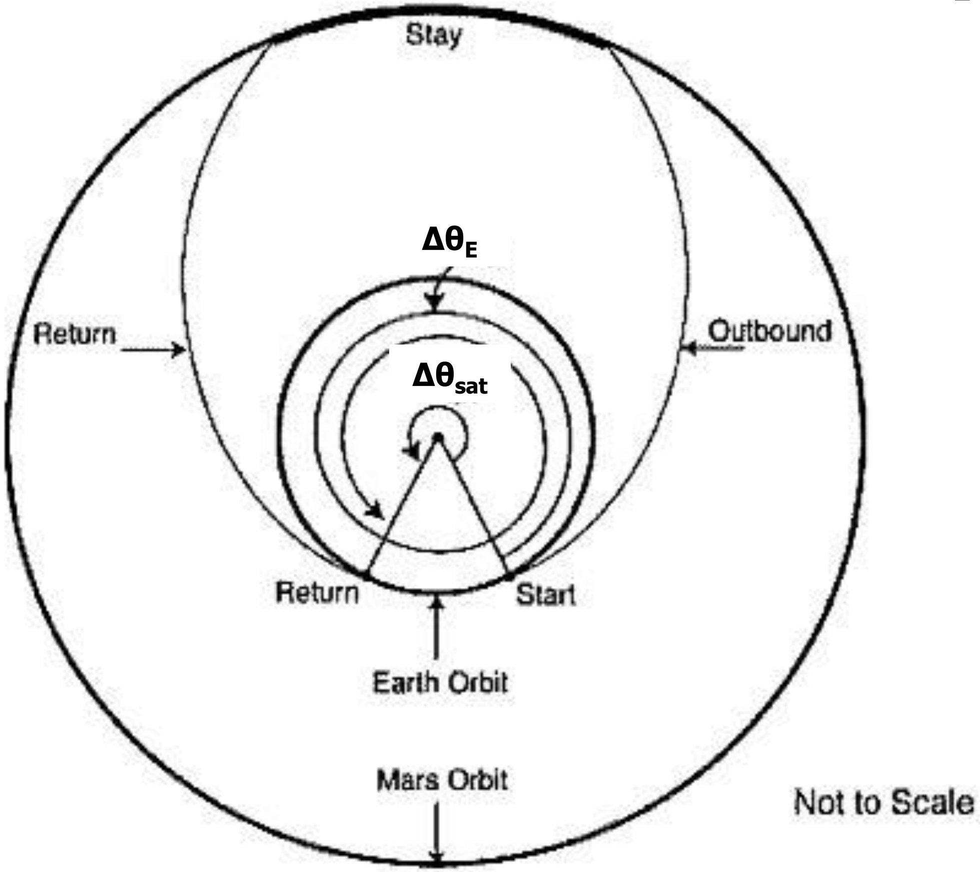
e) Can we change the launch window? If so, how? A qualitative answer is sufficient.

Data: μ_{Sun}=1.3271 × 10^{11} km^3/s^2; distance Earth-Sun = 1 AU; distance Neptune-Sun = 30.1 AU; 1 AU = 149.6 × 10^6 km.

Answers: see footnote below **(BUT TRY YOURSELF FIRST!!)**

Timing round-trip missions

after [Wertz, 2003]



Timing round-trip missions (cnt'd)

after [Wertz, 2011]

angle covered by Earth:

$$\Delta\theta_E = \Delta\theta_{sat} + 2\pi N$$

(N integer; fastest for Mars: N = 1; fastest for Venus: N = −1)

total trip time:

$$T = 2T_H + t_{stay}$$

$$\Delta\theta_E = \omega_E T = \omega_E (2T_H + t_{stay}) = \Delta\theta_{sat} + 2\pi N = \pi + \omega_M t_{stay} + \pi + 2\pi N$$

$$t_{stay} = \frac{2\pi(N+1) - 2\omega_E T_H}{\omega_E - \omega_M}$$

$$T = \frac{2\pi(N+1) - 2\omega_M T_H}{\omega_E - \omega_M}$$

Hohmann

Timing round-trip missions (cnt'd)

Example for round-trip travel times:

Earth → Jupiter → Earth

N [-]	stay time [yrs]	round trip time [yrs]
-4	-9.241	-3.779
-3	-8.149	-2.687
-2	-7.057	-1.595
-1	-5.965	-0.503
0	-4.873	0.589
+1	-3.781	1.681
+2	-2.689	2.773
+3	-1.597	3.866
+4	-0.505	4.958
+5	0.587	6.050
+6	1.679	7.142
+7	2.771	8.234

Verify!!

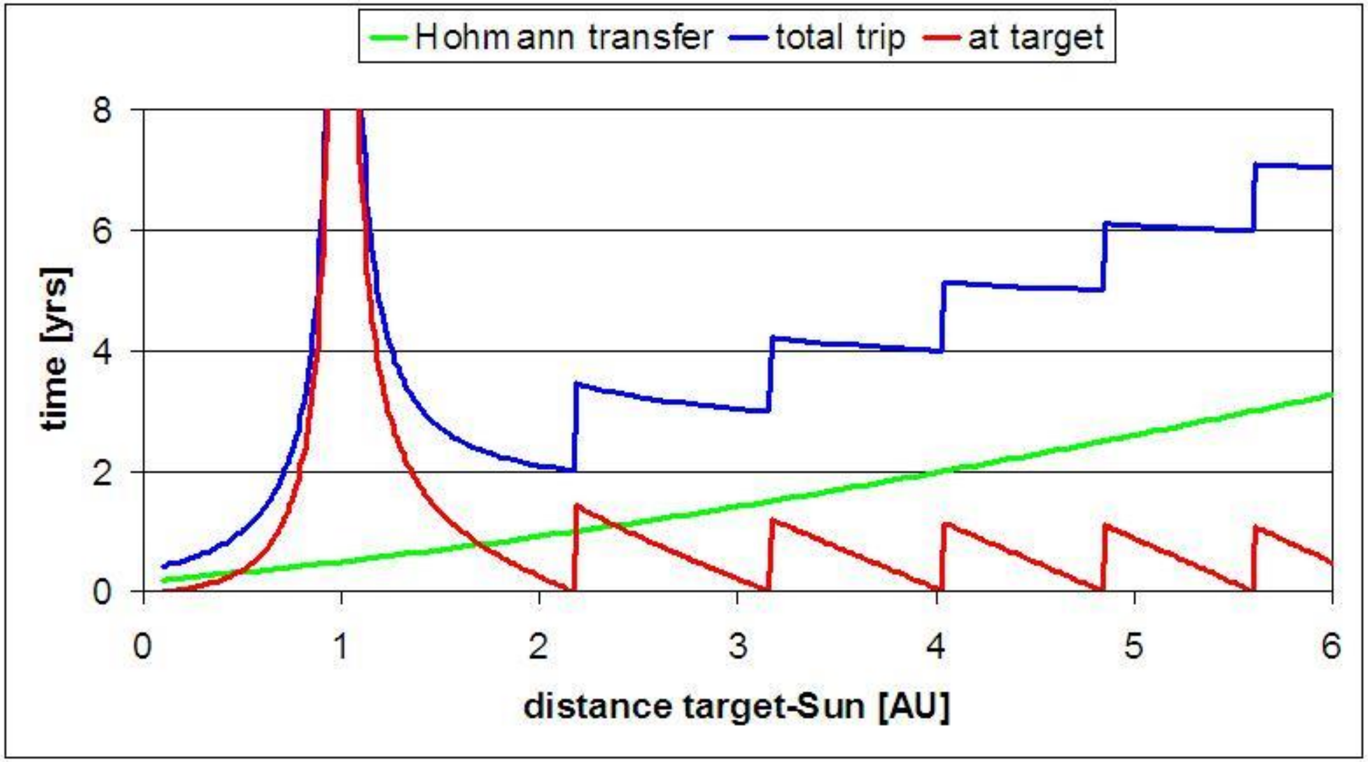
Timing round-trip missions (cnt'd)

Some examples for round-trip travel times:

target planet	mean orbit radius [AU]	angular motion [rad/s]	Hohmann transfer time [yrs]	stay time [yrs]	round trip time [yrs]	N [-]
Venus	0.723	3.236×10^{-7}	0.400	1.279	2.078	-1
Mars	1.524	1.059×10^{-7}	0.709	1.244	2.661	+1
Jupiter	5.203	1.667×10^{-8}	2.731	0.587	6.050	+5

Verify!!

Timing round-trip missions (cnt'd)



Timing round-trip missions (cnt'd)

Question 6:

Consider a round trip mission to Saturn.

a) Determine the Hohmann transfer time for a trip to Saturn.

b) Derive the relation for round trip travel time, for a mission to Saturn.

c) Derive the relation for the stay time at Saturn.

d) What would be the minimum stay time?

Data: μ_{Sun} = 1.3271 × 10^{11} km^3/s^2; distance Earth-Sun = 1.0 AU;
distance Saturn-Sun = 9.537 AU; 1 AU = 149.6 × 10^6 km.

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

Timing round-trip missions (cnt'd)

Question 7:

Consider a round trip mission to Mercury.

a) Determine the Hohmann transfer time for a trip to Mercury.

b) Derive the relation for round trip travel time, for a mission to Mercury.

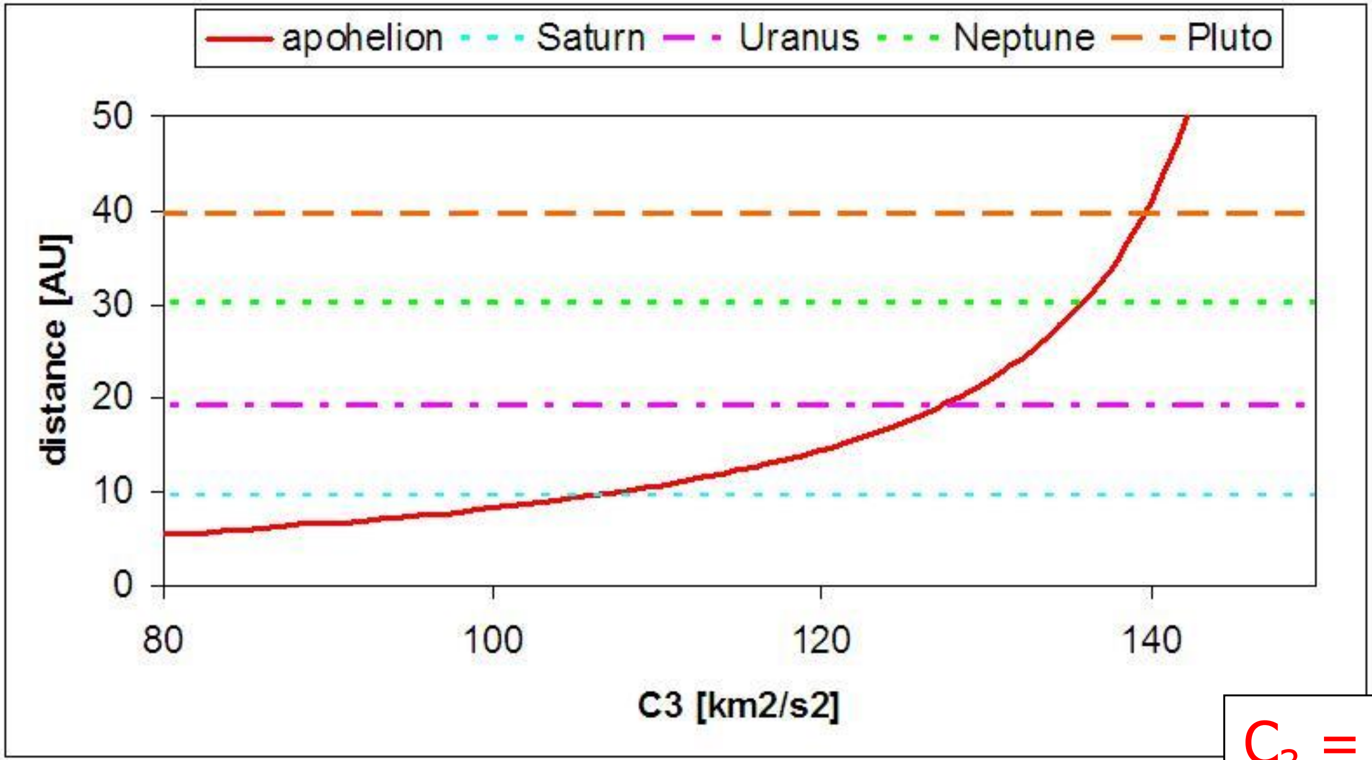
c) Derive the relation for the stay time at Mercury.

d) What would be the minimum stay time?

Data: $\mu_{Sun} = 1.3271 \times 10^{11}$ km^3/s^2; distance Earth-Sun = 1.0 AU; distance Mercury-Sun = 0.387 AU; 1 AU = 149.6×10^6 km.

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

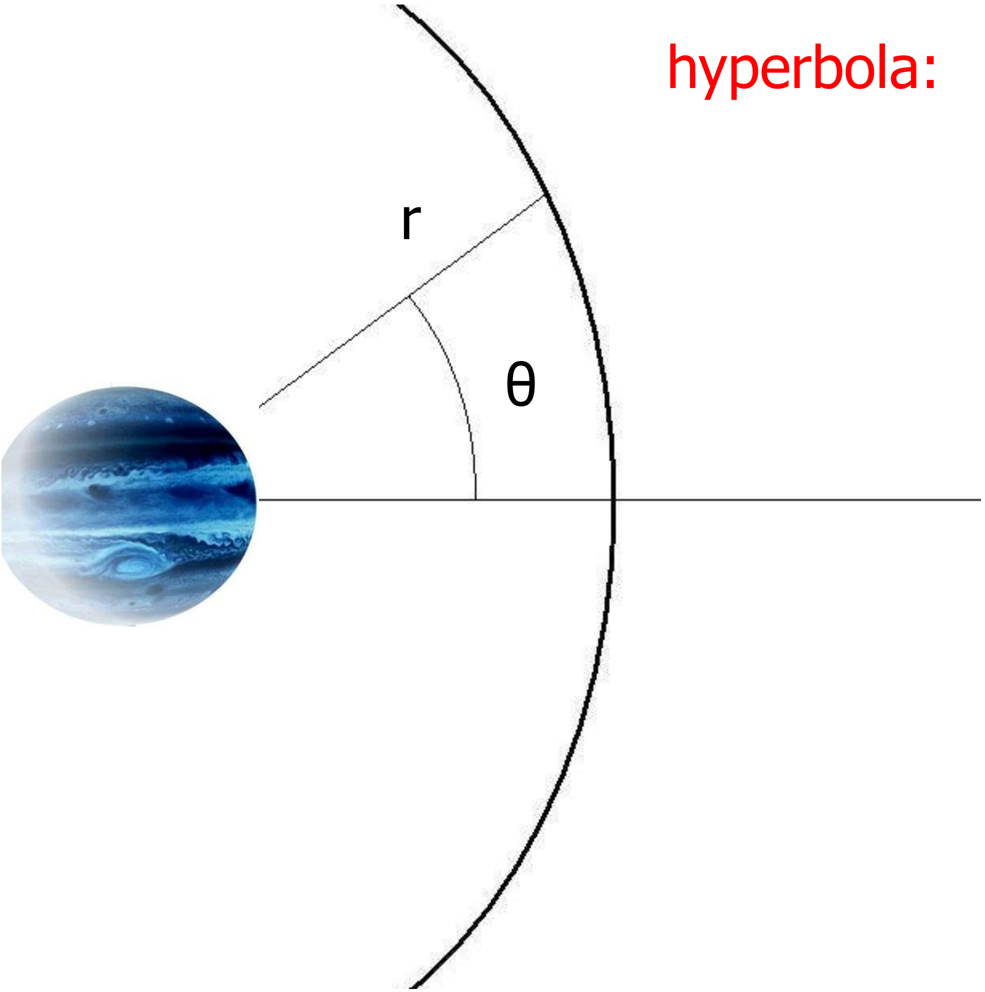
Gravity assist



$C_3 = V_\infty^2$

- Can we reach Saturn? Pluto?

Gravity assist (cnt'd)



hyperbola:

$$r = \frac{a\,(1-e^2)}{1+e\cos\theta}$$

$$a < 0$$

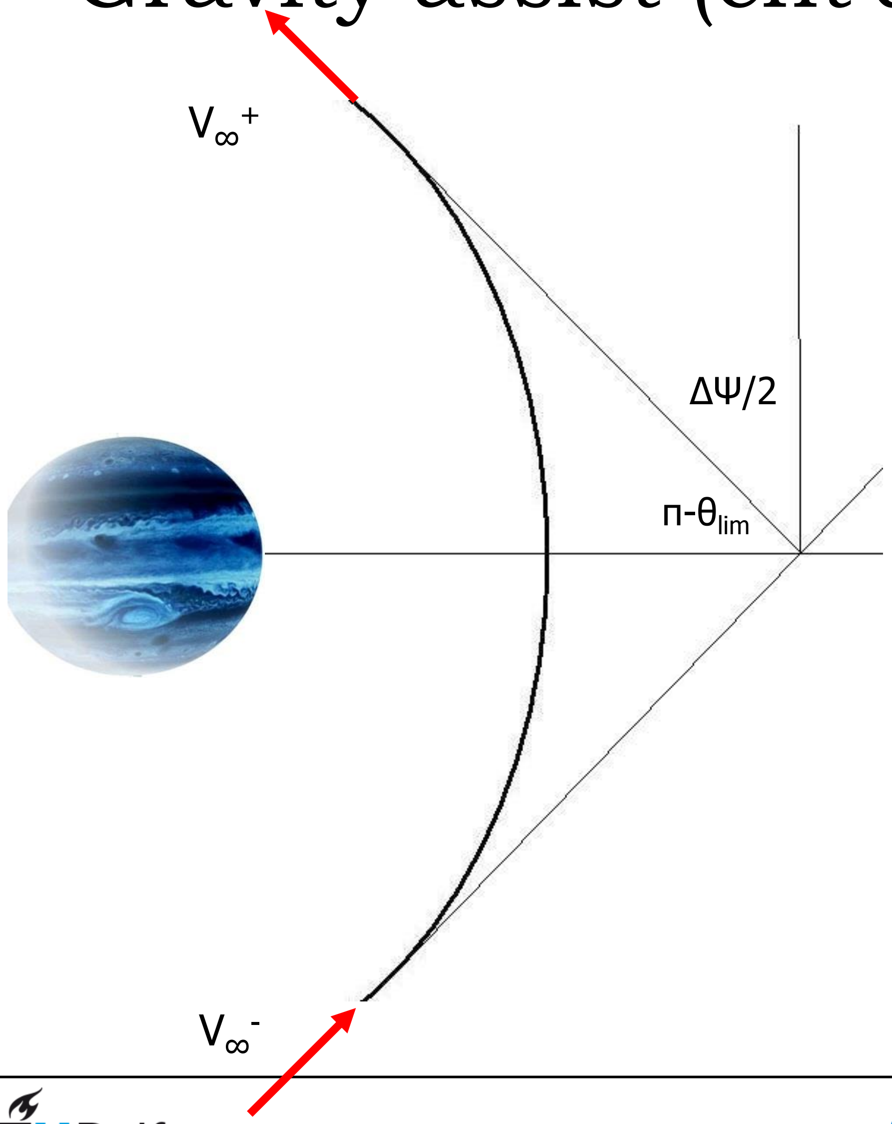
$$e > 1$$

$$\cos(\theta_{\mathrm{lim}}) = -\frac{1}{e}$$

$$\left|\mathbf{V}_\infty^-\right| = \left|\mathbf{V}_\infty^+\right|$$

$$\mathbf{V}_\infty^- \neq \mathbf{V}_\infty^+$$

Gravity assist (cnt'd)



$$\Delta\Psi = 2\left[\frac{\pi}{2} - (\pi - \theta_{lim})\right] = 2\theta_{lim} - \pi$$

$$\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

$$\frac{V_\infty^2}{2} = -\frac{\mu}{2a}$$

at infinite distance, so V → V_∞

so

$$a = -\frac{\mu}{V_\infty^2}$$

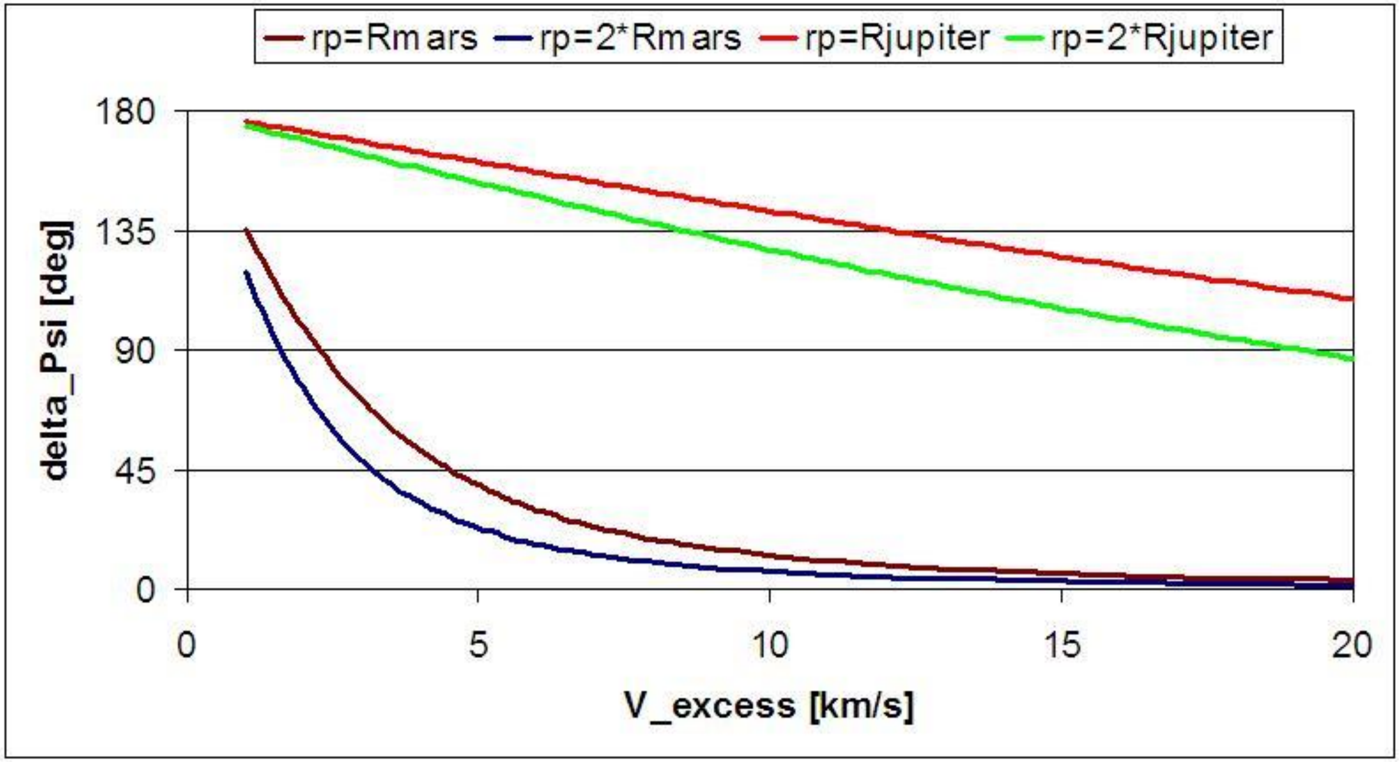
always:

$$r_p = a(1-e) \quad \Rightarrow \quad e = 1 - \frac{r_p}{a}$$

so:

$$a(V_\infty), e(V_\infty, r_p), \theta_{lim}(V_\infty, r_p), \Delta\Psi(V_\infty, r_p)$$

Gravity assist (cnt'd)

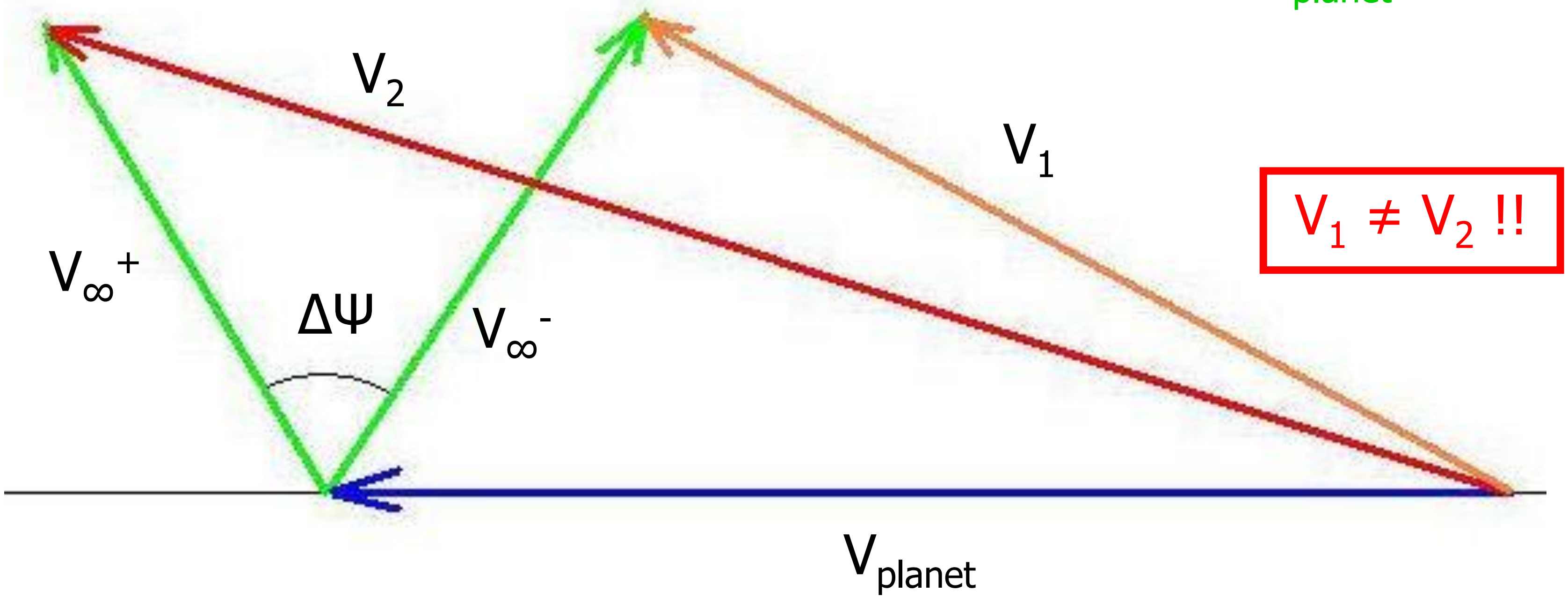


bending:

- increases for heavier planets
- increases for smaller pericenter distances
- decreases with increasing excess velocity

Gravity assist (cnt'd)

assumption (!!): hyperbola symmetric w.r.t. V_{planet}



$V_1 \neq V_2$!!

Gravity assist (cnt'd)

assumption (!!): hyperbola symmetric w.r.t. V_{planet}

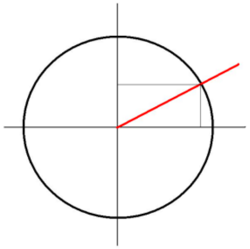
heliocentric velocities satellite (before, after encounter):

$$V_1^2 = V_{planet}^2 + V_\infty^2 - 2V_{planet}V_\infty \cos\left(\frac{\pi}{2} - \frac{\Delta\Psi}{2}\right)$$

$$V_2^2 = V_{planet}^2 + V_\infty^2 - 2V_{planet}V_\infty \cos\left(\frac{\pi}{2} + \frac{\Delta\Psi}{2}\right)$$

with

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha) \quad \text{and} \quad \cos\left(\frac{\pi}{2} + \alpha\right) = -\sin(\alpha)$$

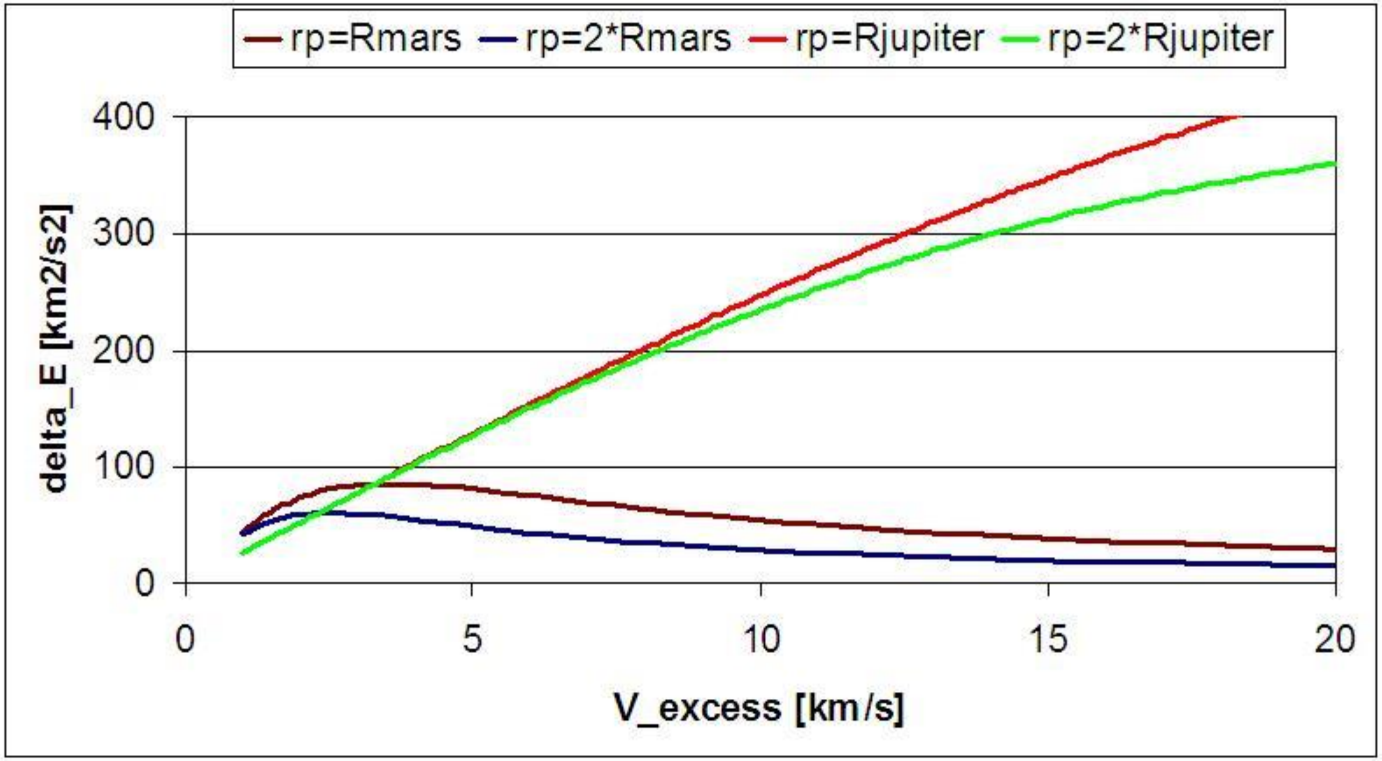


energy gain :

$$\Delta E = \frac{V_2^2}{2} - \frac{V_1^2}{2} = 2V_{planet}V_\infty \sin\left(\frac{\Delta\Psi}{2}\right)$$

Gravity assist (cnt'd)

assumption (!!): hyperbola symmetric w.r.t. V_{planet}



energy gain:

- not proportional to mass of planets
- increases with decreasing pericenter distances
- strong dependence on excess velocity

Gravity assist (cnt'd)

assumption (!!): hyperbola symmetric w.r.t. V_{planet}

Example:

Consider a Gravity Assist along Mars (μ_{Sun}=1.3271 × 10^{11} km^3/s^2; μ_{Mars}=42,832 km^3/s^2; R_{Mars} = 3397 km; distance Mars-Sun = 1.52 AU; 1 AU = 149.6 × 10^6 km). Assume a relative velocity when entering the Sphere of Influence of 4 km/s, and a pericenter distance of 1.1 * R_{Mars}.

- $a=-\mu_{Mars}/(V_\infty^2) \rightarrow a = -2677.0$ km
- $r_p=a(1-e) \rightarrow e = 2.396$
- $r = a(1-e^2)/(1+e*\cos(\theta)) \rightarrow \theta_{lim} = 114.67°$
- $\Delta\Psi=2*\theta_{lim}-\pi \rightarrow \Delta\Psi=49.34°$
- Mars at 1.52 AU → V_{Mars} = 24.158 km/s (heliocentric)
- velocity triangle before GA: $V_{sat,1}$ = 22.780 km/s
- velocity triangle after GA: $V_{sat,2}$ = 26.082 km/s
- $\Delta E=V_2^2/2-V_1^2/2 \rightarrow \Delta E = 80.667$ km^2/s^2

Gravity assist (cnt'd)

assumption (!!): hyperbola symmetric w.r.t. V_{planet}

Question 8:

Consider a Gravity Assist along Jupiter.

a) Assuming a relative velocity when entering the Sphere of Influence of 10 km/s, and a nearest passing distance w.r.t. the center of Jupiter of 200,000 km, compute the value of the semi-major axis and the eccentricity of this orbit (use the vis-viva equation $V^2/2-\mu/r=-\mu/(2a)$, amongst others).

b) Compute the heliocentric velocity of Jupiter.

c) Derive the following relation, which indicates the maximum value for the direction of motion: $\cos(\theta_{lim}) = -1/e$.

d) Derive the following relation for the bending angle around the central body: $\Delta\Psi=2*\theta_{lim}-\pi$.

e) Using the velocity diagrams, compute the heliocentric velocity of the satellite before and after the encounter, respectively.

f) Compute the gain in energy caused by the Gravity Assist.

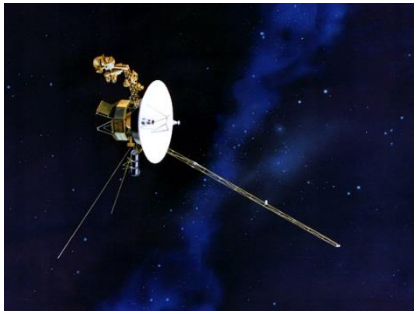
Data: $\mu_{Sun}=1.3271 \times 10^{11}$ km^3/s^2; $\mu_{Jupiter}=1.267 \times 10^8$ km^3/s^2; distance Jupiter-Sun = 5.20 AU; 1 AU = 149.6×10^6 km.

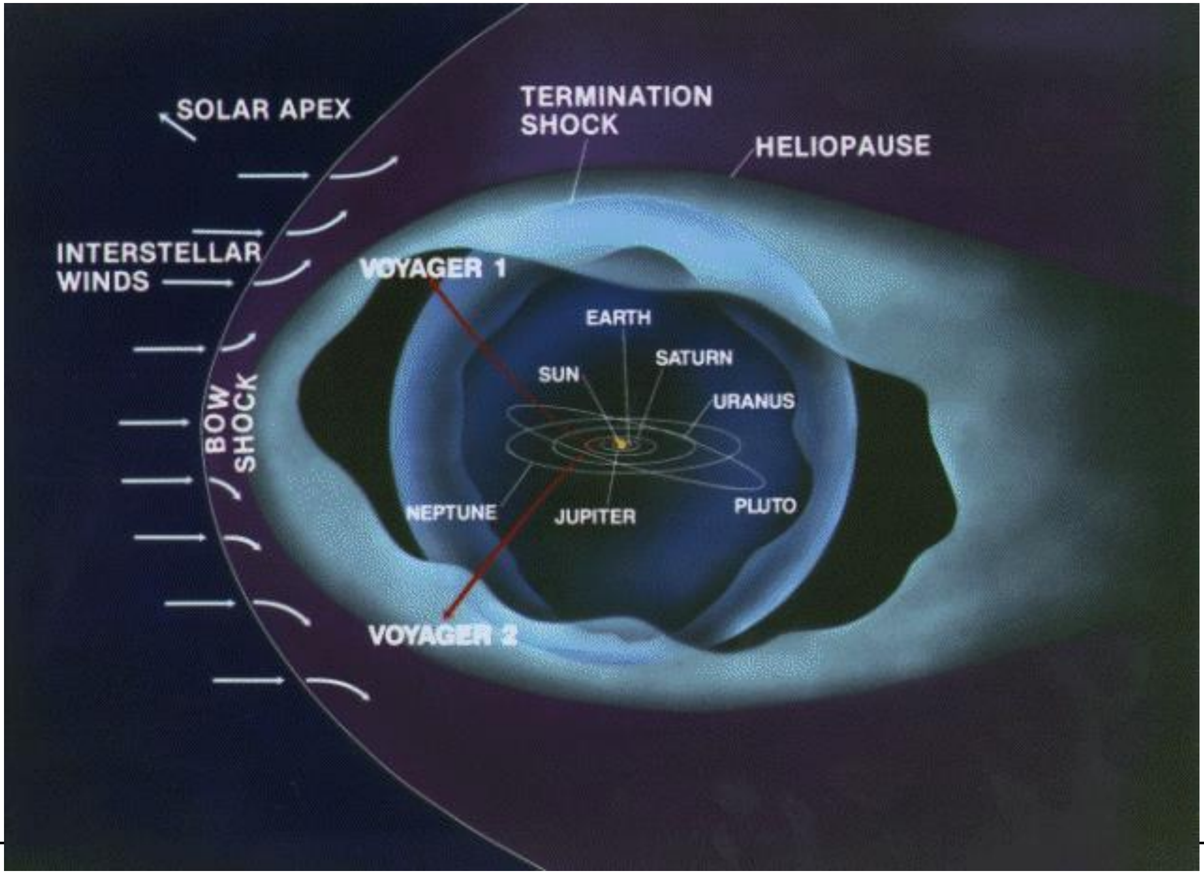
Answers: see footnote below **(BUT TRY YOURSELF FIRST!!)**

Gravity assist (cnt'd)

example 1: Voyager-1, -2

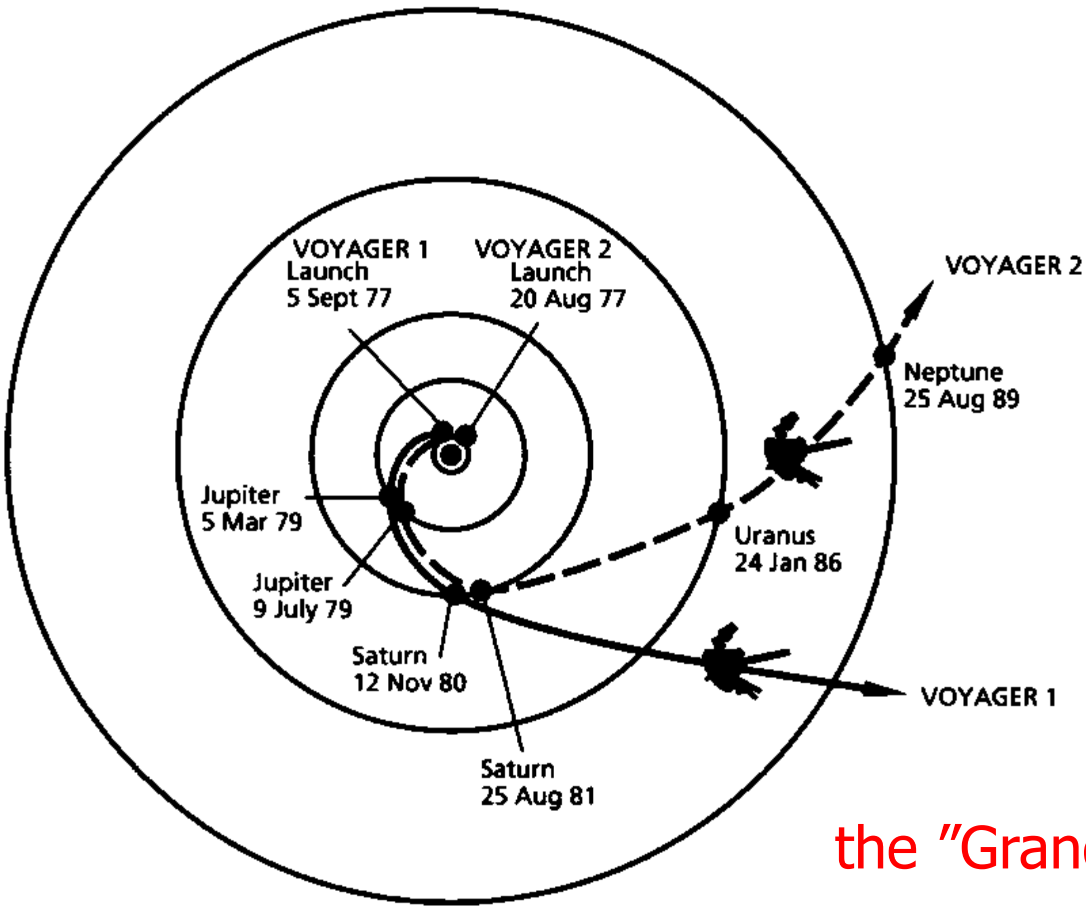
[NASA, 2010]





[NASA, 2010]

Gravity assist (cnt'd)



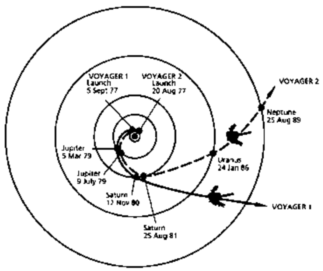
the "Grand Tour"

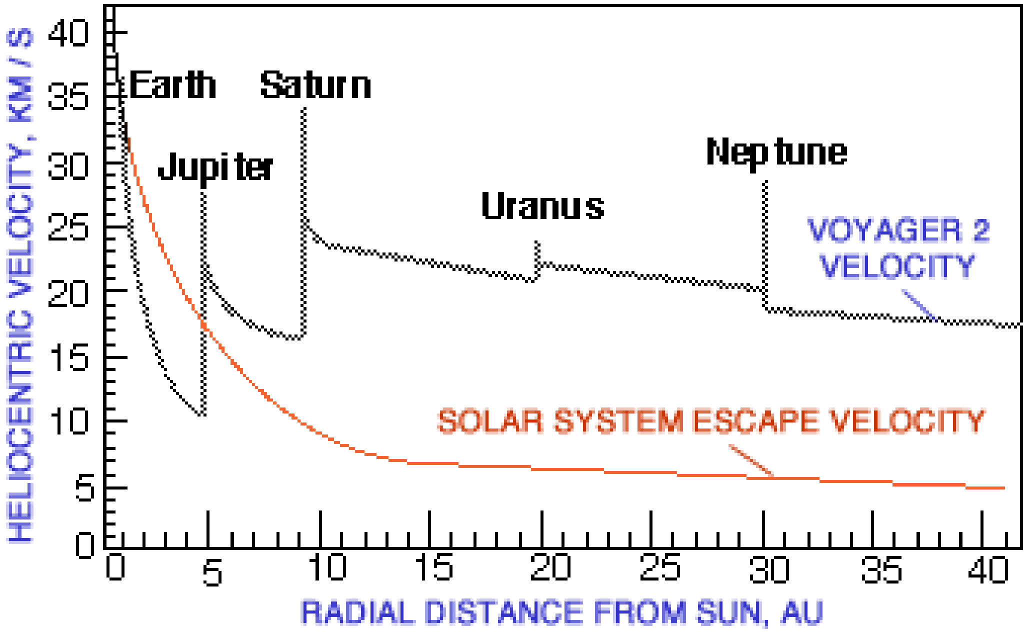
Gravity assist (cnt'd)

	Voyager-1	**Voyager-2**
launch	September 5, 1977	August 20, 1977
Jupiter flyby	March 5, 1979	July 9, 1979
Saturn flyby	November 12, 1980	August 25, 1981
Uranus flyby		January 24, 1986
Neptune flyby		August 25, 1989

[http://voyager.jpl.nasa.gov/mission/fastfacts.html]

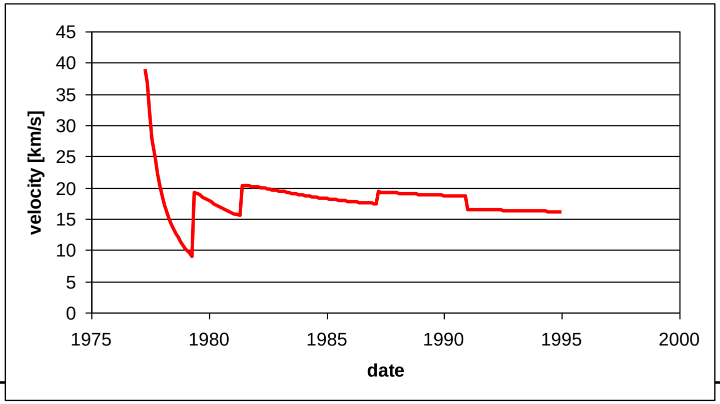
Gravity assist (cnt'd)





reconstructed

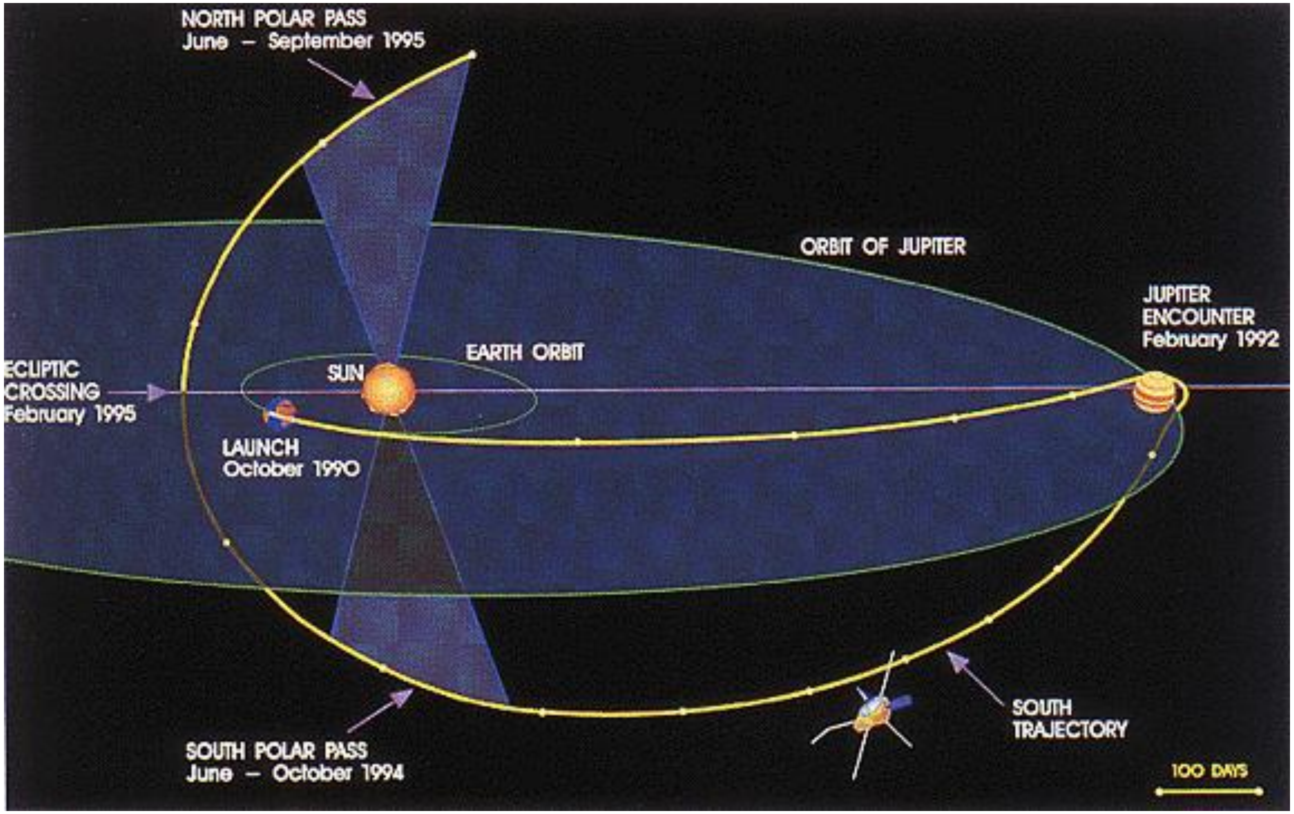
reality



Gravity assist (cnt'd)

example 2: Ulysses

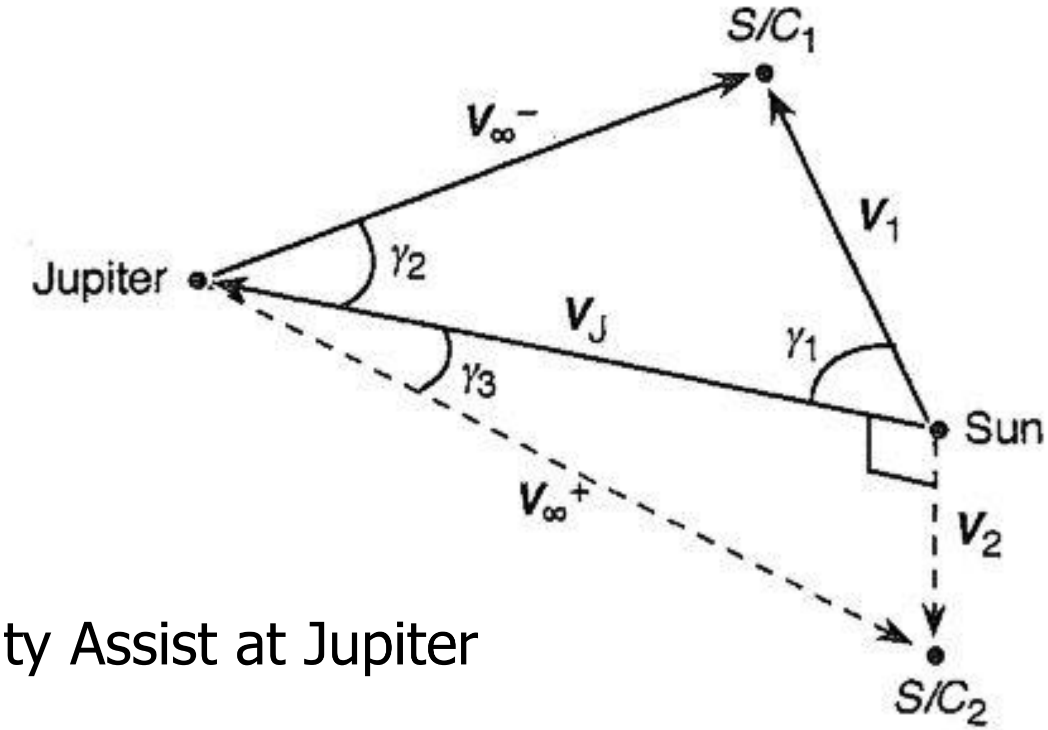
[ESA, 2010]



[Uni-Bonn, 2010]

[Fortescue, Stark & Swinerd, 2003]

Gravity assist (cnt'd)



Solution:

- 3-dimensional Gravity Assist at Jupiter

- heliocentric velocity effectively in ecliptic
- (small component perpendicular to ecliptic)
- swing-by changes heliocentric inclination to 80.2°

Gravity assist (cnt'd)

Some characteristics:

	reality	reconstruction
distance Earth-Sun [AU]	1.0	
distance Jupiter-Sun [AU]	5.4	5.2
heliocentric velocity Jupiter [km/s]	12.6	13.1
heliocentric velocity Ulysses near Earth [km/s]	41.2	
heliocentric velocity Ulysses near Jupiter [km/s]	??	16.4
excess velocity Ulysses w.r.t. Jupiter [km/s]	13.5	15.2
required deflection angle [°]	??	73.1
minimum distance Ulysses w.r.t. center Jupiter	6.3 R_J	5.2 R_J
minimal distance Ulysses w.r.t. Sun [AU]	1.34	1.13
distance Ulysses over poles Sun [AU]	2.3	1.85
travel time to closest approach Sun [yr]	4.4	4.04
time δ_S > 70° [yr]	0.36	0.30

Miscellaneous

Topics not treated here (1):

- *3D ephemerides :*
  How can we model the elements in the real solar system? Consequences for mission design?

- *Lambert targeting * :*
  How can we obtain parameters of trip between arbitrary positions (and epochs) in space?

- *Low-thrust propulsion * :*
  How can we compute orbits with (semi)continuous low-thrust propulsion? Howe to optimize them?

Miscellaneous (cnt'd)

Topics not treated here (2):

- *Aero Gravity Assist :*
  Can we improve the efficiency of a planetary flyby by using the atmosphere of the flyby planet?

- *Local geometry * :*
  What are the geometrical conditions when departing from and arriving at a planet?

- *Optimization * :*
  How can we find the most attractive trajectory (ΔV, time-of-flight, geometry, …)

Appendix: eqs. for Kepler orbits

Appendix: elementary equations for Kepler orbits

Appendix: eqs. for Kepler orbits (cnt'd)

$$r = \frac{a\,(1-e^2)}{1+e\cos\theta} = \frac{p}{1+e\cos\theta} \;;\; r_p = a\,(1-e) \;;\; r_a = a\,(1+e)$$

$$E_{tot} = \frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

$$V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \;;\quad V_{circ} = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{\mu}{a}} \;;\quad V_{esc} = \sqrt{\frac{2\mu}{r}}$$

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

Appendix: eqs. for Kepler orbits (cnt'd)

ellipse $(0 \le e < 1)$:

$$n = \sqrt{\frac{\mu}{a^3}}$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$$

$$M = E - e\sin E$$

$$M = n(t - t_0)$$

$$E_{i+1} = E_i + \frac{M - E_i + e\sin E_i}{1 - e\cos E_i}$$

$$r = a(1 - e\cos E)$$

hyperbola $(e > 1)$:

$$n = \sqrt{\frac{\mu}{(-a)^3}}$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{e+1}{e-1}}\tanh\frac{F}{2}$$

$$M = e\sinh F - F$$

$$M = n(t - t_0)$$

$$r = a(1 - e\cosh F)$$

$$V^2 = V_{esc}^2 + V_\infty^2 = \frac{2\mu}{r} + V_\infty^2$$