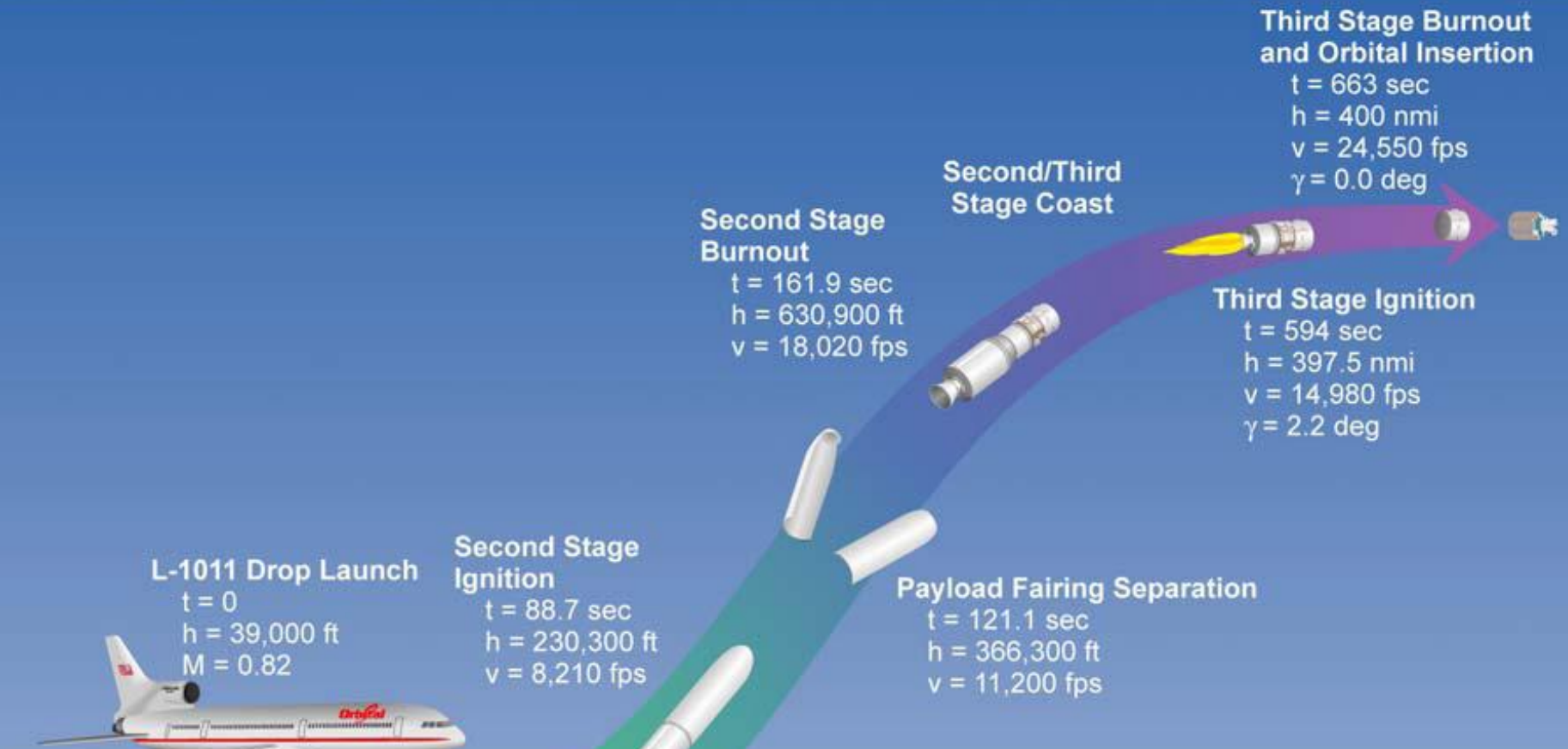


Flight and Orbital Mechanics

Lecture slides



Flight and Orbital Mechanics

AE2-104, lecture hours 25+26: Launchers

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October 1, 2012

PUG-005

Example: Ariane 5



Questions:

- what is the payload of this launcher?
- why does it have 2 stages and 2 boosters?
- what are the characteristics of each stage?
-

[Arianespace, 2010]

Overview

- Ideal single-stage launcher
- Ideal multi-stage launcher
- Real single-stage launcher (gravity, atmosphere)
- Real multi-stage launcher (idem)
- Overall performance (Pegasus)
- Design (Pegasus)

Learning goals

The student should be able to:

- derive, describe and explain Tsiolkovsky's equation
- describe and explain the concept of a multi-stage launcher and quantify its performance
- describe and quantify the performance of a launcher in realistic conditions, *i.e.*, under the influence of gravity and drag
- make a 1st-order design of a new launcher from scratch
-

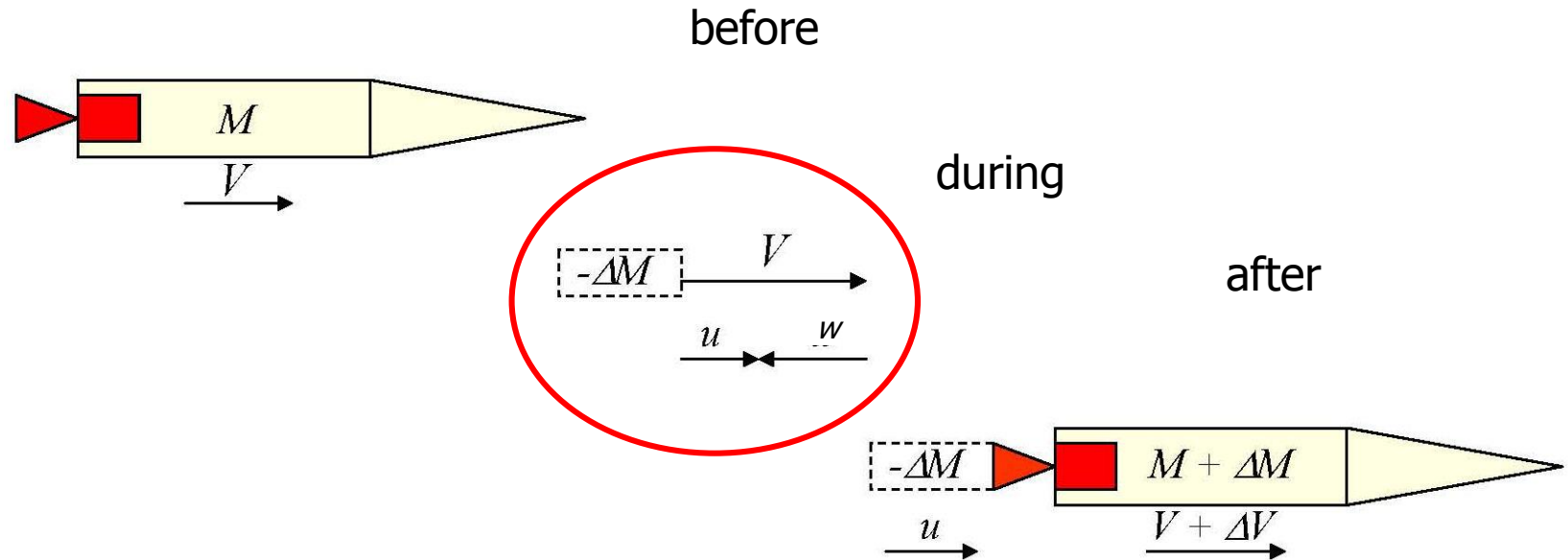
Lecture material:

- these slides (incl. footnotes)

Principles

Principles + performance ideal rocket: partial recap of ae1-102

Principles (cnt'd)



- vehicle contains payload, structure, propellant
- exhaust velocity propellant w
- conservation of momentum of system
- vehicle accelerates

Principles (cnt'd)

- system = launcher + expelled propellant
- momentum system = constant

$$M \frac{dV}{dt} = - \frac{dM}{dt} w$$

Solidification Principle:

$$F = M a = M \frac{dV}{dt} = m w$$

- M = instantaneous mass of rocket [kg]
- m = expelled (gaseous) mass per unit of time, or mass flow [kg/s]
- V = inertial velocity of launcher [m/s]
- w = relative exhaust velocity of expelled propellant [m/s]

Ideal single stage rocket

Equation of motion (vacuum, no gravity):

$$M \frac{dV}{dt} = - \frac{dM}{dt} w$$

Integration:

$$\Delta V = w \ln \left(\frac{M_{begin}}{M_{end}} \right)$$

Tsiolkovsky's Equation (a.k.a. "the rocket equation")

Note: $w = I_{sp} g_0$

Ideal single stage rocket (cnt'd)

Characteristic parameters:

- thrust-to-weight ratio:

$$\Psi_0 = \frac{F}{M_0 g_0}$$

- mass ratio:

$$\Lambda = \frac{M_{begin}}{M_{end}}$$

So:

- burn time:

$$t_b = \frac{M_{begin} - M_{end}}{m} = \frac{I_{sp}}{\Psi_0} \left(1 - \frac{1}{\Lambda}\right)$$

- end velocity

$$V_{end} = I_{sp} g_0 \ln(\Lambda)$$

- burnout altitude:

$$s_{end} = \frac{g_0 I_{sp}^2}{\Psi_0} \left(1 - \frac{\ln(\Lambda) - 1}{\Lambda}\right)$$

Ideal single stage rocket (cnt'd)

	"normal"	impulsive shot *
Λ	$M_{\text{begin}} / M_{\text{end}}$	
t_b	$(I_{\text{sp}} / \Psi_0) (1 - 1/\Lambda)$	0
Ψ_0	$F / (M_0 g_0)$	∞
V_{end}	$I_{\text{sp}} g_0 \ln(\Lambda)$	
S_{end}	$g_0 I_{\text{sp}}^2 / \Psi_0 (1 - (\ln(\Lambda)-1)/\Lambda)$	0

*: impulsive shot: all propellants are ejected in 1 instant

Ideal single stage rocket (cnt'd)

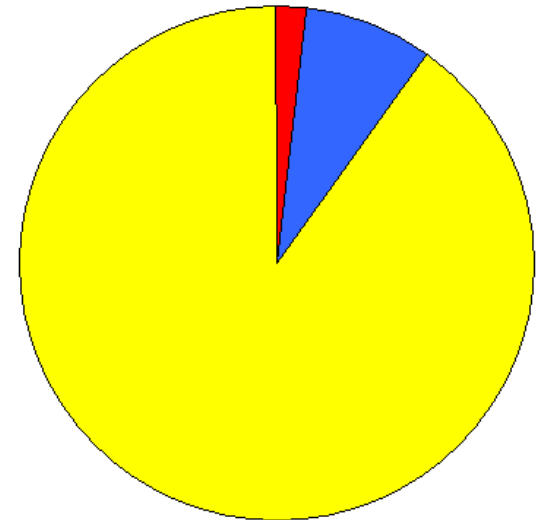
Do not forget (cf. ae1-102):

- $\Psi_0 > 1$
- structural loading at burnout

Ideal multi-stage rocket (cnt'd)

Definition of parameters:

- M_{total} = total mass (*i.e.*, before firing)
 - M_{payload} = payload mass (■)
 - M_{constr} = construction mass (■)
 - M_{prop} = propellant mass (■)
-
- $M_{\text{begin}} = M_{\text{total}} = M_{\text{payload}} + M_{\text{constr}} + M_{\text{prop}}$
 - $M_{\text{end}} = M_{\text{payload}} + M_{\text{constr}}$



Ideal multi-stage rocket

$$\Delta V = g_0 I_{sp} \ln \left(\frac{M_{total}}{M_{total} - M_{prop}} \right)$$

$$\Delta V = g_0 I_{sp} \ln \left(\frac{M_{total}}{M_{constr} + M_{payload}} \right)$$

$$\frac{M_{constr}}{M_{total}} + \frac{M_{payload}}{M_{total}} = \exp \left(\frac{-\Delta V}{I_{sp} g_0} \right)$$

Ideal multi-stage rocket (cnt'd)

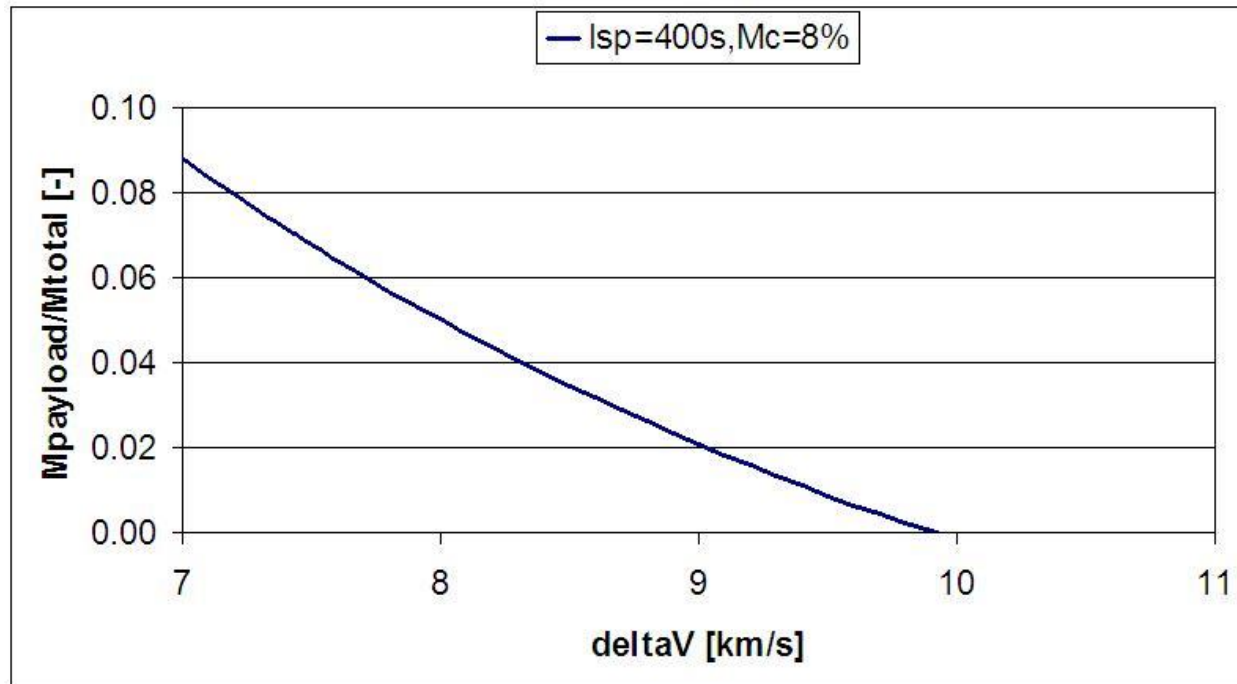
Example 1:

$$\Delta V = 10 \text{ km/s}, I_{sp} = 400 \text{ s}, M_{\text{constr}}/M_{\text{total}} = 8 \%, \\ M_{\text{payload}} = 500 \text{ kg} \rightarrow$$

- $M_{\text{total}} = M_{\text{begin}} = ???$
- $M_{\text{prop}} = ???$
- $M_{\text{constr}} = ???$

Ideal multi-stage rocket (cnt'd)

Example 1 (cnt'd):



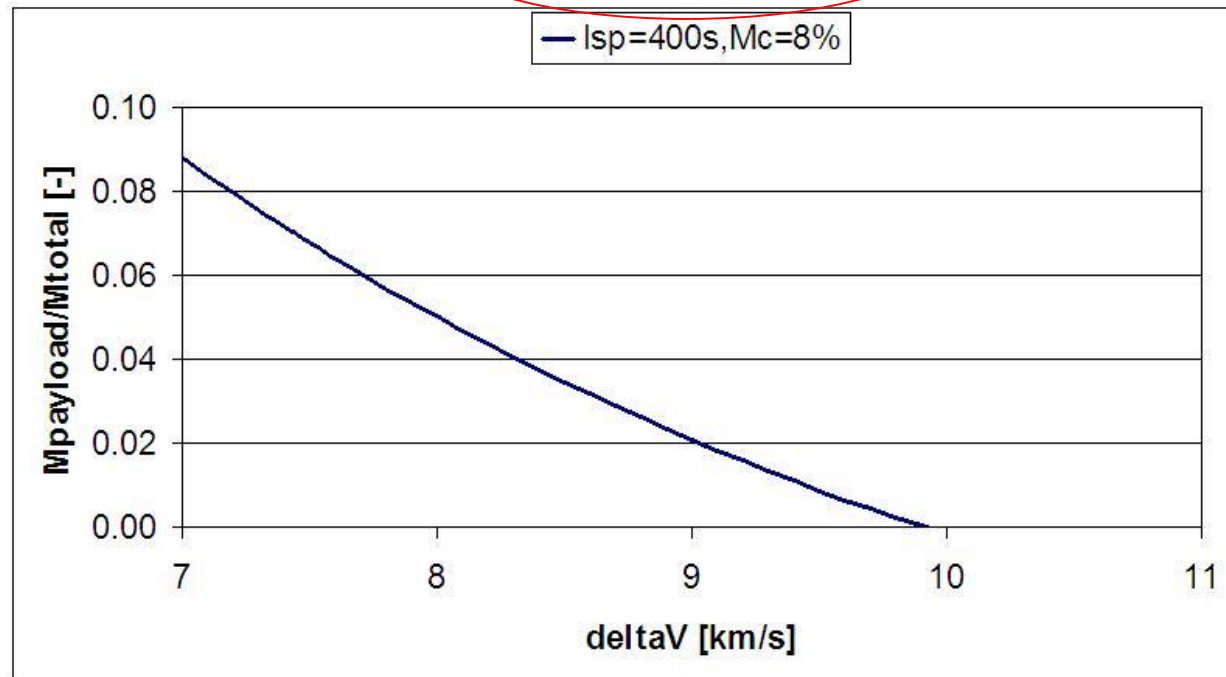
Ideal multi-stage rocket (cnt'd)

Options:

- reduce required M_{payload}
- use engine/propellant with higher I_{sp}
- use lighter construction
- multi-staging

Ideal multi-stage rocket (cnt'd)

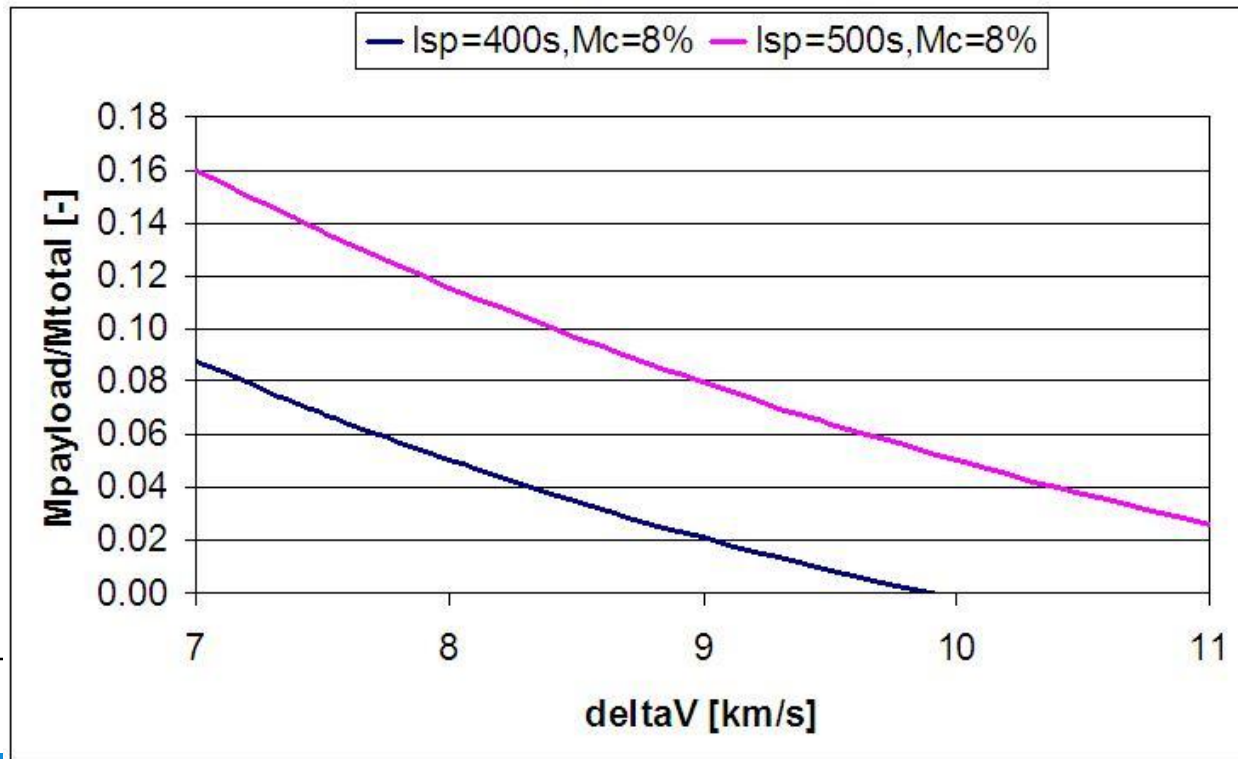
Example 2: $\Delta V = 10 \text{ km/s}$, $I_{sp} = 400 \text{ s}$,
 $M_{constr}/M_{total} = 8 \%$, $M_{payload} = 250 \text{ kg}$



Ideal multi-stage rocket (cnt'd)

Example 3:

$\Delta V = 10 \text{ km/s}$, $I_{sp} = 500 \text{ s}$, $M_{constr}/M_{total} = 8 \%$,
 $M_{payload} = 500 \text{ kg} \rightarrow$



Ideal multi-stage rocket (cnt'd)

Example 3 (cnt'd):

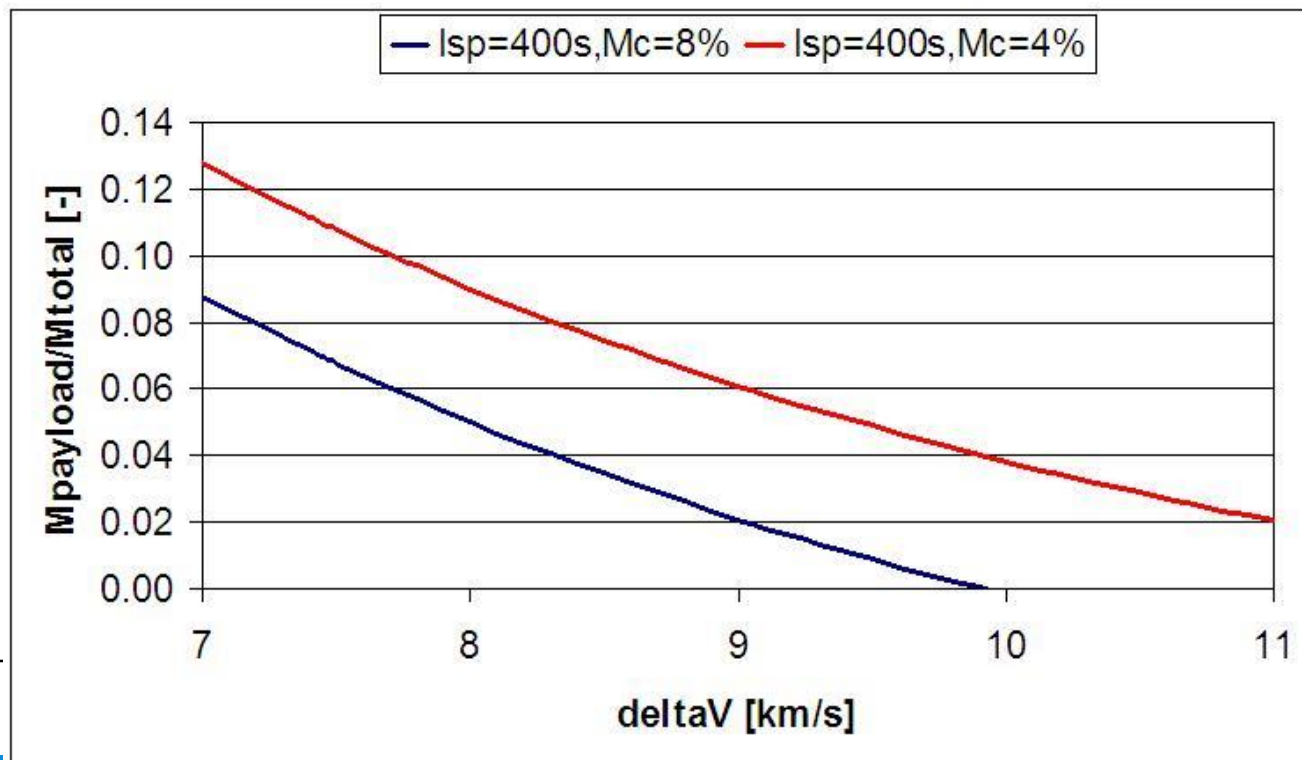
$$\Delta V = 10 \text{ km/s}, I_{sp} = 500 \text{ s}, M_{\text{constr}}/M_{\text{total}} = 8 \%, \\ M_{\text{payload}} = 500 \text{ kg} \rightarrow$$

- $M_{\text{payload}}/M_{\text{total}} = 0.0502$
- $M_{\text{total}} = M_{\text{begin}} = 9960 \text{ kg}$
- $M_{\text{constr}} = 797 \text{ kg}$
- $M_{\text{prop}} = 8663 \text{ kg}$
- $M_{\text{prop}}/M_{\text{total}} = 87.0 \%$

Ideal multi-stage rocket (cnt'd)

Example 4:

$\Delta V = 10 \text{ km/s}$, $I_{sp} = 400 \text{ s}$, $M_{constr}/M_{total} = 4 \%$,
 $M_{payload} = 500 \text{ kg} \rightarrow$



Ideal multi-stage rocket (cnt'd)

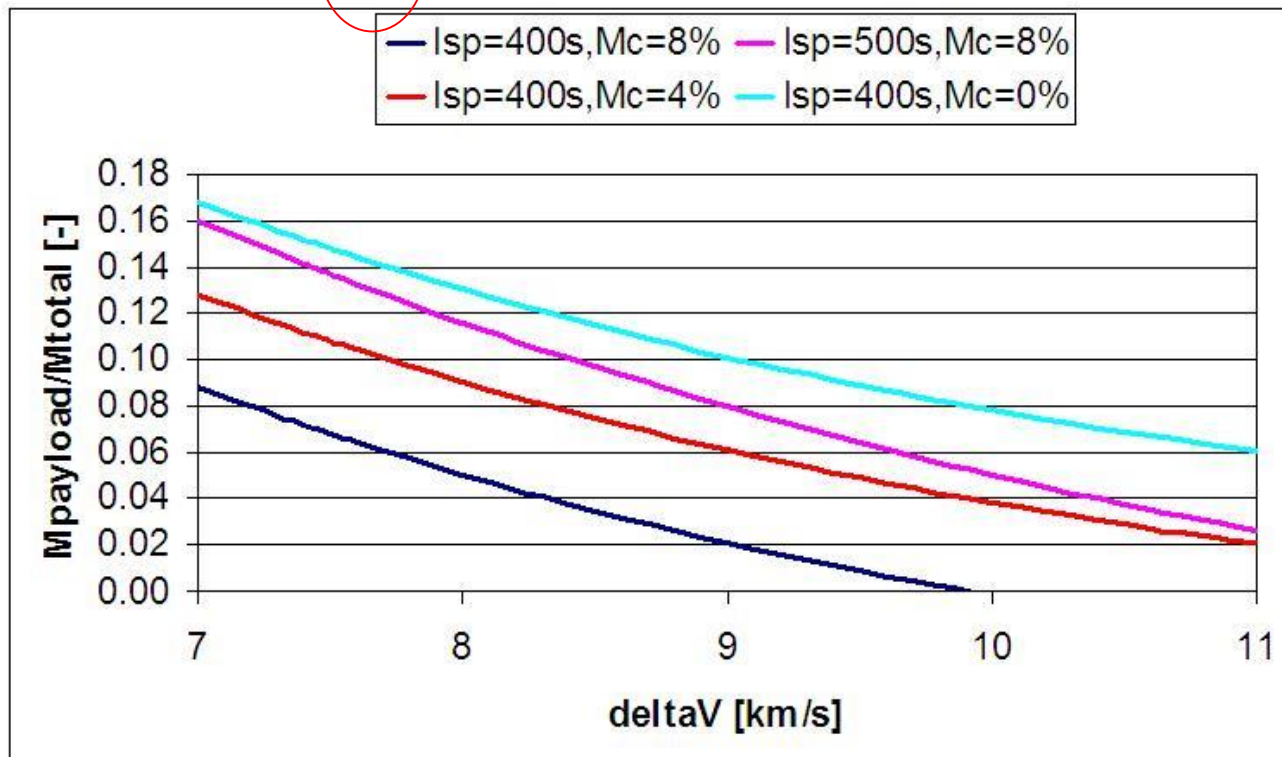
Example 4 (cnt'd):

$$\Delta V = 10 \text{ km/s}, I_{sp} = 400 \text{ s}, M_{\text{constr}}/M_{\text{total}} = 4 \%, \\ M_{\text{payload}} = 500 \text{ kg} \rightarrow$$

- $M_{\text{payload}}/M_{\text{total}} = 0.0382$
- $M_{\text{total}} = M_{\text{begin}} = 13089 \text{ kg}$
- $M_{\text{constr}} = 524 \text{ kg}$
- $M_{\text{prop}} = 12065 \text{ kg}$
- $M_{\text{prop}}/M_{\text{total}} = 92.2 \%$

Ideal multi-stage rocket (cnt'd)

Example 5: no construction mass

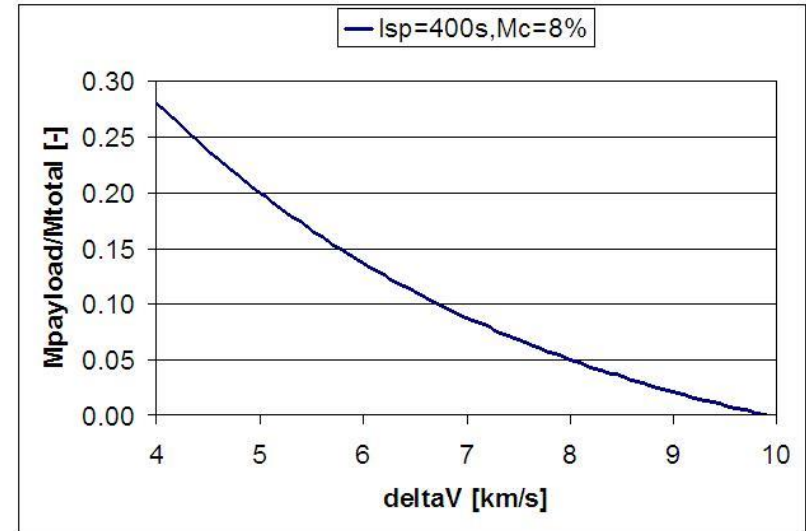


Ideal multi-stage rocket (cnt'd)

Example 6a:

$\Delta V = 5 \text{ km/s}$, $I_{sp} = 400 \text{ s}$, $M_{constr}/M_{total} = 8 \%$,
 $M_{payload} = 500 \text{ kg} \rightarrow$

- $M_{payload}/M_{total} = 0.1997$
- $M_{total} = M_{begin} = 2504 \text{ kg}$
- $M_{constr} = 200 \text{ kg}$
- $M_{prop} = 1804 \text{ kg}$
- $M_{prop}/M_{total} = 72.0 \%$

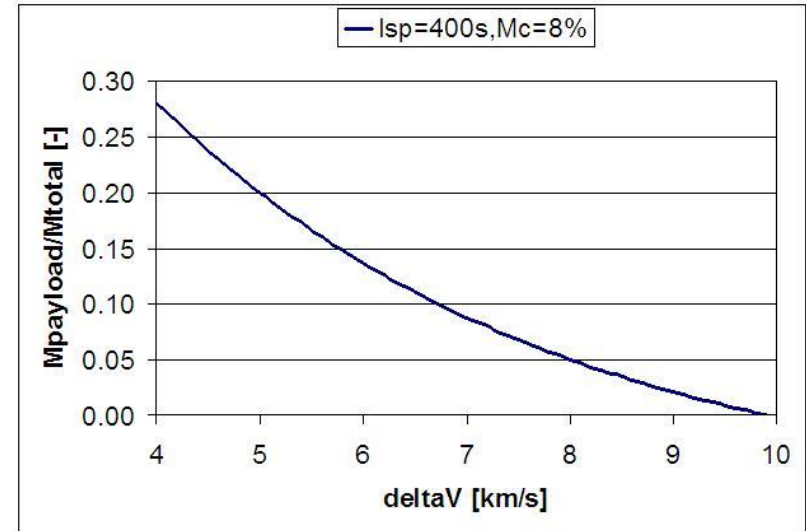


Ideal multi-stage rocket (cnt'd)

Example 6b:

$\Delta V = 5 \text{ km/s}$, $I_{sp} = 400 \text{ s}$, $M_{constr}/M_{total} = 8 \%$,
 $M_{payload} = 2504 \text{ kg} \rightarrow$

- $M_{payload}/M_{total} = 0.1997$
- $M_{total} = M_{begin} = 12,539 \text{ kg}$
- $M_{constr} = 1003 \text{ kg}$
- $M_{prop} = 9032 \text{ kg}$
- $M_{prop}/M_{total} = 72.0 \%$



Ideal multi-stage rocket (cnt'd)

Add numbers examples 6a+b:

	Example 6a	Example 6b	total
ΔV [km/s]	5.0	5.0	10.0
M_{prop} [kg]	1804	9032	10836
M_{constr} [kg]	200	1003	1203
M_{payload} [kg]	500	2504	500
M_{total} [kg]	2504	12539	12539
	stage 2	stage 1	

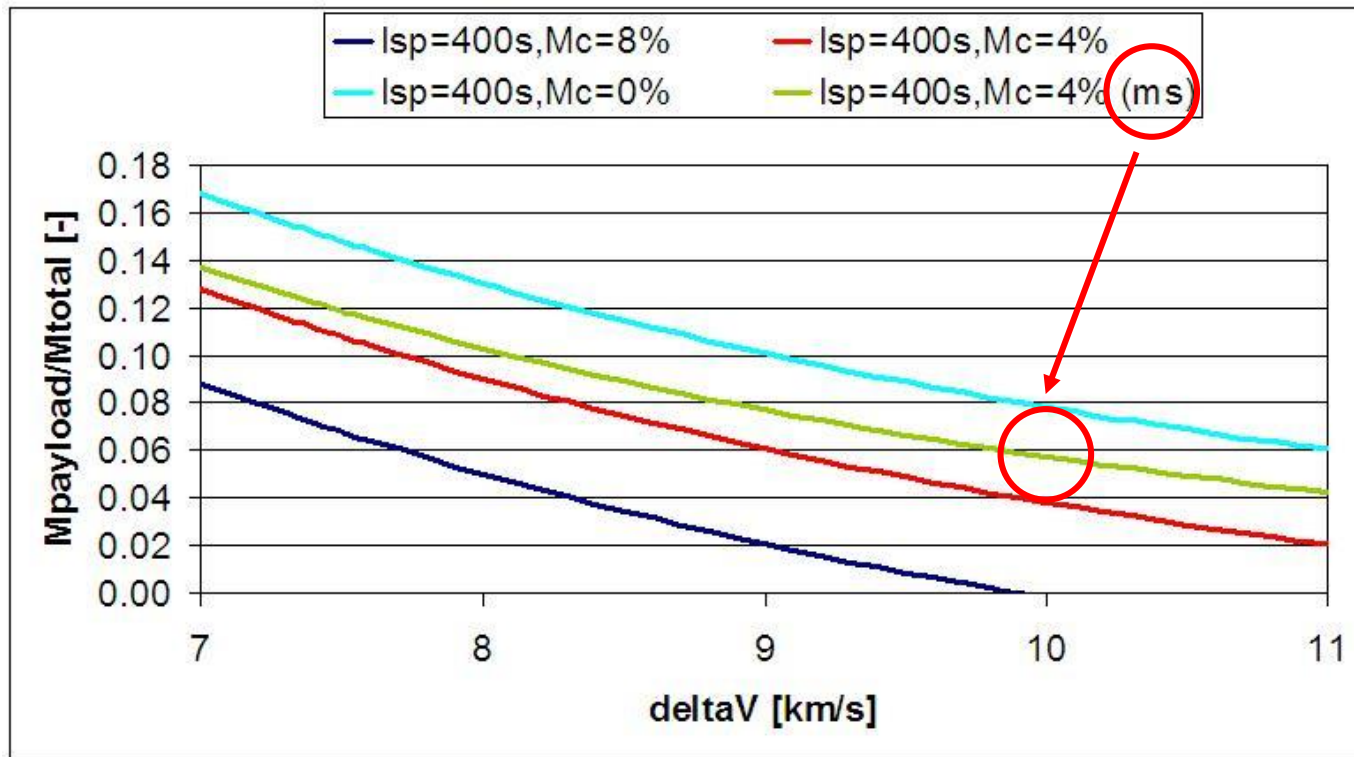
Ideal multi-stage rocket (cnt'd)

Compare examples:

	Example 1	Example 3	Example 4	Example 6a+b
ΔV [km/s]	10.0			
M_{payload} [kg]	500			
I_{sp} [s]	400	500	400	400
$M_{\text{constr}}/M_{\text{total}}$ [%]	8	8	4	8
# stages	1	1	1	2
M_{prop} [kg]	n.a.	8663	12065	10836
M_{constr} [kg]	n.a.	797	524	1203
M_{total} [kg]	n.a.	9960	13089	12539

Ideal multi-stage rocket (cnt'd)

Compare examples:



Conclusion: 50% gain in payload (ratio) !!

Ideal multi-stage rocket (cnt'd)

Multi-staging:

Advantages:

- no need to accelerate total construction mass until final velocity → upper stages perform more efficiently
 - more payload capacity
 - more ΔV capacity

Disadvantages:

- more complexity (engines, piping, ...)
- more risk (jettison, ignition, ...)

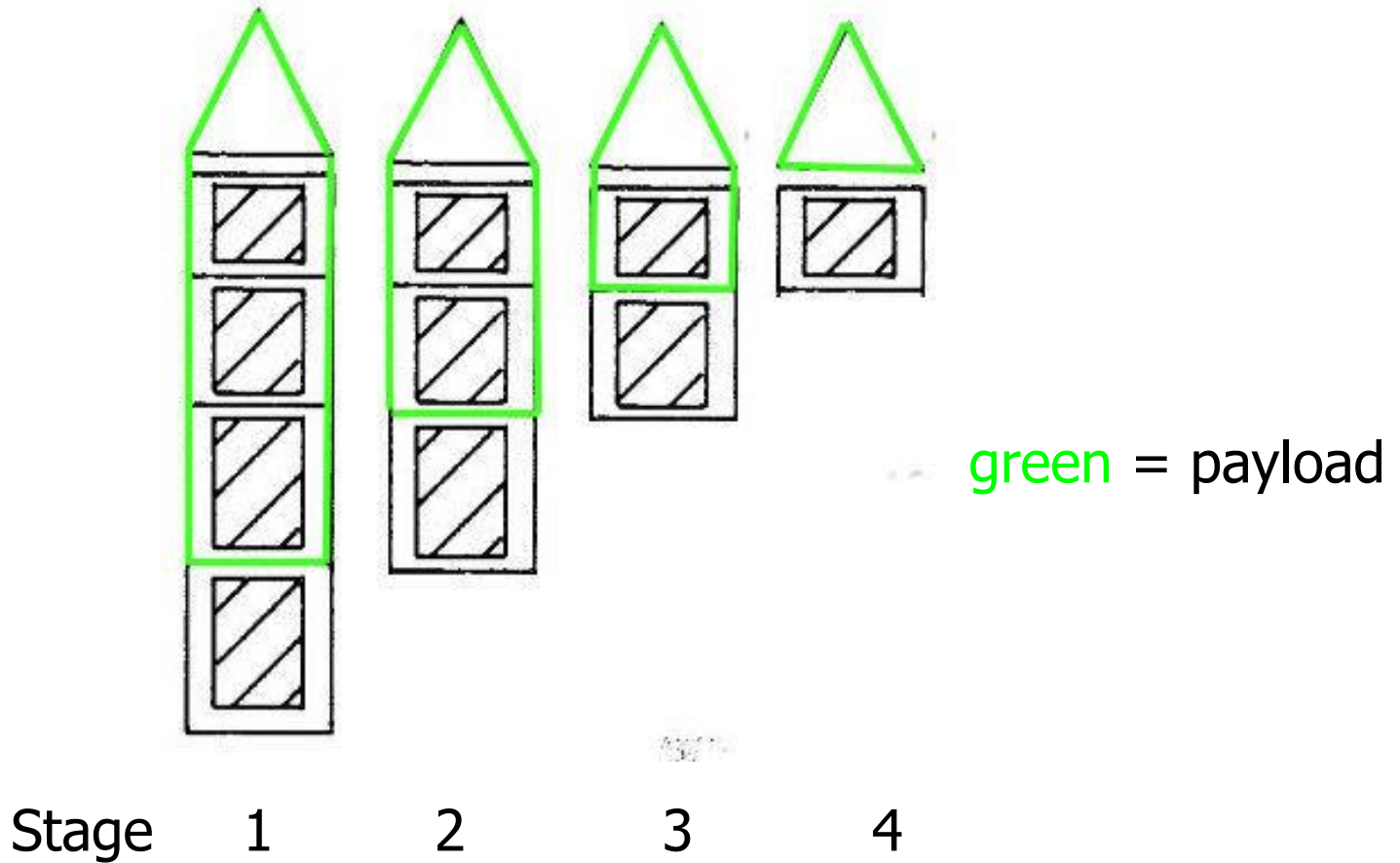
Ideal multi-stage rocket (cnt'd)

Definition of parameters:

- $M_{\text{total},i}$ = total mass of stage "i" (*i.e.*, before firing)
- $M_{\text{payload},i}$ = payload mass of stage "i"
- $M_{\text{constr},i}$ = construction mass of stage "i"
- $M_{\text{prop},i}$ = propellant mass of stage "i"

- Note: $M_{\text{payload},i} = M_{\text{total},i+1}$

Ideal multi-stage rocket (cnt'd)



Ideal multi-stage rocket (cnt'd)

Tsiolkovsky, single stage:

$$\Delta V = I_{sp} g_0 \ln \left(\frac{M_{begin}}{M_{end}} \right)$$

define

$$p = M_{payload} / M_{begin}$$

$$\sigma = M_{constr} / M_{prop}$$

then

$$\frac{M_{begin}}{M_{end}} = \frac{M_{payload} + M_{constr} + M_{prop}}{M_{payload} + M_{constr}} = \frac{1 + \sigma}{p + \sigma}$$

so

$$\Delta V = I_{sp} g_0 \ln \left(\frac{1 + \sigma}{p + \sigma} \right)$$

(after [Fortescue, Stark & Swinerd, 2003])

Ideal multi-stage rocket (cnt'd)

Derivation of 4th equation on previous sheet:

$$M_{payload} = p M_{begin} \quad \text{and} \quad M_{constr} = \sigma M_{prop}$$

so

$$M_{begin} = p M_{begin} + M_{constr} + M_{prop}$$

which becomes

$$(1-p) M_{begin} = M_{constr} + M_{prop} = (\sigma+1) M_{prop}$$

or

$$M_{begin} = \frac{1+\sigma}{1-p} M_{prop}$$

mass ratio for Tsiolkovsky's equation :

$$\frac{M_{begin}}{M_{end}} = \frac{\frac{1+\sigma}{1-p} M_{prop}}{p \frac{1+\sigma}{1-p} M_{prop} + \sigma M_{prop}} = \frac{\frac{1+\sigma}{1-p}}{p \frac{1+\sigma}{1-p} + \sigma} = \frac{1+\sigma}{p(1+\sigma) + \sigma(1-p)} = \frac{1+\sigma}{p + p\sigma + \sigma - p\sigma} = \frac{1+\sigma}{p + \sigma}$$

Ideal multi-stage rocket (cnt'd)

stage "i" only :

$$\Delta V_i = I_{sp,i} g_0 \ln \left(\frac{1 + \sigma_i}{p_i + \sigma_i} \right)$$

total launcher :

$$\Delta V_{tot} = \sum \Delta V_i = \sum I_{sp,i} g_0 \ln \left(\frac{1 + \sigma_i}{p_i + \sigma_i} \right)$$

assume

$$I_{sp,i} = I_{sp}$$

$$\sigma_i = s$$

so

$$\Delta V_{tot} = \sum I_{sp} g_0 \ln \left(\frac{1 + s}{p_i + s} \right)$$

(after [Fortescue, Stark & Swinerd, 2003])

Ideal multi-stage rocket (cnt'd)

by definition :

$$P_{tot} = p_1 \times p_2 \times p_3 \times \dots \times p_N = \prod p_i$$

optimal solution (w.o.derivation):

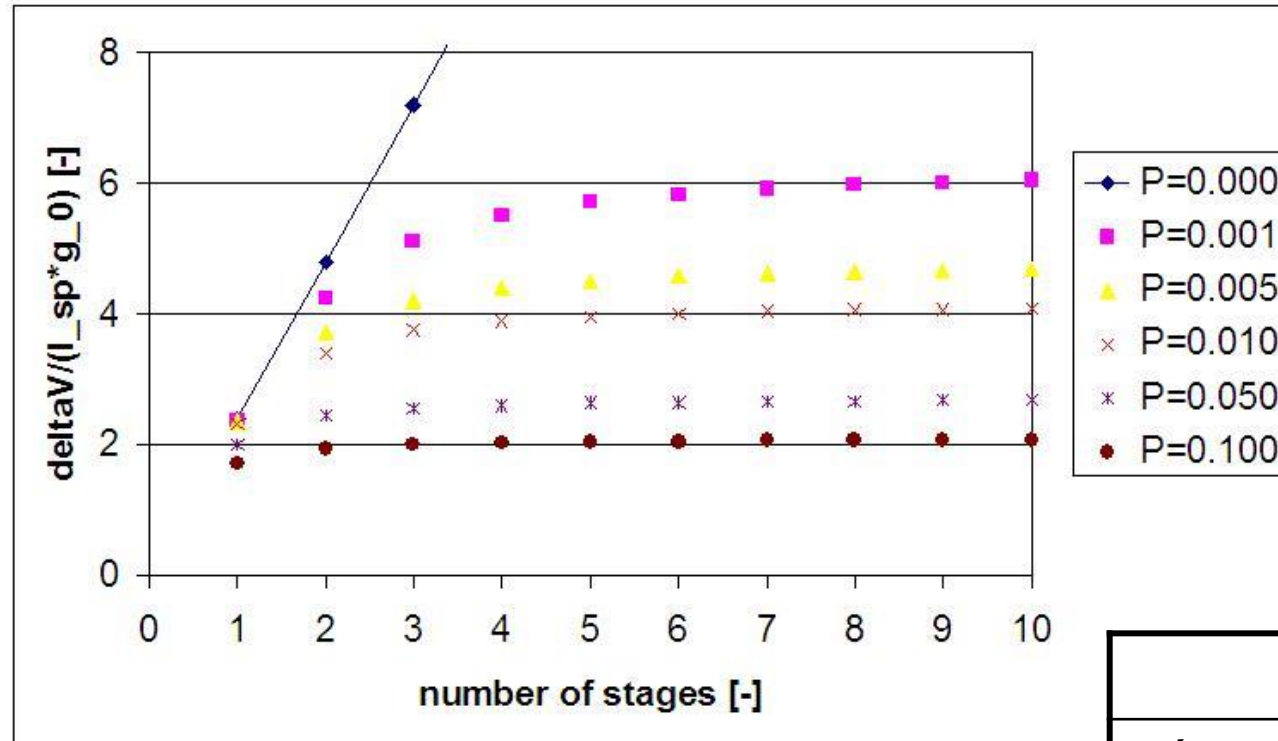
$$p_i = \sqrt[N]{P_{tot}}$$

so

$$\Delta V_{tot} = I_{sp} g_0 N \{ \ln(1 + s) - \ln(s + \sqrt[N]{P_{tot}}) \}$$

(after [Fortescue, Stark & Swinerd, 2003])

Ideal multi-stage rocket (cnt'd)



	N	P
Véronique	1	0.044
Ariane-4	2	0.02-0.03

- #stages typically 3 or 4
- single stage: $\Delta V / (I_{sp} g_0)$ 1.7 – 2.4 (factor 1.4)
- 4 stages: $\Delta V / (I_{sp} g_0)$ 2.0 – 5.5 (factor 2.8)
- staging very attractive (for modest P)
- high P: gain multi-staging limited (\rightarrow 2 stages for Ariane-5, Delta IV, Titan V, ...) **real challenge!**

Ideal multi-stage rocket (cnt'd)

Question 1

The performance of a rocket (*i.e.*, the ΔV that can be obtained) is determined by the ratio $M_{\text{begin}}/M_{\text{end}}$, amongst others. New parameters "p" and "σ" can be defined:
 $\rho = M_{\text{payload}}/M_{\text{begin}}$ and $\sigma = M_{\text{constr}}/M_{\text{prop}}$.

Derive the following equation:

$$M_{\text{begin}}/M_{\text{end}} = (1+\sigma)/(\rho+\sigma)$$

Ideal multi-stage rocket (cnt'd)

Question 2

The performance of a rocket (*i.e.*, the ΔV that can be obtained) is determined by the ratio $M_{\text{begin}}/M_{\text{end}}$, amongst others. New parameters " p_i " and " σ_i " can be defined for each possible stage " i ": $p_i = M_{\text{payload},i}/M_{\text{begin},i}$ and $\sigma_i = M_{\text{constr},i}/M_{\text{prop},i}$.

Derive the following equation which holds for an arbitrary number of stages N (where it is assumed that the parameters σ_i are equal to " s " for all stages, and the payload fractions of all stages p_i are equal to $(N)\sqrt[N]{P_{\text{tot}}}$ (*i.e.*, the N^{th} root of P_{tot}):

$$\Delta V_{\text{tot}} = I_{sp} g_0 N \{ \ln(1 + s) - \ln(s + \sqrt[N]{P_{\text{tot}}}) \}$$

Ideal multi-stage rocket (cnt'd)

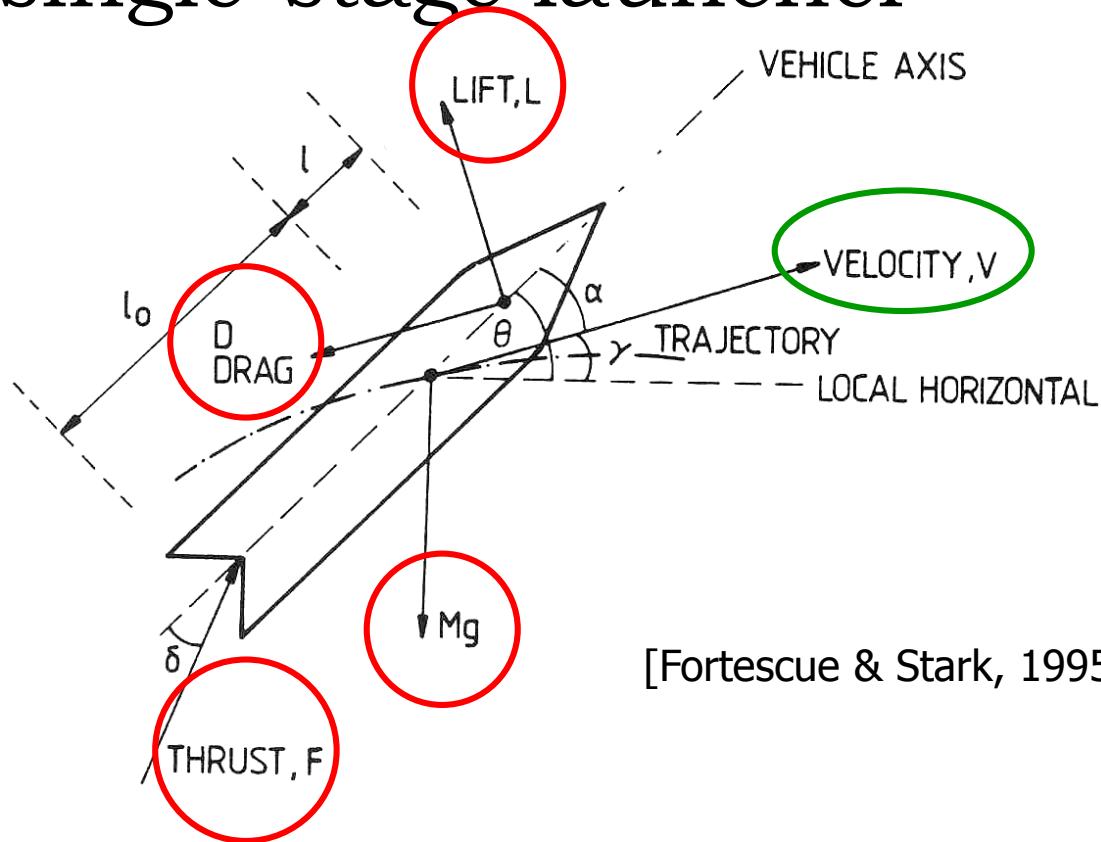
Question 3

Given the equation

$$\Delta V_{tot} = I_{sp} g_0 N \{ \ln(1 + s) - \ln(s + \sqrt[N]{P_{tot}}) \}$$

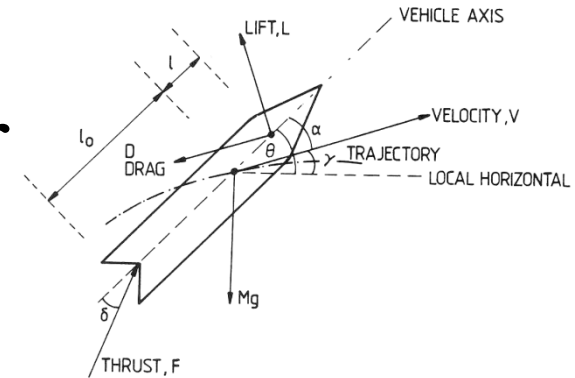
1. What do the various parameters represent?
2. What does the equation express?
3. Make a sketch of the behaviour of $\Delta V_{tot}/(I_{sp} g_0)$ as a function of parameter N , for the case $P_{tot} = 0.001$ and the case $P_{tot} = 0.010$ (parameter "s" is equal to 10%). Clearly indicate the (range of) numerical values for $\Delta V_{tot}/(I_{sp} g_0)$.
4. Discuss the consequences of increasing N for both cases of P_{tot} .

Real single-stage launcher



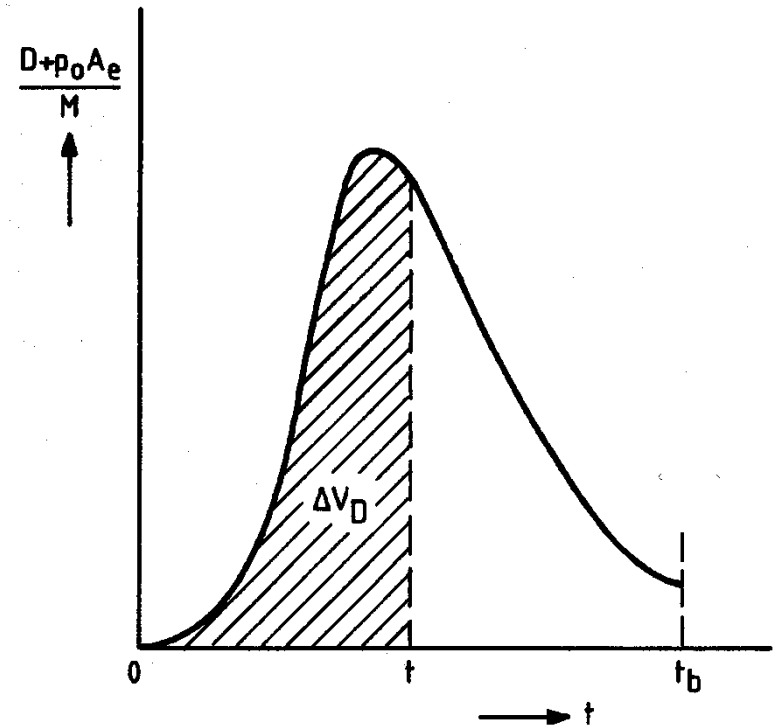
In direction of flight: $M \frac{dV}{dt} = F \cos(\alpha + \delta) - M g \sin(\gamma) - D$

Real single-stage launcher



In direction of flight: $M \frac{dV}{dt} = F \cos(\alpha + \delta) - M g \sin(\gamma) - D$

1. **Thrust misalignment:** $\alpha + \delta \neq 0^\circ$
(α needed to counteract gravity, δ for steering \rightarrow cannot be avoided)
2. **Gravity loss:** $\gamma \neq 0^\circ$ (launcher lifts off in vertical direction \rightarrow unavoidable)
3. **Drag loss:** $D \neq 0$ (first part of trajectory through atmosphere \rightarrow unavoidable)



Real single-stage launcher (cnt'd)

vertical flight :

$$dV = -w \frac{dM}{M} - g dt - \frac{D}{M} dt$$

integration :

$$\begin{aligned} V_{end} &= -\int w \frac{dM}{M} - \int g dt - \int \frac{D}{M} dt = \\ &= V_{end,ideal} - \Delta V_g - \Delta V_d \end{aligned}$$

where

$$V_{end,ideal} = I_{sp} g_0 \ln(\Lambda)$$

$$\Delta V_g = g_0 t_b$$

Real single-stage launcher (cnt'd)

velocity at burnout :

$$V_{burnout} = I_{sp} g_0 \left[\ln(\Lambda) - \frac{1}{\Psi_0} \left(1 - \frac{1}{\Lambda} \right) \right]$$

- including gravity losses
- w/o drag losses

altitude at burnout :

$$h_{burnout} = \frac{I_{sp}^2 g_0}{\Psi_0} \left[\left(1 - \frac{\ln(\Lambda) + 1}{\Lambda} \right) - \frac{1}{2\Psi_0} \left(1 - \frac{1}{\Lambda} \right)^2 \right]$$

altitude at culmination :

$$h_{culm} = \frac{I_{sp}^2 g_0}{\Psi_0} \left(\frac{1}{2} \Psi_0 \ln^2(\Lambda) - \ln(\Lambda) - \frac{1}{\Lambda} + 1 \right)$$

time until culmination :

$$t_{culm} = I_{sp} \ln(\Lambda)$$

Real single-stage launcher (cnt'd)

Data:

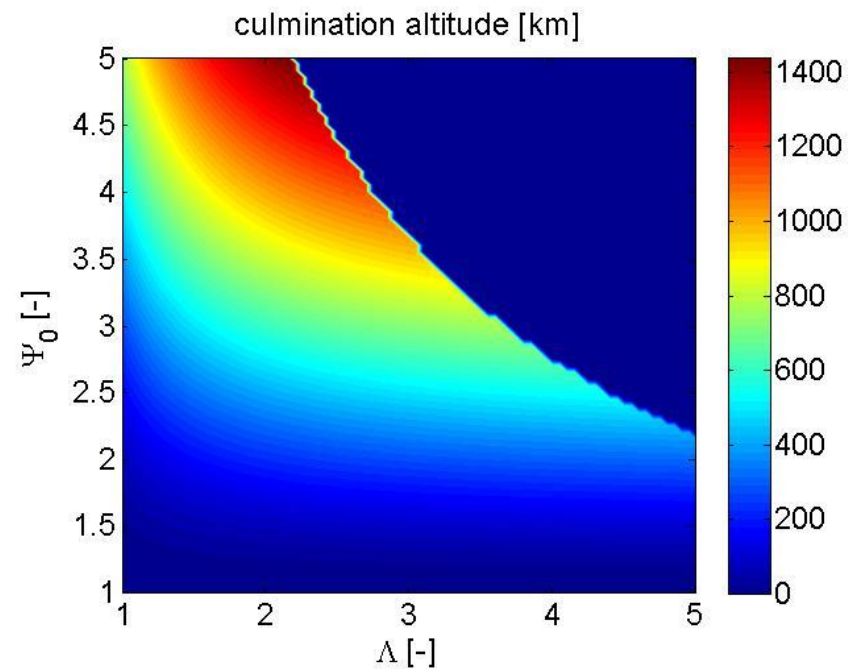
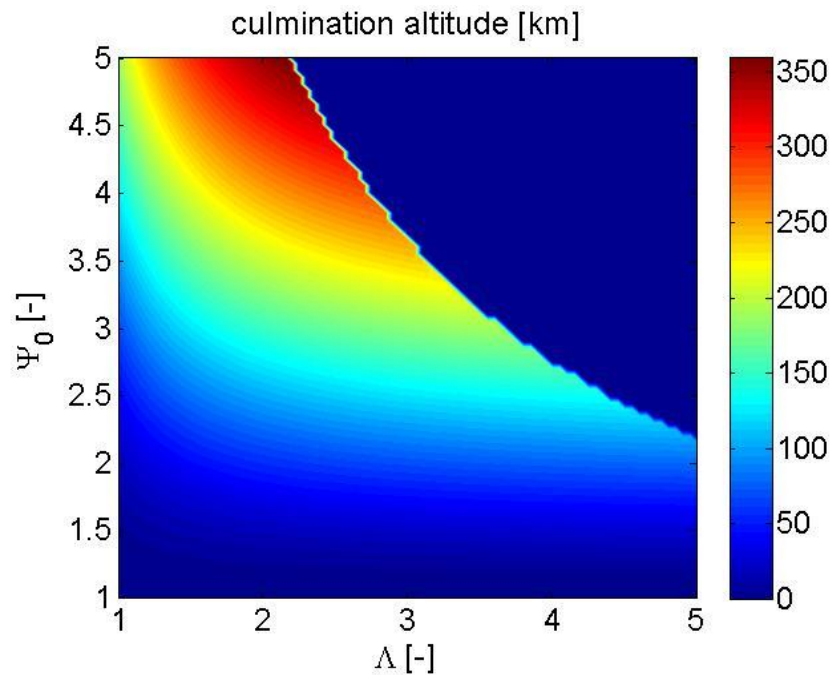
- specific impulse $I_{sp} = 300$ s
- $\Psi_0 = 1.5$
- $\Lambda = 5$

Results:

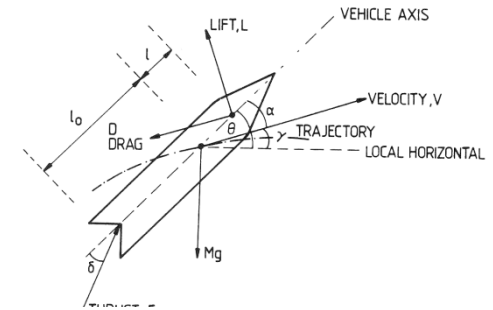
	w/o gravity	with gravity
burn time [s]	160.0	
burnout velocity [m/s]	4736.6	3167.0
burnout height [km]	281.4	155.8
culmination time [s]	-	482.8
culmination height [km]	-	667.0
culmination height for impulsive shot [km]	-	1143.5

Real single-stage launcher (cnt'd)

Culmination altitudes of single-stage launchers, for $I_{sp} = 200$ s (left) and 400 s (right). Maximum acceleration = 10g.



Real single-stage launcher (cnt'd)



Gravity loss: minimize by shifting to horizontal flight a.s.a.p.

Drag loss: minimize by reducing trajectory through atmosphere

CONFLICT !!

Solution 1: start in vertical directory, then turn to (more) horizontal direction.

Solution 2: use air-launched vehicle.

Example: Pegasus

Requirements [OSC, 2003]:

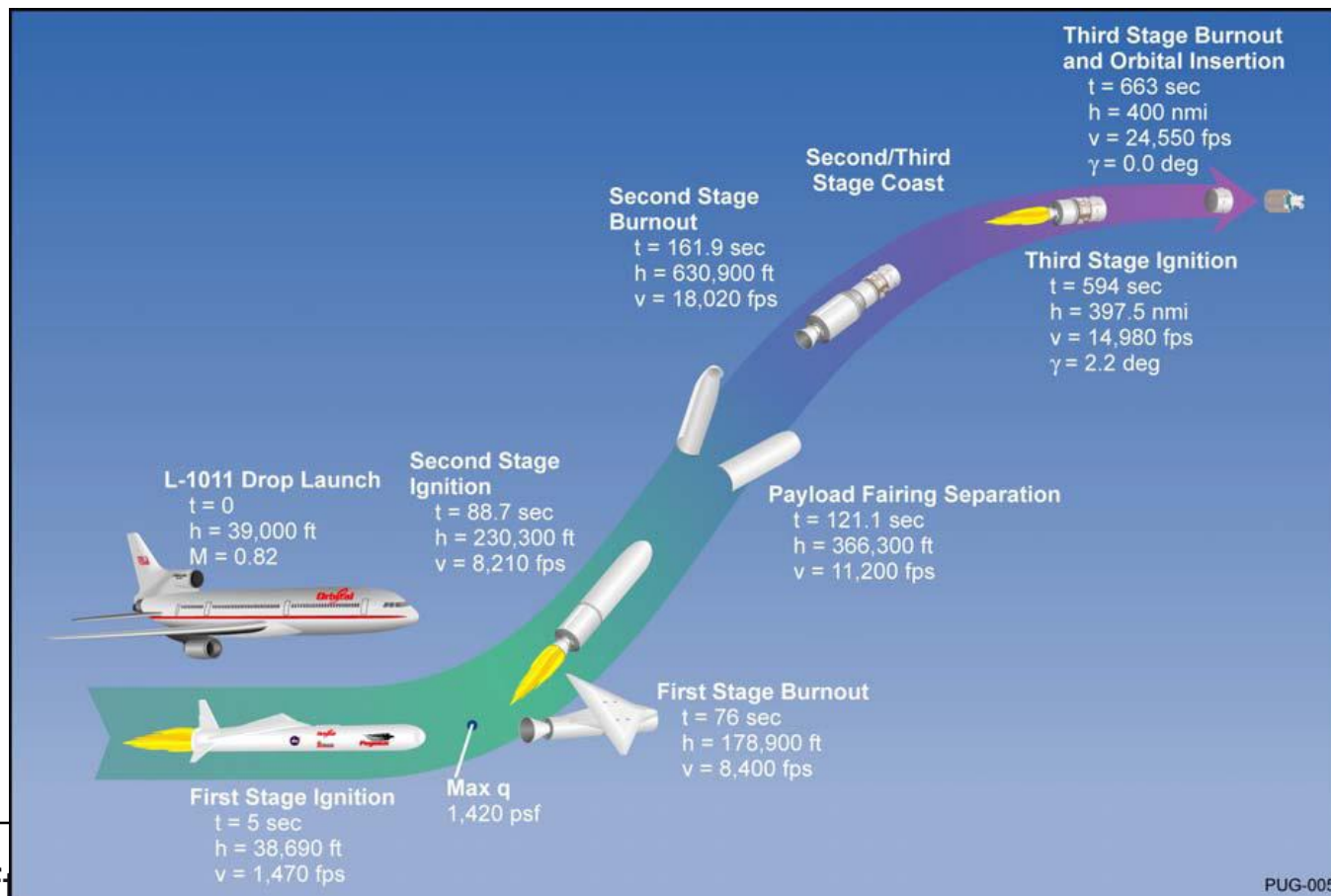
- maximum payload 455 kg into LEO
- cost-effective
- reliable
- flexible
- minimum ground support
- multiple payload capability
- short lead time
- (released at 12 km altitude)



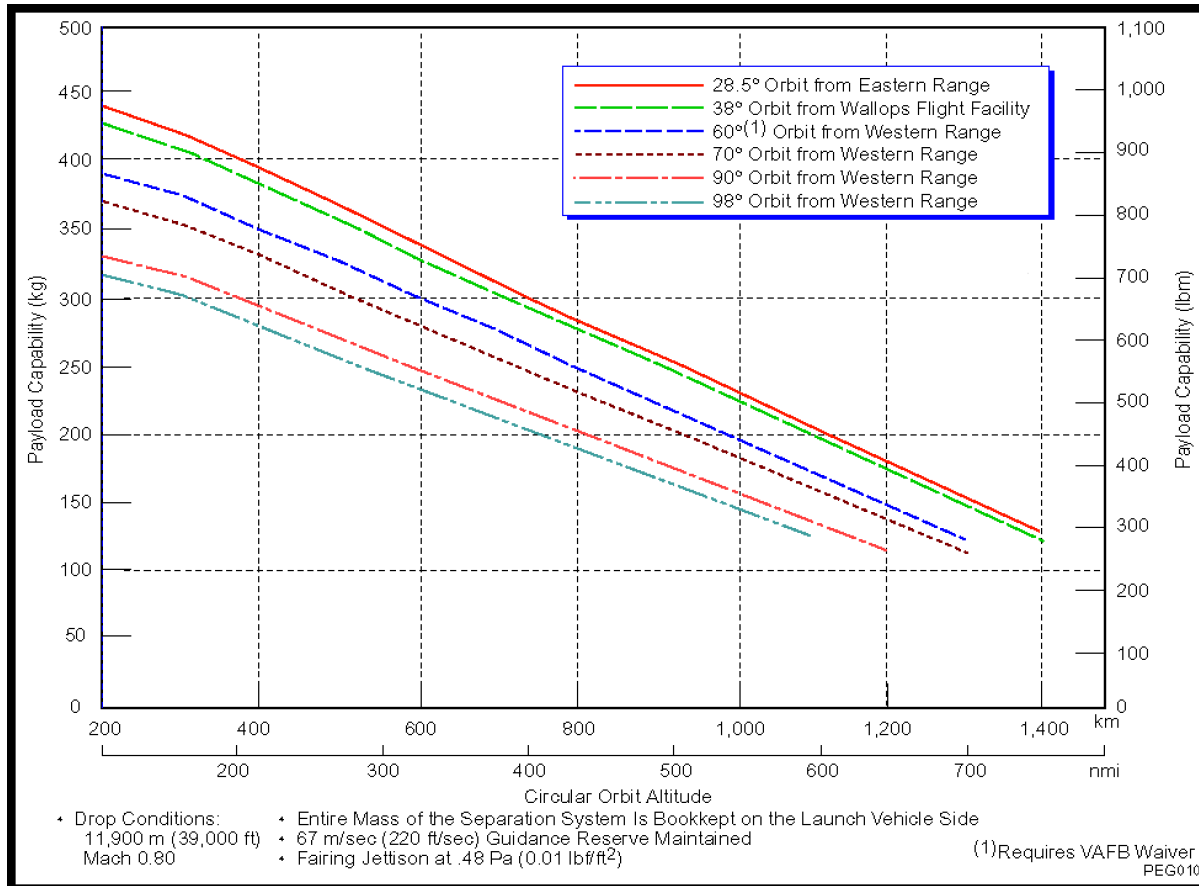
[OSC, 2010]
AE2104 Flight and Orbital Mechanics

Example: Pegasus (cnt'd)

Pegasus XL mission profile [OSC, 2007]



Overall performance



[OSC, 2000]

Can we (**easily**) reproduce these numbers?

Overall performance (cnt'd)

Specific energy (*i.e.*, energy per unit of mass):

$$E_{\text{orbit}} = E_{\text{pot,begin}} + E_{\text{kin,eff}} - \Delta E_{\text{pot}}$$

- E_{orbit} = total energy in orbit (sum of kinetic+potential)
- $E_{\text{pot,begin}}$ = potential energy at launch
- $E_{\text{kin,eff}}$ = effective kinetic energy
- ΔE_{pot} = gain in potential energy

Overall performance (cnt'd)

Substitution:

$$-\frac{\mu}{2a} = -\frac{\mu}{R_e + h_{\text{launch}}} + \frac{1}{2} \left(V_0 + \Delta V_1 + \Delta V_2 + \Delta V_3 - \Delta V_{d+g} \right)^2 - \left(-\frac{\mu}{a} + \frac{\mu}{R_e + h_{\text{launch}}} \right)$$

- μ = gravitational parameter Earth
- a = semi-major axis of orbit
- h_{launch} = altitude of launch platform
- V_0 = velocity of launch platform
- $\Delta V_{1,2,3}$ = velocity increment delivered by stage 1,2,3
- ΔV_{d+g} = velocity loss due to atmosphere and gravity

Overall performance (cnt'd)

Pegasus carrier:

$$V_0 = 0.463 \cos(\delta_{\text{launch}}) \frac{\cos(i)}{\cos(\delta_{\text{launch}})} + 0.222$$

velocity Earth \downarrow

velocity L1011 \swarrow

Pegasus vehicle:

stage	I_{sp} [s]	M_{prop} [kg]	M_{constr} [kg]	M_{begin} [kg]
3	289.3	770	126	$M_{\text{payload}} + M_{\text{prop},3} + M_{\text{constr},3}$
2	291.3	3925	416	$M_{\text{begin},3} + M_{\text{prop},2} + M_{\text{constr},2}$
1	295.9	15014	1369	$M_{\text{begin},2} + M_{\text{prop},1} + M_{\text{constr},1}$

→ $a = f(i, \delta_{\text{launch}}, h_{\text{launch}}, \text{payload mass})$

Overall performance (cnt'd)

[Wertz&Larson, 1991]:

- Drag+gravity losses 1.5-2.0 km/s
- Drag loss: 0.3 km/s

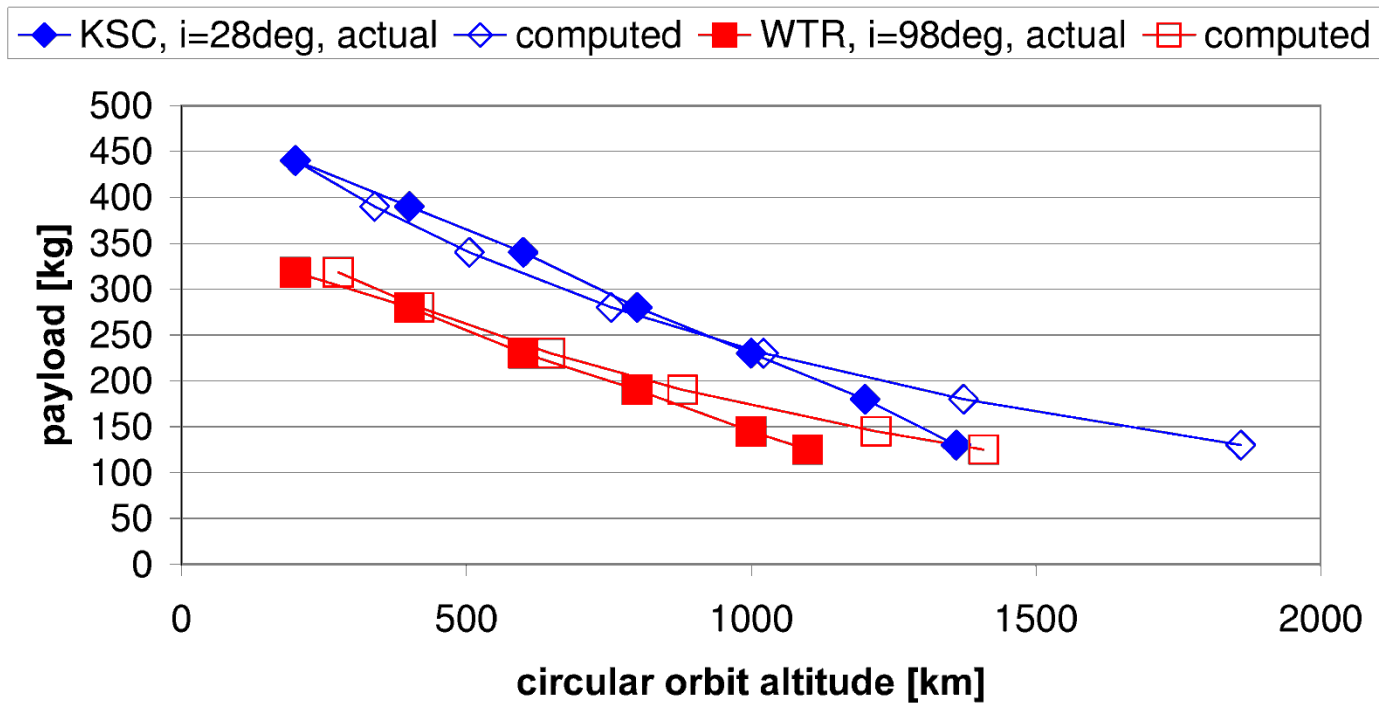
Pegasus: small launcher →

- Drag+gravity losses 1.5 km/s
- Drag loss 0.3 km/s
- Gravity loss $1.5 - 0.3 = 1.2$ km/s

Overall performance (cnt'd)

Results for launches due East from KSC ($\delta=28.5^\circ$) and WTR ($\delta=34.6^\circ$):

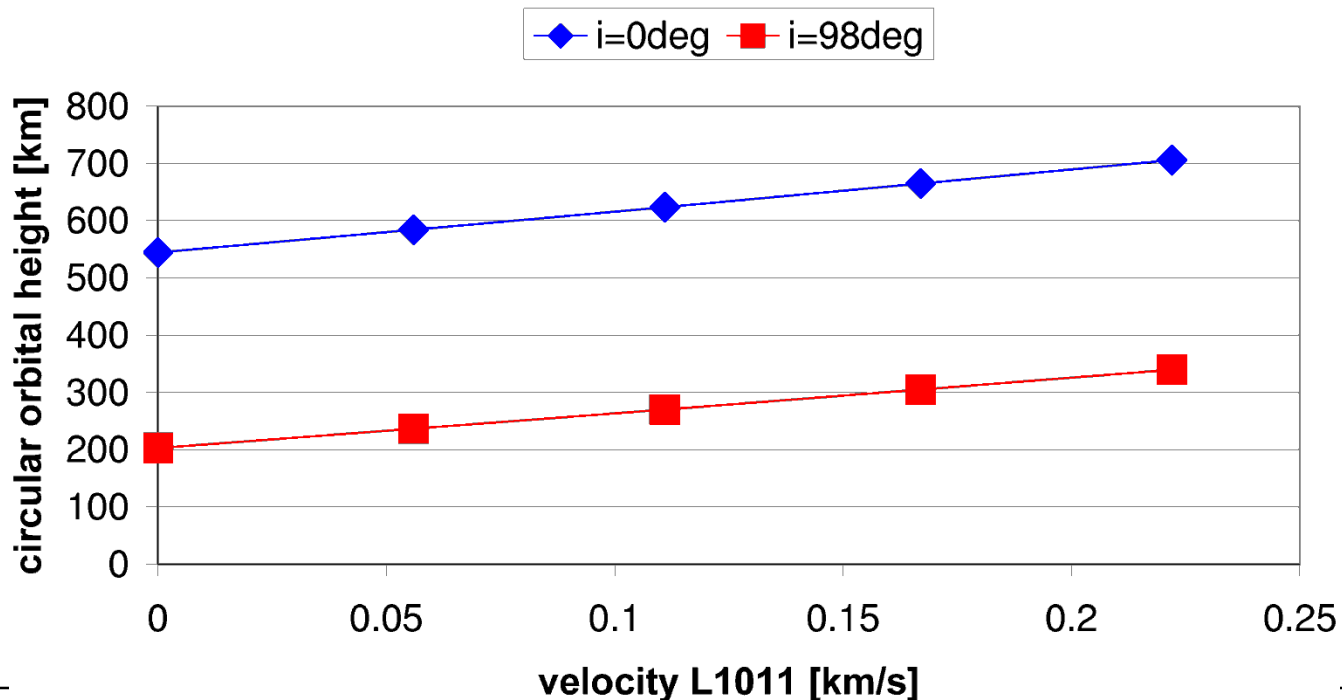
Pegasus payload performance



Overall performance (cnt'd)

Orbit altitude as a function of carrier velocity ($M_{\text{payload}} = 300 \text{ kg}$, launch at equator):

Pegasus capability for 300 kg payload



Design

Can we (**easily**) reproduce the overall layout of a launcher?

Example: Pegasus



Design (cnt'd)

Some data and assumptions:

- 3 stages
- I_{sp} identical for all stages (290 s)
- M_{constr}/M_{total} identical for all stages (0.08)
- $V_c = 7.784$ km/s at $h=200$ km
- $V_{earth} = 0.464$ km/s at equator
- $V_{carrier} = 0.222$ km/s w.r.t. Earth
- $\rightarrow V_{pegasus,initial} = 0.686$ km/s

Design (cnt'd)

- Drag loss 0.3 km/s
- Gravity loss 1.2 km/s

1st order approach:

- ΔV_{ideal} equally distributed over 3 stages
- Drag loss on account of 1st stage
- Gravity loss equally distributed over 3 stages



- Stage 1: $\Delta V = 2.366 + 0.3 + 0.4 = 3.066$ km/s

- Stage 2 and 3: $\Delta V = 2.366 + 0.4 = 2.766$ km/s (each)

Design (cnt'd)

Tsiolkovsky's rocket equation:

$$\Delta V = g_0 I_{sp} \ln \left(\frac{M_{total}}{M_{total} - M_{prop}} \right)$$

$$\Delta V = g_0 I_{sp} \ln \left(\frac{M_{total}}{M_{constr} + M_{payload}} \right)$$

$$\frac{M_{constr}}{M_{total}} + \frac{M_{payload}}{M_{total}} = \exp \left(\frac{-\Delta V}{I_{sp} g_0} \right)$$

Design (cnt'd)

Stage 3:

$$M_{\text{payload}} = 455 \text{ kg}, I_{\text{sp}} = 290 \text{ s}, M_{\text{constr}} / M_{\text{total}} \sim 0.08:$$

so:

- $M_{\text{total}} = 1526 \text{ kg}$
- $M_{\text{constr}} = 122 \text{ kg}$
- $M_{\text{payload}} = 455 \text{ kg}$
- $M_{\text{prop}} = 949 \text{ kg}$

Design (cnt'd)

	stage 3			stage 2			stage 1		
	re-eng [kg]	real [kg]	Δ [%]	re-eng [kg]	real [kg]	Δ [%]	re-eng [kg]	real [kg]	Δ [%]
payload	455	455	0.0	1526	1351	12.9	5116	5692	-10.1
constr.	122	126	-3.1	409	416	-1.6	1572	1369	14.8
prop.	949	770	23.2	3181	3925	-19.0	12961	15014	-13.7
total	1526	1351	12.9	5116	5692	-10.1	19649	22075	-11.0

Further reading

- Koelle, D.E., Cost Analysis of Present Expendable Launch Vehicles as contribution to Low Cost Access to Space Study. In: (2nd ed.), *Technical Note TCS-TN-147 (96)*, TransCostSystems, Ottobrun, Germany (December 1966).
- Parkinson, R.C., Total System Costing of Risk in a Launch Vehicle. In: *44th International Astronautical Congress* (2nd ed.), AA-6.1-93-735 (16–22 Oct., 1993) Graz, Austria .
- Isakowitz, S.J.. In: (2nd ed.), *International Reference Guide to Space Launch Systems*, American Institute for Aeronautics and Astronautics, Washington DC (1991).
- “ESA Launch Vehicle Catalogue”, *European Space Agency, Paris, Revision 8: December 1997*.
- <http://www.orbital.com> info on Pegasus, Taurus and Minotaur
- users.comkey.net/Braeunig/space/specs/pegasus.htm
- http://arianespace.com/english/leader_launches/html
- <http://www.boeing.com/defence-space/space/delta/record.htm>)