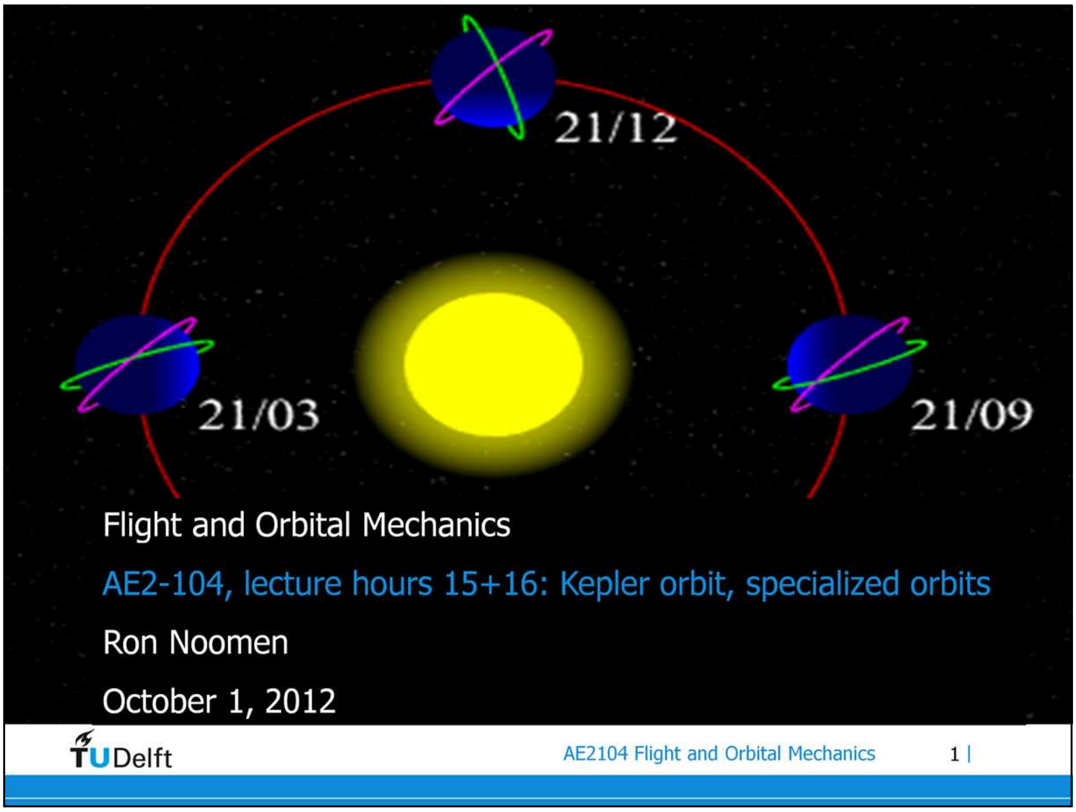
Flight and Orbital Mechanics

Lecture slides



Material for exam: this presentation (*i.e.*, no material from text book).

Sun-synchronous orbit: used for a variety of earth-observing missions.

Example: ENVISAT

[ESA, 2010]

Questions:
- what is the purpose of this mission?
- what is its orbit?
- what would the ΔV budget be?
- ….

Introduction picture.

Learning goals

The student should be able to:

- derive accelerations in arbitrary direction, induced by arbitrary components of the gravity field
- derive, understand and explain the equations that describe Earth-repeat orbits
- quantify relevant parameters of (potential) Earth-repeat orbits
- derive, understand and explain the equations that describe Sun-synchronous orbits
- quantify relevant parameters of Sun-synchronous orbits
- quantify relevant parameters of orbits that satisfy both Earth-repeat and Sun-synchronous conditions
- derive and quantify orbital perturbations on geostationary satellites and their implications

In essence, this is about understanding an important component in the space environment, and dealing with the consequences (in a positive way!).

Overview

- Kepler orbits
- Gravity potential
- Gravitational accelerations
- Earth repeat orbits
- Sun-synchronous orbits
- Geostationary orbit

This topic relies to a certain extent on the theory of Kepler orbits as lectured in ae1-102.

The material covered in this powerpoint presentation needs to be studied for the exam; more (background) information on all topics can be found in “Spacecraft Systems Engineering” by Fortescue, Stark and Swinerd.

Questions

- What are the parameters with which a gravity field can be described?
- How can one compute a gravitational acceleration in arbitrary direction?
- What is the effect of the flattening of the Earth?
- For what purpose do we need repeat orbits?
- What are the characteristics of repeat orbits?
- For what purpose do we need Sun-synchronous orbits?
- What are the characteristics of Sun-synchronous orbits?
- What are the effects of the gravity field on geostationary orbits?
- ....

Some examples of relevant questions that you should be able to answer after having mastered the topics of these lectures.

Introduction

Why gravity field ?

- Satellites provide a unique observation point for wide variety of observations (Earth objects, atmosphere, astronomical targets)
- Location (motion) of satellites is predominantly driven by Earth's gravity, including irregularities

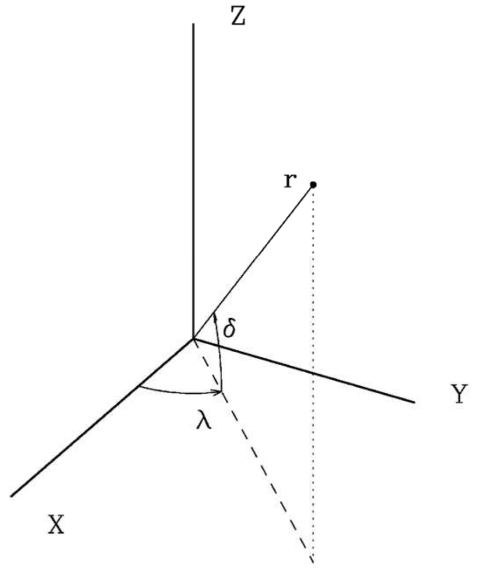
source	central gravity	flattening Earth	atmospheric drag	Solar radiation	Sun (3rd body)
acceleration [m/s^2]	9	1×10^{-2}	3×10^{-6}	5×10^{-9}	5×10^{-7}

- We can use the irregularities of the Earth's gravity field to our advantage.

The values mentioned in this table hold for the GOCE satellite (altitude 250 km), and are meant for illustration purposes only; actual acceleration values depend on specific circumstances (position, level of solar activity, ….). More on perturbations in one of the next lecture hours.

Kepler orbits (1)

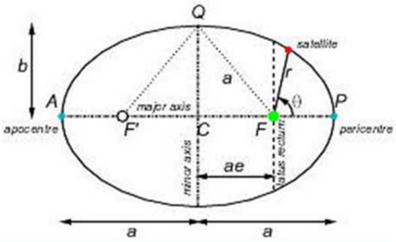
- Partial overlap with "space environment" (ae1-101)
- Coordinates: systems and parameters



Selecting a proper reference system and a set of parameters that describe a position in 3 dimensions is crucial to quantify most of the phenomena treated in this chapter, and to determine what a satellite mission will experience. Option 1: cartesian coordinates, with components x, y and z. Option 2: spherical coordinates, with components r (radius, measured w.r.t. the center-of-mass of the central object; not to be confused with the altitude over its surface), δ (latitude) and λ (longitude).

Kepler orbits (2)

$$r = \frac{a(1-e^2)}{1+e\cos(\theta)}$$

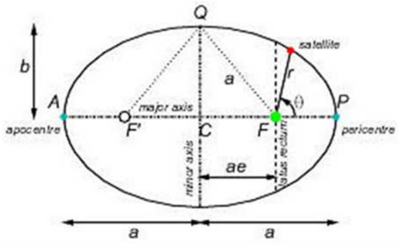


- $\theta = 0°$: $r = r_p = a(1-e)$; point at minimum distance → pericenter (perigee, perihelion, ….)
- $\theta = 90°$: $r = a(1-e^2) = p$
- $\theta = 180°$: $r = r_a = a(1+e)$; point at maximum distance → apocenter (apogee, apohelion, ….)
- $r_p + r_a = 2a$ → $a = (r_p + r_a) / 2$
- $e = (r_a - r_p) / (r_a + r_p)$

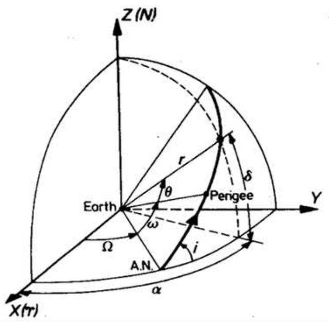
”Pericenter” is the general expression for the position at closest distance to the focal center; when the object orbits the Earth, one generally speaks of the “perigee”, and when it orbits the Sun, it is named ”perihelion”. Similar expressions for the point at farthest distance “apocenter” -> “apogee”, “apohelion”.

Kepler orbits (3)

2-dimensional orbits:
- a – semi-major axis [m]
- e – eccentricity [-]
- t_p , τ – time of pericenter passage [s]



3-dimensional orbits:
- i – inclination [deg]
- Ω - right ascension of ascending node [deg]
- ω – argument of pericenter [deg]



TU Delft AE2104 Flight

The time of passage of a well-defined point in the orbit (*e.g.*, the pericenter) is indicated by "t_p" or, equivalently, "τ" ("tau"). Knowing this value, one can relate the position in the orbit to absolute time (cf. following sheets).

The inclination "i" is the angle between the orbital plane and a reference plane, such as the equatorial plane. It is measured at the ascending node, *i.e.*, the location where the satellite transits from the Southern Hemisphere to the Northern Hemisphere, so by definition its value is between 0° and 180°. The parameters Ω and ω can take any value between 0° and 360°.

Gravity field Earth

Newton: $force = G \frac{mass_1 \times mass_2}{distance^2}$

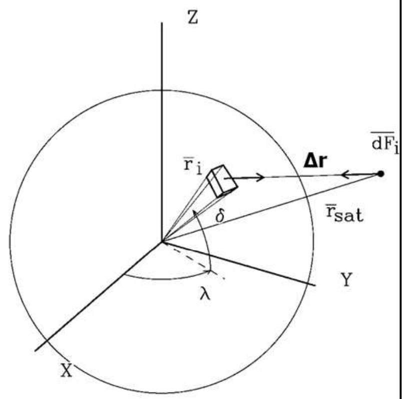
Elementary force: $d\overline{F} = -\frac{G\, m_{sat}}{(\Delta r)^2} \frac{\Delta \overline{r}}{\Delta r} \rho\, dv$

Total force: $\overline{F} = -\int_V \frac{G\, m_{sat}}{(\Delta r)^2} \frac{\Delta \overline{r}}{\Delta r} \rho\, dv$

Total acceleration due to symmetrical Earth: $\ddot{\overline{r}} = -\frac{G\,M}{r^3} \overline{r}$

Radial acceleration: $\ddot{r} = -\frac{G\,M}{r^2} = -\frac{\mu}{r^2}$

Idem, written as function of potential: $\ddot{r} = -\frac{\partial U}{\partial r} = -\frac{\partial}{\partial r}[-\frac{\mu}{r}]$ where $U = -\frac{\mu}{r}$



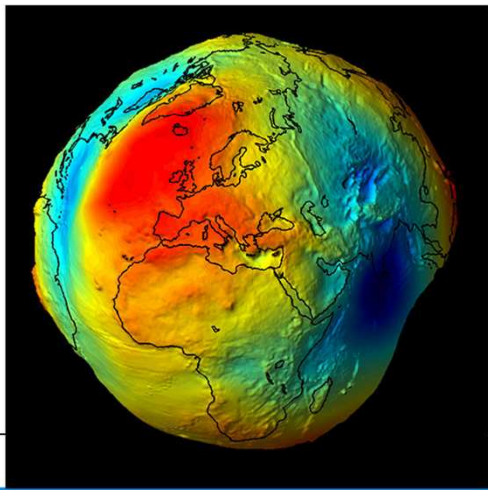
Parameter "G" is the universal gravitational constant (6.67259×10^{-11} m^3/kgs^2), "m_{sat}" represents the mass of the satellite, "Δr" is the distance between a mass element of the Earth and the satellite (written in bold it is the vector, directed from the mass element to the satellite), "r" is the distance between the satellite and the center-of-mass of the Earth (equal to r_{sat} in the picture – apologies for difference in notation), "ρ" is the mass density of an element "dv" of the Earth [kg/m^3], "V" is the total volume of the Earth, "M" is the total mass of the Earth (5.9737×10^{24} kg), "U" is the gravity potential (last equation: for a symmetric Earth), and "μ" is the gravitational parameter of the Earth ($=G \times M_{earth} = 398600.44 \times 10^9$ m^3/s^2).

The last equation shows how to compute the radial acceleration. Similar expressions can be used to derive the acceleration in x, y and z direction (that is: by taking the partial derivatives w.r.t. these parameters, and adding a minus sign).

Gravity field Earth (cnt'd)

real Earth: highly irregular gravity field

- potential
- accelerations
- geoid



[GFZ, 2011]

The Earth is irregular in shape and mass distribution; this picture illustrates the geoid, *i.e.* an equipotential surface w.r.t. a 3D ellipsoid; offsets are about 80 m in either direction.

Gravity field Earth (cnt'd)

Gravity potential:

$$U = -\frac{\mu}{r}\Big[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n(\sin\delta) + \sum_{n=2}^{\infty}\sum_{m=1}^{n} J_{n,m}\left(\frac{R_e}{r}\right)^n P_{n,m}(\sin\delta)\cos(m(\lambda - \lambda_{n,m}))\Big]$$

Associated Legendre function:

$$P_{n,m}(x) = (1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m}$$

Legendre polynomial:

$$P_n(x) = \frac{1}{(-2)^n\, n!}\frac{d^n}{dx^n}(1-x^2)^n$$

TU Delft | AE2104 Flight and Orbital Mechanics | 12 |

The first equation gives the classical description of the gravity field potential, with respectively the main term, the zonal terms (independent of longitude) and the sectorial and tesseral terms (cf. next sheet). Parameter "R_e" is the Earth's equatorial radius (6378×10^3 m); the satellite position is described in an Earth-fixed (as in: co-rotating) reference frame by the radius "r" (w.r.t. the center of Earth), the latitude "δ" (w.r.t. the equator) and the longitude "λ" (w.r.t. an Earth-fixed reference meridian: the 0° meridian crossing Greenwich). The parameters "J_n", "$J_{n,m}$" and "$\lambda_{n,m}$" are scaling and orientation coefficients of the gravity field model, respectively. J_2 is about 1082×10^{-6}, whereas the other J values are $O(10^{-6})$.

When developing a specific term of the gravity field potential, it is strongly recommended to develop the Legendre function or potential as function of the general argument "x" first, and substitute the actual argument "sinδ" at the very end.

The (infinite long) series expansion, with different "frequencies" as determined by the Legendre polynomial/function, is best compared to a Fourier series to describe an arbitrary signal.

Gravity field Earth (cnt'd)

- examples of gravity field coefficients: NASA's EGM96 [Lemoine *et.al.* 1998]
- zonal terms (*i.e.*, independent of longitude λ)
- other terms $O(10^{-6})$
- 1st order: central force only (accurate to 0.1%)

n	J_n $[10^{-6}]$
2	1082.6267
3	-2.5327
4	-1.6196
5	-0.2273

See previous sheet for role of degree "n", order "m" and coefficient "$J_{n,m}$". EGM is the abbreviation of Earth Gravity Model.

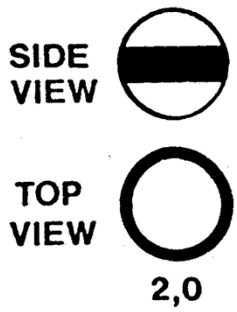
Gravity field Earth (cnt'd)

main term:

$$U_0 = -\frac{\mu}{r}$$

most prominent irregularity: J_2

$$U_2 = \frac{\mu}{r} J_2 \left(\frac{R_e}{r} \right)^2 P_2(\sin\delta)$$



J_2 is about 1082×10^{-6}, whereas the other J values are $O(10^{-6})$. It is related to the equatorial bulge, the ring of extra material around the equator of the Earth. U_2 is the contribution of the flattening of the Earth to the total potential U (so, directly related to the value of J_2). It can also be written as $U_{2,0}$.

Gravity field Earth (cnt'd)

$$U_2 = \frac{\mu}{r} J_2 \left(\frac{R_e}{r} \right)^2 P_2(\sin\delta)$$

step 1 :

$$P_2(x) = \frac{1}{(-2)^2\, 2!} \frac{d^2}{dx^2}(1-x^2)^2 = \frac{1}{8}\frac{d^2}{dx^2}(1-2x^2+x^4) =$$

$$= \frac{1}{8}\frac{d}{dx}(-4x+4x^3) = \frac{1}{8}(-4+12x^2) = -\frac{1}{2}+\frac{3}{2}x^2$$

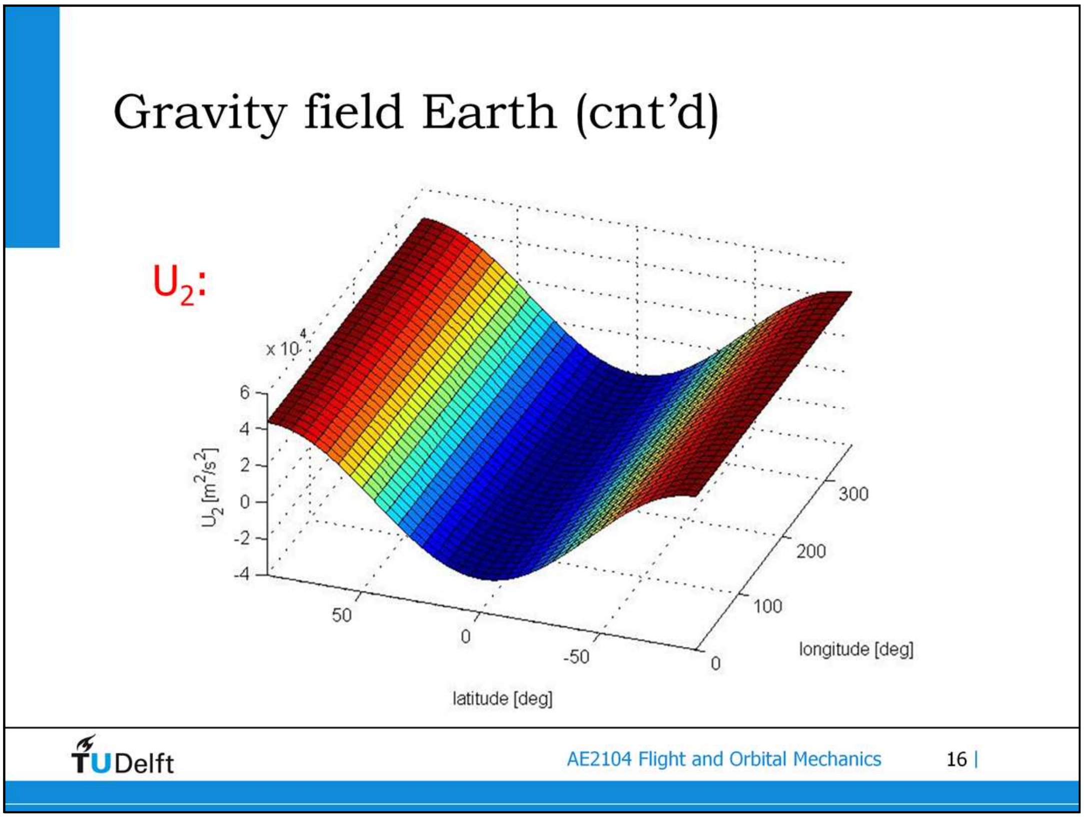
step 2 :

$$U_2 = \mu J_2 R_e^2 r^{-3} \left(-\frac{1}{2}+\frac{3}{2}\sin^2\delta \right)$$

J_2 is about 1082×10^{-6}, whereas the other J values are $O(10^{-6})$. U_2 can also be written as $U_{2,0}$.

First develop P_2 as a function of the general argument "x", and then substitute this with the actual argument "sinδ".

In these equations, parameter "x" does NOT represent the x-position of our spacecraft, but it is a general argument (as in "y=f(x)").



J_2 is about 1082×10^{-6}, whereas the other J values are O(10^{-6}). U_2 can also be written as $U_{2,0}$. It can be interpreted as potential energy (per unit mass). In this illustration, it is evaluated at 1000 km altitude. Clearly, U_2 is independent of longitude and symmetric in latitude (see equation).

Gravity field Earth (cnt'd)

accelerations: take partial derivative w.r.t. direction of interest

$$a_x = -\frac{\partial U}{\partial x}$$

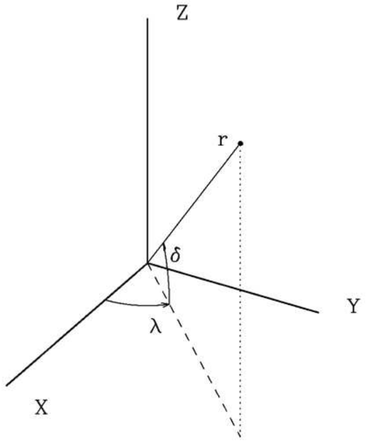
$$a_y = -\frac{\partial U}{\partial y}$$

$$a_z = -\frac{\partial U}{\partial z}$$

$$a_r = -\frac{\partial U}{\partial r}$$

$$a_\delta = -\frac{1}{r}\frac{\partial U}{\partial \delta}$$

$$a_\lambda = -\frac{1}{r\cos\delta}\frac{\partial U}{\partial \lambda}$$



Important: coordinates are taken w.r.t. an Earth-fixed, *i.e.*, co-rotating system!

Straightforward treatment of gravity field components. Parameters r, δ and λ represent radius (w.r.t. center of Earth), geocentric latitude (w.r.t. equatorial plane) and longitude (w.r.t. Earth-fixed reference), respectively.

Accelerations are derived by taking the derivative w.r.t. a parameter that expresses length in the required direction. Since an infinitesimal distance in NS direction is equal to rdφ (angle φ in radians, measured along the great circle), the expression for the acceleration in NS direction contains a scaling factor 1/r. In a similar fashion, an infinitesimal distance in EW direction, at a certain latitude δ, is equal to r cos(δ)dφ (the total length of a latitude circle is equal to 2πrcos(δ)). Therefore, the expression for EW acceleration contains a scaling factor 1/(r cos(δ)).

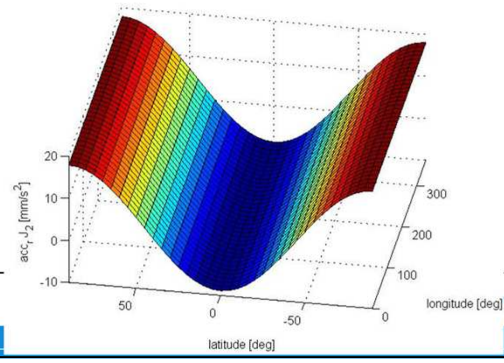
When deriving expressions for the accelerations due to a particular term, always develop the Legendre functions $P_{n,m}$ and P_n as a function of the general parameter "x" first, and not until the very end substitute "x" with the real argument "sin δ".

Gravity field Earth (cnt'd)

$$U_2 \quad = \quad \mu\, J_2\, R_e^{\,2}\, r^{-3} \left(-\frac{1}{2} + \frac{3}{2} \sin^2 \delta \right)$$

example 1: acceleration in radial direction due to J_2:

$$a_{r,2} = -\frac{\partial U_2}{\partial r} \quad = \quad 3\,\mu\, J_2\, R_e^{\,2}\, r^{-4} \left(-\frac{1}{2} + \frac{3}{2} \sin^2 \delta \right)$$



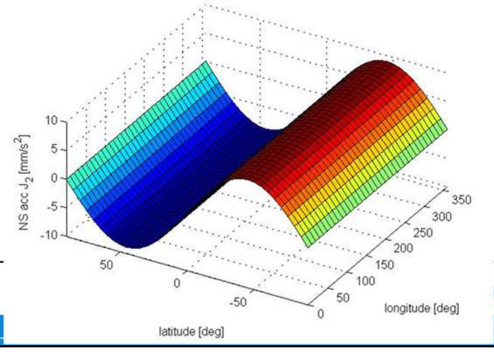
Application of recipe on previous sheet. Again, for altitude of 1000 km. Radial acceleration due to J_2 is independent of longitude, and symmetric in latitude.

Gravity field Earth (cnt'd)

$$U_2 \quad = \quad \mu\, J_2\, \mathrm{R_e}^2\, r^{-3} \left(-\frac{1}{2} + \frac{3}{2}\sin^2\delta \right)$$

example 2: acceleration in latitudinal (*i.e.* North-South) direction due to J_2

$$a_{\delta,2} = -\frac{1}{r}\frac{\partial U_2}{\partial \delta} \quad = \quad -\mu\, J_2\, \mathrm{R_e}^2\, r^{-4} \left(3 \sin\delta \cos\delta \right)$$



Idem. The NS acceleration due to J_2 is also independent of longitude, and anti-symmetric in latitude. Note: the direction "North-South" is generally different from the z-direction, since the former follows the curvature of the surface of the Earth, whereas the latter is always pointed along the rotation axis of the Earth.

Gravity field Earth (cnt'd)

$$U_2 \quad = \quad \mu\, J_2\, \mathrm{R_e}^2\, r^{-3} \left(-\frac{1}{2} + \frac{3}{2} \sin^2 \delta \right)$$

example 3: acceleration in longitudinal (*i.e.* East-West) direction due to J_2

$$a_{\lambda,2} = -\frac{1}{r\cos\delta}\frac{\partial U_2}{\partial \lambda} \quad = \quad 0$$

Idem. The EW acceleration due to J_2 is zero, since U is independent of longitude (it is symmetric around the z-axis, cf. illustration on sheet 14).

Gravity field Earth (cnt'd)

example 4: East – West acceleration due to $J_{3,2}$:

step 1:

$$U_{3,2} = -\frac{\mu}{r}\left[+J_{3,2}\left(\frac{R_e}{r}\right)^3 P_{3,2}(\sin\delta)\cos(2(\lambda-\lambda_{3,2}))\right]$$

step 2:

$$P_3(x) = \frac{1}{(-2)^3\, 3!}\frac{d^3}{dx^3}(1-x^2)^3 = \frac{1}{-8\times 6}\frac{d^3}{dx^3}(1-3x^2+3x^4-x^6) = \ldots\ldots = \frac{5}{2}x^3-\frac{3}{2}x$$

step 3:

$$P_{3,2}(x) = (1-x^2)^{2/2}\frac{d^2P_3(x)}{dx^2} = (1-x^2)\frac{d^2}{dx^2}(\frac{5}{2}x^3-\frac{3}{2}x) = \ldots\ldots = 15\,(1-x^2)\,x$$

This is the most complicated derivation that you can encounter in this course ae2-104 (or what you can expect during an exam….).

When doing such a derivation in an exam, the full derivation should be given (and not with the ”…..” text as shown here, for the sake of brevity).

First develop P_3 and $P_{3,2}$ as function of the general argument “x”, and then substitute this by the actual argument “sinδ”.

Gravity field Earth (cnt'd)

example 4: East – West acceleration due to $J_{3,2}$ (cnt'd):

step 4 :

$$U_{3,2} = -\mu J_{3,2} R_e^3 r^{-4} 15 \cos^2\delta \sin\delta \cos(2(\lambda - \lambda_{3,2}))$$

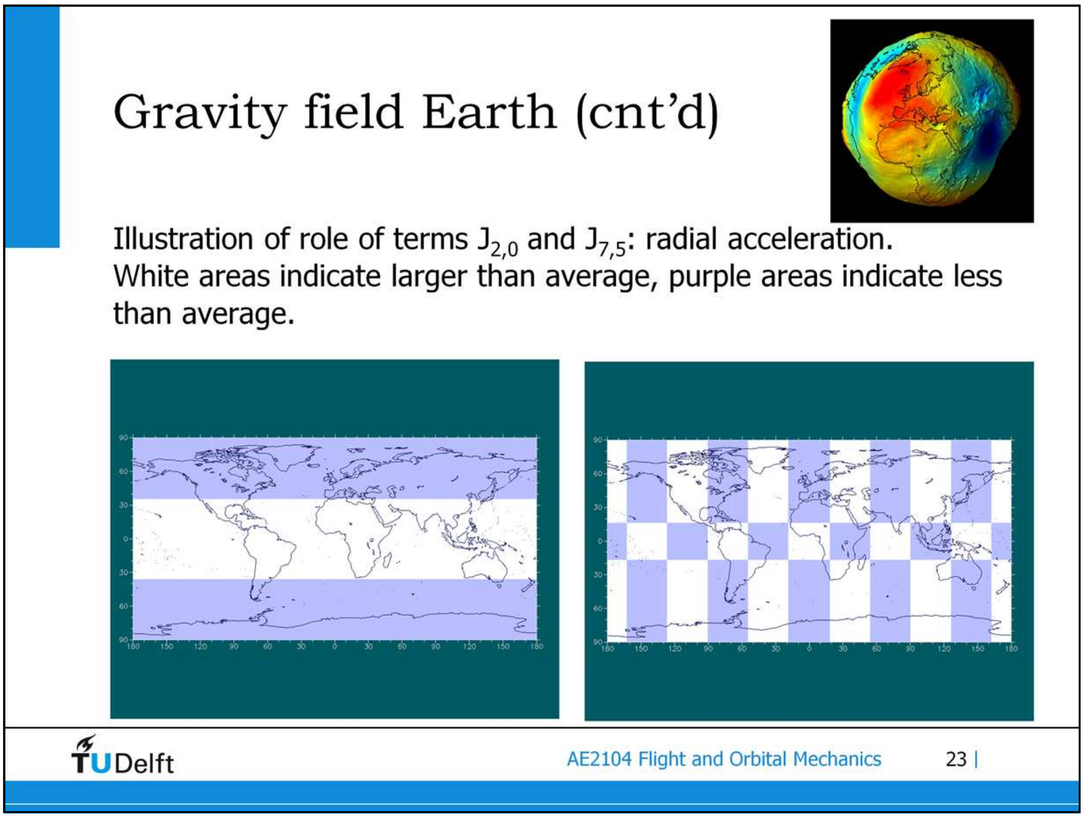
step 5:

$$a_{ew;3,2} = -\frac{1}{r\cos\delta}\frac{\partial U_{3,2}}{\partial\lambda} = \ldots\ldots = -30\,\mu J_{3,2} R_e^3 r^{-5} \cos\delta \sin\delta \sin(2(\lambda - \lambda_{3,2}))$$

STILL TO DO: INCLUDE ILLUSTRATION MATLAB

This is the most complicated derivation that you can encounter in this course ae2-104 (or what you can expect during an exam….).

When doing such a derivation in an exam, the full derivation should be given (and not with the "….." text as shown here, for the sake of brevity).



The collection of these (and other) terms can be regarded as corrections to the 1^{st} order model of a spherical, radially symmetric Earth. The equatorial bulge is represented by the J_2 (or $J_{2,0}$, as depicted in this plot) term, which is the dominant correction term in the Earth's gravity field model. The real gravity field can be approximated by an (in principle infinite) series of individual terms, with different characteristics (cf. a Fourier analysis).

Gravity field Earth (cnt'd)

Effect of irregularities in gravity field on satellite orbit:

First-order: linear perturbation on orbital elements due to J_2 ($p = a\,(1 - e^2)$):

$$\Delta a_{2\pi} = 0$$

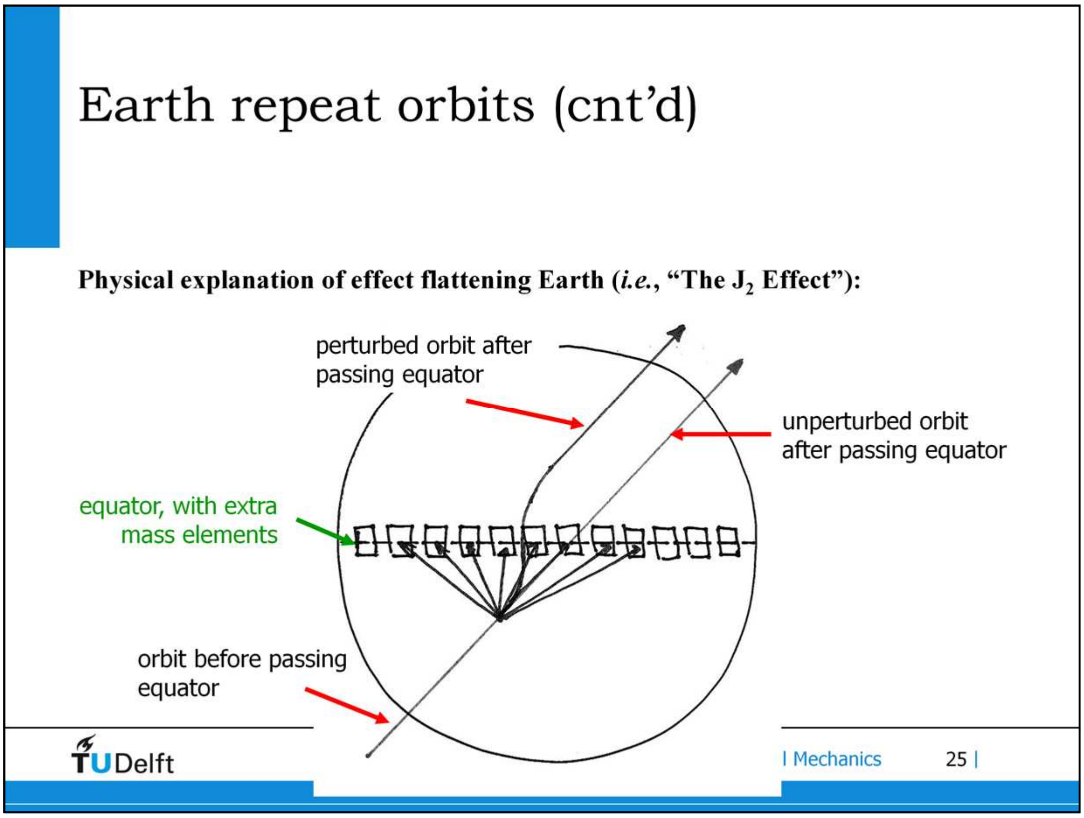
$$\Delta e_{2\pi} = 0$$

$$\Delta i_{2\pi} = 0$$

$$\Delta \Omega_{2\pi} = -\,3\,\pi\;J_2\,(R_e / p)^2 \cos i \qquad \text{[rad]}$$

$$\Delta \omega_{2\pi} = 1.5\,\pi\,J_2\,(R_e / p)^2\,(5 \cos^2 i - 1) \qquad \text{[rad]}$$

Changes in parameters after one complete revolution of the satellite around the central body (Earth). So, no secular J_2-effect on semi-major axis, eccentricity nor inclination.



The net force acting on the satellite when traveling over the Southern hemisphere causes the trajectory to be bent in northern direction (*i.e.*, the inclination of the orbit increases). When over the Northern hemisphere, the same thing happens but with reverse sign → the net effect on the inclination is zero, but the orientation of the orbit has shifted in a western direction (for inclinations smaller than 90º, as sketched here).

Gravity field Earth (cnt'd)

Question 1:

1. What is the definition for the radial acceleration, based on the potential formulation for the gravity field of the Earth?
2. Derive the general equation for the radial acceleration due to the term (2,0) for an arbitrary satellite.
3. What is the numerical expression for the radial acceleration due to $J_{2,0}$ for a satellite at 500 km altitude?
4. Make a sketch of this acceleration as a function of latitude ($-90° \leq \delta \leq 90°$).

Data: μ_{Earth} = 398600.4415 km^3/s^2; R_e = 6378.137 km; $J_{2,0} = 1082 \times 10^{-6}$

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

ANSWERS **(TRY YOURSELF FIRST!!):**

1. $a_r = -\partial U/\partial r$
2. $a_r = 3\,\mu\, J_2\, R_e^2\, r^{-4}\,(-0.5 + 1.5 \sin^2\delta)$
3. $a_r = 0.0235\,(-0.5 + 1.5 \sin^2\delta)$ [m/s^2]
4. Give sketch. Note: sketch is symmetric w.r.t. δ=0°.

Gravity field Earth (cnt'd)

Question 2:

1. What is the definition for the North-South acceleration, based on the potential formulation for the gravity field of the Earth?
2. Derive the general equation for the North-South acceleration due to the term (2,0) for an arbitrary satellite.
3. What is the numerical expression for the North-South acceleration due to $J_{2,0}$ for a satellite at 500 km altitude?
4. Make a sketch of this acceleration as a function of latitude ($-90° \leq \delta \leq 90°$).

Data: μ_{Earth} = 398600.4415 km^3/s^2; R_e = 6378.137 km; $J_{2,0} = 1082 \times 10^{-6}$

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

ANSWERS **(TRY YOURSELF FIRST!!):**

1. $a_{NS} = -1/r \times \partial U/\partial\delta$
2. $a_{NS} = -3\, \mu\, J_2\, R_e^2\, r^{-4} \sin\delta \cos\delta$
3. $a_{NS} = -0.0235 \sin\delta \cos\delta$ [m/s^2]
4. Give sketch. Note: sketch is anti-symmetric w.r.t. δ=0°.

Gravity field Earth (cnt'd)

Question 3:

1. What is the definition for the radial acceleration, based on the potential formulation for the gravity field of the Earth?
2. Derive the general equation for the radial acceleration due to the term (3,0) for an arbitrary satellite.
3. What is the numerical expression for the radial acceleration due to $J_{3,0}$ for a satellite at 500 km altitude?
4. Make a sketch of this acceleration as a function of latitude ($-90° \leq \delta \leq 90°$).

Data: μ_{Earth} = 398600.4415 km^3/s^2; R_e = 6378.137 km; $J_{3,0} = -2.53 \times 10^{-6}$

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

ANSWERS **(TRY YOURSELF FIRST!!):**

1. $a_r = -\partial U / \partial r$
2. $a_r = 4\, \mu\, J_3\, R_e^3\, r^{-5}\, (2.5 \sin^3\delta - 1.5 \sin\delta)$
3. $a_r = -6.80 \times 10^{-5}\, (2.5 \sin^2\delta - 1.5 \sin\delta)$ [m/s^2]
4. Give sketch. Note: sketch is anti-symmetric w.r.t. δ=0°.

Gravity field Earth (cnt'd)

Question 4:

1. What is the definition for the North-South acceleration, based on the potential formulation for the gravity field of the Earth?
2. Derive the general equation for the North-South acceleration due to the term (3,0) for an arbitrary satellite.
3. What is the numerical expression for the North-South acceleration due to $J_{3,0}$ for a satellite at 500 km altitude?
4. Make a sketch of this acceleration as a function of latitude ($-90° \leq \delta \leq 90°$).

Data: $\mu_{Earth} = 398600.4415$ km^3/s^2; $R_e = 6378.137$ km; $J_{3,0} = -2.53\times10^{-6}$

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

ANSWERS **(TRY YOURSELF FIRST!!):**

1. $a_{NS} = -1/r \times \partial U/\partial \delta$
2. $a_{NS} = -\mu J_3 R_e^3 r^{-5} (7.5 \sin^2\delta \cos\delta - 1.5 \cos\delta)$
3. $a_{NS} = 1.70\times10^{-5} (7.5 \sin^2\delta \cos\delta - 1.5 \cos\delta)$ [m/s^2]
4. Make sketch. Note: sketch is symmetric w.r.t. δ=0°.

Earth repeat orbits

What?

The ground track of the satellite repeats after a certain period (*i.e.*, the satellite overflies a particular spot on Earth exactly after a certain time interval)

Why?

- To have exactly the same viewing geometry of a target on Earth
- To be able to correlate sea-level observations taken at one and the same point on Earth

How?

Use of J_2 effect

The ground track of a satellite is the succession of sub-satellite points (*i.e.*, the projection of the satellite position on the surface of the Earth; altitude drops out).

Raw sea-level observations need to be corrected for many phenomena, such as local tides, effects of local sea-floor geometry, etcetera. When evaluated at a single, well-defined spot, systematic errors in such corrections will drop out (whereas they can introduce errors when comparing observations at *e.g.* 30 km distance).

Earth repeat orbits (cnt'd)

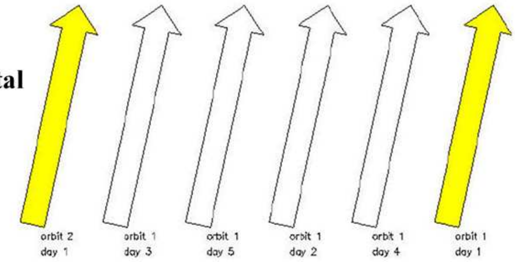
satellite	period	Earth-repeat	Sun-synch.
Skylab	1973-1974	no	no
GEOS-3	1975-1978	??	no
Seasat	1978-1978	yes	no
Geosat	1985-1990	yes	no
ERS-1	1991-2000	yes	yes
TOPEX/Poseidon	1992-2005	yes	no
ERS-2	1995-2003*	yes	yes
GFO	1998-2008	yes	no
Jason-1	2001-current	yes	no
ENVISAT	2002-2012	yes	yes
ICESat	2003-2009	yes	no
Jason-2	2008-current	yes	no
Cryosat-2	2010-current	yes	no

Overview of satellites equipped with altimeter instruments. Note the "families" Seasat - Geosat - GFO, ERS-1 – ERS-2 – ENVISAT, and TOPEX/Poseidon – Jason-1 – Jason-2. Satellites in red are US, satellites in green are European. GFO is the abbreviation for Geosat Follow-On. Examples of other satellites in Earth-repeat orbits (but not necessarily equipped with an altimeter instrument): SMOS, Cartosat, Sentinel-3.

Earth repeat orbits (cnt'd)

Req 1: ground track repeats after *j* orbital revolutions and *k* "days"

Req 2: effects are measured w.r.t. Earth's surface



approach: consider effects on position where satellite crosses equatorial plane in S-N direction (in Earth-fixed coordinates!)

$$\Delta L_1 = -2\pi \frac{T}{T_E} \quad \text{[rad/rev]}$$ (contribution of Earth rotation)

$$\Delta L_2 = -\frac{3\pi J_2 R_e^2 \cos i}{a^2 (1-e^2)^2} \quad \text{[rad/rev]}$$ (contribution of J_2)

1st order: $e = 0$

$$j \left|\Delta L_1 + \Delta L_2\right| = k\,2\pi$$

"L" is longitude: measured in co-rotating Earth-fixed reference frame, positive in Eastward direction.

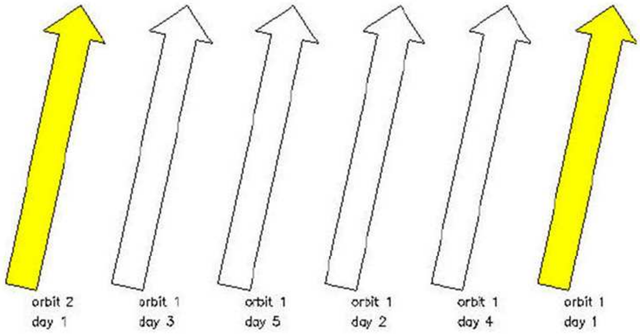
Verify direction of shift in plot and definition of signs in equations.

Eccentricity ~0 holds for 95% of actual spacecraft.

T is the orbital period of the satellite [s]. T_E is the rotational period of the Earth (sidereal day, [s]).

Effect ΔL_2: cf. expressions on sheet 24.

Earth repeat orbits (cnt'd)



Refinements:

- **effect of J_2 on ω and θ (treated in ae4-878)**
- **effect of terms other than J_2 on Ω, ω and θ**
- **effect of atmospheric drag**

Neither of these aspects treated here

Earth repeat orbits (cnt'd)

$$j\left|-2\pi\frac{2\pi\sqrt{a^3/\mu}}{T_E}-\frac{3\pi J_2 R_e{}^2\cos(i)}{a^2(1-e^2)^2}\right|=k\,2\pi$$

Usual assumption: e = 0

Possible solution strategies:

1. assume i → a(i) (iterations needed)
2. assume a → i(a) (direct solution)

The value for "k" follows from the top-level requirements. The value for "j" is a reasonable guess, knowing that the real satellite is expected to fly at a a certain altitude. What are the options for (a,e,i)?

Earth repeat orbits (cnt'd)

(j,k)=(14,1)

$a = 7200\,\text{km}:$

$$14\left| -2\pi\frac{2\pi\sqrt{7200^3/398600.4415}}{86164} - \frac{3\pi 1082\times10^{-6}\,6378.137^2\cos(i)}{7200^2(1-0^2)^2}\right| = 1\times2\pi$$

$\Rightarrow \quad i = 47.2°$

$a = 7300\,\text{km}:$

$$14\left| -2\pi\frac{2\pi\sqrt{7300^3/398600.4415}}{86164} - \frac{3\pi 1082\times10^{-6}\,6378.137^2\cos(i)}{7300^2(1-0^2)^2}\right| = 1\times2\pi$$

$\Rightarrow \quad i = 119.5°$

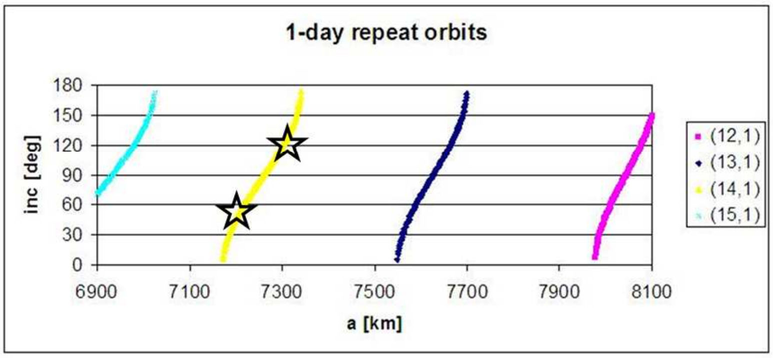
$a = 7500\,\text{km}:$

$$14\left| -2\pi\frac{2\pi\sqrt{7500^3/398600.4415}}{86164} - \frac{3\pi 1082\times10^{-6}\,6378.137^2\cos(i)}{7500^2(1-0^2)^2}\right| = 1\times2\pi$$

$\Rightarrow \quad \cos(i) = -3.06$; no feasible solution

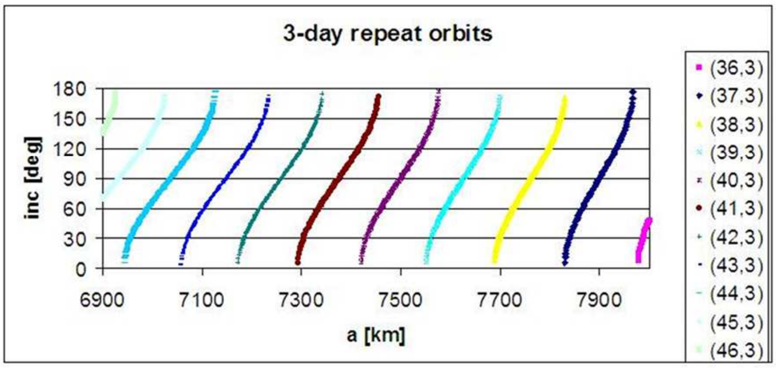
3 examples for (j,k) = (14,1). Here we follow the approach of assuming a value for the semi-major axis, and solving for inclination. Can we identify possible combinations (a,i) (eccentricity e is assumed to be 0) that satisfy the requirements?

Earth repeat orbits (cnt'd)



Examples for zero-eccentricity orbits, with a 1-day repeat cycle. A higher altitude corresponds with a longer orbital period, which means less revolutions per day. Circular orbits: altitude is semi-major axis minus Earth radius (6378.137 km). The black stars indicate the two possible solutions as derived on the previous sheet.

Earth repeat orbits (cnt'd)



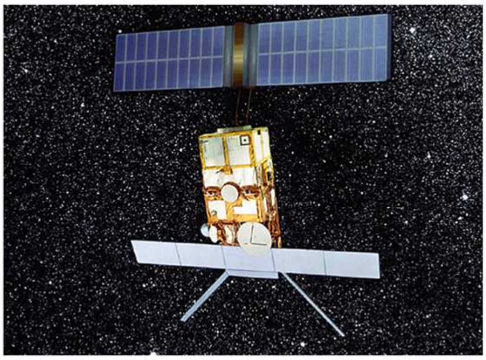
Examples for zero-eccentricity orbits, with a 3-day repeat cycle. A higher altitude corresponds with a longer orbital period, which means less revolutions per day. Circular orbits: altitude is semi-major axis minus Earth radius (6378.137 km). Clearly, going from a 1-day repeat period to a 3-day repeat period increases the range of options. Compare some lines with the plot on the previous sheet: 45 revolutions in 3 days is exactly identical to 15 revolutions in 1 day….

Earth repeat orbits (cnt'd)

Example 1:

ERS-1

Mission purpose: Earth observation (instruments: SAR, altimeter, …)
Launch date July 17, 1991
Retired March 10, 2000
Altitude ~780 km
Semi-major axis ~7158 km
Eccentricity ~0
Inclination 98.52°
Repeat period reference orbit 35 days



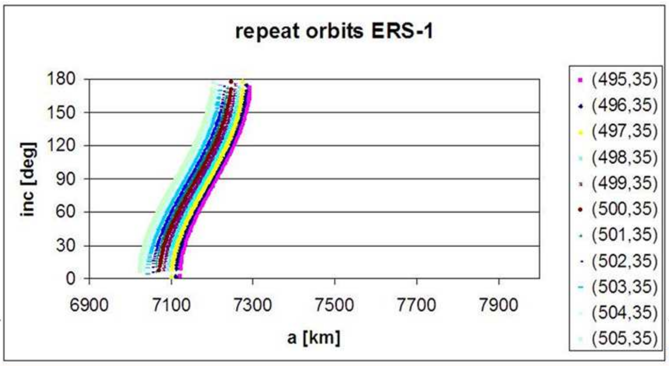
[Dutchspace, 2010]

ERS-1 is the abbreviation for European Remote-Sensing satellite. ESA was extremely proud of this one, it's a classical one!

Earth repeat orbits (cnt'd)

ERS-1: e=0, k=35, j=495,…,505

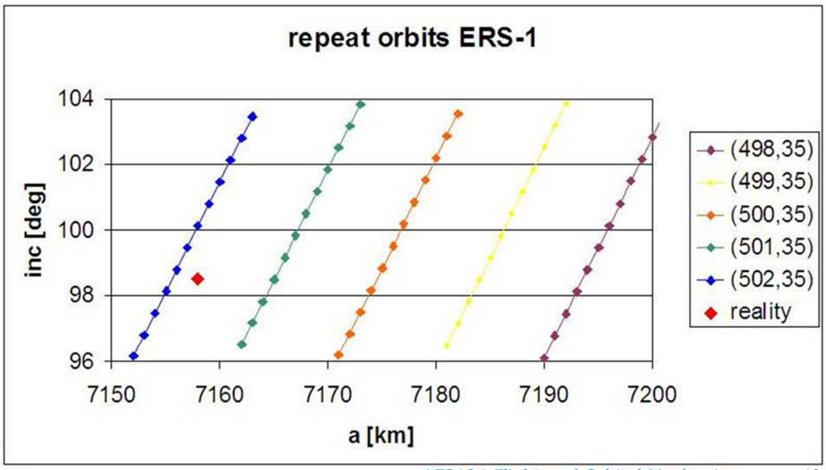
$$j\left|-2\pi\frac{2\pi\sqrt{a^3/\mu}}{T_E}-\frac{3\pi J_2 R_e^2\cos(i)}{a^2(1-e^2)^2}\right|=k\,2\pi$$



In standard configuration, the repeat period is 35 days. The repeat interval is typically traded against geometric resolution (*i.e.*, the distance between neighboring ground tracks).

Earth repeat orbits (cnt'd)

ERS-1: e=0, k=35, j=498,...,502



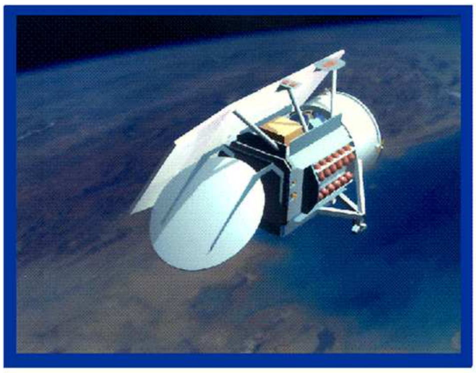
Zooming in on plot on previous sheet. 1st–order estimate misses real value by about 2 km…..

Earth repeat orbits (cnt'd)

Example 2:

Geosat Follow-On (GFO)

Mission purpose: observation sea-state
Launch date February 10, 1998
Retired October 2008
Altitude ~784 km
Semi-major axis ~7163 km
Eccentricity ~0
Inclination 108.0448°
Repeat period 17 days

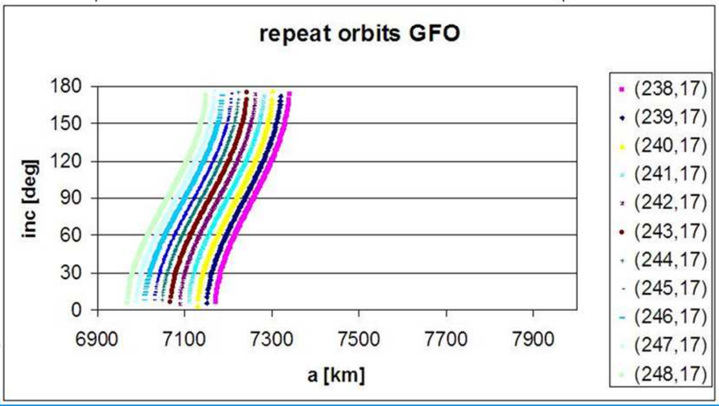


[NASA, 2010]

Earth repeat orbits (cnt'd)

GFO: e=0, k=17, j=238,…,248

$$j\left|-2\pi\frac{2\pi\sqrt{a^3/\mu}}{T_E}-\frac{3\pi J_2 R_e^2\cos(i)}{a^2(1-e^2)^2}\right|=k\,2\pi$$

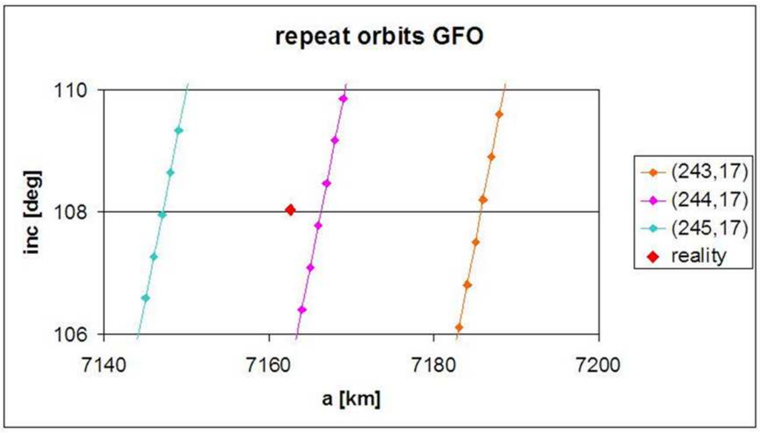


TUDelft

42 |

Earth repeat orbits (cnt'd)

GFO: e=0, k=17, j=243,244,245



Zooming in on plot on previous sheet. 1st–order estimate also misses real value by about 2 km… Agreement w.r.t. reality is comparable with that of ERS-1 (same orbit altitude → more or less the same additional disturbances of perturbations, drag, …).

Earth repeat orbits (cnt'd)

Question 5:
The following equation describes an arbitrary Earth-repeat orbit:

$$j \left| -2\pi \frac{2\pi\sqrt{a^3/\mu}}{T_E} - \frac{3\pi J_2 R_e^2 \cos(i)}{a^2(1-e^2)^2} \right| = k\,2\pi$$

a) Consider the situation where the semi-major axis has a value of 7300 km, j = 41 and k = 3. What is the required inclination for the satellite to be in a circular Earth-repeat orbit?

b) Consider the situation where the semi-major axis has a value of 7300 km, j = 42 and k = 3. What is the required inclination for the satellite to be in a circular Earth-repeat orbit?

c) What is the orbital period of a satellite with a semi-major axis of 7300 km?

d) What is the repeat period of the situation of question (a)? What is it of question (b)? How do you explain the difference?

Data: μ_{Earth} = 398600.4415 km^3/s^2; R_e = 6378.137 km; J_2 = 1082×10^{-6}
Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

ANSWERS **(TRY YOURSELF FIRST!!):**

a) i = 24.0°

b) i = 119.5°

c) T = 6207 s = 103.45 min.

d) Question a): T_{repeat} = 4241.6 min = 70.69 hr = 2.9455 days. Question b): T_{repeat} = 4345.0 min = 72.417 hr = 3.017 days. Earth's flattening has opposite effects on orbits with inclination < 90° (J_2 precession acts in Westward direction) and orbits with inclination > 90° (J_2 precession acts in Eastward direction).

Sun-synchronous orbits

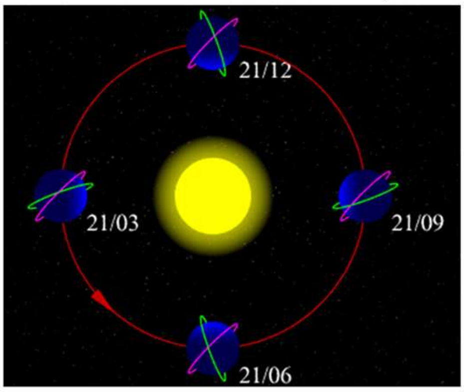
What?
The orientation of the satellite orbit follows the direction towards the Sun

Why?
The satellite will encounter the same pattern of lighting conditions throughout an orbit, both on the satellite as well as on targets on the Earth

How?
Use of J_2 effect

Having a satellite in a Sun-synchronous orbit does NOT mean that it is in constant sunlight throughout its mission!

Sun-synchronous orbits (cnt'd)



Orientation of orbital plane remains approximately constant w.r.t. direction to Sun (ignoring north-south motion Sun, effects of eccentricity of Earth orbit, etc.)

Note: effects are measured w.r.t. inertial reference system

Inertially stable orbit: no perturbations, so no changes in orientation. Sun-synchronous orbit: effect is fully due to J_2.

Sun-synchronous orbits (cnt'd)

satellite	period	Earth-repeat	Sun-synch.
Skylab	1973-1974	no	no
GEOS-3	1975-1978	??	no
Seasat	1978-1978	yes	no
Geosat	1985-1990	yes	no
ERS-1	1991-2000	yes	yes
TOPEX/Poseidon	1992-2005	yes	no
ERS-2	1995-2003*	yes	yes
GFO	1998-2008	yes	no
Jason-1	2001-current	yes	no
ENVISAT	2002-2012	yes	yes
ICESat	2003-2009	yes	no
Jason-2	2008-current	yes	no
Cryosat-2	2010-current	yes	no

Overview of satellites orbiting Earth in Sun-synchronous orbits. Note the "families" Seasat - Geosat - GFO, ERS-1 – ERS-2 – ENVISAT, and TOPEX/Poseidon – Jason-1 – Jason-2. Satellites in red are US, satellites in green are European. GFO is the abbreviation for Geosat Follow-On.

Sun-synchronous orbits (cnt'd)

always:

$$\dot{\Omega} = \frac{\Delta\Omega_{2\pi}}{T} = -3\,\pi\,J_2\left(\frac{R_e}{p}\right)^2 \cos i\,\frac{1}{T}$$

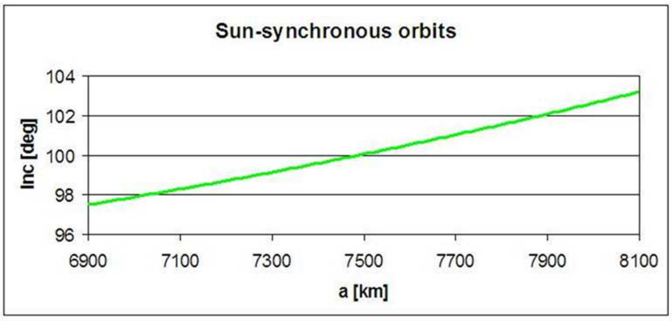
with:

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

requirement:

$$\dot{\Omega} = \frac{2\pi}{T_{ES}}$$

1st order: $e = 0$



Direct, 1-on-1 relation between semi-major axis and inclination.

T is the orbital period of the satellite [s]; T_E is the length of a sidereal day [s]; T_{ES} is the orbital period of the Earth around the Sun [s].

Again, compare with equations on sheet 24.

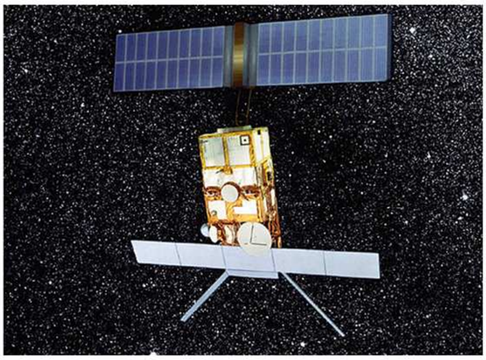
Note: inclination is always larger than 90°.

Sun-synchronous orbits (cnt'd)

Example:

ERS-1

Launch date July 17, 1991
Retired March 10, 2000
Altitude ~780 km
Semi-major axis ~7158 km
Eccentricity ~0
Inclination 98.52°
Repeat period reference orbit 35 days



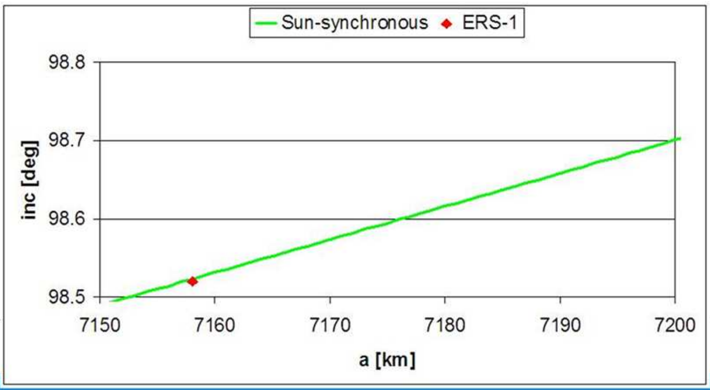
[Dutchspace, 2010]

ERS-1 is the abbreviation for European Remote-Sensing satellite.

Sun-synchronous orbits (cnt'd)

$$\frac{d\Omega}{dt} = -3\pi J_2 \left(\frac{R_e}{a(1-e^2)} \right)^2 \cos(i) \frac{1}{2\pi} \sqrt{\frac{\mu}{a^3}} = \frac{2\pi}{T_{ES}}$$

ERS-1: e=0, a=6378.137+780=7158.137 km ➔ i = 98.52°



No significant difference.

Sun-synchronous orbits (cnt'd)

Question 6:

a) What is the main characteristic of a Sun-synchronous orbit?

b) The parameters of a Sun-synchronous orbit have to satisfy the following equation:

$$\frac{d\Omega}{dt} = -3\pi J_2 \left(\frac{R_e}{a(1-e^2)} \right)^2 \cos(i) \frac{1}{2\pi} \sqrt{\frac{\mu}{a^3}} = \frac{2\pi}{T_{ES}}$$

Derive an equation for the semi-major axis as a function of inclination (assume a zero eccentricity).

c) Make a sketch of the inclination as function of the semi-major axis.

Data: μ_{Earth} = 398600.4415 km^3/s^2; R_e = 6378.137 km, J_2 = 1082×10^{-6}; T_{ES} = 365.25×86400 s.

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

ANSWERS **(TRY YOURSELF FIRST):**

a) The orientation of the orbital plane w.r.t. the direction towards the Sun is constant over time.

b) $a^{3.5} = -(3/4\pi)\, J_2\, R_e^2\, \mathrm{sqrt}(\mu)\, T_{ES} \cos(i)$

c) $a = (-2.0936 \times 10^{14} \cos(i))^{-3.5}$ [km] → make table with some numbers → sketch

Earth-repeat and Sun-synchronous

Earth – repeat: $j\left|\Delta L_1 + \Delta L_2\right| = k\,2\pi$

Sun – synchronous: $\dfrac{d\Omega}{dt} = \dfrac{\Delta L_2}{T} = \dfrac{2\pi}{T_{ES}}$

combine: $j\left| -2\pi\dfrac{T}{T_E} + 2\pi\dfrac{T}{T_{ES}} \right| = k\,2\pi$

or:

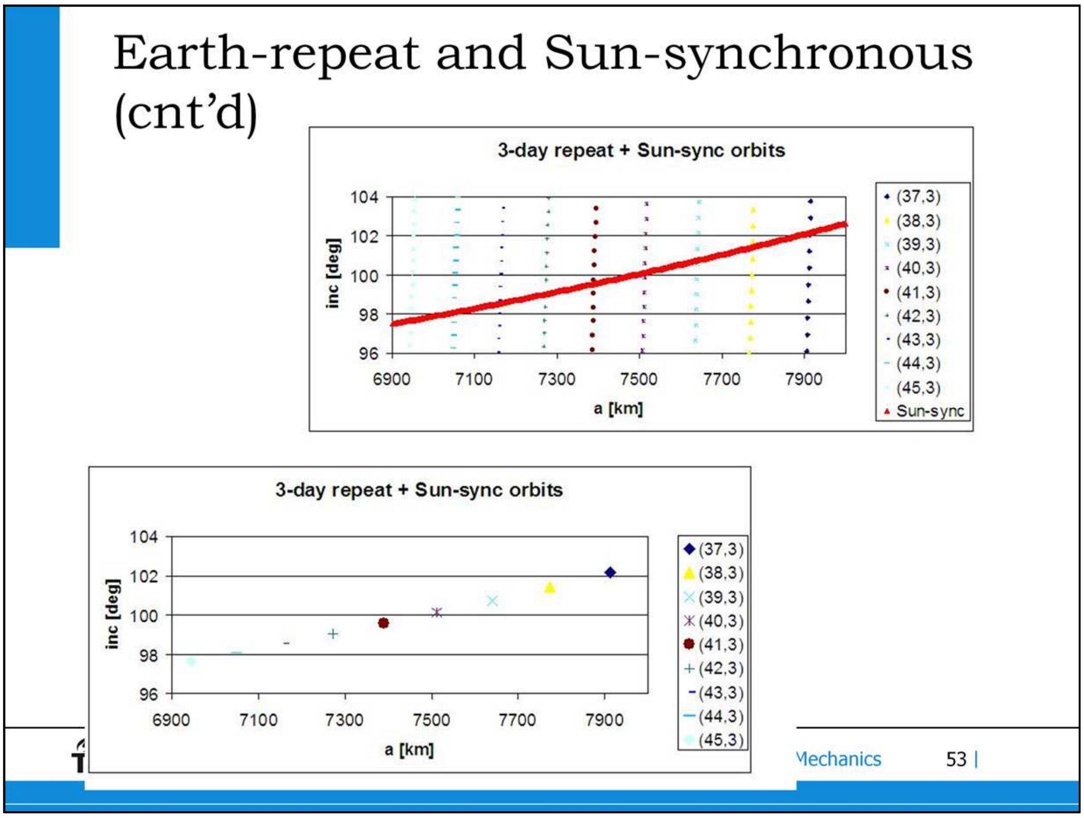
$$jT\left(\frac{1}{T_E} - \frac{1}{T_{ES}}\right) = k \quad \Rightarrow \quad a(j,k,T_E,T_{ES},\mu) \quad (\text{since } T = 2\pi\sqrt{\frac{a^3}{\mu}})$$

finally: $-3\pi J_2\left(\dfrac{R_e}{a(1-e^2)}\right)^2 \cos(i)\,\dfrac{1}{T} = \dfrac{2\pi}{T_{ES}} \quad \Rightarrow \quad i$

Direct, 1-on-1 relation between semi-major axis and (j,k) combination; inclination follows directly from solution for semi-major axis.

T is orbital period of satellite [s]; T_E is the length of a sidereal day [s]; T_{ES} is the orbital period of the Earth around the Sun [s].

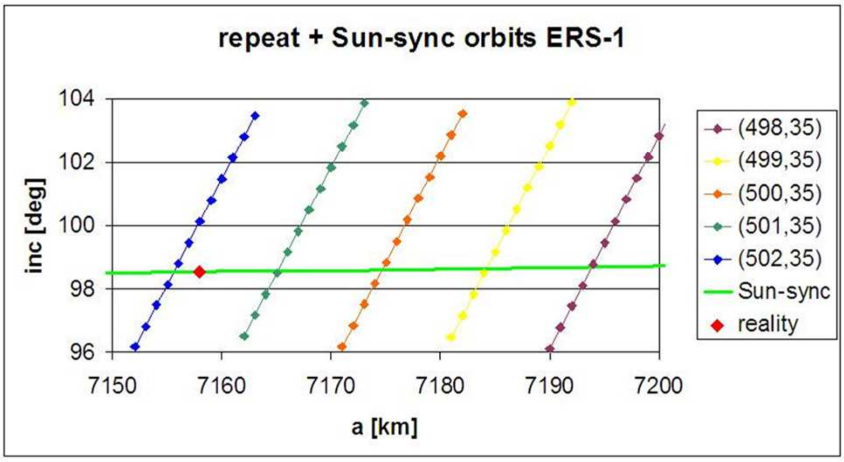
Again, compare with equations on sheet 24.



Direct, 1-on-1 relation between semi-major axis and (j,k) combination; inclination follows directly from solution for semi-major axis.

Earth-repeat and Sun-synchronous (cnt'd)

ERS-1: e=0, k=35, j=498,…502:



Direct, 1-on-1 relation between semi-major axis and (j,k) combination; inclination follows directly from solution for semi-major axis.

Earth-repeat and Sun-synchronous (cnt'd)

Question 7:

a) Derive an equation for the semi-major axis of a satellite orbit that satisfies requirements on Earth-repeat and Sun-synchronous orbits simultaneously.

b) Compute the value for the semi-major axis for such an orbit for the Earth-repeat conditions (43,3).

c) Compute the corresponding orbital inclination.

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

ANSWERS (**TRY YOURSELF FIRST**):

a) See 3rd-previous sheet.

b) a = 7158.748 km.

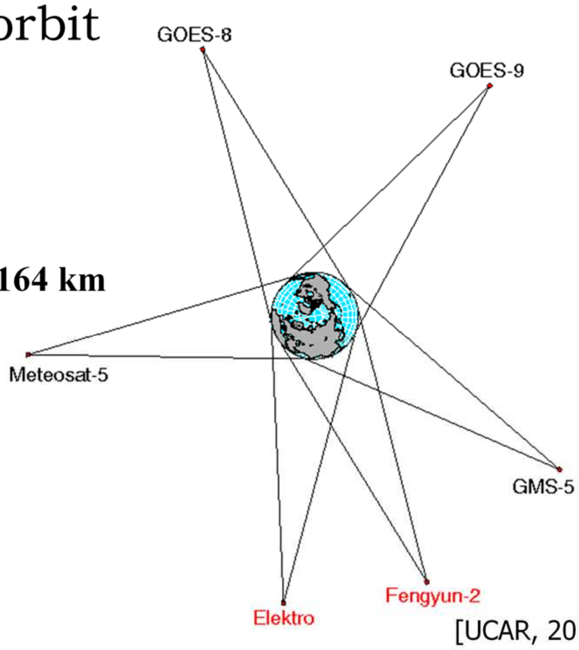
c) i = 98.53°

Geostationary orbit

Main characteristics:

- **$T = 23^h 56^m 4^s \rightarrow a = 42164$ km**
- **$e = 0$**
- **$i = 0°$**

GMS-5
[NASDA, 2011]



[UCAR, 2011]

Geostationary orbit (cnt'd)

effects of gravity field on geostationary satellites:

$J_{2,0}$:

$$U_{2,0} = -\frac{\mu}{r}\left[-J_2\left(\frac{R_e}{r}\right)^2 P_{2,0}(\sin\delta)\right]$$

$$P_2(x) = \frac{1}{(-2)^2\,2!}\frac{d^2}{dx^2}(1-x^2)^2$$

$J_{2,2}$:

$$U_{2,2} = -\frac{\mu}{r}\left[J_{2,2}\left(\frac{R_e}{r}\right)^2 P_{2,2}(\sin\delta)\cos(2(\lambda-\lambda_{2,2}))\right]$$

$$P_{2,2}(x) = (1-x^2)^{2/2}\frac{d^2 P_2(x)}{dx^2}$$

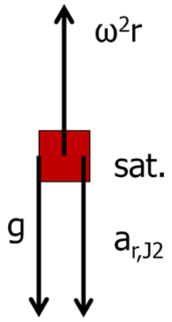
$$P_2(x) = \frac{1}{(-2)^2\,2!}\frac{d^2}{dx^2}(1-x^2)^2$$

Straightforward application of recipe.

Geostationary orbit (cnt'd)

effects of gravity field on geostationary satellites:

R_e = 6378 km, r = 42164 km → $R_e^2/r^4 = 1.3 \times 10^{-11}$ km^{-2}



$J_{2,0}$:

Value: 1082.6×10^{-6}

Important? yes

$J_{2,2}$:

Value: 1.816×10^{-6}

Important? yes

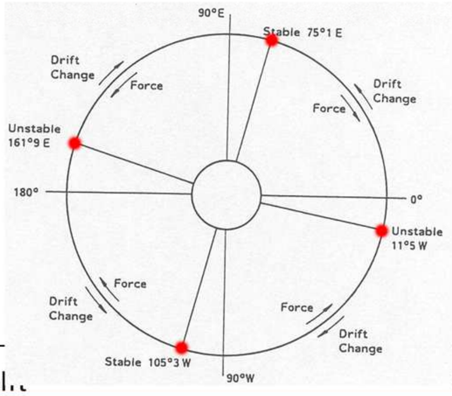
The J_2 contribution is important for the exact determination of the altitude for which the orbital period is equal to the length of a sidereal day (compared to an unperturbed Kepler orbit, the difference is about 500 m……): the value of the semi-major axis "a" has to be chosen such that there is equilibrium between the centripetal acceleration $\omega^2 r$ and the physical accelerations "g" (due to the central gravity field of Earth) and "$a_{r,J2}$" (the radial acceleration due to J_2). $J_{2,2}$ is important for station keeping (see next sheets).

Geostationary orbit (cnt'd)

$J_{2,2}$ effect: constant East-West acceleration on GEO satellites (λ in [deg]):

$$a_\lambda = -1/(r\cos\delta)\,(\partial U_{2,2}/\partial\lambda) = -6\,\mu\,J_{2,2}\,R_e^2\,r^{-4}\cos\delta\,\sin(2(\lambda-\lambda_{2,2})) =$$

$$-5.6\times 10^{-8}\sin(2(\lambda+14.9))\quad [m/s^2]$$



ΔV budget [m/s/yr]:

- **$J_{2,2}$: 1.7 sin (2(λ-75))**
- **Sun + Moon: 51.4**

$J_{2,2}$-effect is in many cases ignored for LEO satellites, but is elementary for GEO spacecraft (note opposite direction of acceleration and effect; why?). What defines equilibrium points? What defines stability/instability of equilibrium points? Can we ignore J_2 ? Here, the effect of $J_{2,2}$ in along-track direction is evaluated only; it of course has a full, 3-dimensional effect (r,δ,λ or x,y,z).

Geostationary orbit (cnt'd)

Question 8:

1. What is the definition for the East-West acceleration, based on the potential formulation for the gravity field of the Earth?
2. Derive the general equation for the East-West acceleration due to the term (2,2) for an arbitrary satellite.
3. Compute the orbit radius of a geostationary satellite.
4. What is the equation for the East-West acceleration due to $J_{2,2}$ for a geostationary satellite?
5. What are the locations of the equilibrium points?
6. Are these stable or unstable?

Data: μ_{Earth} = 398600.4415 km^3/s^2; T_E = 23^{h}56^{m}4^s; $J_{2,2}$ = 1.816×10^{-6}; $\lambda_{2,2}$ = -14.9°.

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

ANSWERS **(TRY YOURSELF FIRST!!):**

1. $a_{EW} = -1/(r \cos\delta) \times \partial U/\partial\lambda$
2. $a_{EW} = -6\, \mu\, J_{2,2}\, R_e^2\, r^{-4} \cos \delta \sin (2(\lambda-\lambda_{2,2}))$
3. r_{GEO} = 42164.14 km
4. $a_{EW} = -5.6 \times 10^{-8} \sin (2(\lambda+14.9))$ [m/s^2]
5. Located at -14.9°, 75.1°, 165.1° and 255.1°. (East longitude)
6. Stable: 75.1° and 255.1°; unstable: -14.9° and 165.1°.