AE4520: Advanced Structural Analysis
Take Home Exam

Due date January 16^{th} 2014

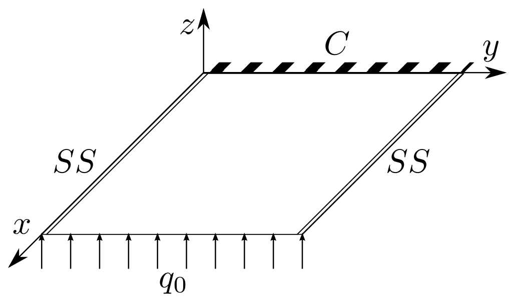

Figure 1: Rectangular Plate under Edge Load.

1. (30 points) The square plate shown in figure 1 has a side length a and a constant thickness t. The plate is made of an isotropic material having Young's modulus E and Poisson's ratio $\nu = 0.3$. The plate is simply supported (SS) on two opposite sides. One of the two others is clamped (C) and the other is free (F). The plate is subject to a uniform edge line load q_0.

   (a) Assume that the deflection is expanded in a series of the form

   $$w = \sum_i \phi_n(x) \sin \frac{n\pi y}{a}$$

   Show, by integrating only over y, that the total potential energy can be written in the form,

   $$\Pi = \sum_n \Pi_n,$$

   where Π_n depends only on $\phi_n(x)$.

   (b) Using the principle of minimum potential energy find the governing fourth order ordinary differential equations for ϕ_n and their boundary conditions.

   (c) Plot the deflection of the plate indicating maximum displacement.

   (d) Plot the variation of the von Mises stress over the plate and indicate the critical areas.

2. (40 points) In modern aerospace structures, there is a relentless drive to reduce weight and cost. One concept that is currently under research is the use of iso-grid stiffened structures. Such structures are constructed from a triangular grid of stiffeners supporting a thin skin. While most of the loads are taken by the supporting grid, the skin is also non-negligibly loaded and need to be properly designed. One possible failure mode for the skin is buckling.

   A single traingular cell is modelled as a simply supported equilateral triangular plate of side length a. The notation for the plate is shown in figure 2.

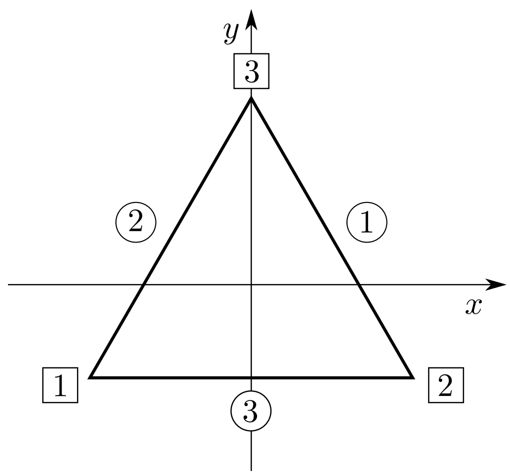

Figure 2: Triangular Panel.

(a) Show that the x and y locations of a general point in the triangle is given by,

$$x = \sum_{i=1}^{3} x_i \xi_i, \text{and,} \quad y = \sum_{i=1}^{3} y_i \xi_i,$$

where x_i and y_i are the coordinates of the ith vertex and $0 \leq \xi_i \leq 1,\ i = 1 \ldots 3$ are barycentric coordinates satisfying,

$$\xi_1 + \xi_2 + \xi_3 = 1.$$

Find the corresponding equations for ξ_i in terms of the x and y coordinates.

(b) Show that functions of the form,

$$\phi_i = \xi_1^{p_{1i}} \xi_2^{p_{2i}} \xi_3^{p_{3i}}$$

satisfy the simply supported conditions for all integer $p_{1i}, p_{2i}, p_{3i} > 1$.

(c) Assume a Rayleigh-Ritz series of the form,

$$w = \sum_i c_i \phi_i$$

containing all the terms such that $p_{1i} + p_{2i} + p_{3i} = p$. Where $p \geq 3$ is the polynomial degree of the approximation.

Use the Rayleigh-Ritz method to find the first four buckling modes and buckling loads under equal biaxial compression. Use Poisson's ratio $\nu = 0.3$. Obtain results for $p = 3, 4, 5, 6$, and comment on convergence.

[Hint: All derivatives with respect to x and y may be converted to derivatives with respect to the barycentric coordinates. Integrations can be reduced to simpler integrations using the $\xi_1 \xi_2$ plane. You might want to use a symbolic manipulator, e.g., maple, to calculate the integrals.]