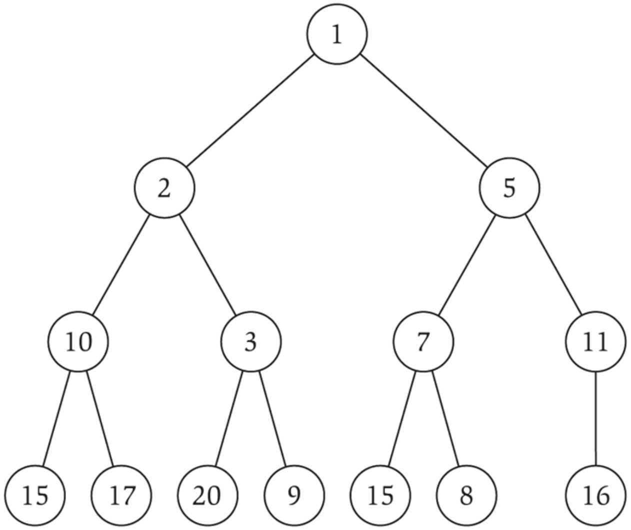
3.1 Graphs:
Basic Definitions and Applications

From trees to graphs

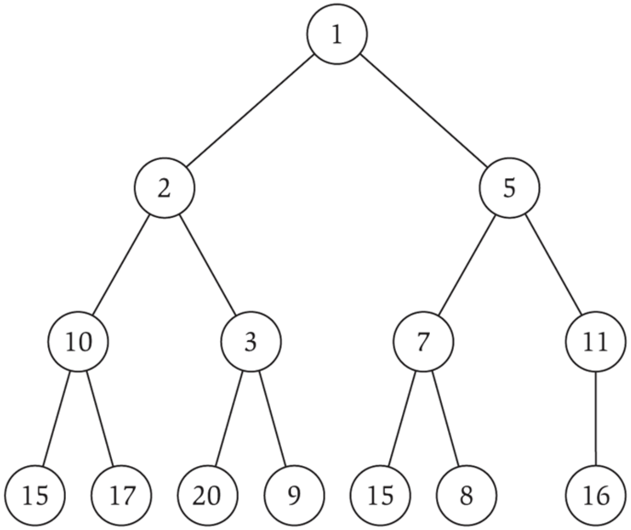
Q. How many edges does a tree with n nodes (internal+leaves) have?



From trees to graphs

Q. How many edges does a tree with n nodes (internal+leaves) have?

A. n-1

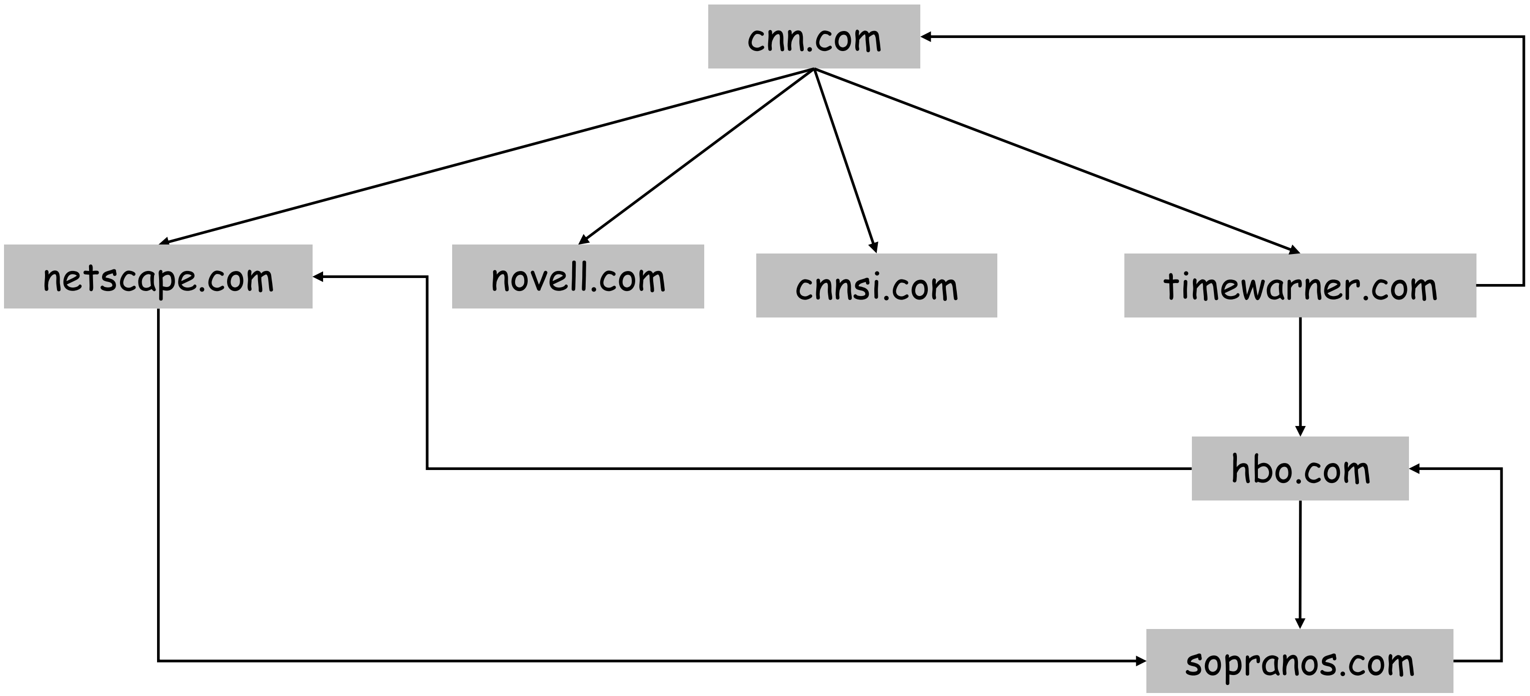


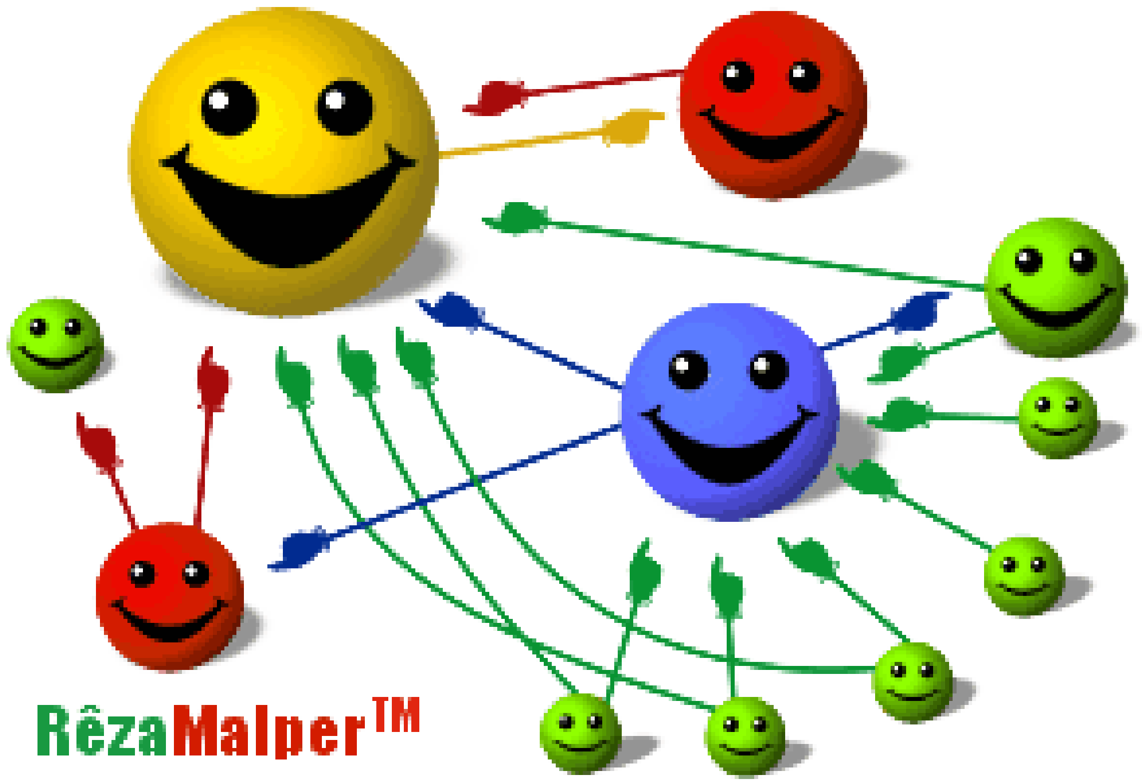

NAVMAN
CAROLINE DRIVE
270 ft
A329
ROTHERFIELD AVENUE
BROOKSIDE
CAROLINE DRIVE
25 ARTHUR ROAD
1200 ft
ESC

World Wide Web

Web graph.

- Node: web page.
- Edge: hyperlink from one page to another.





http://www.kurdishworld.com/projects/google/

TU Delft

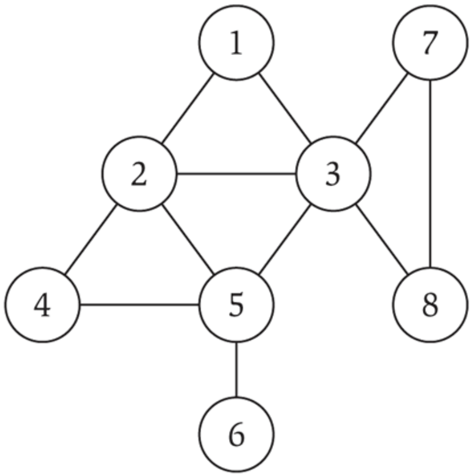
Why Graphs?

Graph	*Nodes*	*Edges*
transportation	street intersections	highways
communication	computers	fiber optic cables
World Wide Web	web pages	hyperlinks
social	people	relationships
food web	species	predator-prey
software systems	functions	function calls
scheduling	tasks	precedence constraints
circuits	gates	wires

Undirected Graphs

Undirected graph. G = (V, E)

- V = set of nodes (=vertices). ("knopen")
- E = set of edges between pairs of nodes. ("kanten")
- Captures pairwise relationship between objects.
- Graph size parameters: n = |V|, m = |E|.



V = { 1, 2, 3, 4, 5, 6, 7, 8 }

E = { 1-2, 1-3, 2-3, 2-4, 2-5, 3-5, 3-7, 3-8, 7-8, 4-5, 5-6 }

n = 8

m = 11

Q. How to implement a graph? Which datastructure to use? (1 min.)

- How much space do you need?
- How much time to check whether node 2 and 4 neighbors?
- How much time to list all edges?

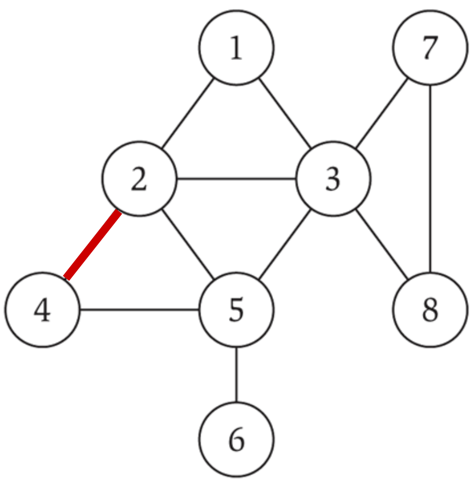
TU Delft

Graph Representation: Adjacency Matrix

Adjacency matrix. n-by-n matrix with A_{uv} = 1 if (u, v) is an edge.

- Two representations of each edge.
- Space proportional to n^2.
- Checking if (u, v) is an edge takes $\Theta(1)$ time.
- Identifying all edges takes $\Theta(n^2)$ time.

Q. How is this edge between 2 and 4 represented in the matrix?



	1	2	3	4	5	6	7	8
1	0	1	1	0	0	0	0	0
2	1	0	1	1	1	0	0	0
3	1	1	0	0	1	0	1	1
4	0	1	0	0	1	0	0	0
5	0	1	1	1	0	1	0	0
6	0	0	0	0	1	0	0	0
7	0	0	1	0	0	0	0	1
8	0	0	1	0	0	0	1	0

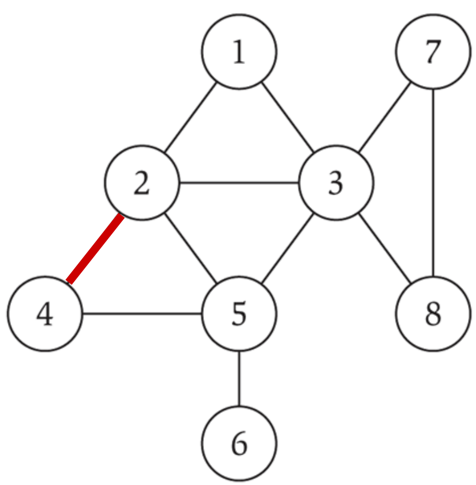
Graph Representation: Adjacency List

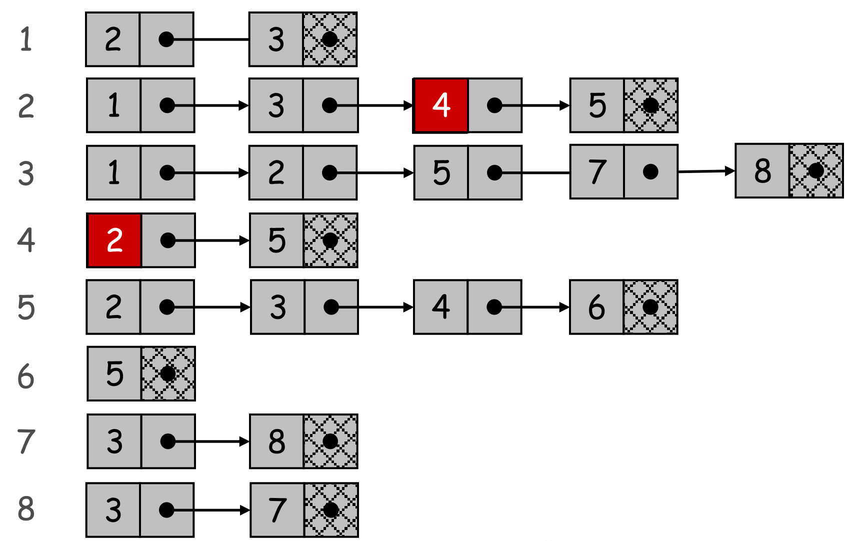
Adjacency list. Node indexed array of lists.

- Two representations of each edge.
- Space proportional to m + n.
- Checking if (u, v) is an edge takes O(deg(u)) time.
- Identifying all edges takes Θ(m + n) time.

degree = number of neighbors of u

Q. How is this edge between 2 and 4 represented in the adjacency list?





Paths and Connectivity

Def. A path in an undirected graph G = (V, E) is a sequence P of nodes v_1, v_2, …, v_{k-1}, v_k with the property that each consecutive pair v_i, v_{i+1} is joined by an edge in E.

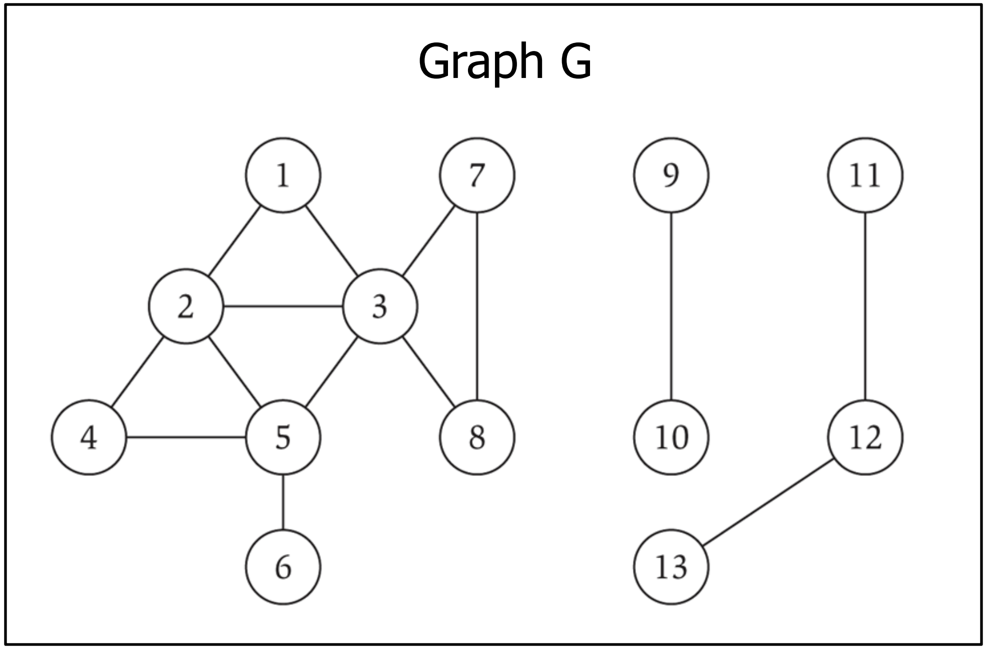
Def. A path is simple if all nodes are distinct.

Def. An undirected graph is connected if for every pair of nodes u and v, there is a path between u and v.

Q. Is 1-3-8-7-3-5 a path?

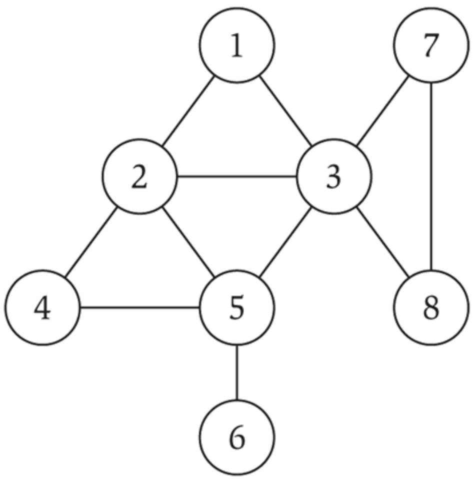
Q. Is it a simple path?

Q. Is this graph connected?



Cycles

Def. A cycle is a path $v_1, v_2, \ldots, v_{k-1}, v_k$ in which $v_1 = v_k$, $k > 2$, and the first k-1 nodes are all distinct.



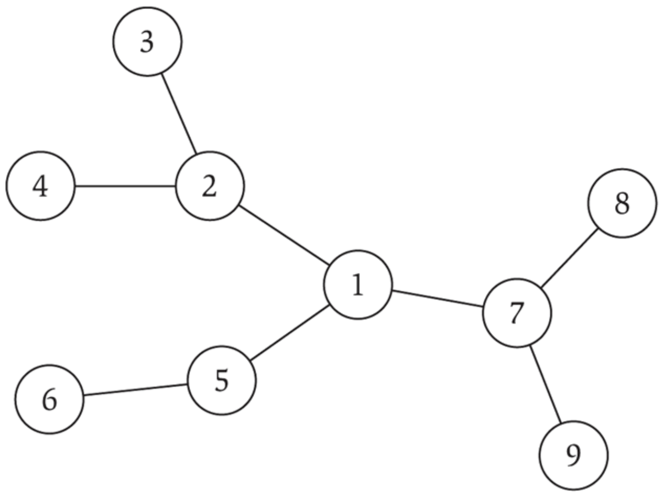
cycle *C* = 1-2-4-5-3-1

Trees

Def. An undirected graph is a tree if it is connected and does not contain a cycle.

Theorem. Let G be an undirected graph on n nodes. Any two of the following statements imply the third.

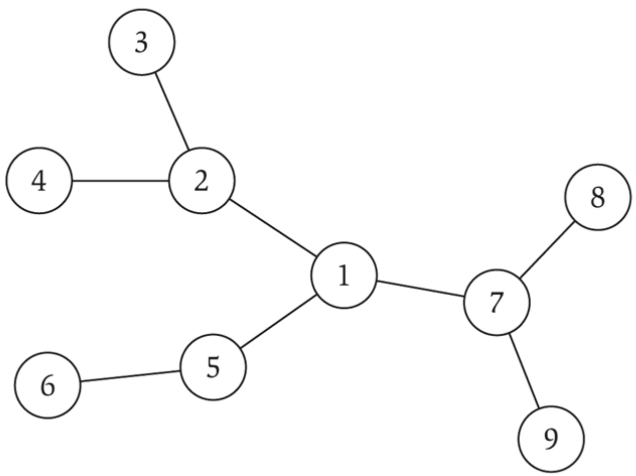
- G is connected.
- G does not contain a cycle.
- G has n-1 edges.



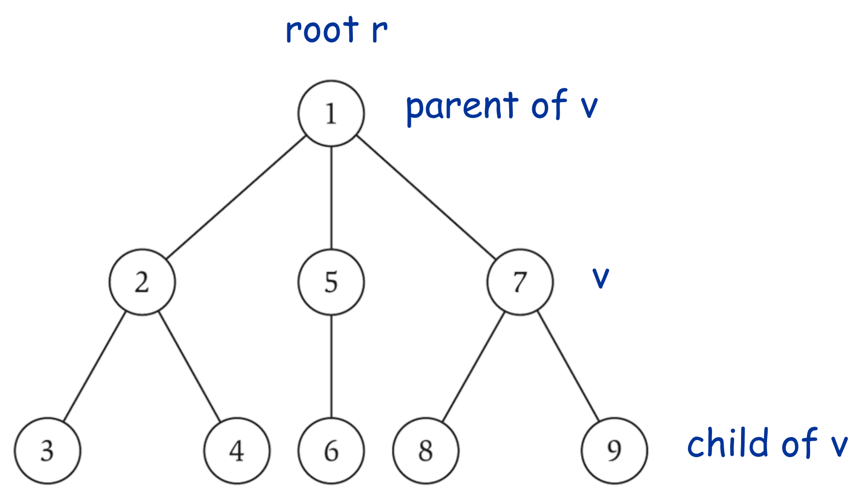
Rooted Trees

Rooted tree. Given a tree T, choose a root node r and orient each edge away from r.

Importance. Models hierarchical structure.



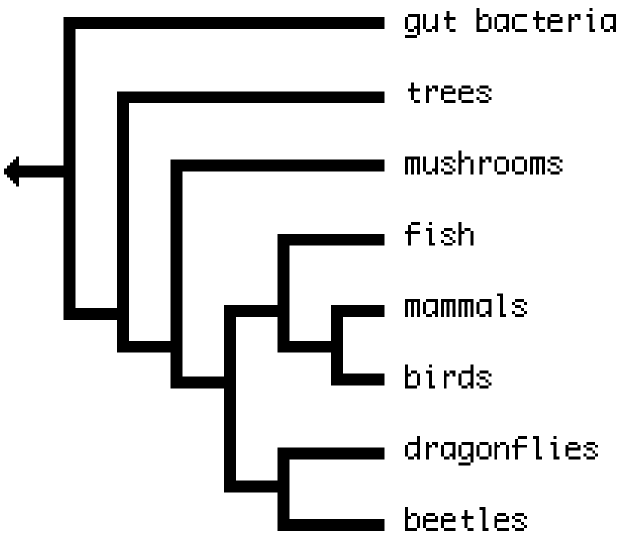
a tree



the same tree, rooted at 1

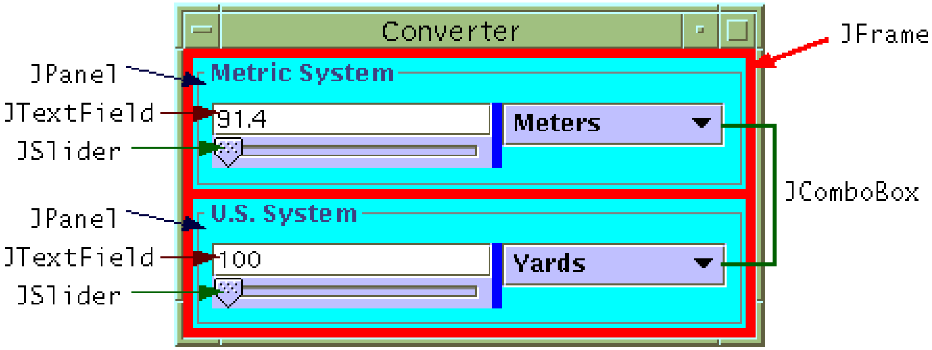
Phylogeny Trees

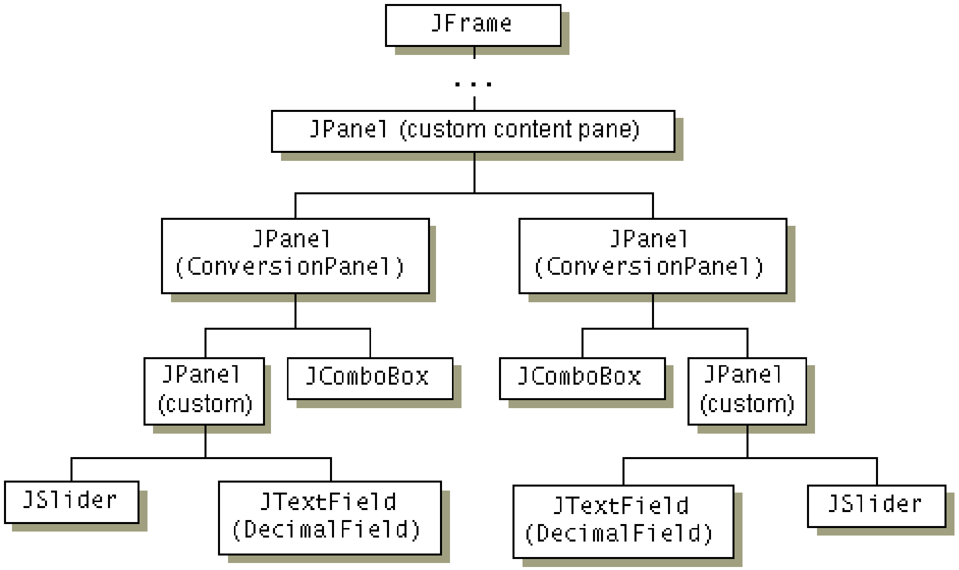
Phylogeny trees. Describe evolutionary history of species.



GUI Containment Hierarchy

GUI containment hierarchy. Describe organization of GUI widgets.





Reference: http://java.sun.com/docs/books/tutorial/uiswing/overview/anatomy.html