7.11 Project Selection

- Project Selection Problem
- Translation to Network Flow
- Correctness Proof of Translation

Project Selection

can be positive or negative ↓

Projects with prerequisites.

- Set P of possible projects. Project v has associated revenue p_v.
  - some projects generate money: create interactive e-commerce interface, redesign web page
  - others cost money: upgrade computers, get site license
- Set of prerequisites E. If $(v, w) \in E$, can't do project v and unless also do project w. (Arrow from v to w.)
- A subset of projects $A \subseteq P$ is feasible if the prerequisite of every project in A also belongs to A.

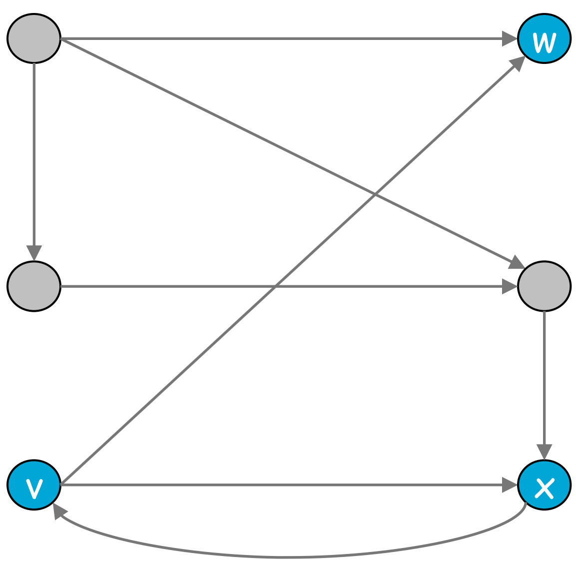
Project selection. Choose a feasible subset of projects to maximize revenue.

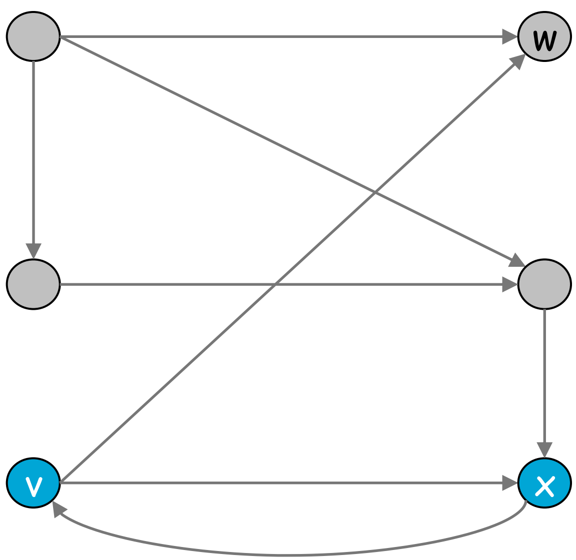
TU Delft

Project Selection: Prerequisite Graph

Prerequisite graph.
- Include an edge from v to w if we can't do v without also doing w.

Q. Which project selection is infeasible: {v,w,x}, {v,x}, both, or none ?





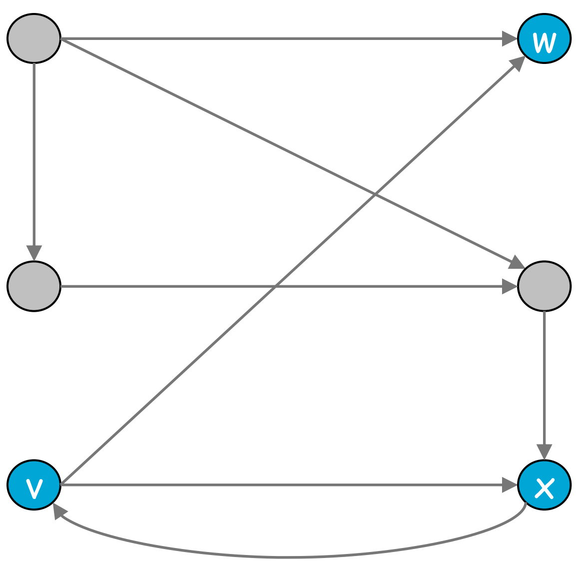
Project Selection: Prerequisite Graph

Prerequisite graph.

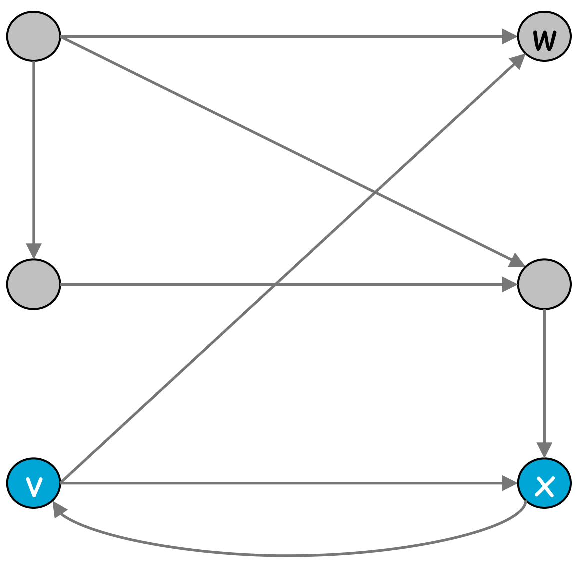
- Include an edge from v to w if can't do v without also doing w.

Q. Which project selection is infeasible: {v,w,x}, {v,x}, both, or none ?

A. Only {v, x} is an infeasible subset of projects, because v needs w



feasible



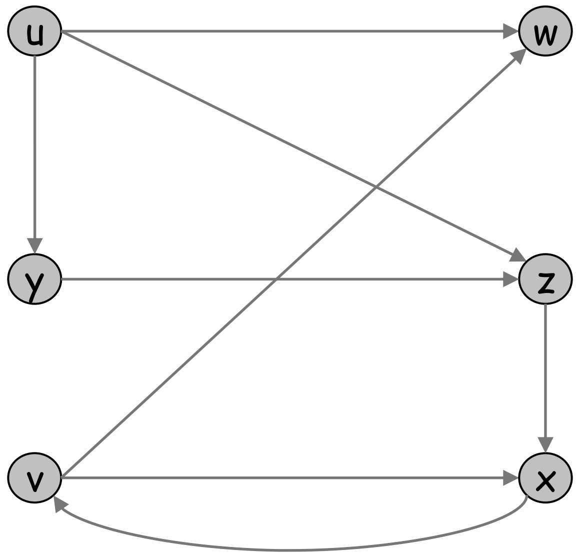
infeasible

Project Selection

- Project v has associated revenue p_v: if $p_v > 0$: profit, $p_v<0$ costly
- Prerequisites E: If $(v, w) \in E$, if project v then also project w.

Q*. How can we formulate this as a max-flow/min-cut problem? (1 min)

1. Do you want the max matching to be the max flow or the min cut?
2. Which nodes should be s and t? (Existing or new?)
3. Do we need more edges, what should be the direction?
4. What should be the capacities?

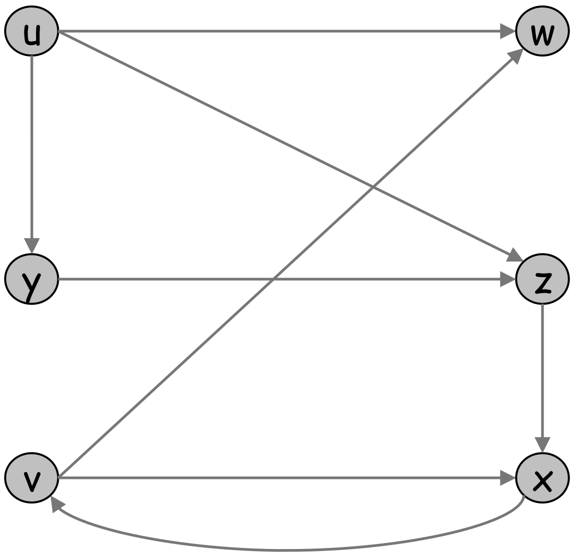


Project Selection: Min Cut Formulation

Min cut formulation

- such that (A, B) is min cut iff A – { s } is optimal set of projects.

Q. How to ensure that A – { s } is feasible?



formulation: cut (A,B):
A-{s}: selected projects
B: unselected projects

Project Selection: Min Cut Formulation

Min cut formulation
- such that (A, B) is min cut iff A – { s } is optimal set of projects.
- infinite capacity edges ensure A – { s } is feasible.

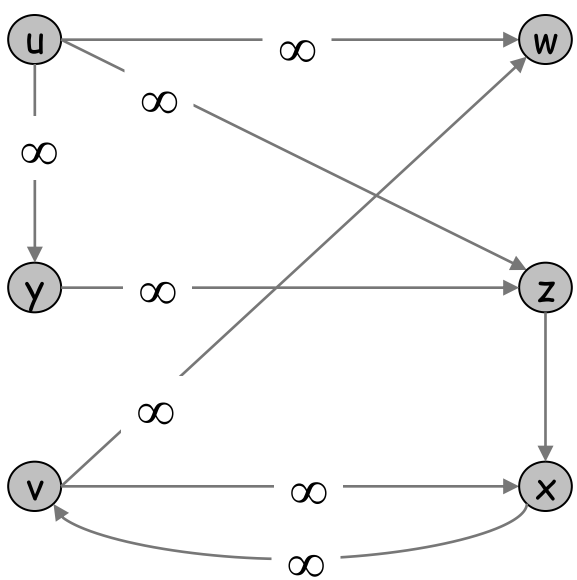
Q. How to ensure that capacity of min cut relates to profit?

A. minimize selected costly and unselected profitable projects:

$$cap(A,B) = \sum_{v \in B : p_v > 0} p_v + \sum_{v \in A : p_v < 0} (-p_v)$$

unselected profitable projects

selected costly projects



formulation: cut (A,B):
A-{s}: selected projects
B: unselected projects

Project Selection: Min Cut Formulation

Min cut formulation.

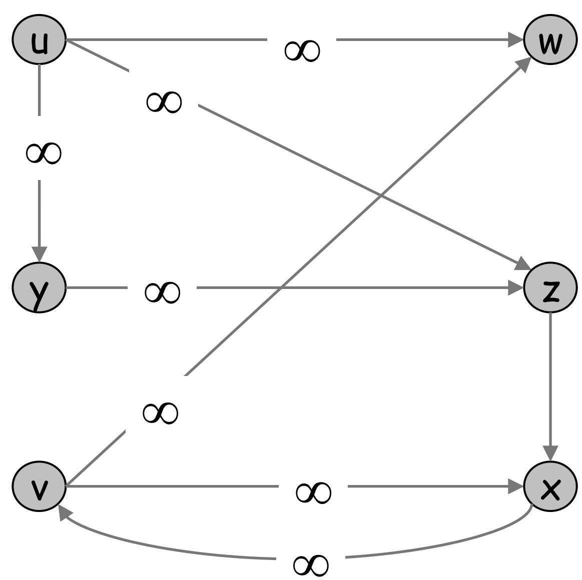
- Assign capacity ∞ to all prerequisite edges.

Q. How to assign other capacities to ensure that

$$cap(A,B) = \sum_{v \in B \,:\, p_v > 0} p_v + \sum_{v \in A \,:\, p_v < 0} (-p_v)$$

unselected profitable projects

selected costly projects

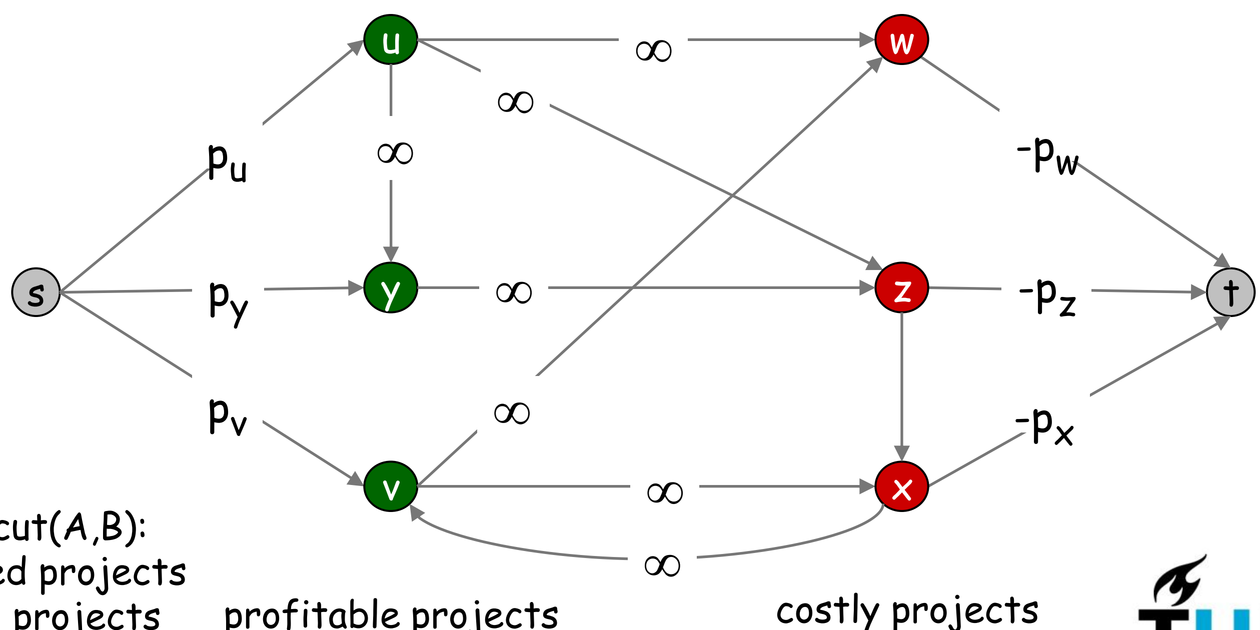


formulation: cut (A,B):
A-{s}: selected projects
B: unselected projects

Project Selection: Min Cut Formulation

Min cut formulation.

- Assign capacity ∞ to all prerequisite edges.
- Add edge (s, v) with capacity p_v if $p_v > 0$.
- Add edge (v, t) with capacity $-p_v$ if $p_v < 0$.
- For notational convenience, define $p_s = p_t = 0$.



formulation: cut(A,B):
A-{s}: selected projects
B: unselected projects

Project Selection: Min Cut Formulation

Claim. If (A, B) is min cut then A – { s } is optimal set of projects.

- (i) Infinite capacity edges ensure A – { s } is feasible.
- (ii) Max revenue because:

$$cap(A,B) = \sum_{v \in B:\, p_v > 0} p_v + \sum_{v \in A:\, p_v < 0} (-p_v)$$

unselected profitable projects — selected costly projects