5.1 Mergesort

Mergesort

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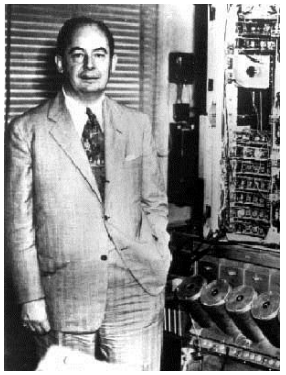
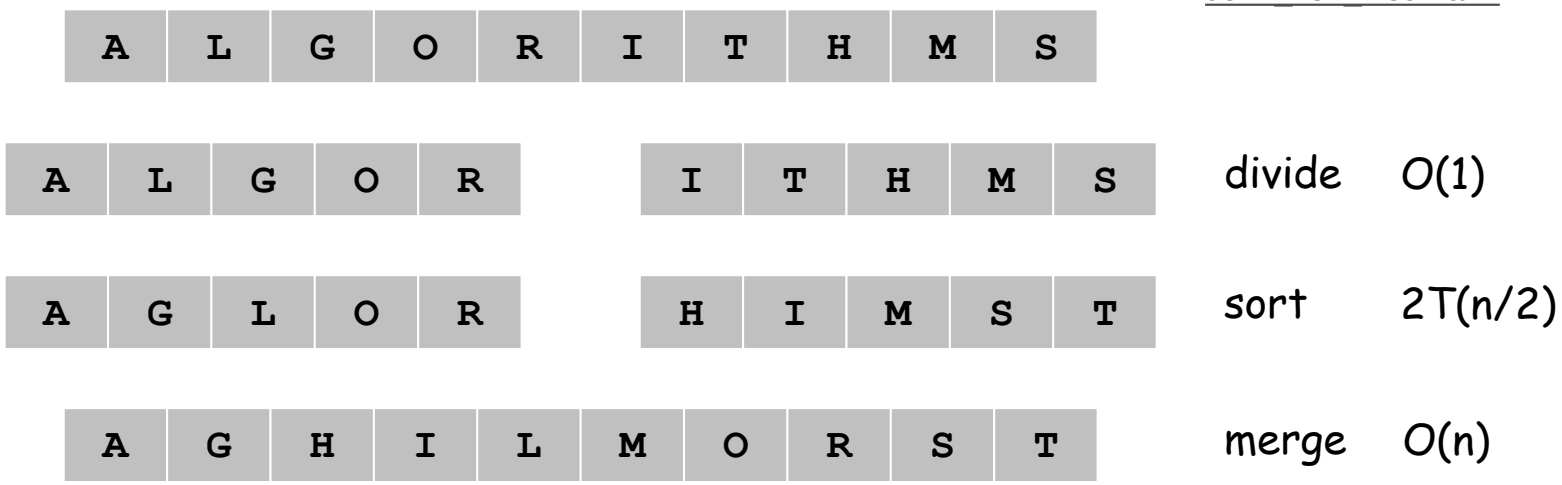
- Divide array into two halves.
- Recursively sort each half.
- Merge two halves to make sorted whole.



Jon von Neumann (1945)

http://en.wikipedia.org/wiki/John_von_Neumann



A Useful Recurrence Relation

Def. T(n) = number of comparisons to mergesort an input of size n.

Mergesort recurrence. (Dutch: "recurrente betrekking")

$$T(n) \leq \begin{cases} 0 & \text{if } n = 1 \\ \underbrace{T(\lceil n/2 \rceil)}_{\text{solve left half}} + \underbrace{T(\lfloor n/2 \rfloor)}_{\text{solve right half}} + \underbrace{n}_{\text{merging}} & \text{otherwise} \end{cases}$$

Q. How do we derive that T(n) = O(n $\log_2$ n) ?

TU Delft

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Assorted proofs. We describe several ways to prove this recurrence. Initially we assume n is a power of 2 and replace $\leq$ with =.

Proof by Recursion Tree (when n is power of 2)

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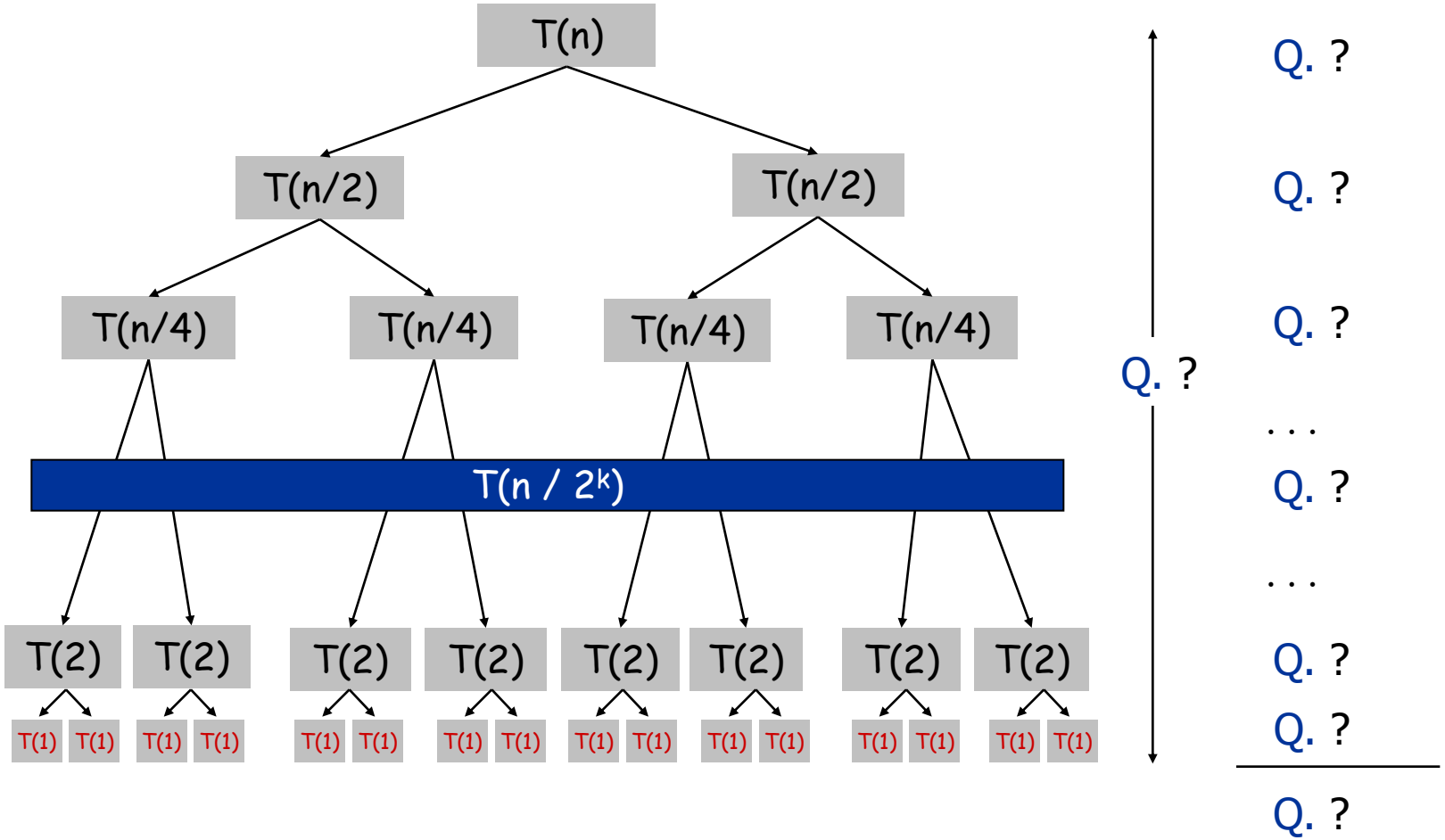
#nodes ·(merge time):



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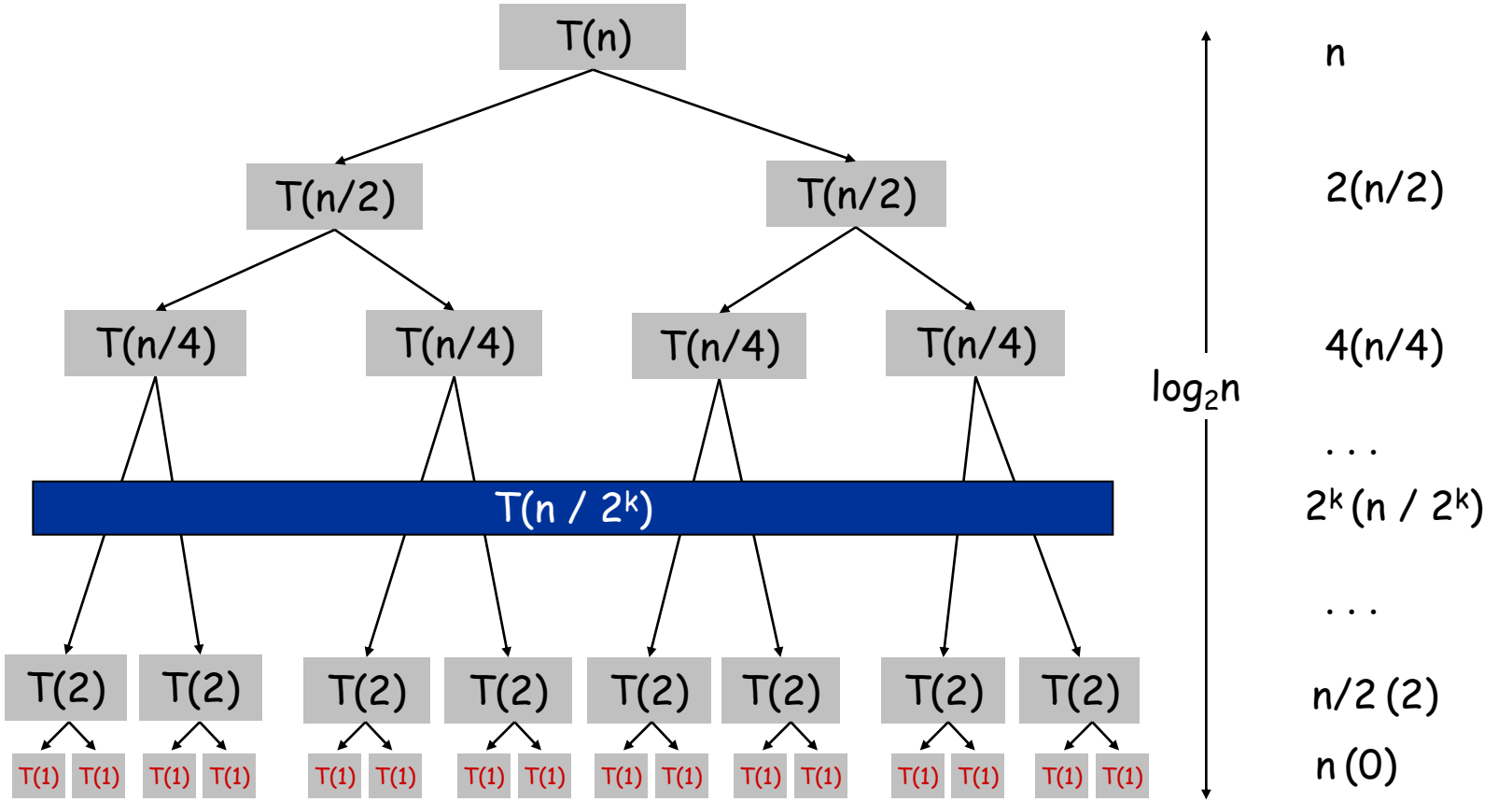

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