Appendix A

TABLES

A.1 Greek Symbols

Classic Greek symbols			
name	letter	capital letter	English pronounciation
alpha	α	A	*'alpha'*
beta	β	B	*'vita'*
gamma	γ	Γ	*'gama'*
delta	δ	Δ	*'delta'*
epsilon	ϵ , ε	E	*'epsilon'*
zeta	ζ	Z	*'zita'*
eta	η	H	*'ita'*
theta	ϑ , θ	Θ	*'thita'*
iota	ι	I	*'giota'*
kappa	κ	K	*'kapa'*
lambda	λ	Λ	*'lambda'*
mu	μ	M	*'mi'*
nu	ν	N	*'ni'*
xi ("ksi")	ξ	Ξ	*'xi'*
omikron	o	O	*'onikron'*
pi	ϖ , π	Π	*'pi'*
rho	ρ	P	*'ro'*
sigma	σ	Σ	*'sigma'*
tau	τ	Υ	*'taph'*
upsilon	υ	Y	*'ipsilon'*
phi	φ , ϕ	Φ	*'phi'*
chi	χ	X	*'chi'*
psi	ψ	Ψ	*'psi'*
omega	ω		*'omega'*

Greek symbols used here		
name	letter	capital letter
alpha	α	
beta	β	
gamma	γ	Γ
delta	δ	Δ
epsilon	ε	
zeta	ζ	
eta	η	
theta	θ	
iota		
kappa	κ	
lambda	λ	Λ
mu	μ	
nu	ν	
xi ("ksi")	ξ	
omikron		
pi	π	Π
rho	ρ	
sigma	σ	Σ
tau	τ	Υ
upsilon		
phi	φ , ϕ	Φ
chi	χ	
psi	ψ	Ψ
omega	ω	

[0] J.M.J. Journée and W.W. Massie, *"OFFSHORE HYDROMECHANICS"*, First Edition, January 2001, Delft University of Technology. For updates see web site: http://www.shipmotions.nl.

A.2 Water Constants

Temp. t (°C)	Density Fresh Water ρ_{fresh} (kg/m^3)	Density Salt Water ρ_{salt} (kg/m^3)	Kinematic Viscosity Fresh Water ν_{fresh} (m^{2}s $\cdot 10^{-6}$)	Kinematic Viscosity Salt Water ν_{salt} (m^{2}s $\cdot 10^{-6}$)	Vapor Pressure Water p_v (N/m^2)
0	999.8	1028.0	1.78667	1.82844	608
1	999.8	1027.9	1.72701	1.76915	
2	999.9	1027.8	1.67040	1.71306	706
3	999.9	1027.8	1.61665	1.65988	
4	999.9	1027.7	1.56557	1.60940	814
5	999.9	1027.6	1.51698	1.56142	
6	999.9	1027.4	1.47070	1.51584	932
7	999.8	1027.3	1.42667	1.47242	
8	999.8	1027.1	1.38471	1.43102	1069
9	999.7	1027.0	1.34463	1.39152	
10	999.6	1026.9	1.30641	1.35383	1226
11	999.5	1026.7	1.26988	1.31773	
12	999.4	1026.6	1.23495	1.28324	1402
13	999.3	1026.3	1.20159	1.25028	
14	999.1	1026.1	1.16964	1.21862	1598
15	999.0	1025.9	1.13902	1.18831	1706
16	998.9	1025.7	1.10966	1.15916	1814
17	998.7	1025.4	1.08155	1.13125	
18	998.5	1025.2	1.05456	1.10438	2059
19	998.3	1025.0	1.02865	1.07854	
20	998.1	1024.7	1.00374	1.05372	2334
21	997.9	1024.4	0.97984	1.02981	
22	997.7	1024.1	0.95682	1.00678	2638
23	997.4	1023.8	0.93471	0.98457	
24	997.2	1023.5	0.91340	0.96315	2981
25	996.9	1023.2	0.89292	0.94252	
26	996.7	1022.9	0.87313	0.92255	3364
27	996.4	1022.6	0.85409	0.90331	
28	996.2	1022.3	0.83572	0.88470	3785
29	995.9	1022.0	0.81798	0.86671	
30	995.6	1021.7	0.80091	0.84931	4236

Appendix B

MODELING AND MODEL SCALES

B.1 Introduction and Motivations

This appendix provides a rather complete discussion of the design of various sorts of models used in offshore engineering. Certain specific elements - such as Reynolds or Froude scaling, for example - are also summarized in the various chapters of the main text. In its most abstract sense, a model is some form of representation of an object. In a few cases, it is the object itself, but it is usually a more convenient representation (in some form) of an actual or proposed situation.
Models are used for many purposes; the main reasons for using models include the facts that they are:
- easier,
- faster,
- safer,
- cheaper.

It is generally easier, faster, safer, and cheaper to work with a model than to make all discoveries when a 'real thing' is used. For example, in the days before Computer Aided Design (CAD) software became trustworthy, it was common practice to build a plastic model of an offshore platform topsides - from the design drawings - in order to make sure that the piping all fits - or even worse - a major structural element conflicts with the piping or something else. While such a model may have cost tens of thousands of guilders, it was obviously much less expensive to correct errors in the design phase rather having to correct them in the field.

B.2 Model Types

There are many types of models that can be used.
CAD models in which the form of an object is modeled in a computer have been mentioned in the example above. Such models are used as well to generate the form of a ship or floating structure, for example. Since volumes and weights can easily be calculated, such systems can conveniently be extended to interface with hydrodynamic models or to check floating stability, for example.

[0] J.M.J. Journée and W.W. Massie, *"OFFSHORE HYDROMECHANICS"*, First Edition, January 2001, Delft University of Technology. For updates see web site: http://www.shipmotions.nl.

Computer Simulations are a special category within the group of Computer Models. These represent the dynamic behavior of an object - such as a ship in waves or even the spreading of an oil spill - effectively by solving the related differential equations.

Computer models - of whatever type - have several advantages, especially in relation to their alternatives:

- They are reasonably inexpensive to use, although their initial development costs may be significant.
- Computers are so universally available today that they can be used most anywhere.
- A computer model is safe; they present not danger to humans or the environment.
- All included phenomena can be represented at full scale. (The signifcance of this will become apparent below.)

On the other hand, a computer model and especially a simulation has one important drawback: it is only as good as the mathematics which is used to (approximately) describe the phenomena which actually take place. In the mid nineteen-eighties, Petroski reported on the comparison of measured and computed loads in a large transmission line tower.

> "Computer predicitions of structural behavior were within only sixty percent of the actual measured values only ninety-five percent of the time, ..."

Computer users should never allow blind trust to replace reasonable engineering thinking!

Of course it is possible to carry out testing using a 'real thing': a full-sized and complete prototype. Such testing can be expensive - certainly if it leads to required changes. Even without that, measuring a ship's behavior in waves at sea requires a significant mobilization of test equipment and personnel and one has no control over the input - the weather-dependent waves - either. Many field test programs have been unsuccessful because nature provided the wrong (either too much or too little) input! In some cases it can even be downright dangerous to carry out measurements on a full-scale basis.
This is not to say that prototype testing is always bad; indeed not. Such measurements can be very valuable as well for diagnostic purposes such as to determine why relatively many persons on a particular platform complain of motion sickness. Field measurements are often used as well to verify the correctness of a different model - of whatever type. Indeed, all physical phenomena are (obviously) properly modeled in a full-scale prototype situation.

One way to avoid the disadvantages of a (series of) prototype(s) is to use a small scaled model and test it in a laboratory instead. Such a model is obviously cheaper than a prototype and the laboratory conditions under which it is tested can be carefully controlled. There is seldom risk to life and limb involved either!
Compared to a computer model, a physical model is slower in use but it has at least the potential to work with a more accurate representation of nature. Indeed, more of the actual physical phenomena are inherently included in a physical model.
A disadvantage of a physical model, however, is that a specialized (and therefore scarce and expensive) test faciltiy is often needed in order to carry out its testing.

B.3 Basic Phenomena and Scales

Given that one wants to work with a model (any type will do, but it is conceptually easier to think in terms of a physical model for now), one must decide what phenomena to include in the model. These are discussed in this section.

Geometric

The first requirement of any model is that it reproduce the geometry of the situation in some consistent way. This usually means that all physical dimensions of the model are represented at the same length scale, α_L. An exception to this will be mentioned in section 9, however.

Kinematic

Secondly, it is convenient if the kinematics (velocities and accelerations) are reproduced in the model at consistent scales. The velocity scale is denoted by α_V and since the acceleration of gravity is an acceleration, this scale is denoted by α_g.

Dynamic or Kinetic

Additionally, it is even handy if the dynamics - the forces - are also modeled consistently to some scale, α_F. Fluid density is usually scaled by a factor α_ρ, and kinematic viscosity is scaled by a factor α_ν.
The following table summarizes the scale factors found so far. Each factor relates a prototype phenomena (subscript, p) and a model phenomena (subscript, m):

Unit	Scale	Relationship
Length:	α_L,	$L_p = \alpha_L \cdot L_m$
Velocity:	α_V,	$V_p = \alpha_V \cdot V_m$
Acceleration of gravity:	α_g,	$g_p = \alpha_g \cdot g_m$
Density:	α_ρ,	$\rho_p = \alpha_\rho \cdot \rho_m$
Fluid kinematic viscosity:	α_ν,	$\nu_p = \alpha_\upsilon \cdot \nu_m$
Fluid dynamic viscosity	α_η,	$\eta_p = \alpha_\eta \cdot \eta_m$

Note that all α values in this table are greater than or equal to 1.0 .

B.4 Derived Scales

With these, the scale factors for the areas S, the volumes ∇, the masses M and the mass moments of inertia I, respectively, are then found easily:

$$\alpha_S = \alpha_L^2 \qquad \alpha_\nabla = \alpha_L^3 \tag{B.1}$$

$$\alpha_M = \alpha_\rho \cdot \alpha_\nabla = \alpha_\rho \cdot \alpha_L^3 \tag{B.2}$$

$$\alpha_I = \alpha_\rho \cdot \alpha_L^5 \tag{B.3}$$

The velocity of a body or a (water) particle is defined as a displacement per unit of time, so the scale factor for the time becomes:

$$\alpha_T = \frac{\alpha_L}{\alpha_V} \tag{B.4}$$

The acceleration of a body or a (water) particle is defined as an increase of the velocity per unit of time, so the scale factor for the acceleration becomes:

$$\alpha_A = \frac{\alpha_V}{\alpha_T} = \frac{\alpha_V^2}{\alpha_L} \tag{B.5}$$

This should be same, by the way, as the scale factor for the acceleration of gravity, α_g. According to Newton's law, inertia forces are defined as a product of mass and acceleration, so the scale factor for the inertia forces (and also the resulting pressure forces) is:

$$\alpha_F = \alpha_M \cdot \alpha_A = \left(\alpha_\rho \cdot \alpha_L^3\right) \cdot \left(\frac{\alpha_V^2}{\alpha_L}\right) = \tag{B.6}$$

$$= \alpha_\rho \cdot \alpha_V^2 \cdot \alpha_L^2 \tag{B.7}$$

Then, the relation beteen the forces F_p on the prototype and the forces F_m on the model are:

$$F_p = \alpha_\rho \cdot \alpha_V^2 \cdot \alpha_L^2 \cdot F_m \tag{B.8}$$

or:

$$\alpha_F = \frac{F_p}{F_m} = \frac{\rho_p \cdot V_p^2 \cdot L_p^2}{\rho_m \cdot V_m^2 \cdot L_m^2} \tag{B.9}$$

From this, it is obvious that these forces can be expressed as:

$$F_p = C \cdot \frac{1}{2}\rho_p V_p^2 \cdot L_p^2 \quad \text{and} \quad F_m = C \cdot \frac{1}{2}\rho_m V_m^2 \cdot L_m^2 \tag{B.10}$$

in which the coefficient or constant C does not depend on the scale. Further, one recognizes the term $\frac{1}{2}\rho V^2$ as the **stagnation pressure**.

B.5 Forces to Model

Since forces and kinetic similarity are so important in offshore engineering, extra attention to specific forces is given in this section.

Inertia

Whenever velocities and accelerations are involved - and that is the usual case in offshore engineering - modeling will involve inertia forces.

Gravity

Gravity forces are important for problems involving buoyancy and more generally for flow situations in which a free water surface is involved. Gravity forces are therefore important for waves and for all sorts of open channel flow. They also play an important part in soil mechanics as well. They are not usually very important, on the other hand, for pipeline flow.

Viscous or Damping

Viscous forces are important when friction is significantly involved. Pipeline pressure losses are an example of this. Damping forces in structural dynamics are a different example; they only become important when a structure has a near-resonant response.

Surface Tension

Surface tension forces are important when capillary action - between a fluid and a wall - or other deformations of a fluid surface are involved. It is only occasionally that surface tension forces become very important in offshore engineering. One example involves the later phases of the spreading of an oil on water; this is driven by surface tension forces.

Internal Stresses

Internal stresses can often be as important as the external shape in a design. Indeed, a sleek and fast ship which breaks when it encounters its first wave is of little practical use.

B.6 Force Scaling

A casual reader can conclude that there are no real problems with physical modeling yet. This impression may change if we compare the ways in which the various forces are scaled in a model.

Force	Scale
Inertia	$\alpha_\rho \cdot \alpha_V^2 \cdot \alpha_L^2$
Gravity	$\alpha_\rho \cdot \alpha_g \cdot \alpha_L^3$
Viscous	$\alpha_\eta \cdot \alpha_V \cdot \alpha_L$
Surface Tension	$\alpha_\sigma \cdot \alpha_L$
Internal Stresses	$\frac{\alpha_\rho \cdot \alpha_g \cdot \alpha_L^3 \cdot \alpha_L \cdot \alpha_L}{\alpha_L^4} = \alpha_\rho \cdot \alpha_g \cdot \alpha_L$
Bulk Strain	$\alpha_K \cdot \alpha_L^2$

in which the following scale factors are used:

$$
\begin{aligned}
\alpha_g &= \text{gravitational acceleration} \\
\alpha_K &= \text{bulk modulus of the fluid} \\
\alpha_L &= \text{length} \\
\alpha_V &= \text{velocity} \\
\alpha_\eta &= \text{dynamic viscosity} \\
\alpha_\rho &= \text{mass density} \\
\alpha_\sigma &= \text{surface tension}
\end{aligned}
$$

One can see from this table that not all forces are scaled equally; this is unfortunate. As stated above, only a computer model has the potential to include all forces in their proper relative scales.
The fact that not all of the above forces are identically reproduced in a physical scale model of a prototype, means that some forces will become (relatively) more important in the model than they actually are in the prototype. What must the model designer do? The

most common way to minimize the effects of this problem is to keep the **ratio of the two most important forces** - such as inertia and gravity (or possibly some other important phenomena) for example - the same in the model as they are in the prototype. Such ratios are dimensionless and many have special names as outlined in the following section.

B.7 Dimensionless Ratios

The following ratios - not all of which are strictly defined in terms of forces, by the way - are relatively common:

Name	Symbol	Ratio
Froude	Fn	$\sqrt{\frac{\text{Inertia Force}}{\text{Gravity Force}}}$
Reynolds	Rn	$\frac{\text{Inertia Force}}{\text{Viscous Force}}$
Keulegan-Carpenter	KC	$2\pi \cdot \frac{\text{Water Displacement Amplitude}}{\text{Cylinder Diameter}}$ *
Sarpkaya Beta	β	$\frac{\text{Re}}{\text{KC}}$
Reduced Velocity	V_r	$\frac{\text{Steady Flow Velocity}}{\text{Oscillating Flow Velocity Amplitude}}$
Strouhal	St	$2\pi \cdot \frac{\text{Cylinder Diameter}}{\text{Water Displacement Amplitude}}$ *
Kenn	Ke	$\frac{\text{Viscous Forces}}{\text{Surface Tension Forces}}$
Weber	We	$\frac{\text{Inertia Forces}}{\text{Surface Tension Forces}}$

*While St and KC look quite similar, they are associated with entirely different phenomena and are therefore quite different.

One sees from this table that:

- Not all dimensionless ratios involve forces directly.
- Some ratios - such as β above - can be expressed in terms of others.

If one includes additional engineering fields, then one can compile what seems like an endless list containing several hundred entries.

Scaling Consequences

Once one has chosen to use a given dimensionless ratio as the basis for a model, then this provides additional information for use in modeling in many practical situations.

It is usually impractical to replace water with another liquid in a physical model. Indeed, one author knows of cases when water has replaced other more exotic fluids - such as supercritical steam or even molten sodium - for very special testing or modeling purposes. Generally in offshore engineering however, one is confronted with water in both the model and prototype - even though one will usually be saltier than they other! A consequence of this is that the densities and other fluid properties will have a scale which is quite close to unity. This means that: $\alpha_K, \alpha_\eta, \alpha_\rho,$and α_σ are all very close to unity.

Similarly, except in modern soil mechanics, it is seldom convenient to change the acceleration of gravity; α_g will be identically equal to 1. (Centrifuges are often used in soil mechanics to create an artifically higher 'gravitational' acceleration.) The only other know way to change the acceleration of gravity is to work in a space lab or on the moon; both of these alternatives are (still) too expensive for routine use!

This information can now be used in combination with a scaling law to derive additional relations.

Froude Scaling

Given that the Froude Number in the model must be same as that in the prototype, and that the Froude Number is the ratio of inertia to gravity forces, one may conclude that - using information from the tables above:

$$\alpha_\rho \cdot \alpha_V^2 \cdot \alpha_L^2 = \alpha_\rho \cdot \alpha_g \cdot \alpha_L^3 \tag{B.11}$$

and since:

$$\alpha_V = \frac{\alpha_L}{\alpha_T} \text{ and } \alpha_g = 1. \tag{B.12}$$

then equation B.11 can be re-written as so that:

$$\frac{\alpha_L}{\alpha_T^2} = 1. \tag{B.13}$$

Time One can conclude from this that the time scale, $\alpha_T = \sqrt{\alpha_L}$. Thus if a model is built with a length scale of 100 (1 meter in the prototype is 1 centimeter in the model) then one second in the model will correspond to 10 seconds in the prototype when Froude scaling is used.

Velocity A quick check will show, too, that velocities in the model will also be scaled with $\sqrt{\alpha_L}$; a prototype current of 1 m/s will correspond to 10 cm/s in the model.

Reynolds Number and Viscous Forces Since viscous forces may also be involved in a physical model designed to Froude Scale, it can be interesting to check how the Reynolds Number is scaled.
Since Re is the ratio of inertia to viscous forces, then:

$$\alpha_{\mathrm{Re}} = \frac{\alpha_\rho \cdot \alpha_V \cdot \alpha_L}{\alpha_\eta} \tag{B.14}$$

Since α_ρ and α_η are both equal to 1, then $\alpha_{\mathrm{Re}} = \alpha_L^{1.5}$. This means that the Reynolds Number in the model will be $\frac{1}{100^{1.5}}$ or 1000 times too small. In other words, the viscous forces in the model will be 1000 times more important in the model than in the field.

Keulegan Carpenter Number The Keulegan Carpenter number, $KC = \frac{\hat{u}T}{D}$, is scaled according to:

$$\alpha_{KC} = \frac{\alpha_V \cdot \alpha_T}{\alpha_L} = \frac{\sqrt{\alpha_L} \cdot \sqrt{\alpha_L}}{\alpha_L} \equiv 1 \tag{B.15}$$

KC is not changed when Froude scaling is used.

Internal Forces Internal forces - such as bending stresses in the hull of a ship - are scaled according to:
$\alpha_\rho \cdot \alpha_g \cdot \alpha_L$ - from the table above. Since α_ρ and α_g are both equal to 1 , then internal stresses are scaled in the same way as the length. Now the example model will be (relatively) 100 times as strong as the prototype. Failure to recognize this scaling little detail has led to

the failure (in the field) of concrete armor units for rubble mound breakwaters. Concrete units used in the model were strong enough, but the prototype scale units in the field did not have a significantly higher allowable stress than in the model. As a consequence, the concrete units in the sea broke up in a storm and the whole breakwater - as well as the infrastructure it was protecting - was severely damaged.

Reynolds Scaling

One way to avoid the distortion of the viscous forces in a Froude Scale Model is to use Reynolds Scaling, instead. Now the ratio of inertia to viscous forces is kept constant. This means that:

$$\alpha_\rho \cdot \alpha_V^2 \cdot \alpha_L^2 = \alpha_\eta \cdot \alpha_V \cdot \alpha_L \tag{B.16}$$

Since α_ρ and α_η are still equal to 1, then $\alpha_V \cdot \alpha_L = 1$. so that $\alpha_V = \frac{1}{\alpha_L}$. This means that if the model has a scale (as above) of 100, then the velocities in the model will have to be 100 times larger than in the prototype. This is essentially impossible to achieve in practice!

Other Scaling Laws

Most any of the dimensionless ratios listed in the table above can be used as a basis for a scaling law. Froude Scaling is the most common in offshore engineering hydromechanics simply because gravity plays a dominant role in the behavior of the free surface of the ocean. Reynolds scaling is often used for pipe flows (under pressure) such as can be found in the topsides of an offshore production platform.
The following figure compares a variety of scaling laws.

B.8 Practical Compromises

Consider for the moment a physical model of the entire North Sea (to stay offshore!) or even of a few kilometers of a broad river. Since the free water surface is important, Froude scaling would be most appropriate, but one quickly becomes concerned about the strongly increased influence of the viscous forces. Indeed, the model can become so shallow that boundary layer effects become too dominant.

Distorted Scale

One way to reduce the viscous influence in an open channel model is to use a smaller length scale for vertical dimensions than that used for horizontal dimensions. One author has worked on such a model with a vertical scale of 40 and a horizontal scale of 60. This makes the model relatively 1.5 times as deep as would be indicated from the field. This keeps the Reynolds numbers 1.5 times as large (relative to the undistorted model); they are still small relative to the prototype or field situation, however.

Added Roughness

A quite opposite problem occurs with ship models towed in a towing tank. Because the models are relatively smooth, the laminar boundary layer which forms near the bow extends

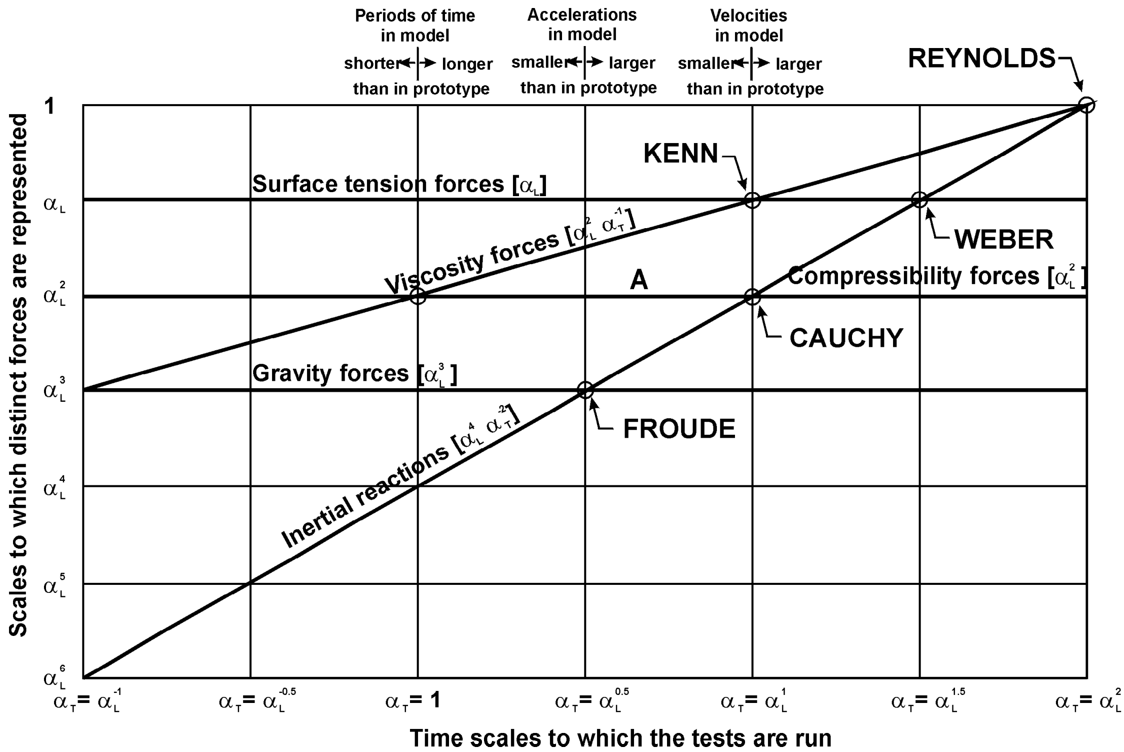

Figure B.1: Graphical Comparison of Scaling Laws

much too far aft (in the model) and thus distorts its skin friction resistance. In this case, this laminar boundary layer is forcefully broken up by attaching strips of rough material (It looks like coarse sandpaper.) to the model hulls a bit aft of the bow.

Adjust Gravity

The use of centrifuges has already been mentioned above in connection with geotechnical work. By artificially increasing g, one can use a relatively thin soil layer to model a much thicker one. Such models have been used - for example - to study the behavior of deeply penetrating offshore anchors in soft clay soils.

Adjusting gravity is never inexpensive! It can run into a number of very practical problems associated with carrying out the test, too - especially when free surfaces of liquids are involved.

Distort All Scales

Some have suggested that instead of keeping one dimensionless ratio constant, experiments might be designed so that all (more than one) important dimensionless ratios are distorted more or less equally. This idea may be better in theory than in practice, however. The author is aware of no specific examples of its application. This could mean, for example, that one would choose scaling corresponding to point A in the above figure.

B.9 Conclusion

Any physical scale model will distort the relative importance of various forces and other physical phenomena involved. On the other hand, a physical model **does include** all these physical phenomena.

The only way to avoid the distortions associated with physical models is to use a computer model or simulation. This has the disadvantage, however, that its representation of the physical situation is only as good as the mathematician has been able to make it.

Appendix C

FOURIER SERIES APPROXIMATIONS

C.1 Basic Form

Baron Jean Baptiste Jouseph Fourier, a French mathematician who died in 1830, concluded in that any time dependent signal, $F(t)$, which repeats itself with period, T, can be expressed as:

$$F(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)] \tag{C.1}$$

in which:

$F(t)$	=	Arbitray periodic function
a_n	=	Coefficients; $n = 0, 1, 2, ...$
b_n	=	Coefficients; $n = \quad 1, 2, ...$
n	=	An integer
t	=	Time
$\omega = 2\pi/T$	=	Frequency
T	=	Period of the function

Further, the coefficients a_n and b_n can be computed from $F(t)$ using:

$$a_0 = \frac{1}{T} \int_0^T F(t)\, dt \tag{C.2}$$

$$a_n = \frac{2}{T} \int_0^T F(t) \cdot \cos(n\omega t)\, dt \quad \text{with: } n > 0 \tag{C.3}$$

$$b_n = \frac{2}{T} \int_0^T F(t) \cdot \sin(n\omega t)\, dt \tag{C.4}$$

[0] J.M.J. Journée and W.W. Massie, *"OFFSHORE HYDROMECHANICS"*, First Edition, January 2001, Delft University of Technology. For updates see web site: http://www.shipmotions.nl.

These equations express a Fourier series in its basic form.
The above integrals must be carried out over one period, T, of the measured signal. It does not matter when, exactly, that period begins or ends. The limits of 0 and T above (as well as in the rest of this appendix) can be replaced by t and $t+T$ respectively if desired. The basic version will be used in this appendix, however.

C.2 Derived Form

Equation C.1 can be expressed in another way as well:

$$F(t) = a_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega t + \phi_n) \tag{C.5}$$

in which:

$$c_n = \sqrt{a_n^2 + b_n^2} \tag{C.6}$$

and:

$$\phi_n = \arctan\left(\frac{b_n}{a_n}\right) \tag{C.7}$$

(The arctan function denotes the angle whose tangent is ....)

C.3 Limits

Fourier's theory indicates explicitly that his series includes an infinite number of terms; this is never practical, however.
One approach is to bluntly limit the order of the Fourier Series to $n = 1$. This is often used as a means of linearizing a peridoic signal, by the way. If one wants to average this linearization over several periods of the signal, one can do this by treating some integer, k number of periods of that signal and then computing only the $n = k^{th}$ harmonic.
While use of a single periodic Fourier Series component is very attractive from both a computational and linearization points of view, one can become worried about whether such a simplification is really justified.
Fourier showed that the following relation also holds:

$$\frac{2}{T}\int_0^T [F(t)]^2 = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} c_n^2 \tag{C.8}$$

so that in practice the error, E_N (represented by all of the terms of order higher than N) is given by:

$$E_N = \frac{2}{T}\int_0^T [F(t)]^2 - \frac{a_0^2}{2} - \sum_{n=1}^{N} c_n^2 \tag{C.9}$$

This can evaluated readily, and if one is lucky, E_N will decrease rapidly as N increases.

C.4 Application Example

One function for which a Fourier Series linearization is commonly used that for quadratic drag in the Morison equation. This means that one wants to express a function of the form:

$$F(t) = A\cos(\omega t) \cdot |\cos(\omega t)| \tag{C.10}$$

as a Fourier series. (See chapter 12 for a discussion of the Morison Equation.)
Using equations C.3 and C.4 one finds that:

$$\begin{aligned}
a_0 &\equiv 0 \\
a_1 &= \tfrac{8}{3\pi} = 0.849A \\
b_n &\equiv 0 \quad \text{for all } n \\
a_n &\equiv 0 \quad \text{for all even } n \\
a_3 &= \tfrac{8}{15\pi} = 0.170A = \tfrac{a_1}{5} \\
a_5 &= \tfrac{8}{105\pi} = 0.024A = \tfrac{a_1}{35} \\
& \text{etcetera}
\end{aligned}$$

If a linearization is used in this particular case, then the amplitude of the linear equivalent component should be chosen equal to a_1. Note, however, that even though the second harmonic is absent, the amplitude of the third harmonic, a_3, is still 20% of the amplitude of the first harmonic; linearization may not be all that precise in this case.

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