ET4119 Electronic Power Conversion 2012/2013
Solutions Exam 19 April 2013

1. A single-phase diode rectifier is shown in the figure below. The rms value of the grid voltage is Vs = 230V. Assume that the load is represented by a constant dc current, I_d = 10A. The grid frequency is equal to 50 Hz.

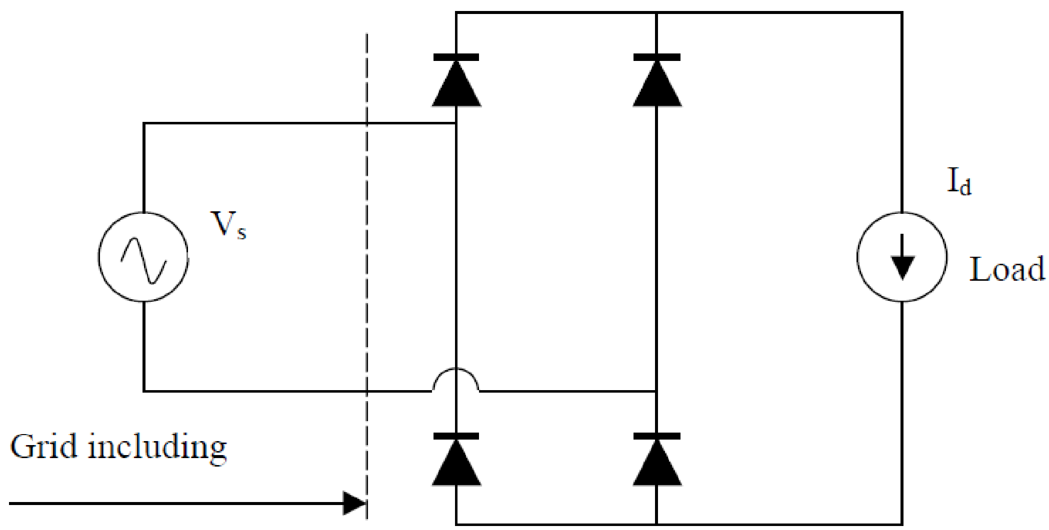
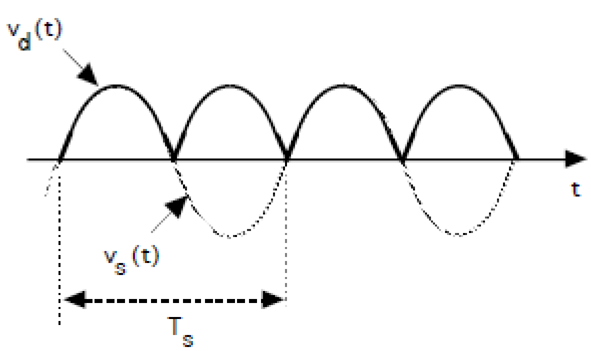


Figure 1 Rectifier circuit

Assume an ideal grid (L_s=0).

1.1

Waveform shown below.



$$V_{do} = \frac{2}{T_s}\int_0^{T_s/2} \sqrt{2}V_s \sin(\omega t)dt = \frac{2\sqrt{2}V_s}{\omega T_s}\{\cos((0) - \cos(\omega T_s/2)\} = \frac{2\sqrt{2}V_s}{\pi}$$

1.2.

$i_s(t)$ is an odd function of time. Hence even numbered harmonics are zero.

$$I_{s1,peak} = \frac{2}{\pi}\int_0^{\pi} I_d \sin(\omega t)d(\omega t) = \frac{4\,I_d}{\pi}\ ;\ I_{s1,rms} = I_{s1} = \frac{I_{s1,peak}}{\sqrt{2}} = \frac{4\,I_d}{\sqrt{2}\pi}$$

Power Factor $PF = \frac{P}{I_s V_s}$; $P = V_s I_{s1}$ average power at fundamental frequency delivered by grid. $V_s I_s$ = rms power delivered by the grid.

$$PF = \frac{I_{s1}\,V_s}{I_s\,V_s} = \frac{4I_d}{\sqrt{2}\pi I_d} = 0.9$$

1.3 See lecture slides 13 & 14from lecture 4 (the lecture is based on Chapter 5 of the book).

2. In a buck converter, consider all components to be ideal. The inductance of L is 50 mH and C is so large that the output voltage can be considered to be constant. The switching frequency is 50kHz.

Note: Derive the formulae from circuit waveforms, do not use pre-made formulae.

2.1

At the edge of CCCM-DCM:

$V_o = DV_d$. $D = \frac{10V}{40V} = 0.25$. Average output current $I_o = \frac{DT_s(V_d - V_o)}{2L}$

$$I_o = \frac{(0.25)(2x10^{-5})(40 - 10)}{(2)(5x10^{-5})} = 1.5 \text{ A}$$

2.2

For $I_o = 1.5A/10 = 0.15$ A, converter operating in DCM. Output voltage given by

$$V_o = \frac{V_d D^2}{D^2 + \frac{2I_o L}{T_s V_d}} \text{ : Eqs. (7-7) and (7-17) ; Solving for } D = \sqrt{\frac{2I_o L V_o}{T_s V_d (V_d - V_o)}}$$

$$\text{Evaluating: } D = \sqrt{\frac{(2)(0.15)(5x10^{-5})(10)}{(2x10^{-5})(40)(40 - 10)}} = 0.079$$

2.3

$$V_o(0.15 + 1\%) = \frac{(40)(0.079)^2}{(0.079)^2 + \frac{(2)(0.1515)(5x10^{-5})}{(2x10^{-5})(40)}} = 9.91 \text{ V}$$

2.4 For a duty cycle of 25%, Vo is independent of Io and thus appears as an ideal voltage source of 10 V. For a duty cycle of 7.9%, the voltage changes by 10.06 - 9.91 = 0.015 V for a current change of 3 mA. (0.015V/(.003A) = 5 ohms. Converter appears as an ideal voltage source of 10V in series with a 5 ohm resistor.

3. Design a flyback converter operating in the discontinuous conduction mode (DCM) shown shown in figure below for the following specifications:

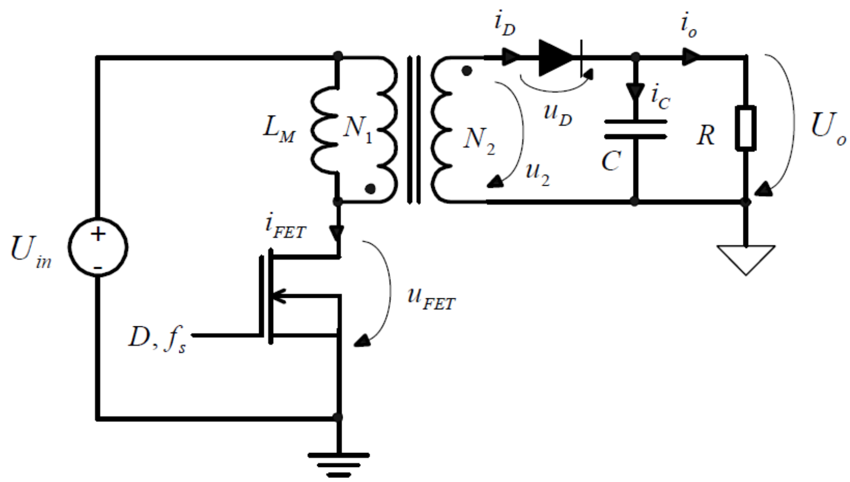
$300V < V_{in} < 400V$ (nominal 400V)

$0W < P_o < 50W$ (nominal 50W)

$V_o = 27V$

$f_s = 50kHz$

$\Delta v_{Op\_p} \leq 20mV$



3.1, 3.2, 3.3:

1) The design of the converter starts with defining the turns ratio $\frac{N_2}{N_1}$ at the minimum input voltage, maximum output voltage, and maximum load assuming that the converter enters into boundary mode or BCM. We may assume that the diode forward voltage is 1 V. In BCM, the modulus of the flyback converter is $\frac{U_o + U_D}{U_{in}} = \frac{N_2}{N_1} \cdot \frac{D}{D'}$. In the case of a flyback converter, the pulse width may be utilized up 100 % without saturation of the 'transformer' but better results (i.e., peak currents and voltages are lower) may be achieved using lower maximum duty ratio. If a PWM circuit is used a typical value is a 50 % of ideal pulse width giving practical maximum as e.g. 48 %. Therefore,

$$\frac{N_2}{N_1} = \frac{1-D_{\max}}{D_{\max}} \cdot \frac{U_{\mathrm{omax}} + U_D}{U_{\mathrm{inmin}}} = \frac{0.52}{0.48} \times \frac{30\mathrm{V}+1\mathrm{V}}{300\mathrm{V}} \approx \underline{0.112}$$

Now that the "flyback transformer" turns ratio is defined, we will define the magnetizing inductance (i.e. the inductance of the primary winding) so that the converter operates in DCM regardless of the operating point. This is done by assuming BCM operation and forming the design table out of which the maximum value for magnetizing inductance can be found.

In BCM the peak magnetizing current must equal twice the *average magnetizing current*. By applying the ampere-second balance to the output capacitor, we obtain that average current, which turns out to be

$$I_{\mathrm{LM}} = \frac{1}{D'} \frac{N_2}{N_1} I_{\mathrm{o}}.$$

Next, we express the peak magnetizing current using the familiar inductor element equation and we obtain

$$i_{\mathrm{M,peak}} = \frac{U_{\mathrm{in}} D T_{\mathrm{s}}}{L_{\mathrm{M}}}.$$

As a result we can write the BCM equality

$$i_{\mathrm{M,peak}} = 2I_{\mathrm{LM}} \rightarrow \frac{U_{\mathrm{in}} D T_{\mathrm{s}}}{L_{\mathrm{M}}} = \frac{1}{D'} \frac{N_2}{N_1} I_{\mathrm{o}} \rightarrow L_{\mathrm{M}} = \frac{DD' U_{\mathrm{in}} T_{\mathrm{s}}}{2 \frac{N_2}{N_1} I_{\mathrm{o}}}.$$

NOTE: The average magnetizing current at the primary side is the *on-time average* of the input current and therefore the *average input current* (over a complete switching period) is found by multiplying the average magnetizing current with *D*. This way the input/output relationships between currents and voltages are consistent.

Since the converter should always operate in DCM, we use the design table to calculate the corresponding BCM magnetizing inductances, then find the smallest one and select the actual value to be slightly smaller. The design table is presented in a separate excel-file and will not be shown here.

Defining the output capacitance to meet the voltage ripple requirement:

To find the value for output capacitance, we have to look at the output capacitor current waveform, since we are interested in the stored and released charge of the capacitor. The following figure depicts the capacitor current waveform with important values shown symbolically:

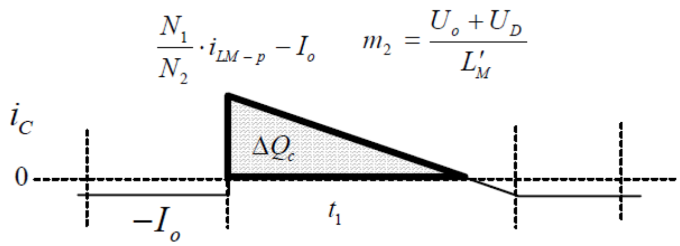


Fig. 2 The output capacitor current waveform

The peak capacitor current equals peak diode current minus the output current. Respectively, the peak diode current equals the peak primary magnetizing current multiplied by the turns ratio, which is shown in the fig. 2. The falling slope of the diode/capacitor current (m_2) is defined by the magnetizing inductance and its off-time voltage. The rate of falling current is expressed in the figure now with respect to the secondary side, i.e.

$$L'_{\mathrm{M}} = \left(\frac{N_2}{N_1}\right)^2 L_{\mathrm{M}}$$

Hence the important time value of t_1, which equals the time when the capacitor current has fallen to zero, can be given as follows:

$$t_1 = \left(\frac{N_2}{N_1}\right)^2 L_{\mathrm{M}} \frac{\frac{N_1}{N_2} i_{\mathrm{M,pk}} - I_{\mathrm{o}}}{U_{\mathrm{D}} + U_{\mathrm{o}}}$$

Finally, the output capacitance defined by the change in the charge and the specified voltage ripple:

$$C = \frac{dQ}{du} = \frac{\Delta Q}{\Delta u_{\mathrm{o,pp}}} = \frac{1}{2} \frac{\frac{N_1}{N_2} i_{\mathrm{M,pk}} - I_{\mathrm{o}}}{\Delta u_{\mathrm{o,pp}}} t_1$$

The value given by the previous equation now equals the minimum capacitance to meet the ripple voltage requirement and therefore one should again consider all limiting operation points and select the maximum of the four different values of C.

Solution: The solution below is generalised for the situation that output diodes are not ideal which means they have certain voltage drop U_D. In the exam question the diodes are assumed to be ideal so you can replace U_D with 0.

The associated waveforms of the defined voltages and currents are shown in Fig. 3.

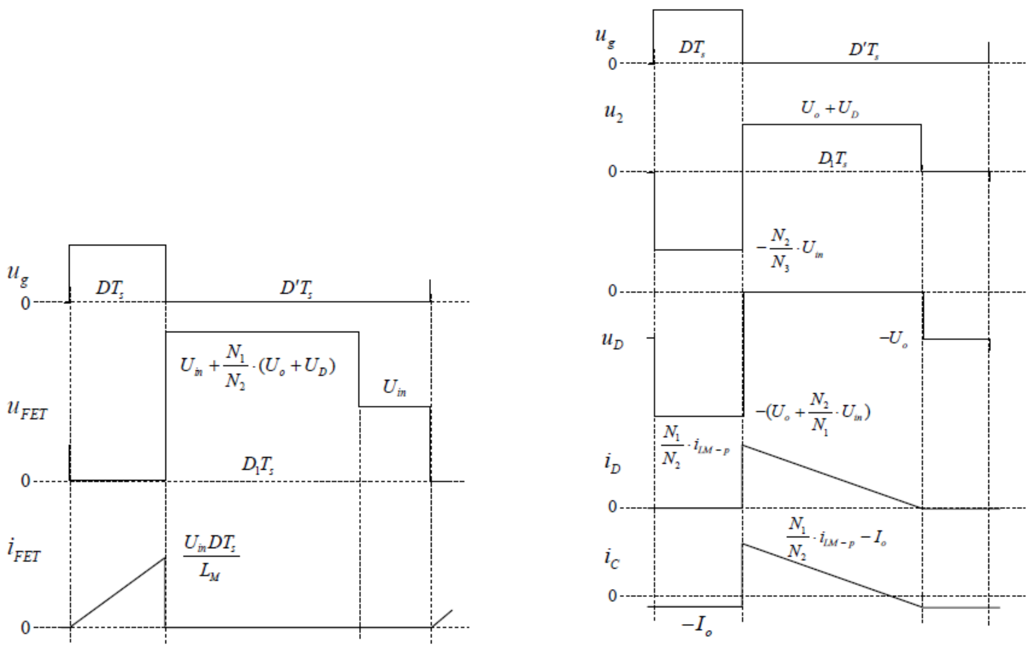
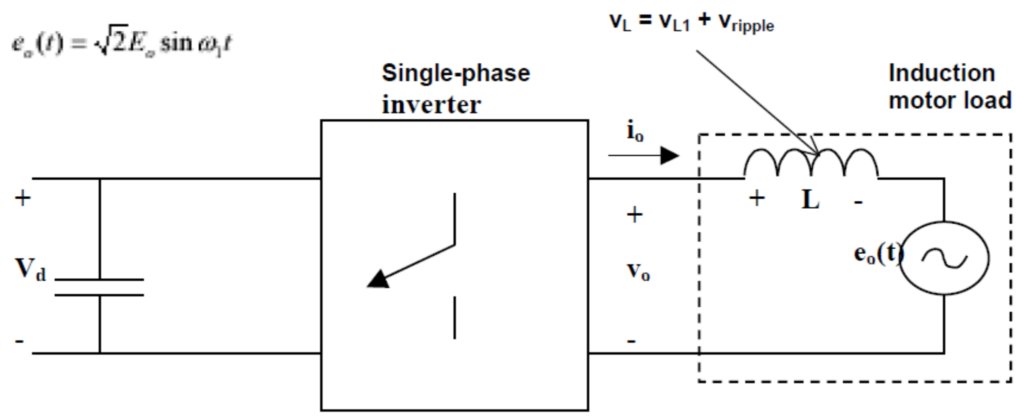


Fig. 3 a) Primary waveforms b) Secondary waveforms

The solution is generalised for the situation that output diodes are not ideal which means they have certain voltage drop U_D. In the exam question the diodes are assumed to be ideal so you can replace U_D with 0.

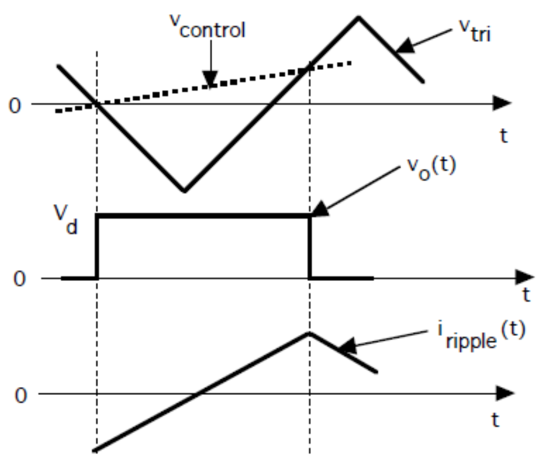
4. The problem with ripple in the output current from a single-phase full bridge inverter is to be studied. The first harmonic of the output voltage is given by V_{o1}=220V at f = 47 Hz. The load is given in the figure as L = 100 mH in series with an ideal voltage source $e_o(t)$. The converter works in square wave mode. The converter operates in sinusoidal PWM-mode, bipolar modulation m_f =21 and m_a = 0.8.



4.1 See the book, Chapter 8 on Inverters or Lecture presentation Chapter 8, slide 13
4.2

c) $m_a = 0.8$; $V_{o1,peak} = (0.8)V_d = (0.8)(220) = \sqrt{2}\,220$; $V_d = 389$ V

4.3



The switching frequency, f_s, is $f_s = 21x47 = 987$ Hz. Period $T_s \approx 1$ msec. As can be seen in the figure above, the ovoltage across the inductor is positive for approximately one half of a period about 0.5 ms. In this interval the inductor voltage is approximately constant at V_d since $v_o(t) \approx 0$.

The peak of the ripple in the output current will be $I_{ripple,peak} \approx \frac{V_d T}{4L} = 1$ A