ME1633: Integration of Physics, Motion and Metrology

Assignment 5: Dynamics

6 December 2013

- You can do this assignment on your own or in groups, as long as you hand in your own solutions and it becomes clear that you understand your solutions. Formulate your solutions step-by-step, carefully pointing out the logical structure of your answer, but keep your answers brief.
- If in some question you happen to need an answer from a previous question that you don't know the answer to, assume an answer or at least explain the method you would use when you would have had that previous answer.
- *Only for the students who follow the PME track's course ME1633:* Your solution to this assignment must be submitted in the lecture or via Blackboard (in pdf format) *before* 29 January 2014, 23:59h.
- Your solution may be formulated in either *English* or *Dutch*.

Satellite shaker

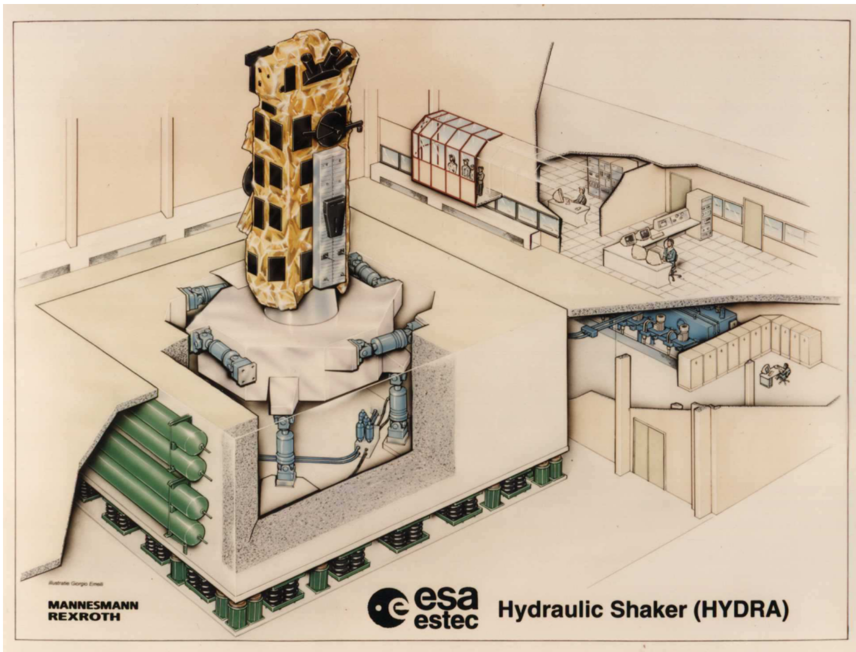
Satellites only experience light vibrations in space. During the launch, however, they experience large forces and are put heavily to the proof. To be sure that no damage will occur when launching the satellites, they are extensively tested. Amongst others, their dynamic behaviour is examined by measuring the response to vibrations applied by a shaker.

As modern satellites can easily weigh tens of tons, one can imagine that such a shaker will have extreme specifications. Figure 1a shows satellite shaker 'Hydra', which is located in the (huge) cleanroom facilities of the ESA in Noordwijk.[1] Satellites are mounted on the test table, which can be actuated in six degrees of freedom using a hydraulic system. The hydraulic cylinders are mounted in between the table and a very large seismic foundation body, whose purpose it is to reduce the forces transmitted to the ground. This allows people in neighbouring buildings to keep on working comfortably when the shaker is used. The seismic mass is supported by big springs that are connected to the ground.

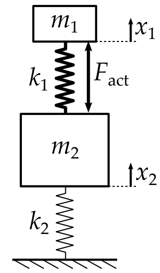
[1] http://www.european-test-services.net/services-mechanical-Hydra-Vibration.html

Table 1: Mass and stiffness parameters of the Hydra satellite shaker.

Parameter	Value
Mass of the shaker table (m_1)	$23.5 \cdot 10^3$ kg
Mass of the seismic body (m_2)	$1.4 \cdot 10^6$ kg
Stiffness of the hydraulic system (k_1)	10^9 N/m
Stiffness of the connection between the seismic body and the ground (k_2)	$7.5 \cdot 10^7$ N/m



(a) Drawing of the Hydra satellite shaker (source: ESA).



(b) Simplified one-dimensional model of a satellite shaker.

Figure 1: The satellite shaker system.

The satellite shaker is used for enforcing acceleration to the table with the satellite under test. In this assignment we aim at getting basic understanding of the system's dynamics and the possible control issues this dynamics imposes. We will only consider the vertical movement of the system and model the shaker as a simple, one-dimensional, two-body mass-spring system, see Figure 1b. m_1 represents the mass of the table and m_2 represents the mass of the seismic foundation body. Stiffness k_1 represents the stiffness of the hydraulic system, which works between the table and the foundation mass. The seismic body is coupled to the ground with a stiffness k_2. Finally, F_{act} is the actuation force that is applied by the hydraulic system. The mass and stiffness values are listed in Table 1. Unless explicitly stated otherwise, damping may be neglected.

Eigenmodes

The simplified one-dimensional model of the shaker possesses two bodies, with coordinates x_1 and x_2, so that two eigenmodes can be found. The modeshape, $\boldsymbol{\varphi}_1$, corresponding to the lowest eigenfrequency can be written as

$$\boldsymbol{\varphi}_1 = \left\{ \begin{array}{c} x_1 \\ x_2 \end{array} \right\} = \left\{ \begin{array}{c} 1 \\ 1-\alpha \end{array} \right\}, \tag{1}$$

where $\alpha = 0.0012$. Clearly, both bodies move with almost equal amplitudes in the first eigenmode.

(8p) **1a.** Explain that this is the modeshape belonging to the *lowest* eigenfrequency of the system. Use a reasoning based on modal mass and the modal stiffness to answer this question.

(6p) **1b.** To find a good estimate for the eigenfrequency corresponding to the modeshape of Equation (1) it is sufficient to use the following approximation of the modeshape:

$$\boldsymbol{\varphi}_1 = \left\{ \begin{array}{c} x_1 \\ x_2 \end{array} \right\} \simeq \left\{ \begin{array}{c} 1 \\ 1 \end{array} \right\}. \tag{2}$$

Find a simple approximate expression for the first eigenfrequency ω_1. For answering this, you should not need too extensive math.

The modeshape corresponding to the highest eigenfrequency, $\boldsymbol{\varphi}_2$, is

$$\boldsymbol{\varphi}_2 = \left\{ \begin{array}{c} x_1 \\ x_2 \end{array} \right\} = \left\{ \begin{array}{c} 1 \\ -\beta \end{array} \right\}, \tag{3}$$

where β a *positive* number much smaller than 1 ($\beta \ll 1$).

(8p) **2a.** Explain that this is the modeshape belonging to the *highest* eigenfrequency of the system. Again, use a reasoning based on modal mass and modal stiffness to answer this question.

(6p) **2b.** To find a good estimate for the eigenfrequency corresponding to the modeshape of Equation (3) it is sufficient to use the following approximate of the modeshape:

$$\boldsymbol{\varphi}_2 = \left\{ \begin{array}{c} x_1 \\ x_2 \end{array} \right\} \simeq \left\{ \begin{array}{c} 1 \\ 0 \end{array} \right\}. \tag{4}$$

Find a simple approximate expression for the second eigenfrequency ω_2. For answering this, you should not need too extensive math.

Actually, the modeshapes in Equations (2) and (4) are approximations which are useful for approximating the eigenfrequencies of the system. For understanding the transfer functions from force to displacement or acceleration, however, we need to know the modeshapes more accurately.

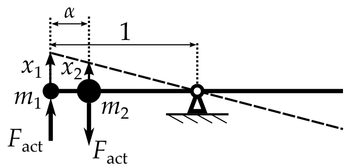
(6p) **2c.** Calculate an estimate of β, using the assumption that the suspension stiffness between the seismic body and the ground has a negligible contribution to the modal stiffness of the second eigenmode, i.e. $k_2 \simeq 0$.
Hint for this question: What is the movement of the system's centre of mass?

(4p) **2d.** Explain that an approximation as in Equation (4) could lead to big mistakes, when, for example, calculating the transfer from actuation forces to the force exerted to the ground.

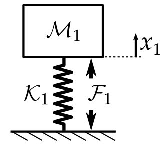
Modal lever representation

As we know, the equations of motion of this dynamic system can be written in the matrix form

$$\left[\begin{array}{cc} m_1 & 0 \\ 0 & m_2 \end{array} \right] \left\{ \begin{array}{c} \ddot{x}_1 \\ \ddot{x}_2 \end{array} \right\} + \left[\begin{array}{cc} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{array} \right] \left\{ \begin{array}{c} x_1 \\ x_2 \end{array} \right\} = \left\{ \begin{array}{c} F_1 \\ F_2 \end{array} \right\}, \tag{5}$$



(a) Modal lever representation.



(b) Modal mass-spring system corresponding to coordinate x_1.

Figure 2: Representations of the first eigenmode.

where $x_1(t)$ and $x_2(t)$ are the physical coordinates of the system, in general referred to as 'nodal coordinates', and $F_1(t)$ and $F_2(t)$ are the forces on each of the bodies, which are for the satellite shaker

$$\left\{ \begin{array}{c} F_1 \\ F_2 \end{array} \right\} = \left\{ \begin{array}{c} 1 \\ -1 \end{array} \right\} F_{\text{act}}.$$

These equations of motion can be rewritten in terms of the so called 'modal coordinates', $q_1(t)$ and $q_2(t)$. The modal coordinates describe the amplitude of each of the modeshapes as a function of time. The equations can be rewritten in terms of modal coordinates as follows:

$$\left[\begin{array}{cc} \mathcal{M}_1 & 0 \\ 0 & \mathcal{M}_2 \end{array} \right] \left\{ \begin{array}{c} \ddot{q}_1 \\ \ddot{q}_2 \end{array} \right\} + \left[\begin{array}{cc} \mathcal{K}_1 & 0 \\ 0 & \mathcal{K}_2 \end{array} \right] \left\{ \begin{array}{c} q_1 \\ q_2 \end{array} \right\} = \left\{ \begin{array}{c} \mathcal{F}_1 \\ \mathcal{F}_2 \end{array} \right\},$$

where $\mathcal{M}_i$, $\mathcal{K}_i$ and $\mathcal{F}_i$ are respectively the modal mass, the modal stiffness and the modal force of the ith mode. Note that the modal mass and stiffness matrices are diagonal, the two equations are not coupled any more. The response of the total system is now simply the summation of the response of each of the individual modes:

$$\left\{ \begin{array}{c} x_1 \\ x_2 \end{array} \right\} = \boldsymbol{\varphi}_1 q_1 + \boldsymbol{\varphi}_2 q_2. \tag{6}$$

(4p) **3a.** Explain the fact that each of the modal responses $q_i/\mathcal{F}_i$ can be regarded as the response of a single mass-spring system.

Figure 2a shows the modal lever representation of the first eigenmode of the Hydra satellite shaker. Such a modal lever is a powerful tool for understanding the system's dynamics, for several reasons. First, a lever representation helps understanding that the scaling factor of a modeshape is arbitrary. Furthermore, it helps to more easily find the modal mass and modal force corresponding to a nodal coordinate. Figure 2b shows the modal mass-spring system corresponding to coordinate x_1 of the shaker's first eigenmode.

(8p) **3b.** Find, using the modal lever representation of Figure 2, the modal mass of the first modeshape ($\mathcal{M}_1$) and the modal stiffness ($\mathcal{K}_1$). Also, show that the modal force ($\mathcal{F}_1$) equals

$$\mathcal{F}_1 = \alpha F_{\text{act}}.$$

Hint for this question: The modal lever representation works not too intuitively for calculating modal stiffness; for this reason, use the modal mass in combination with the eigenfrequency to calculate $\mathcal{K}_1$.

(12p) **3c.** Sketch the modal lever representation corresponding to the second eigenmode. Then, sketch the modal mass-spring system corresponding to coordinate x_1 with its modal mass $\mathcal{M}_2$, modal stiffness $\mathcal{K}_2$ and modal force $\mathcal{F}_2$.

(8p) **3d.** Calculate, using the modal lever representation of Question 3c, the modal mass ($\mathcal{M}_2$) and the modal stiffness ($\mathcal{K}_2$) for the second eigenmode. Also, calculate the modal force ($\mathcal{F}_2$) in terms of F_{act}.

(4p) **3e.** Could we call the modal masses and modal stiffnesses as calculated in 3b and 3d '*effective*'? Explain your answer.

The system's response

Once the system's modal parameters (the modal masses, stiffnesses and forces) are known, it is possible to find the transfer of each of the eigenmodes and of the full system.

(18p) **4a.** Sketch *by hand, without using your computer,* the Bode plots of both the transfer q_1/F_{act} and the transfer q_2/F_{act}.

(8p) **4b.** Now use the Bode plots found in Question 4a to sketch the Bode plot of the transfer from the actuator force to the displacement of the shaker table, x_1/F_{act}. Note that you can find this transfer by adding the transfers of each of the modeshape contributions. Mind the phase!

$\longleftarrow$ *Here ends the obligatory part of this assignment.*
The following part, bridging to the next subject (active control), is optional and will not be graded. $\longrightarrow$

Changing the dynamics

Now that we found the system's transfer in terms of its modeshapes, we could use this knowledge to modify the dynamic behaviour. For this, both the shaker's physical parameters and the parameters of the active control system to be added could be tuned.

5a. The transfer from actuator force to displacement of the first body (the shaker table), x_1/F_{act}, possesses an anti-resonance. Which practical problem does this anti-resonance impose? Is it possible to overcome the problem with active control?

5b. Our system engineer wants to solve the anti-resonance problem by adding damping to the system. Explain, using the Bode plots of Question 4a, where he should add damping to the satellite shaker.

5c. Sketch the Bode plot of each of the modal transfers q_1/F_{act} and q_2/F_{act}, as you expect them to become after adding the damping of Question 5b, in one figure. Also sketch the resulting transfer to the shaker table (x_1/F_{act}). Neglect modal coupling due to damping.

Please indicate, for each question, the following:

This question was:

Difficult	$\longleftrightarrow$		Doable
◯	◯	◯	◯

I spent:

Hours