ME1633: Integration of Physics, Motion and Metrology

Assignment 8: Power Electronics

12 February 2014

- You can do this assignment on your own or in groups, as long as you hand in your own solutions and it becomes clear that you understand your solutions. Formulate your solutions step-by- step, carefully pointing out the logical structure of your answer, but keep your answers brief.
- If in some question you happen to need an answer from a previous question that you don't know the answer to, assume an answer or at least explain the method you would use when you would have had that previous answer.
- *Only for the students who follow the PME track's course ME1633:* Your solution to this assignment must be submitted via Blackboard (in pdf format) *before* May 19 2014, 23:59h.

Introduction

Power electronics are used in mechatronic systems to control the force of actuators. The most widely applied actuators are electromagnetic types, where an electric current in a coil serves as the representative term for the force with a more or less linearly proportional characteristic, depending on the type of actuator.

This assignment will use the electrical characteristics of actuators due to their impact on the amplifier.

First the amplifier specifications will be investigated, followed by design choices and stability related issues.

Specifications (1)

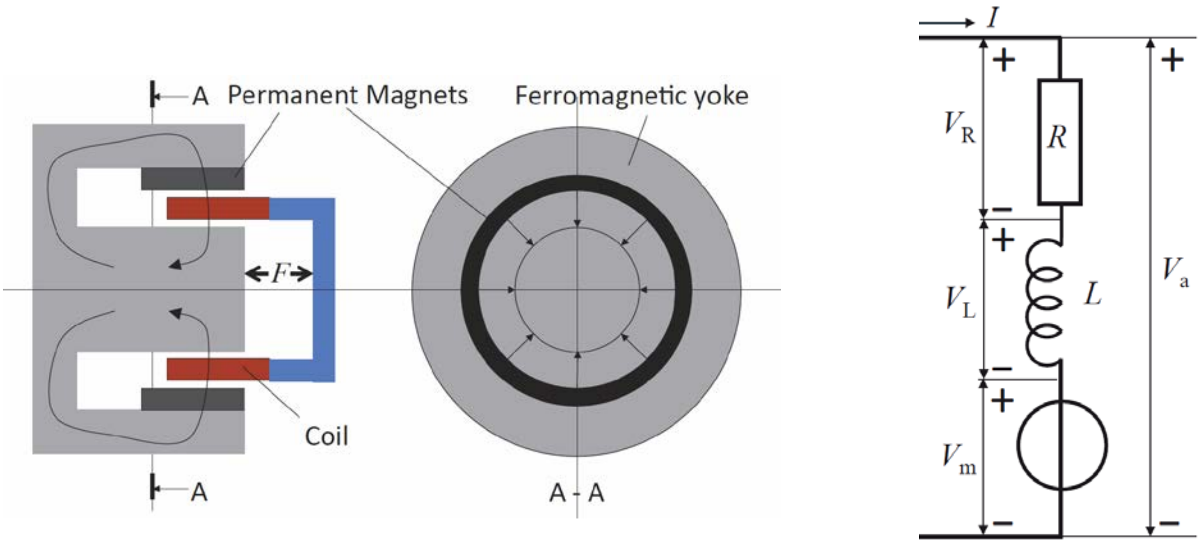


Figure 1: A Lorentz type actuator with its electrical equivalent circuit diagram

Task 1.1 (15 points): Determine the maximum (positive and negative peak) voltage $\pm V_{a,p}$ that an amplifier has to deliver when the following is given:

- The Lorentz actuator, as shown in figure 1, has a constant (ideal) motor constant $F = BI\ell = 10I \ (B\ell = 10)$
- The actuator is mounted on a spring (not shown) which creates a slightly damped resonator with the mass of the actuator at a resonance frequency of 100 Hz
- The coil resistance $R = 10\Omega$
- The coil inductance $L = 100 \text{ mH}$
- The actuator is driven from an amplifier with a sinusoidal current of 1A amplitude (=positive and negative peak level!) at 100Hz.
  - This current will drive the actuator into resonance up to a motion amplitude of $x = 10 \text{ mm}$, limited by the internal damping of the spring.

Give your answer by means of calculations of the voltage contributions of all elements in the equivalent circuit diagram and make a drawing of these voltage contributions of the three elements and the total voltage over one full period of the current, starting at the zero-crossing positive transition of the current.

Indicate which of the three elements determines the maximum voltage and derive the maximum positive and negative peak voltage $\pm V_{a,p}$.

Answer 1.1

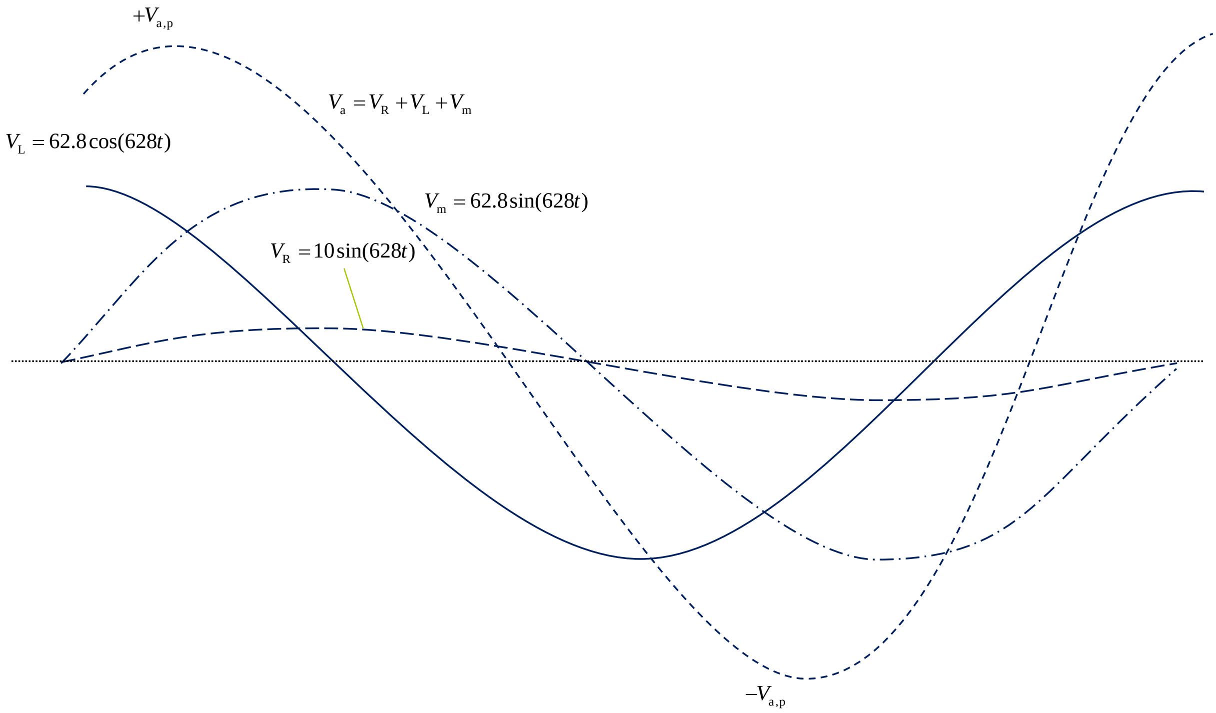
- The current equals: $I = 1\sin(\omega t) = 1\sin(2\pi ft) = 1\sin(628t)$
- The voltage over the resistor equals $V_R = 10\sin(\omega t) = 10\sin(628t)$

- The voltage over the inductor equals:

$$V_{\mathrm{L}} = L\frac{\mathrm{d}I}{\mathrm{d}t} = 1L\omega\cos(\omega t) = 628\cdot 0.1\cos(628t) = 62.8\cos(628t)$$

- The motor voltage is proportional to the velocity. At resonance the position lags 90° to the force ($\propto$ current) ($x(t) = -x\cos(\omega t)$ and thus the motor voltage equals equals:

$$V_{\mathrm{m}} = B\ell v = B\ell\frac{\mathrm{d}x}{\mathrm{d}t} = 10\cdot x\omega\sin(\omega t) = 10.628\cdot 0.01\sin(628t) = 62.8\sin(628t)$$



$V_{\mathrm{a,p}} \approx 100V$

Task 1.2 (15 points): Determine the efficiency of the amplifier when the system of task 1.1 is driven by a linear amplifier.

You may assume for the calculation that the linear amplifier is supplied by two (positive and negative) power supplies with a voltage equal to the above determined maximum positive $+V_{\mathrm{a,p}}$ and negative $-V_{\mathrm{a,p}}$ values for V_{a} .

For this answer you need to calculate all values of the power delivered by the amplifier to the (three elements of the!) actuator and the power delivered by the power supply to the amplifier.

Give a conclusion on the benefit of a switched-mode (PWM) amplifier when applied in this system.

Answer 1.2:

The power that is delivered by the amplifier to the actuator is spread over the three elements:

1. **Power in inductor:** The current is out-of phase with V_{L} which means that no average power is delivered to the inductor.
2. **Power in resistor:** The power in the resistor is dissipated into heat:
   $P_{\mathrm{R}} = I_{\mathrm{RMS}}^2 R = 0.5 I_{\mathrm{p}}^2 R = 5\ \mathrm{W}$
3. **Power in motor voltage:** The power in the motor voltage is the mechanical driving power of the actuator, which will be dissipated in the damper of the slightly damped resonator, hence also dissipated into heat:
   $P_{\mathrm{m}} = I_{\mathrm{RMS}} V_{\mathrm{m,RMS}} = 0.5\sqrt{2} I_{\mathrm{p}} \cdot 0.5\sqrt{2} V_{\mathrm{m,p}} = 0.5 \cdot 62.8 \approx 32\mathrm{W}$

Which gives a total power of 8.14 W delivered to the actuator

The power delivered by the power supply to the amplifier is equal to the maximum supply voltage times the average current delivered to the actuator, which is equal to the average current delivered by the power supply. In the positive current cycle of the sinusoidal current, the current is delivered from the positive voltage $+V_{\mathrm{a,p}}$ and in the negative current cycle from the negative supply voltage $-V_{\mathrm{a,p}}$: both multiplications are positive and equal for a sinusoidal current hence the average power can be calculated from the average value of the sinusoidal current times the maximum voltage:

$$P_{\mathrm{A}} = I_{\mathrm{av}} V_{\mathrm{a,p}} = \frac{2}{\pi} I_{\mathrm{p}} 100 \approx 64\ \mathrm{W}$$

Clearly much energy ($\approx 32\ \mathrm{W}$) is lost in the amplifier. The efficiency equals $\eta \approx \frac{32}{64} \approx 0.5$

A switched-mode PWM amplifier will not show this energy loss.

Switched mode amplifier design (2)

A switched mode amplifier with pulse-width-modulation (PWM) utilises the fact that the average value of a squarewave voltage signal is directly determined by the ratio between the positive and negative cycle. After low-pass filtering the signal below the switching frequency, only the average voltage value remains.

For filtering only a combination of inductors and capacitors can be used due to the high power of the signal that has to be filtered. Most often this filter consists of a combination of an inductor and a capacitor, hence forming a second-order low-pass filter, while especially at higher power levels often only the self-inductance of the actuator is used to average out the squarewave current, hence forming a first-order low-pass filter.

For this assignment a voltage source amplifier with resistive load is analysed first. In a second step this amplifier is transformed into a current source output amplifier by current feedback.

Task 2.1 (20 points): Determine the minimum switching frequency of a voltage source PWM amplifier.

The PWM amplifier is meant to replace a linear amplifier for efficiency reasons in a feedback controlled positioning system. With the original linear amplifier the feedback loop has a unity-gain cross-over frequency of 500Hz at a phase margin of 55 degrees.

The following aspects need to be considered in this replacement:

- The phase margin is not allowed to decrease to less than $\approx 45^{\circ}$ by the use of a PWM amplifier, as caused by its second-order output filter
- The load of the amplifier (the actuator) is assumed purely resistive.
- The second-order output filter is designed such that it has $Q = 1$ when combined with the resistive load.
- The voltage ripple over the load is not allowed to be larger than 1% of the magnitude of the PWM squarewave signal.

Tip: First determine the minimum corner frequency of the output filter that gives not more than the allowed phase shift for the feedback system and then determine the switching frequency that would give the allowed maximum ripple voltage with that filter. For this calculation you have to consider the Fourier decomposition of a square waveform and make a conclusion about which harmonics of the switching signal will contribute to the ripple.

As always you may use figures from the book for your answer. Phase values may be estimated from these figures as exact calculations are not required.

Answer 2.1:

Using Figure 6.20 of the book, a frequency at approximately 0.2 * the corner frequency will give less than 10° phase shift. This means that the corner frequency of the filter has to be >2.5 kHz.

A second order filter attenuates with a -2 slope. This means that the higher harmonics of the squarewave will be attenuated with a factor 4,9,16 etc. relative to the first harmonic, hence these higher harmonics can be neglected.

The first harmonic has an amplitude of $\frac{4}{\pi} = 1.27$ of the squarewave signal. This has to be attenuated by the filter to 0.01. This means that the switching frequency should equal at least:

$f_{\text{sw}} > \sqrt{127} \cdot 2500 \approx 30\ \text{kHz}$. In practice a factor 10 (2.5 kHz) is often taken.

Task 2.2 (20 points): Impact on stability of output filter in current source power amplifier.

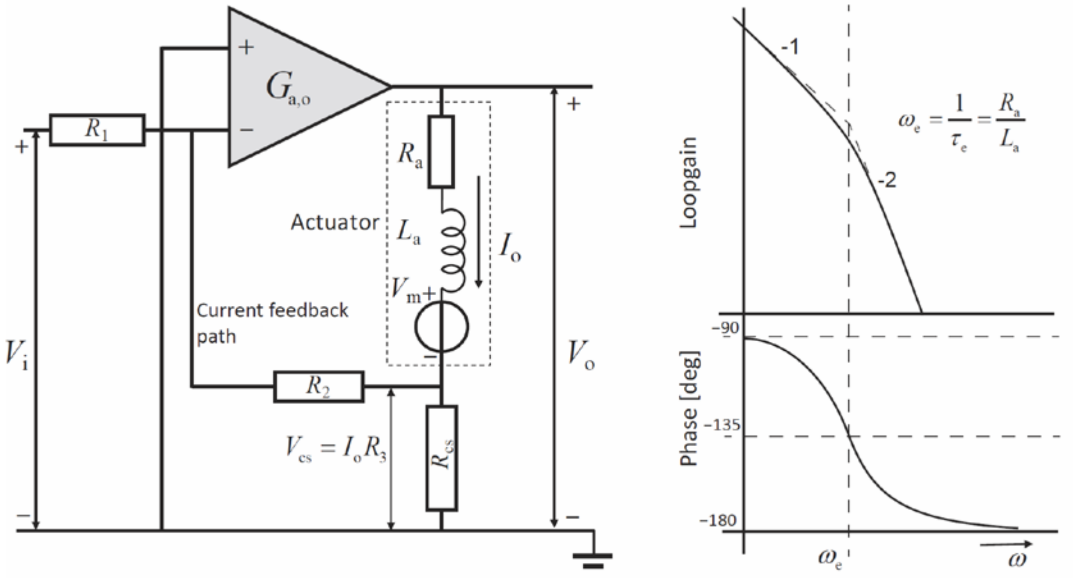


Figure 2 Current source output power amplifier, realised by means of current feedback. The internal first pole of the amplifier gives a -1 slope while the self-inductance of the actuator steepens the slope to -2 with a phase approaching -180^o at higher frequencies.

A current source amplifier is most often made by applying current feedback on a voltage-source amplifier as shown for a linear amplifier in Figure 2, which is a slightly updated version of Figure 6.64 from the first edition of the book and equal to Figure 6.67 from the second edition. The section that these figures refer to deals with stability issues that are caused by the self-inductance of the actuator, creating an additional pole in the feedback loop.

In the same way a switched-mode PWM voltage amplifier, as analysed in task 2.1 can be used in this circuit, replacing the triangle op-amp in the circuit diagram.

Now the task is to **draw the complete Bode-plot of the open-loop frequency response function of the feedback loop**, when the following is given:

- V_{i} is delivered by an ideal voltage source.
- $R_1 = 10\ \text{k}\Omega,\ R_2 = 1\ \text{k}\Omega,\ R_{\text{cs}} = 0.1\ \Omega,\ R_{\text{a}} = 3\ \Omega,\ L_{\text{a}} = 1\ \text{mH},$

- You may neglect the impact of the non-resistive parts of the load impedance on the damping of the output filter. This means that you may calculate the filter as having $Q = 1$ as in task 2.1
- The amplifier has an open-loop gain of 10^6 and a first dominant pole on 1 Hz.

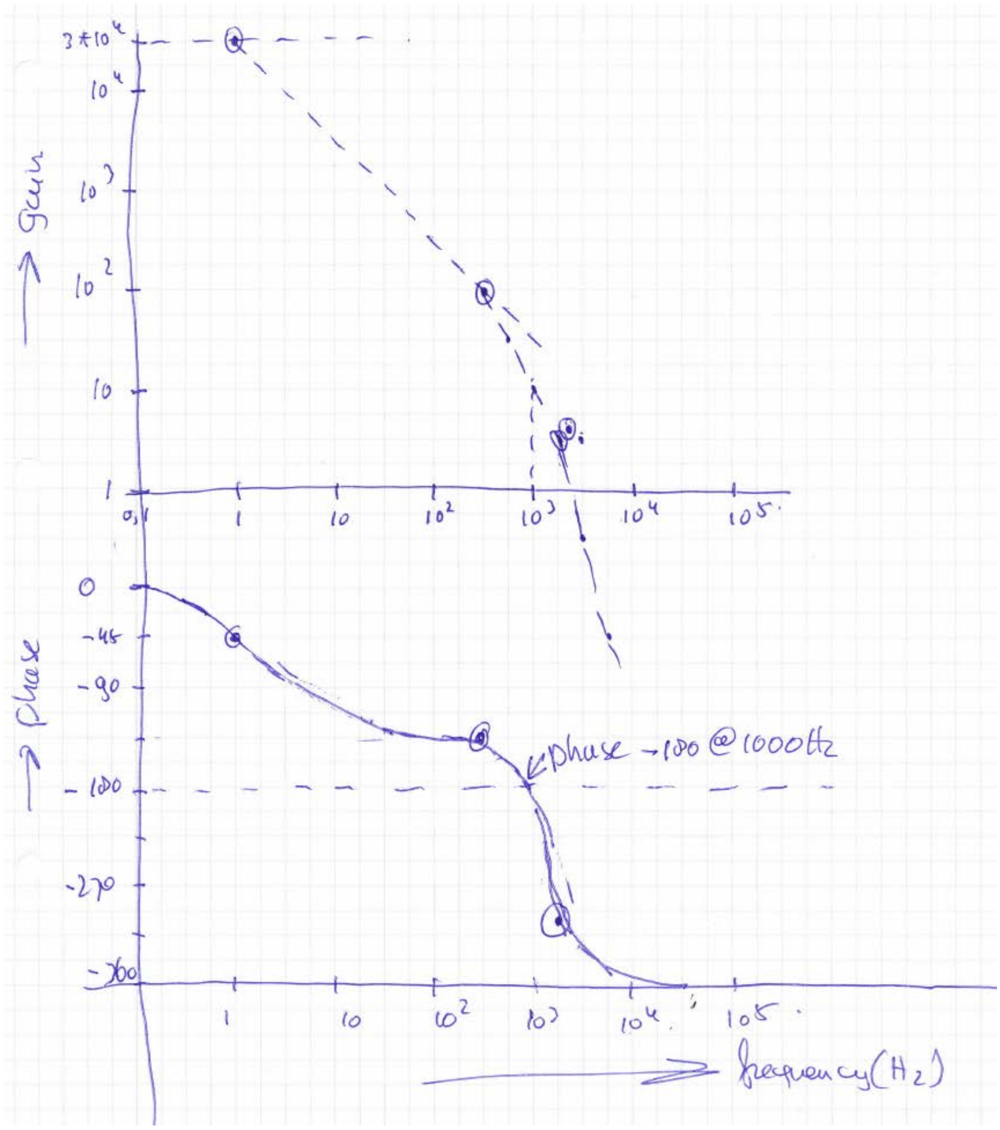
Be aware that the open-loop frequency response <u>of the feedback loop</u> is determined by cutting the feedback loop at any position and determine the frequency response from the input of the loop to the output. So start with determining the feedback loop.

You should make a hand drawing, starting with the asymptotes and finishing with the smoothed, more realistic curves.

Answer 2.2:

First we can conclude that we have only poles in the loop. One at 1 Hz, by the amplifier itself, one at ≈ 500 Hz , due to the actuator inductance and two well damped poles at 5 kHz by the output filter.

The DC loopgain (at 0 Hz) is determined by the amplifier 10^6 and the attenuation of the resistors. R_a and R_{cs} form an attenuator of ≈ 0.03. R_2 and R_1 form an attenuator of ≈ 0.9 . This means that the loopgain starts at 0Hz with $3 \cdot 10^4$



Task 2.3 (5 points):

Comment on the stability of the amplifier and which measure can be taken to increase the phase margin without changing the external components or the output filter.

Answer 2.3:

The amplifier has a unity-gain cross-over frequency at ≈ 4000 Hz and the phase passes -180 degrees @1000 Hz, which means the amplifier is unstable

The phase margin can than only be improved by lowering the open-loop gain of the amplifier until at a unity-gain cross-over frequency of ≈ 500 Hz the phase margin is 45 degrees.

Task 2.4 (5 points):

Comment on the impact of the self-inductance of the actuator on the Q level of the output filter and the drawn Bode-plot.

Answer 2.4

The self-inductance will increase the impedance of the actuator, thereby reducing the damping (higher Q). The Bode plot will show a resonance peak in the magnitude with a steeper phase transition around 2.5 kHz.

Task 2.5 (10 points):

Make a rough qualitative estimation of the measures that can be taken on the output filter, switching frequency, open-loop gain and frequency response when this amplifier has to be used in the position feedback system with 500 Hz unity-gain cross-over frequency of Task 2.1. Address the following points:

1. Closed-loop response of the amplifier with phase and amplitude that influence the position control loop, like in task 2.1. This leads to a required minimum open-loop unity-gain cross-over frequency of the amplifier.
2. Output filter parameters to guarantee stability of the feedback loop at the settings under 1.
3. Switching frequency according to the requirements on the voltage ripple with the parameters under 2.

Answer 2.5:

1. After closing the feedback loop of the amplifier, it will behave as a second-order low-pass filter, with a damping depending on the phase margin. This means that the unity-gain cross-over frequency of the amplifier has to be >~2.5 kHz to contribute no more than 10 degrees to the phase of the position feedback loop at 500 Hz, as was found in task 2.1.
2. The phase margin of the amplifier requires that the corner frequency of the output filter has to be at least a factor 5 higher than 2.5 kHz = >12.5 kHz
3. The switching frequency has to be in the order of 125 kHz when the ripple requirements are equal.