1.

BASIC PRINCIPLES OF FLOW OF LIQUID AND PARTICLES IN A PIPELINE

1.1 LIQUID FLOW

The principles of the flow of a substance in a pressurised pipeline are governed by the basic physical laws of conservation of mass, momentum and energy. The conservation laws are expressed mathematically by means of balance equations. In the most general case, these are the differential equations, which describe the flow process in general conditions in an infinitesimal control volume. Simpler equations may be obtained by implementing the specific flow conditions characteristic of a chosen control volume.

1.1.1 Conservation of mass

Conservation of mass in a control volume (CV) is written in the form: the rate of mass input = the rate of mass output + the rate of mass accumulation. Thus

$$\frac{d\left(mass\right)}{dt}=\sum\left(q_{outlet}-q_{inlet}\right)$$

in which q [kg/s] is the total mass flow rate through all boundaries of the CV.

In the general case of unsteady flow of a compressible substance of density ρ, the differential equation evaluating mass balance (or continuity) is

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}.\left(\rho\vec{V}\right)=0 \tag{1.1}$$

in which t denotes time and $\vec{V}$ velocity vector.

For incompressible (ρ = const.) liquid and steady ($\partial\rho/\partial t = 0$) flow the equation is given in its simplest form

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0 \tag{1.2}$$

The physical explanation of the equation is that the mass flow rates $q_m = \rho VA$ [kg/s] for steady flow at the inlet and outlet of the control volume are equal. Expressed in terms of the mean values of quantities at the inlet and outlet of the control volume, given by a pipeline length section, the equation is

$$q_m = \rho VA = \text{const.} \qquad (1.3).$$

Thus

$$(\rho VA)_{inlet} = (\rho VA)_{outlet} \qquad (1.4)$$

q_m	mass flow rate	[kg/s]
ρ	density of flowing liquid	[kg/m^3]
V	mean velocity in a pipe cross section	[m/s]
A	area of a pipe cross section	[m^2].

In practice volumetric flow rate Q is often used in place of mass flow rate q. The volumetric flow rate $Q = q/\rho = VA$. For a circular pipeline of two different diameters D_1 and D_2 (see Fig.1.1) the mass balance claims $V_1 D_1^2 = V_2 D_2^2$.

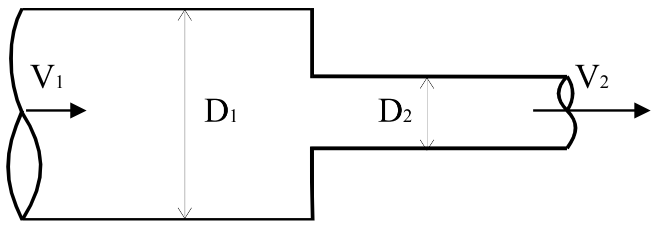


Figure 1.1. Application of continuity equation.

1.1.2 Conservation of momentum

A momentum equation is an application of Newton's second law of motion. The summation of all external forces on a control volume filled with a substance is equal to the rate of change of momentum of the substance in the control volume. The sum of the external forces acting on the control volume is counterbalanced by the inertial force proportional to the momentum flux of the control volume

$$\frac{d(momentum)}{dt} = \sum F_{external} \,.$$

The external forces are
- body forces due to external fields (gravity, magnetism, electric potential) which act upon the entire mass of the matter within the control volume,
- surface forces due to stresses on the surface of the control volume which are transmitted across the control surface.

Gravity is the only body force relevant to the description of the flow of a substance in a conduit. Surface forces are represented by the force from the pressure gradient and by friction forces from stress gradients at the control volume boundary.

In an *infinitesimal control volume* filled with a substance of density ρ the force balance between inertial force, on one side, and pressure force, body force, friction force, on the other side, is given by a differential linear momentum equation in vector form

$$\frac{\partial}{\partial t}\left(\rho\vec{V}\right)+\vec{V}.\vec{\nabla}\left(\rho\vec{V}\right)=-\vec{\nabla}P-\rho g\vec{\nabla}h-\vec{\nabla}.\vec{T} \qquad (1.5)$$

where h denotes the elevation above a datum, $\vec{V}$ the velocity vector and $\vec{T}$ the stress tensor.

To apply the momentum equation to pipeline flow it is convenient to replace the infinitesimal control volume by a macroscopic one given by *a straight piece of pipe of the differential distance dx*, measured in the downstream direction (Fig. 1.2). The momentum equation written for this control volume is simpler because quantities in the equation are averaged over the pipeline cross section. The momentum equation is obtained by integrating the differential linear momentum equation over the pipe cross section. For the one-dimensional liquid flow it has the form (Longwell, 1966 or Shook & Roco, 1991)

$$\rho\left(\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+g\frac{\partial h}{\partial x}\right)+\frac{\partial P}{\partial x}+4\frac{\tau_o}{D}=0 \qquad (1.6).$$

ρ	density of flowing liquid	[kg/m^3]
V	mean velocity in a pipe cross section	[m/s]
g	gravitational acceleration	[m/s^2]
h	elevation above a datum	[m]
P	mean pressure in a pipe cross section	[Pa]
τ_o	shear stress at the pipe wall	[Pa]
D	pipe diameter	[m]

The shear stress at the pipe wall, τ_o, is defined below by Eq. 1.15.

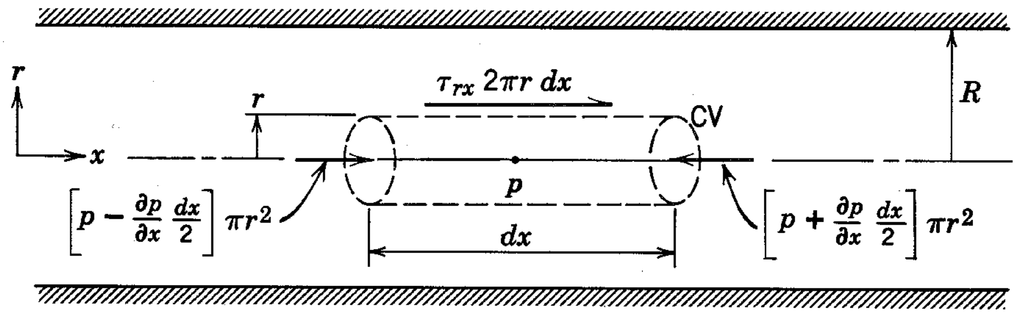


Figure 1.2. Control volume (CV) for analysis of force balance in flow in a circular pipe.

Additional conditions (incompressible liquid, steady and uniform flow in a horizontal straight pipe) make it possible to obtain a simple form of the linear momentum equation for liquid flow. Under the chosen conditions, the momentum flux at the control volume inlet is equal to that at the control volume outlet and the inertial force in the control volume is zero. In this case the integrated form of the linear momentum equation relates the driving force generated by the pressure gradient over the pipe distance dx and the cross section area A (and the perimeter O) to the resisting force due to viscous friction at the flow boundary, which is a pipe wall. The balance is

$$-\frac{dP}{dx}A = \tau_O O \qquad (1.7),$$

i.e. for *a pipe of a circular cross section* and internal diameter D

$$-\frac{dP}{dx} = 4\frac{\tau_O}{D} \qquad (1.8).$$

This equation shows that the wall shear stress must be correlated with the flow conditions to solve the pressure drop due to friction in pipeline flow.

1.1.3 Friction in pipeline flow of liquid

The Eq. 1.8 is not only valid for a pipe flow boundary; it can also be generalized to flow within each cylinder of radius r coaxial with a cylindrical pipe. It then provides an equation for shear stress distribution in the pipe cross section (see Fig. 1.3) that is valid for both laminar and turbulent liquid flow. This is

$$-\frac{dP}{dx} = \tau\frac{2}{r} \qquad (1.9).$$

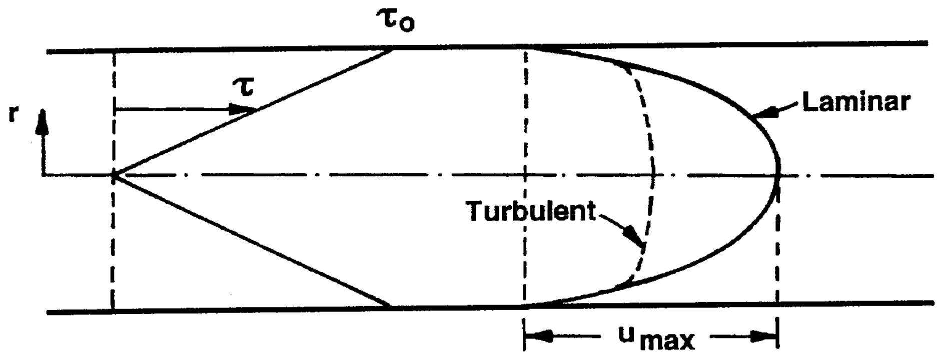


Figure 1.3. Stress and velocity distributions in pipe flow of liquid.

Newton's law of liquid viscosity is

$$\tau = \mu_f\left(-\frac{dv_x}{dr}\right) \qquad (1.10),$$

τ	local shear stress within liquid stream	[Pa]
v_x	local liquid velocity in the pipe-axis direction	[m]
μ_f	dynamic viscosity of liquid	[Pa.s]

where τ and v_x are at the position given by the radius r in a pipe cross section.

In laminar flow, the equation for a shear stress distribution (Eq. 1.9) and Newton's law of liquid viscosity (Eq. 1.10) determine a velocity profile $v_x(r)$ of liquid flow. Its integration over a pipe cross section

$$V_f = \frac{1}{A}\iint_A v_x dA = \frac{8}{D^2}\int_0^{D/2} v_x r dr \qquad (1.11)$$

provides a relationship between pressure drop dP/dx and mean velocity V_f

$$V_f = \frac{D^2}{32\mu_f}\left(\frac{dP}{dx}\right) \qquad (1.12)\cdot$$

Shear stress at the pipe wall is thus determined as

$$\tau_o = \mu_f \frac{8V_f}{D} \qquad (1.13).$$

This procedure cannot be used for *turbulent flow* because the relation between shear stress and strain rate in the turbulent flow is not fully described by the Newtonian viscous law. In a turbulent stream, the local velocity of the liquid fluctuates in magnitude and direction. This causes a momentum flux between liquid laminae in the stream. The momentum exchange has the same effect as a shear stress applied to the flowing liquid. These additional stresses set up by the turbulent mixing process are called apparent shear stresses or Reynolds stresses. They predominate over the Newtonian, purely viscous stresses in the turbulent core of the liquid flow. In a fully developed turbulent flow the turbulent core usually occupies almost the entire pipe cross section, excepting only the near-wall region. A turbulent flow regime is typical for pipelines of an industrial scale.

Thus shear stress τ_0 for turbulent flow cannot be determined directly from Newton's law of viscosity and the force balance equation (Eq. 1.9). Instead, it is formulated by using dimensional analysis. A function

$$\tau_0 = fn(\rho_f, V_f, \mu_f, D, k) \qquad (1.14)$$

τ_o	shear stress at the pipe wall	[Pa]
ρ_f	density of liquid	[kg/m^3]
V_f	mean velocity in a pipe cross section	[m/s]
μ_f	dynamic viscosity of liquid	[Pa.s]
D	pipe diameter	[m]
k	absolute roughness of the pipeline wall	[m]

is assumed. This provides a relation between dimensionless groups

$$\frac{\tau_o}{\frac{1}{2}\rho_f V_f^2} = \text{fn}\left(\text{Re}, \frac{k}{D}\right) \qquad (1.15).$$

The dimensionless group Re, Reynolds number of the pipeline flow, relates the inertial and viscous forces in the pipeline flow

$$\text{Re} = \frac{V_f D \rho_f}{\mu_f} = \frac{V_f D}{\nu_f} \qquad (1.16).$$

Re	Reynolds number of the pipeline flow	[-]
ν_f	kinematic viscosity of liquid μ_f/ρ_f	[m^2/s]

The dimensionless parameter on the left side of the equation 1.15 is called the friction factor. It is the ratio between the wall shear stress and kinetic energy of the liquid in a control volume in a pipeline

$$f_f = \frac{\tau_o}{\frac{1}{2}\rho_f V_f^2} \qquad (1.17).$$

The parameter f is known as Fanning friction factor. Darcy obtained a friction coefficient (called sometimes Darcy-Weisbach friction coefficient)

$$\lambda_f = \frac{8\tau_o}{\rho_f V_f^2} \qquad (1.18).$$

Thus the friction coefficient $\lambda_f = 4f_f$.
The equation for the Darcy-Weisbach friction coefficient, combined with the integrated linear momentum equation for pipeline flow (Eq. 1.8), gives the equation first published by Weisbach in 1850

$$-\frac{dP}{dx} = \frac{\lambda_f}{D}\frac{\rho_f V_f^2}{2} \qquad (1.19)$$

that is for $-\frac{dP}{dx}$ written as $\frac{\Delta P}{L} = \frac{P_1 - P_2}{L}$ (see Fig. 1.4)

$$\boxed{P_1 = P_2 + \frac{\lambda_f}{D}\frac{\rho_f V_f^2}{2} L \qquad (1.20).}$$

P_1	absolute pressure at beginning of pipe section	[Pa]
P_2	absolute pressure at end of pipe section	[Pa]
λ_f	Darcy-Weisbach friction coefficient	[-]
L	length of pipe section	[m]

This equation is known as the Darcy-Weisbach equation for the determination of the frictional head loss I_f in liquid flow in a pipeline.

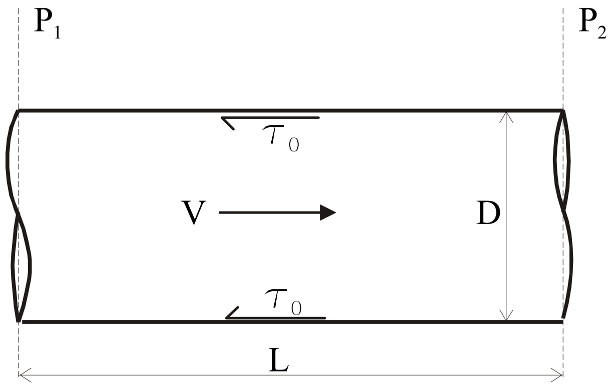


Figure 1.4. Flow in a straight horizontal pipe of constant diameter.

For *laminar flow*, an equation for friction coefficient λ_f (or f_f) is calculated theoretically from the equation for pressure drop (Eq. 1.12) giving

$$\lambda_f = \frac{64}{Re} \qquad (1.21)\cdot$$

In *turbulent flow* there is no simple expression linking the velocity distribution with the shear stress (and so with the pressure gradient) in the pipe cross section. Over the years an empirical approach has provided a number of correlations λ_f = fn(Re, k/D) for different pipe flow regimes. The regimes are: hydraulically smooth, transitional and hydraulically rough (Fig. 1.5). The λ_f = fn(Re, k/D) correlations have been derived from empirical expressions for a velocity profile in the turbulent flow in a pipeline. The λ_f = fn(Re, k/D) values can be determined also from the Moody diagram (Fig. 1.6) or its computational version (Churchill, 1977)

$$\lambda_f = 8\left[\left(\frac{8}{Re}\right)^{12} + (X+Y)^{-1.5}\right]^{\frac{1}{12}} \qquad (1.22)$$

where

$$X = \left\{-2.457\ln\left[\left(\frac{7}{Re}\right)^{0.9} + \frac{0.27k}{D}\right]\right\}^{16} \qquad (1.23)$$

and

$$Y = \left(\frac{37530}{Re}\right)^{16} \qquad (1.24)$$

λ_f	Darcy-Weisbach friction coefficient	[-]
Re	Reynold number for liquid flow	[-]

k	absolute roughness of pipe wall	[m]
D	pipe diameter	[m].

The value λ_f = 0.010 - 0.012 is usually appropriate for an initial estimation of water-flow friction losses in industrial pipelines (Fig. 1.6).

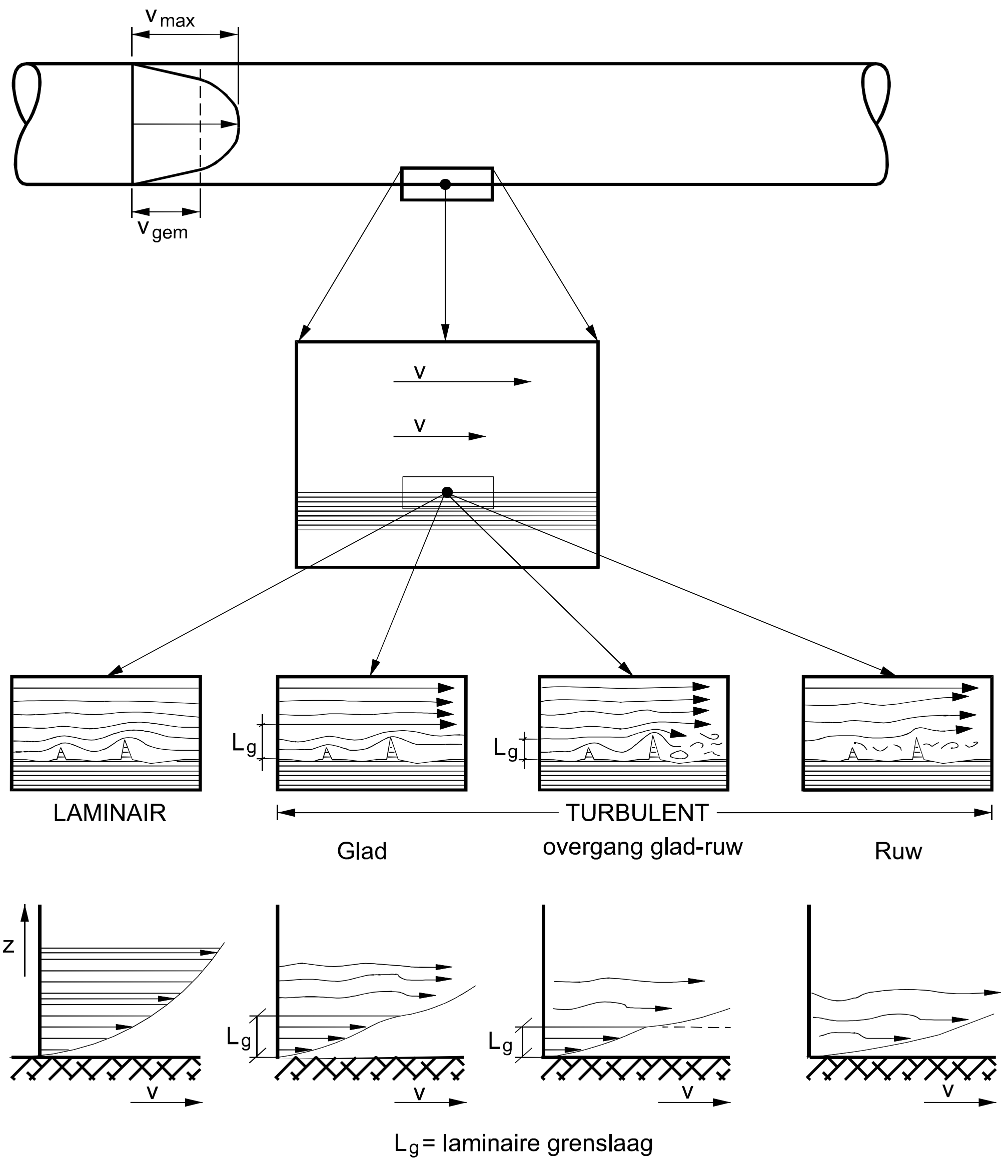


Figure 1.5. Regimes of flow over a pipeline wall. Regimes for λ_f determination.

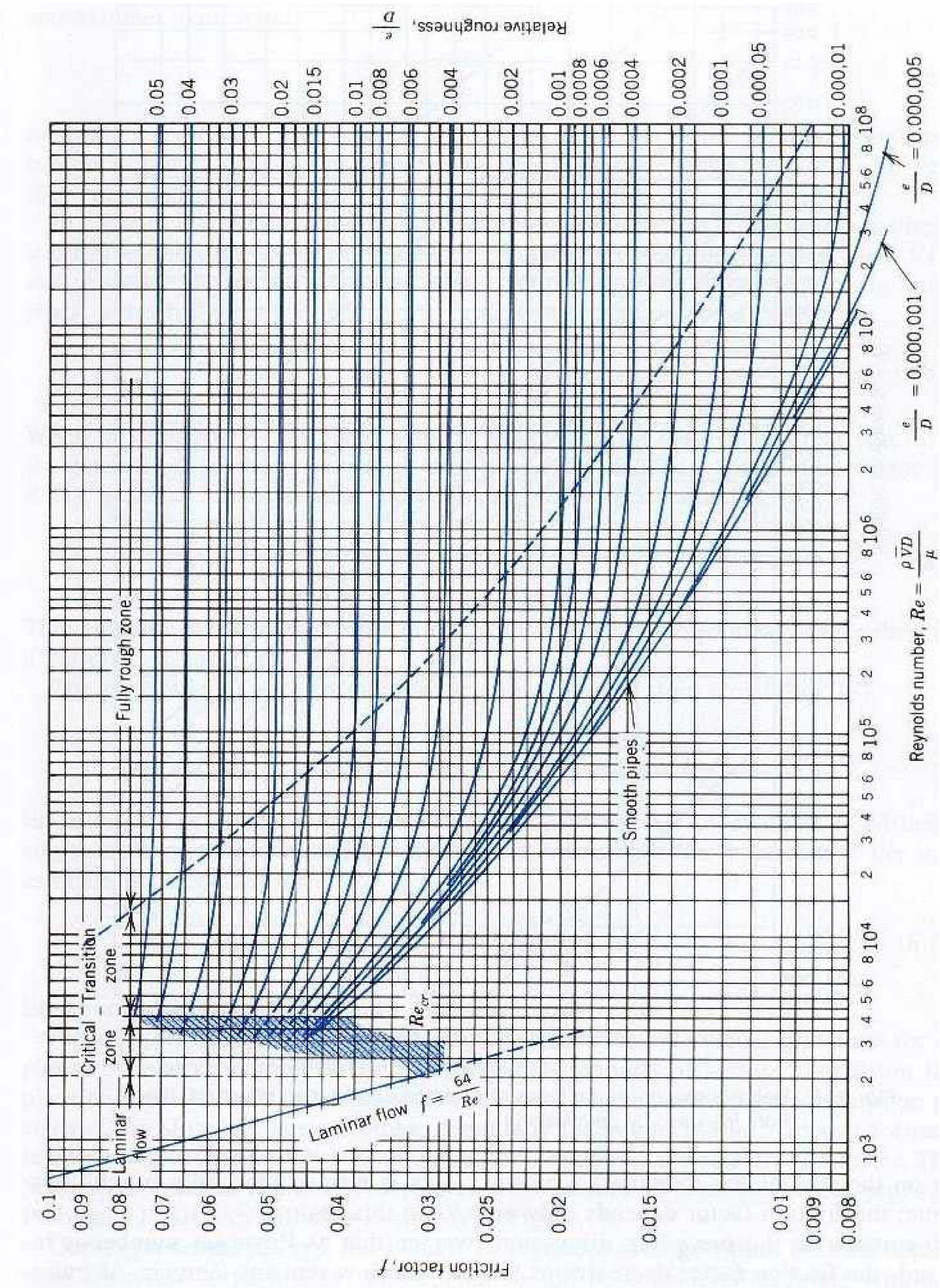
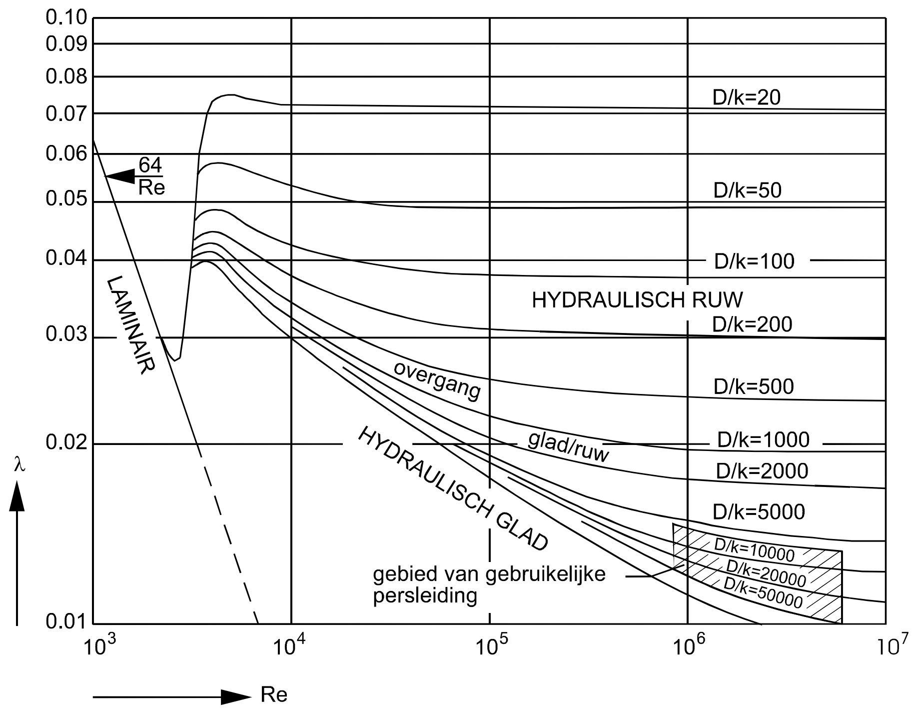


Figure 1.6a. Moody diagram for a determination of Darcy-Weisbach friction coefficient f_f (λ_f).



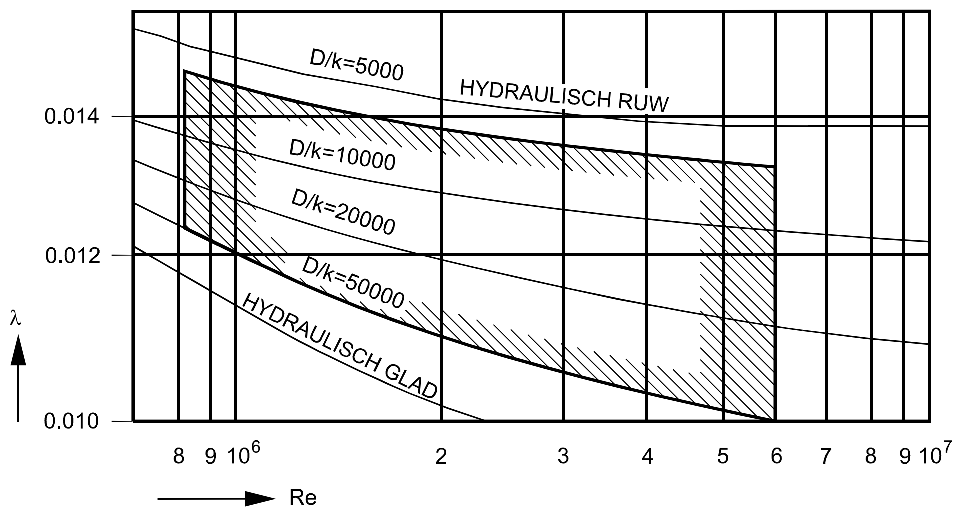


Figure 1.6b. Moody diagram for a determination of Darcy-Weisbach friction coefficient λ_f, zoom to most used region.

CASE STUDY 1

Frictional head loss in flow of water through a straight horizontal pipeline.

Determine the pressure drop due to friction for flow of water at room temperature (for calculations take water density ρ_f **= 1000 kg/m^3** and kinematic viscosity ν_f **= 10^{-6} m^2/s**) in a **1000 m** long pipeline of the diameter **D = 900 mm**. The absolute roughness of a pipeline wall **k = 10^{-5} m**. Mean velocity of water in a pipeline cross section **V = 4.5 m/s**.

Solution:

a. Friction coefficient, λ_f

The friction coefficient is dependent on the roughness *k* and Reynolds number *Re* of the water flow. For the above given values of parameters Re = (4.5 x 0.9)/10^{-6} = 4.05 x 10^6. The ratio k/D = 1.1 x 10^{-5}.

The numerical approximation of the Moody diagram by Churchill (Eqs. 1.22 – 1.24) gives for these values of *Re* and *k/D* the friction coefficient value λ_f = 0.0099 = 0.010. The same value of λ_f should be obtained directly from the Moody diagram (for D/k = 91000 in Fig. 1.6).

b. Frictional head loss, I_f

The frictional head loss for water flow in a pipeline is determined using the Darcy-Weisbach equation (Eq. 1.20). This gives a parabolic relationship between the hydraulic gradient I_f and mean mixture velocity V_m. For our inputs

$$I_f = \frac{0.010}{0.900}\frac{V_m^2}{19.62} = \frac{0.010}{0.900}\frac{4.50^2}{19.62} = 0.01147\,.$$

Thus the friction loss I_f = 0.01147 meter water column over 1 meter pipeline length, i.e. 11.47 meter water column over 1 kilometer pipeline length. This represents the pressure drop due to friction ΔP = 0.01147 x 1000 x 9.81 = 112.5 Pa/m' or 112.5 kPa over the one kilometer long pipeline.

Summary of the results:

friction coefficient :	***λ_f = 0.010 [-]***
frictional head loss per unit meter of a pipeline length :	***I_f = 0.01147 [-]***
pressure drop due to friction over 1 000-meter long straight pipeline :	***ΔP = 112.5 [kPa / 1000 m]***

1.2 SOLID PARTICLES IN A CARRYING LIQUID

Forces acting on solid particles submerged in a liquid have their origin either in a particle-liquid interaction or in a particle-particle interaction. Particles moving in a conduit may also interact with a conduit boundary. The forces acting on a single particle in a dilute suspension are the body forces. The particle-liquid body forces are the buoyancy force, drag force and lift force. When a solid particle is transported in the turbulent flow of a carrying liquid the turbulent diffusive force from carrier eddies is an additional particle-liquid force. Forces acting on solid particles due to particle-particle interaction are transmitted as the interparticle stress via the particle contacts. Coulombic stresses occur in a granular body occupied by particles in continuous contact. When a granular body is sheared and interparticle contacts are only sporadic, Bagnold stresses are transmitted through the granular body.

1.2.1 Gravitational and buoyancy force

The body force due to gravitational acceleration is determined from the solid particle volume and density. The gravitational force on a spherical solid particle of diameter d is

$$F_{Gp} = \rho_s g \frac{\pi d^3}{6} \qquad (1.25).$$

According Archimedes law, a solid particle immersed in a liquid obeys a buoyancy effect, which reduces its weight in the carrying medium. The submerged weight of the solid particle is a result of gravitational and buoyancy effects on the solid particle immersed in the liquid. For a spherical particle the submerged weight is determined by the expression

$$F_{wp} = (\rho_s - \rho_f) g \frac{\pi d^3}{6} \qquad (1.26).$$

F_{Gp}	gravitational force on a spherical particle	[N]
F_{Wp}	submerged weight of a spherical particle	[N]
ρ_s	density of solid particle	[kg/m^3]
ρ_f	density of liquid	[kg/m^3]
g	gravitational acceleration	[m/s^2]
d	diameter of a particle	[m]

1.2.2 Drag force

When the surrounding liquid moves relative to a solid particle, an additional force is exerted from the liquid onto the submerged particle. The drag force, F_D, acts in the direction of the relative velocity $v_r = v_f - v_s$ between the liquid and the solid particle. The magnitude of the drag force is expressed in terms of the drag coefficient C_D. This comes from dimensional analysis of the function

$$F_D = fn(\rho_f, \mu_f, d, v_r) \qquad (1.27).$$

It provides two dimensionless groups of parameters:
drag coefficient

$$C_D = \frac{8F_D}{\pi d^2 v_r |v_r| \rho_f} \qquad (1.28)$$

and particle Reynolds number

$$Re_p = \frac{\rho_f |v_r| d}{\mu_f} \qquad (1.29)$$

giving $C_D = \text{fn}(Re_p)$.

A balance of the gravitational, buoyancy and drag forces on the submerged solid body determines a settling velocity of the body.

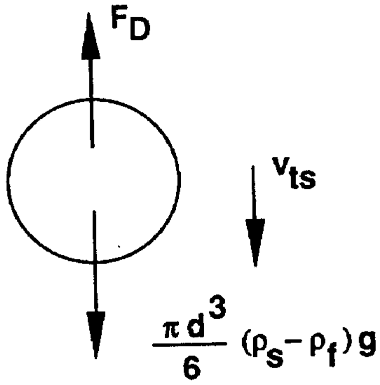


Figure 1.7. Force balance on a solid body submerged in a quiescent liquid.

An experimental determination of the drag coefficient is based on measurement of the terminal settling velocity of a spherical particle, v_{ts}, in a quiescent liquid. Measured v_{ts} is the relative velocity v_r.

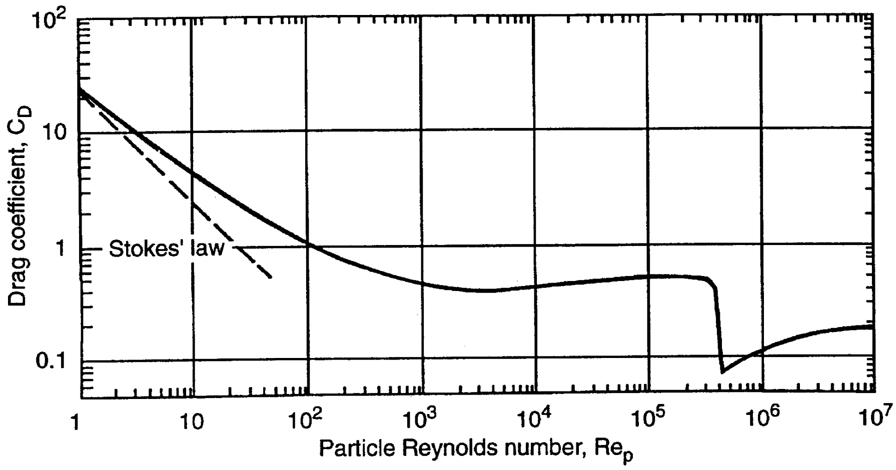
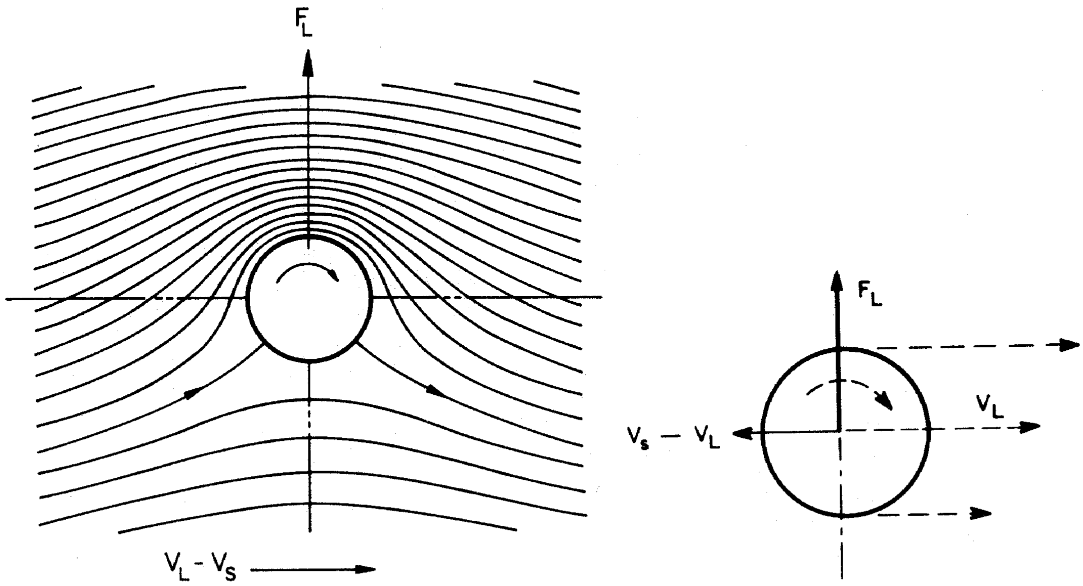


Figure 1.8. Drag coefficient as a function of particle Reynolds number.

Methods for a determination of particle settling velocity are discussed in *Intermezzo I.*

1.2.3 Lift force

The lift force, F_L, on a single solid particle is a product of simultaneous slip (given by relative velocity $v_r = v_f - v_s$) and particle rotation. The force (sometimes called the Magnus force) acts in a direction normal to both the relative velocity v_r and the particle rotation vector. A particle rotation combined with a slip results in a lower hydrodynamic pressure in flow above the particle than in that below the particle. Lift force is due to this pressure gradient.



(a) (b)

Figure 1.9. Lift force on a rotating solid body. (a) lift force on a rotating cylinder, (b) the Saffman force, i.e. lift force due to shear and slip.

The lift force is most active near a pipeline wall where the velocity gradient is high. However, the lift forces due to particle spin play a minor role in the majority of mixture flow regimes compared to the Bagnold and Coulombic forces.

1.2.4 Turbulent diffusive force

Solid particles are also subject to additional liquid-solids interactions when they are transported in a turbulent stream of the carrying liquid. An intensive exchange of momentum and random velocity fluctuations in all directions are characteristic of the turbulent flow of the carrying liquid in a pipeline. Scales of turbulence are attributed to properties of the turbulent eddies developed within the turbulent stream. According to Prandtl's picture of turbulence, the length of the turbulent eddy is given as the distance over which the lump of liquid transports its momentum without losing its identity, i.e. before the lump is mixed with liquid in a new location. This distance is called the mixing length and since it is supposed to represent a mean free path of a pulse of liquid within a structure of turbulent flow it is considered a length scale of turbulence. A turbulent eddy is responsible for the transfer of momentum and mass in a liquid flow. The instantaneous velocity of liquid at any point in the flowing liquid and in arbitrary direction (x, y or z) is given by $v = \overline{v} + v'$ where $\overline{v}$ is the time-averaged velocity and v' is the instantaneous fluctuation velocity. The turbulent fluctuating component v' of the liquid velocity v is associated with a turbulent eddy.

It is well known that turbulent eddies are responsible for solid particle suspension. The intensity of liquid turbulence is a measure of the ability of a carrying liquid to suspend the particles. The size of the turbulent eddy and the size of the solid particle are also important to the effectiveness of a suspension mechanism. The characteristic size of turbulent eddies is assumed to depend on the pipeline diameter.

A low concentration suspension is described by using a classical turbulent diffusion model of Schmidt and Rouse. The model was constructed as a flux balance per unit area perpendicular to the vertical direction in a flow balancing the volumetric settling rate (characterized by settling velocity v_t) in a quiescent liquid and the diffusion flux (characterized by the liquid velocity fluctuation in a vertical direction v'_y, associated with the length of a turbulent eddy, ML) (see Fig. 1.10). A characteristic value of the turbulent pulsative velocity $\tilde{v}'_y = \sqrt{\overline{v'^2_y}}$, i.e. the root mean square of velocity fluctuations in the y-direction.

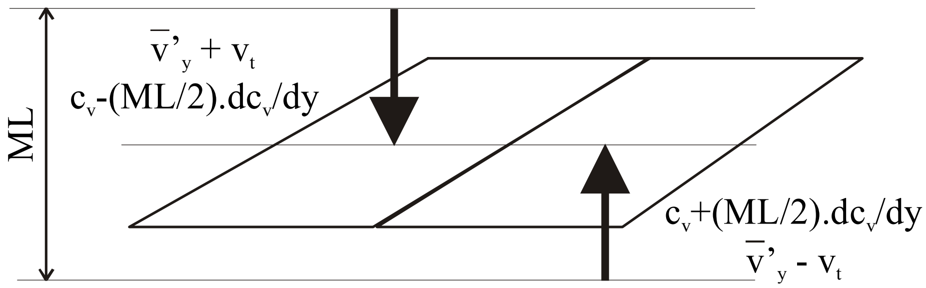


Figure 1.10. Mixing length model of particle exchange by turbulence.

The balance of

the upward flux per unit area = $\frac{1}{2}\left[c_V + \left(\frac{ML}{2}\right)\frac{dc_V}{dy}\right]\left(\tilde{v}'_y - v_t\right)$ and

the downward flux per unit area = $\frac{1}{2}\left[c_V - \left(\frac{ML}{2}\right)\frac{dc_V}{dy}\right]\left(\tilde{v}'_y + v_t\right)$

gives an equation

$$-\varepsilon_S \frac{dc_V}{dy} = v_t . c_V \qquad (1.30)$$

when solids dispersion coefficient $\varepsilon_S = \frac{ML}{2}\tilde{v}'_y$.

Integration of Eq. 1.30 with ε_S considered constant gives an <u>exponential concentration profile $c_V(y)$</u> as

$$c_v(y) = C_{vb}.\exp\left[-\frac{v_t}{\varepsilon_s}\left(y - y_b\right)\right] \quad (1.31)$$

i.e. an exponential concentration variation with height, y, in a flow above a boundary characterized by a position y_b and a concentration c_{vb}.

c_v	local concentration at the height y	[-]
c_{vb}	known local concentration at the position y_b	[-]
v_t	terminal settling velocity of a particle	[m/s]
ε_s	solids dispersion coefficient	[m^2/s]
y	vertical distance from pipe wall defining a position in a pipe cross section	[m]
y_b	vertical distance from pipe wall to boundary	[m]
ML	mixing length	[m]
$\tilde{v}'_y$	turbulent pulsative velocity in the y-direction	[m/s]

A turbulent diffusive force exerted on particles by turbulent eddies is obtained by rewriting the Eq. 1.30 as a force balance between the turbulent diffusive force and the submerged weight of the particles in a unit volume of slurry in a horizontal pipe. The submerged weight is $\rho_f g(S_s\text{-}1)c_v$ so the turbulent dispersive force

$$F_t = -\rho_f g\left(S_s - 1\right)\frac{\varepsilon_s}{v_t}\frac{dc_v}{dy} \quad (1.32).$$

How to determine the solids dispersion coefficient, ε_s, is a major problem connected with the application of the turbulent diffusive model. The effect of distance from a boundary and of the presence of solid particles in a turbulent stream on a local value of ε_s cannot be neglected. Further, the neighboring particles also affect the particle settling velocity handled in the model.

1.2.5 Coulombic contact force

Sand/gravel particles are transported in dredging pipelines often in a form of a granular bed sliding along a pipeline wall at the bottom of a pipeline. A mutual contact between particles within a bed gives arise to intergranular forces transmitted throughout a bed and via a bed contact with a pipeline wall also to the wall

Stress distribution in a granular body occupied by non-cohesive particles in continuous contact is a product of the weight of grains occupying the body. The intergranular pressure (or stress) from the weight of grains is transmitted within the granular body via interparticle contacts. The stress has two components: an intergranular normal stress and an intergranular shear stress. According to Coulomb's law these two stresses are related by the coefficient of friction. Du Boys (1879) applied Coulomb's law to sheared riverbeds. He related the intergranular normal stress, σ_s, and intergranular shear stress, τ_s, at the bottom of a flowing bed by a coefficient

$$\tan\phi = \frac{\tau_s}{\sigma_s} = \frac{\tau_s}{\rho_f g (S_s - 1) C_{vb} H_s} \qquad (1.33)$$

ϕ	angle of repose of the grains	[-]
σ_s	intergranular normal stress	[Pa]
τ_s	intergranular shear stress	[Pa]
S_s	specific gravity of solids, $S_s = \rho_s/\rho_f$	[-]
Y_{sh}	thickness of the sheared bed	[m]
C_{vb}	maximum solids volume fraction of solids in the granular bed, it is considered to be valid for the sheared bed	[-]
ρ_f	density of liquid	[kg/m^3]
g	gravitational acceleration	[m/s^2]

The angle of repose, ϕ, is considered to be the angle at the internal failure of a static granular body (Fig. 1.11). The value of this internal-friction coefficient is basically dependent on the nature of the surface over which the grains start to move, i.e. primarily on a grain size. When the granular bed motion takes place over a pipe wall, the value of the bed-wall friction coefficient can be determined by a tilting tube test.

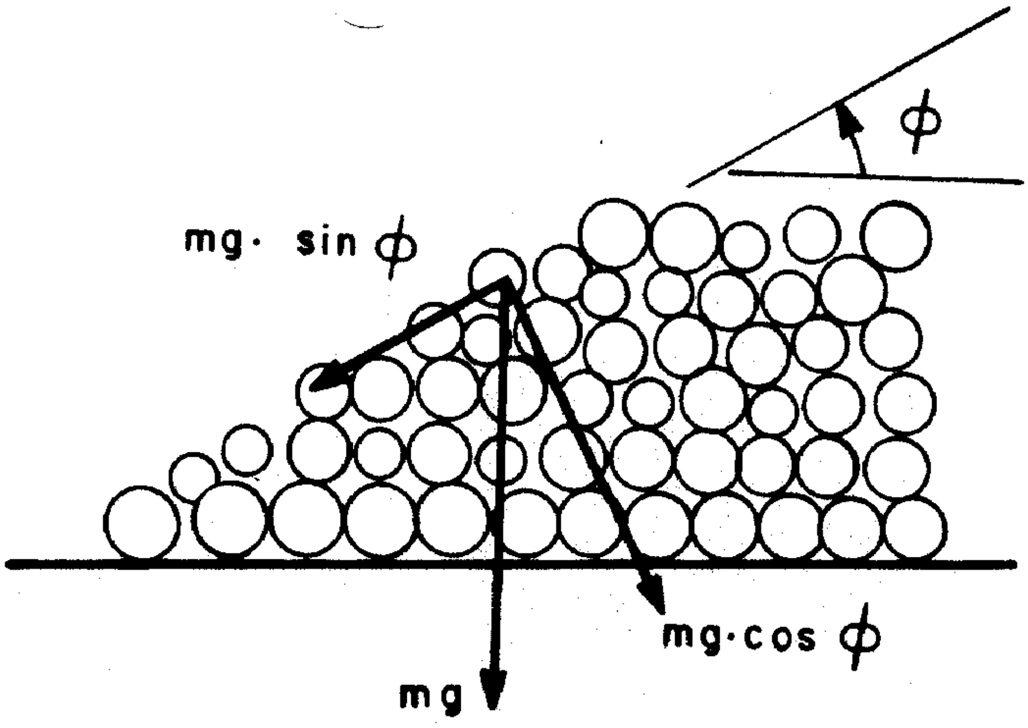


Figure 1.11. The angle of repose of a granular material.

1.2.6 Bagnold dispersive force

Sheared-bed particles flowing in the region of high shear rate maintain sporadic, rather than continuous contact with each other, provided that solids concentration in the sheared bed is considerably lower than the loose-poured bed concentration C_{vb}.

The nature of an interparticle contact influences the relationship between the intergranular stress components. It is appropriate to relate the particulate shear and normal stresses in a granular body experiencing the rapid shearing by using a coefficient of dynamic friction $\tan\phi'$ instead of its static equivalent $\tan\phi$. Bagnold (1954. 1956) measured and described the normal and tangential stresses in mixture flows at high shear rates.

Bagnold's dispersive force is a product of intergranular collisions (particle - particle interactions) in a sheared layer rich in particles. The direction of the force is normal to the layer boundary on which it is acting. The force increases with increasing solids concentration and shear rate in the sheared layer.

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