Intermezzo I.
SETTLING VELOCITY OF SOLID PARTICLE IN A LIQUID

I.1 TERMINAL SETTLING VELOCITY OF A SPHERICAL PARTICLE, v_{ts}

A balance of the gravitational, buoyancy and drag forces on the submerged solid body determines a settling velocity of the body.

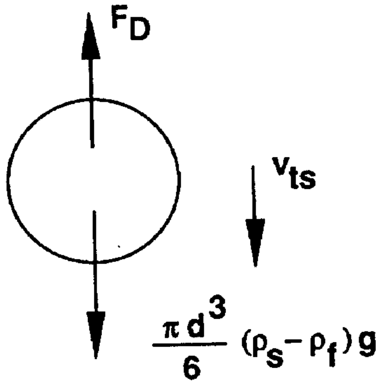

Figure 1.7. Force balance on a solid body submerged in a quiescent liquid.

I.1.1 General equation for settling velocity

The terminal settling velocity of a spherical particle, v_{ts}, in a quiescent liquid is a product of a balance between the submerged weight of a solid particle in the liquid the drag force of the liquid acting onto the settling particle in the direction opposite to the settling velocity vector:

GRAVITY FORCE - BUOYANCY FORCE = DRAG FORCE, i.e.
$m_s g$ - $m_f g$

SUBMERGED WEIGH = DRAG FORCE
Volume.ρ_sg - Volume.ρ_fg

$$\frac{\pi d^3}{6}\left(\rho_s - \rho_f\right)g = \frac{C_D}{8}\pi d^2 v_{ts}^2 \rho_f \qquad \text{(I.1)}$$

and the terminal settling velocity is

$$v_{ts} = \sqrt{\frac{4}{3}\frac{(\rho_s - \rho_f)}{\rho_f}\frac{gd}{C_D}} \qquad \text{(I.2)}$$

v_{ts}	terminal settling velocity of a spherical solid particle	[m/s]
ρ_s	density of solid particle	[kg/m^3]
ρ_f	density of liquid	[kg/m^3]
g	gravitational acceleration	[m/s^2]
d	diameter of a particle	[m]
C_D	drag coefficient of flow round settling particle	[-]

The drag coefficient C_D is sensitive to a regime of the liquid flow round the settling solid particle and this can be expressed by a relationship C_D = fn(Re_p). Re_p is the particle Reynolds number $Re_p = \frac{v_{ts}d}{\nu_f}$.

I.1.2 Application of the general equation to different regimes of particle settling

In a *laminar regime* (obeying Stokes' law and occurring for $Re_p < 0.1$, i.e. sand-density particles of d < 0.05 mm approximately) the relation is hyperbolic, $C_D = 24/Re_p$, so that

$$v_{ts} = \frac{(\rho_s - \rho_f)}{18\rho_f}\frac{gd^2}{\nu_f} \qquad \text{(I.3).}$$

In a *turbulent regime* (obeying Newton's law and occurring for Re_p >500, i.e. sand-density particles of d > 2 mm approximately) the drag coefficient is no longer dependent on Re_p because the process of fast settling of coarse particles is governed by inertial rather than viscous forces. $C_D = 0.445$ for spherical particles and the terminal velocity is given by

$$v_{ts} = 1.73\sqrt{\frac{(\rho_s - \rho_f)}{\rho_f}gd} \qquad \text{(I.4).}$$

These two regimes are connected via a *transition regime*. In this regime a C_D value might be determined from the empirically obtained curve C_D = fn(Re_p) (see Figure 1.8) or its mathematical approximation (Turton & Levenspiel, 1986)

$$C_D = \frac{24}{Re_p}\left(1 + 0.173\,Re_p^{0.657}\right) + \frac{0.413}{1 + 1.63\text{x}10^4\,Re_p^{-1.09}} \qquad \text{(I.5).}$$

The determination of v_{ts} for the transition regime requires an iteration [v_{ts} = fn(C_D) is an implicit equation]. Grace (1986) proposed a method for a determination of v_{ts} without necessity to iterate.

I.1.3 The Grace method for the solution of the settling-velocity equation

The Grace method uses two dimensionless parameters

- the dimensionless particle size: $d^* = d.\sqrt[3]{\frac{\rho_f(\rho_s-\rho_f)g}{\mu_f^2}}$

- the dimensionless settling velocity: $v_{ts}^* = v_{ts}\sqrt[3]{\frac{\rho_f^2}{\mu_f(\rho_s-\rho_f)g}}$

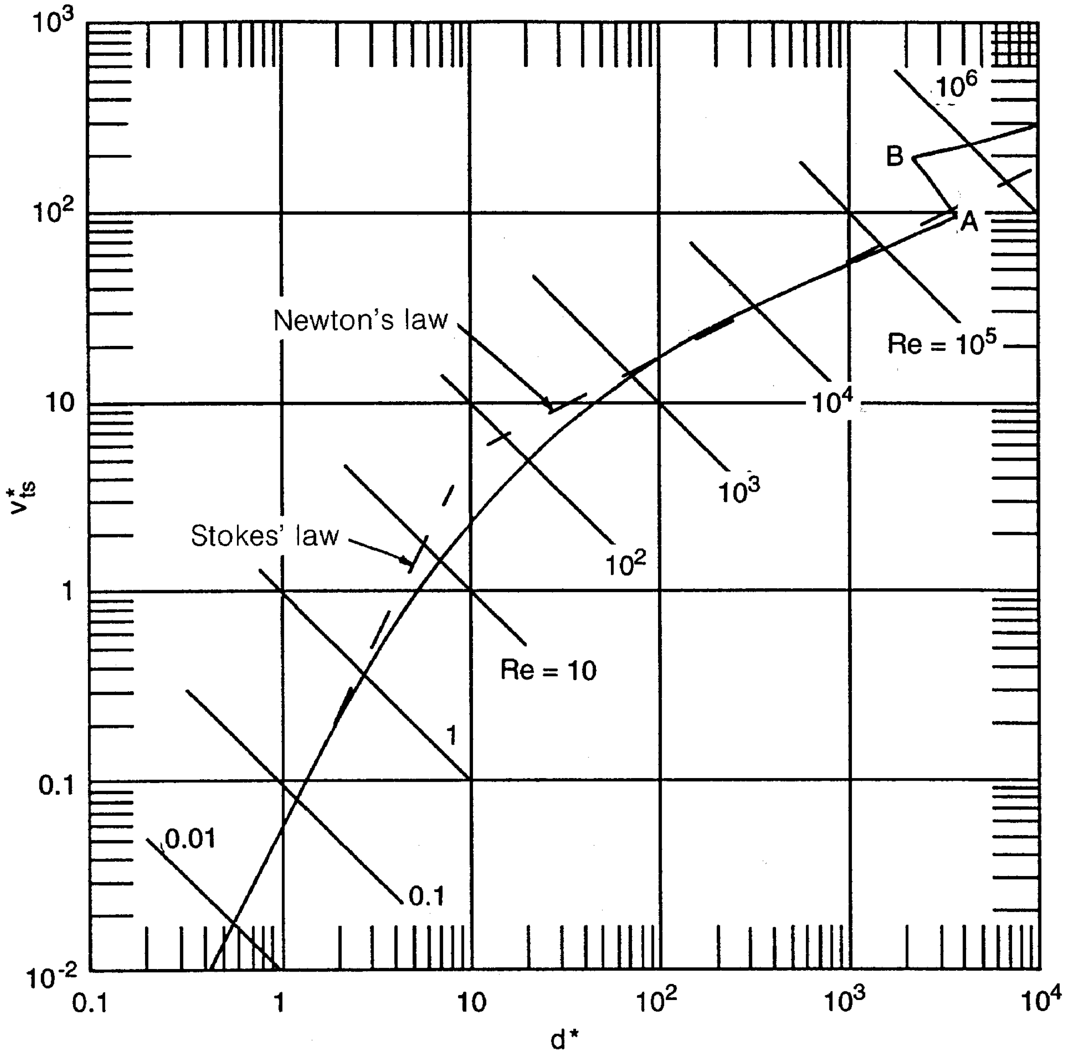

Figure I.1. Dimensionless terminal velocity, v^*_{ts}, as a function of dimensionless particle diameter, d^*, for rigid spheres, after Grace (1986).

Those are mutually related as shown in Figure I.1. Thus using the curve and rearranging gives directly the velocity v_{ts} as a function of particle diameter d. *No iteration is required.*

The curve in Figure I.1. can be also described by analytic expressions (Table I.1.) appropriate for a computational determination of v_{ts} according to Grace method.

Table I.1. Correlations for v_{ts}^* as a function of d^*, after Grace (1986).

Range	Correlation
$d^* \leq 3.8$	$v_{ts}^* = (d^*)^2/18 - 3.1234x10^{-4}(d^*)^5 + 1.6415x10^{-6}(d^*)^8$
$3.8 < d^* \leq 7.58$	$\log v_{ts}^* = -1.5446 + 2.9162(\log d^*) - 1.0432(\log d^*)^2$
$7.58 < d^* \leq 227$	$\log v_{ts}^* = -1.64758 + 2.94786(\log d^*) - 1.0907(\log d^*)^2 + 0.17129(\log d^*)^3$
$227 < d^* \leq 3500$	$\log v_{ts}^* = 5.1837 - 4.51034(\log d^*) + 1.687(\log d^*)^2 - 0.189135(\log d^*)^3$

I.2 TERMINAL SETTLING VELOCITY OF A NON-SPHERICAL PARTICLE, v_t

The above equations are suitable for the spherical particles. The particles of dredged solids are not spherical. The non-spherical shape of a particle reduces its settling velocity.

I.2.1 Shape factor of a solid particle

The reduction of settling velocity due to a non-spherical shape of a solid particle is quantified by the velocity ratio

$$\xi = \frac{v_t}{v_{ts}} \qquad \text{(I.6)}$$

ξ	shape factor of a solid particle	[kg/m^3]
v_t	terminal settling velocity of a non-spherical solid particle	[m/s]
v_{ts}	terminal settling velocity of a spherical solid particle	[m/s].

I.2.1.1 The Grace method for a determination of the shape factor

ξ is a function of the volumetric form factor K (K=0.26 for sand, gravel) and the dimensionless particle diameter $d^* = \sqrt[3]{\frac{\rho_f(\rho_s - \rho_f)g}{\mu_f^2}}.d$ (see Figure I.2.). The terminal velocity for sand particles is typically 50-60 % of the value for the sphere of the equivalent diameter (see Table I.3 in paragraph I.4).

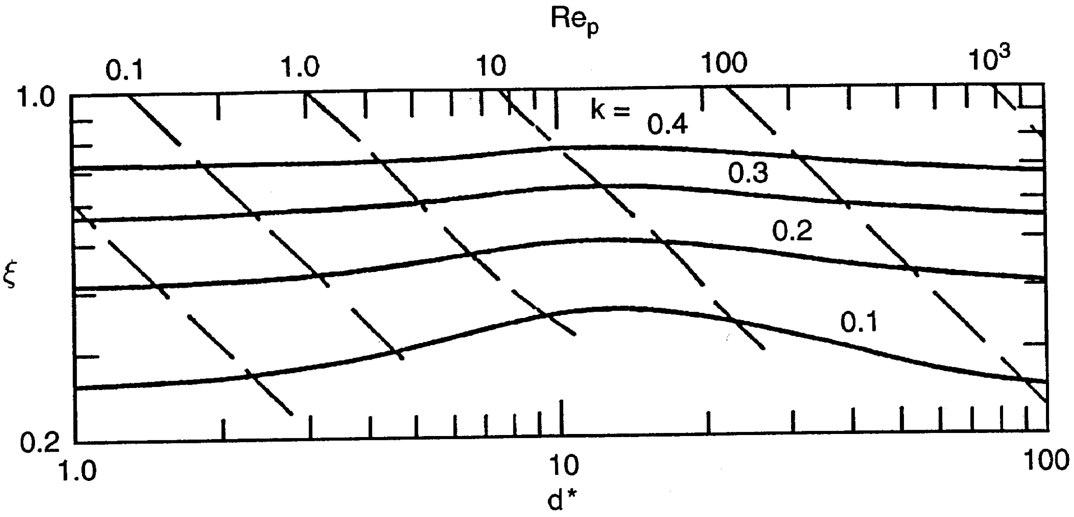

Figure I.2. The shape factor, ξ, as a function of the dimensionless particle diameter d^* and the volumetric form factor K.

The curves in Fig. I.2 can be analytically approximated by the following correlation

$$\log_{10}\xi = -0.55 + K - 0.0015K^{-2} + 0.03(1000)^{K-0.524} + \frac{-0.045 + 0.05K^{-0.6} - 0.0287(55000)^{K-0.524}}{\cosh\left(2.55\left[\log_{10}d^* - 1.114\right]\right)}$$

This equation takes a simpler form for sand-shape particles (K=0.26)

$$\log_{10}\xi = -0.3073 + \frac{0.0656}{\cosh\left(2.55\left[\log_{10}d^* - 1.114\right]\right)} \quad \text{(I.7).}$$

I.2.1.2 The estimation of the shape factor value for sand and gravel

A different method suggests the ξ value of about 0.7 for sand and gravel so that v_t is simply

$v_t = 0.7 v_{ts}$.

I.2.2 Empirical equations for settling velocity of sand and gravel particles

The above methods for a determination of the terminal velocity of spherical and non-spherical particles are supposed to be valid for all sorts of solids settling in an arbitrary fluid. In a dredging practice the terminal settling velocity is often determined using simple equations found specifically for sand particles. These equations need no iteration.

Warning: in the following equations the particle diameter, d, is in mm, and the settling velocity, v_t, in mm/s.

For the *laminar region*, considered for sand particles smaller than 0.1 mm, the **Stokes equation** is written as

$$v_t = 424 \frac{(S_s - S_f)}{S_f} d^2 \qquad \text{(I.8).}$$

For the *transition zone*, 0.1 mm < d < 1 mm, the **Budryck equation** is used

$$v_t = \frac{8.925}{d} \left[\sqrt{1 + 95 \frac{(S_s - S_f)}{S_f} d^3} - 1 \right] \qquad \text{(I.9).}$$

The *turbulent regime* of settling is estimated to occur for sand particles larger than 1 mm. This is described by the **Rittinger equation**

$$v_t = 87 \sqrt{\frac{(S_s - S_f)}{S_f} d} \qquad \text{(I.10).}$$

S_s	specific density of solids, ρ_s/ρ_w	[-]
S_f	specific density of liquid, ρ_f/ρ_w	[-]

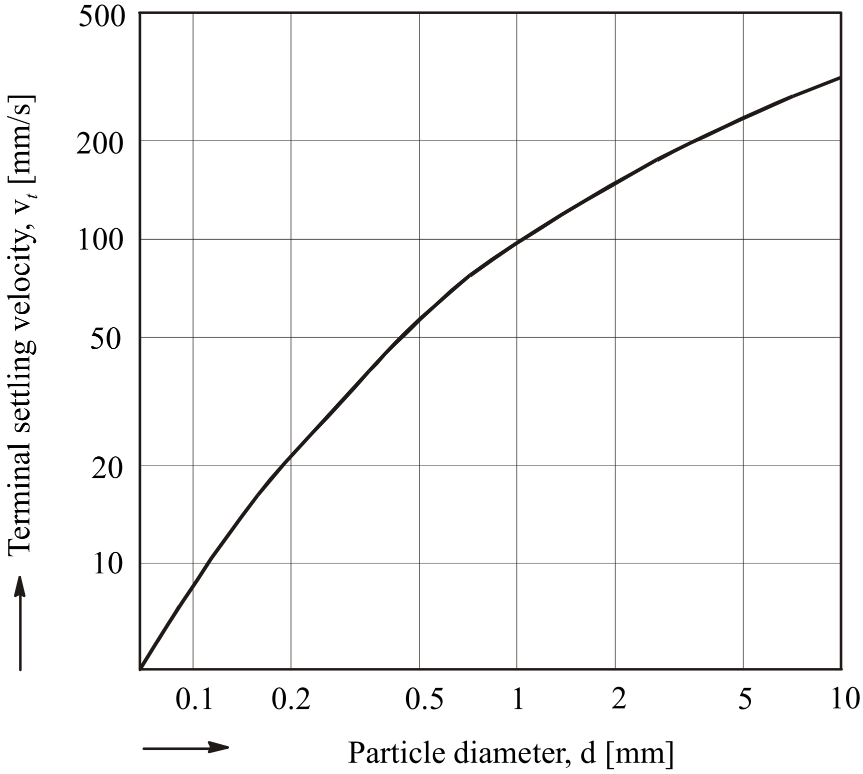

Figure I.3a. Terminal settling velocity of sand & gravel particles using Stokes, Budryck and Rittinger equations.

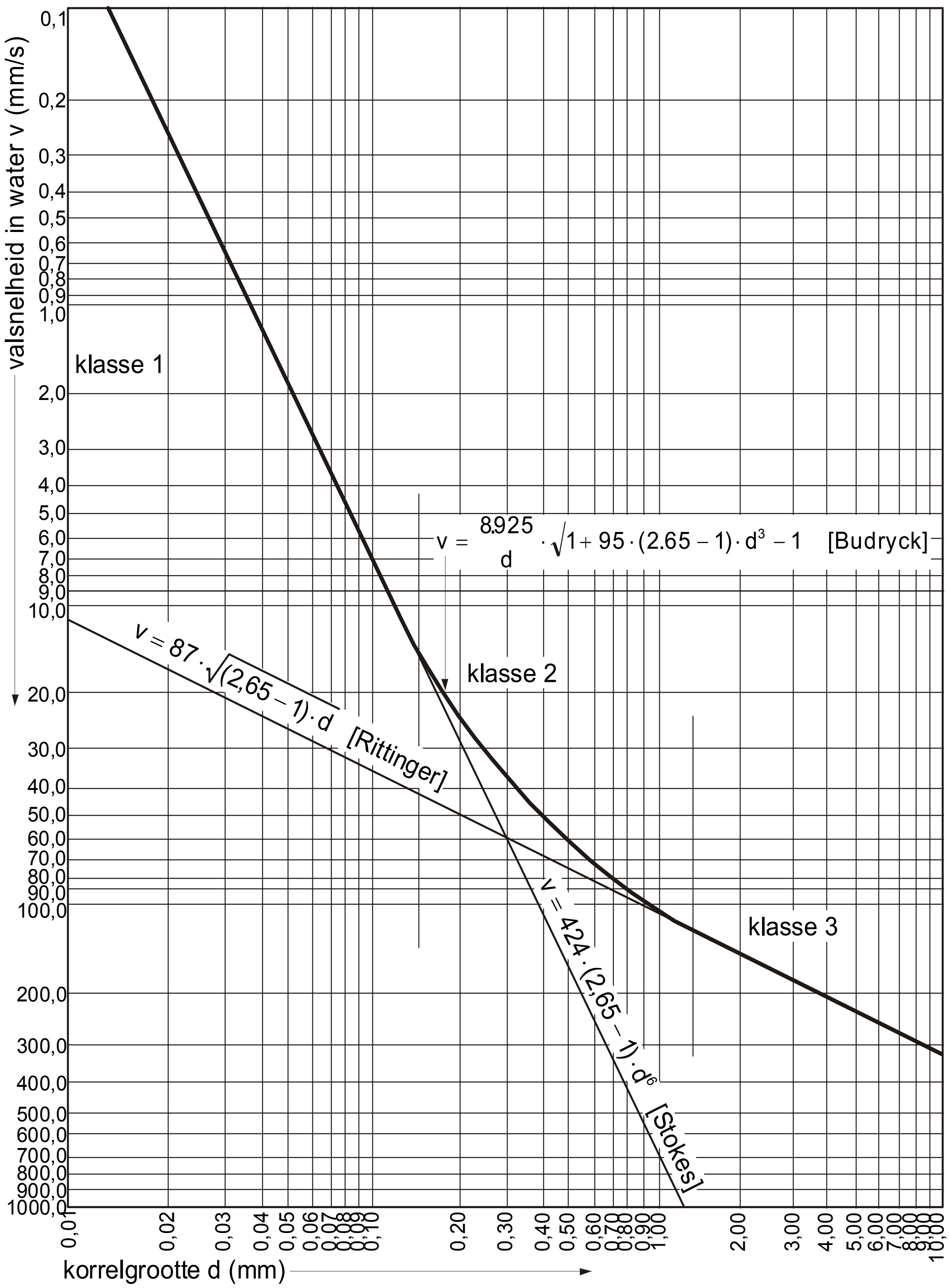

Figure I.3b. Terminal settling velocity of sand & gravel particles using Stokes, Budryck and Rittinger equations.

I.3 HINDERED SETTLING VELOCITY OF A PARTICLE, v_{th}

When a cloud of solid particles is settling in a quiescent liquid additional hindering effects influence its settling velocity. These are the increased drag caused by the proximity of particles within the cloud and the upflow of liquid as it is displaced by the descending particles. The hindering effects are strongly dependent on the volumetric concentration of solids in the cloud, C_v, (**Richardson & Zaki**, 1954)

$$v_{th} = v_t (1 - C_v)^m \qquad \text{(I.11)},$$

in which v_{th} is the hindered settling velocity of solid particle, v_t is the terminal settling velocity of the solid particle and m is the empirical exponent related to the particle Reynolds number Re_p (see Table I.2).

Table I.2. Values for the Richarson-Zaki index m

$Re_p = \frac{v_t d}{\nu_f}$	m
$Re_p \leq 0.2$	4.6
$0.2 < Re_p < 1$	$4.4 Re_p^{-0.03}$
$1 < Re_p < 500$	$4.4 Re_p^{-0.1}$
$500 \leq Re_p$	2.4

Wallis (1969) suggested determining the index m by the approximation

$$m = \frac{4.7\left(1 + 0.15\,Re_p^{0.687}\right)}{1 + 0.253\,Re_p^{0.687}} \qquad \text{(I.12)}$$

covering all ranges of Re_p values.

Theoretically, the validity of the Richardson & Zaki equation is limited by the maximum solids concentration that permits solids particle settling in a particulate cloud. This maximum concentration corresponds with the concentration in an incipient fluidized bed (C_v of about 0.57). Practically, the equation was experimentally verified for concentrations not far above 0.30.

I.4. TYPICAL VALUES OF PARAMETERS DESCRIBING A SETTLING PROCESS FOR SAND PARTICLE

Table I.3. Values of parameters associated with a particle settling for quartz sand particles (ρ_S = 2650 kg/m^3, shape factor K = 0.26) of various sizes in water at room temperature.

d [mm]	v_{ts} [mm/s] Grace (1986)	Re_p	ξ	m Table I.2.
0.074	4.75	0.35	0.51	4.54
0.149	15.6	2.32	0.53	4.04
0.297	41.0	12.1	0.57	3.43
0.595	96.7	57.4	0.59	2.93
1.000	160	160	0.55	2.65
2.000	276	551	0.52	2.40

I.5. REFERENCES

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I.6. RECOMMENDED LITERATURE

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CASE STUDY I

Terminal and hindered settling velocities of grains of sand and gravel.

Estimate the terminal settling velocity, v_t, and hindered settling velocity, v_{th}, for grains of *fine sand* (grain size **d = 0.120 mm**) and *gravel* (**d = 6.0 mm**) in the quiescent column of water at room temperature (for calculations take water density $\boldsymbol{\rho_f}$ **= 1000 kg/m^3** and kinematic viscosity $\boldsymbol{\nu_f}$ **= 10^{-6} m^2/s**). For both sand and gravel grains the following values are typical:

solids density $\boldsymbol{\rho_s}$ **= 2650 kg/m^3**

volumetric shape factor **K = 0.26**.

For hindered settling consider a cloud of particles settling in a water column. Settling particles occupy 27% per cent of the total volume of the column so that the volumetric concentration of solids $\mathbf{C_v}$ **= 0.27**.

Solution:

a. Terminal settling velocity according to Grace

The Grace fitting correlations (Table I.1.) give for the terminal settling velocity (v_{ts}) of a sphere of d = 0.120 mm the value v_{ts} = 11.21 mm/s while for a sphere of d = 6 mm the value v_{ts} = 554.13 mm/s. A very similar result is reached if the Grace graph (Figure I.1.) is used:

$d^* = 0.000120\sqrt[3]{\dfrac{1000x1650x9.81}{0.000001}} = 3.04$ gives $v^*_{ts} =$ 0.44 for the 0.120 mm sand

and

$d^* = 0.006\sqrt[3]{\dfrac{1000x1650x9.81}{0.000001}} =$ 152 gives $v^*_{ts} =$ 22 for the 6 mm gravel.

Then $v_{ts} = v^*_{ts}\sqrt[3]{\dfrac{0.001x1650x9.81}{1000^2}}$ that gives

v_{ts} = 11 mm/s for <u>fine sand</u> and *v_{ts} = 557 mm/s* for <u>medium gravel</u>.

The velocity ratio ξ is determined using the fitting correlation (I.7). This gives ξ = 0.522 for the fine sand and ξ = 0.503 for the studied gravel. Again, very similar results should be obtained using the ξ graph (Figure I.2.).

Using $v_t = \xi v_{ts}$ gives

for <u>fine sand</u> of d = 0.120 mm the velocity *v_t = 5.85 mm/s* and

for <u>medium gravel</u> of d = 6 mm the velocity *v_t = 278.52 mm/s*.

b. Terminal settling velocity according to Budryck, Rittinger respectively

According to this method (see Eqs. I.8-I.10) the terminal settling velocity of a solid grain v_t is calculated directly, thus without intermediation of the velocity v_{ts}. The equations

$$v_t = \frac{8.925}{0.120}\left[\sqrt{1+95\frac{2650-1000}{1000}0.120^3}-1\right] = \boldsymbol{9.47 \;\; mm/s}$$ for the <u>fine sand</u> of diameter d = 0.120 mm (Budryck, Eq. I.9) and

$$v_t = 87\sqrt{\frac{2650-1000}{1000}6} = \boldsymbol{273.74 \; mm/s}$$ for the <u>medium gravel</u> of diameter d = 6 mm (Rittinger, Eq. I.10).

For gravel the result obtained from the Rittinger equation is very similar to that given by Grace. For fine sand, however, the Grace method predicts considerably lower terminal settling velocity than the Budryck equation.

c. Hindered settling velocity according to Richardson & Zaki

If volumetric concentration of solids is known, the hindered settling velocity can be determined using the Richardson & Zaki equation (Eq. I.11). The index m in this equation is related to the particle Reynolds number Re_p. A use of the v_t values according to Budryck and Rittinger (see above) gives

$Re_p = 120 \times 10^{-6} \times 9.47 \times 10^{-3} / 10^{-6} = 1.14$ for our fine sand and
$Re_p = 6 \times 10^{-3} \times 0.274 / 10^{-6} = 1642$ for our gravel.

Wallis correlation (Eq. I.12) provides m = 4.286 (fine sand) and m = 2.832 (gravel).

The Richardson-Zaki equation gives
for <u>fine sand:</u> $v_{th} = 9.47(1-0.27)^{4.286}$ **= 2.46 mm/s** and
for <u>medium gravel:</u> $v_{th} = 273.74(1-0.27)^{2.832}$ ***= 112.27 mm/s***.

Summary of the results:

<u>Fine sand (d = 0.12 mm):</u>	
terminal settling velocity:	v_t ***= 9.47 mm/s***
hindered settling velocity (C_V = 0.27):	v_{th} ***= 2.46 mm/s***
<u>Medium gravel (d = 6.00 mm):</u>	
terminal settling velocity:	v_t ***= 273.74 mm/s***
hindered settling velocity (C_V = 0.27):	v_{th} ***= 112.27 mm/s***