Hydrology of catchments, rivers and deltas (CIE5450)

Prof.dr.ir. Savenije

Lecture ‘Salinity and tides in alluvial estuaries’

Salinity and Tides in Alluvial Estuaries

by Hubert H.G. Savenije
Delft University of Technology

Salinity and Tides in Alluvial Estuaries

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"Salinity and Tides in Alluvial Estuaries" presents an integrated theory for one-dimensional flow and transport in estuaries with an active morphology. The book provides a new approach with numerous case illustrations and a comprehensive overview of the literature to date.

The author has many years of field experience in estuaries in Africa, Asia and Europe. He worked from 1978-1985 in Mozambique where he carried out a large number of surveys in four different estuaries and laid the foundation for the general theory presented in this book. Subsequently he worked as a consultant, mostly in Asia, where he verified the generality of the theory in estuaries in Thailand, Indonesia and Vietnam. He joined the UNESCO Institute for Water Education (IHE-Delft) in 1990, where he completed his PhD on the subject of this book. He became Professor of Water Resources Management at UNESCO-IHE in 1994 with a focus on river basin modelling and global water resources issues. He has been Professor of Hydrology at Delft University of Technology since 2000.

This book is a valuable theoretical resource for graduate students specialising in estuary processes and for researchers in related disciplines. It is also a useful guide for practitioners and consultants with its wide range of analytical equations, describing hydraulic, mixing and salt intrusion processes in estuaries.

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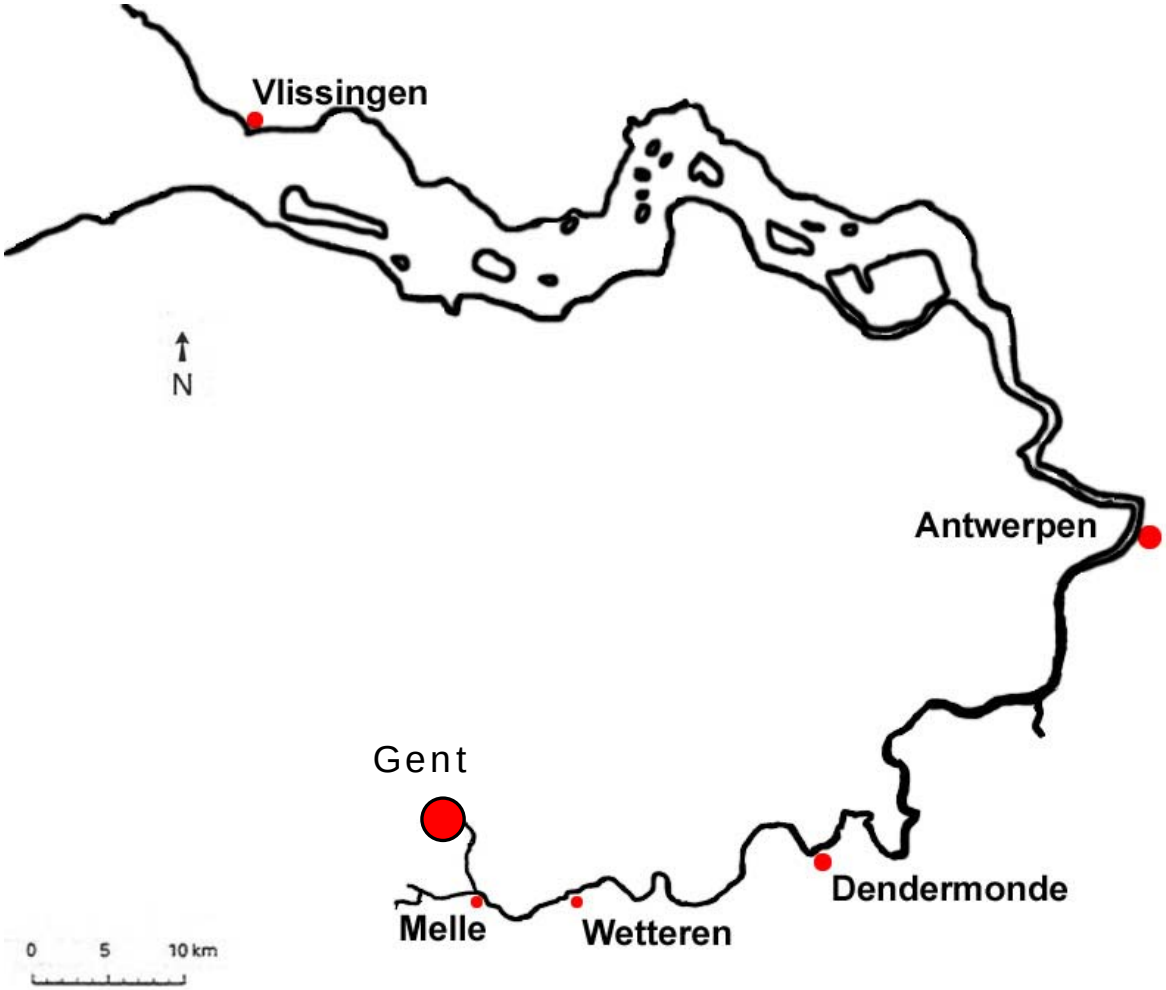
Hubert H. G. Savenije

Contents

- Estuary shape
- 5 new equations
- The role of the phase lag
- The role of tidal damping/amplification
- New versus “Classical” equations

Estuary Shape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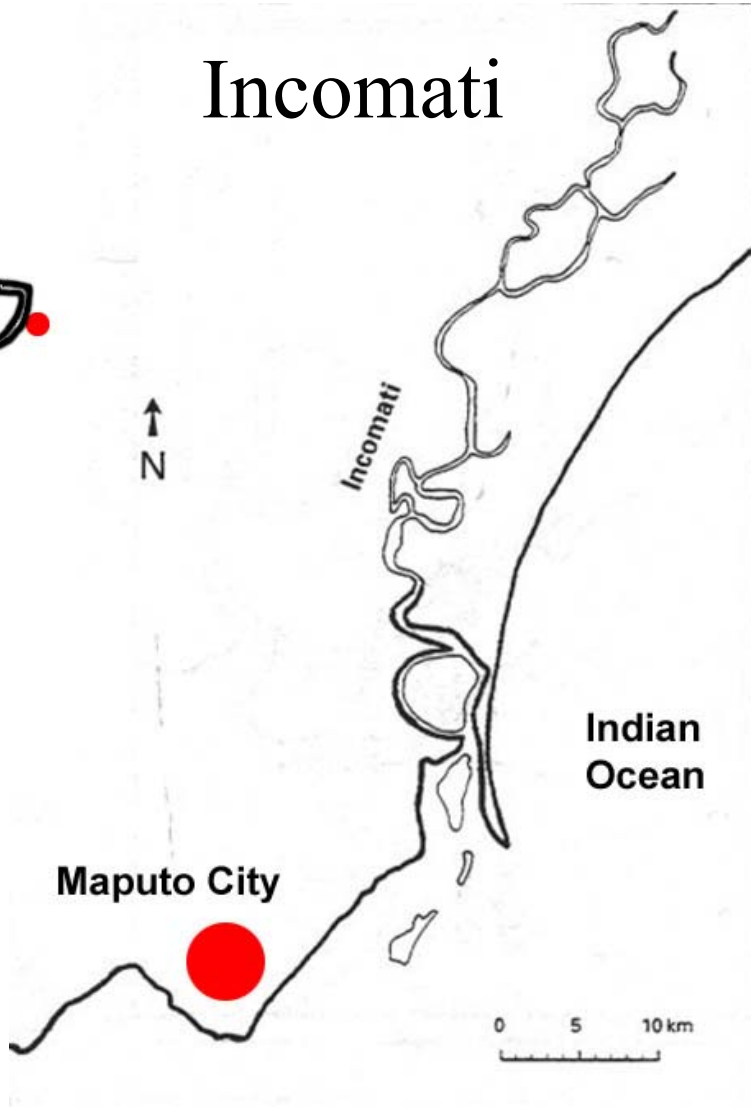
Schelde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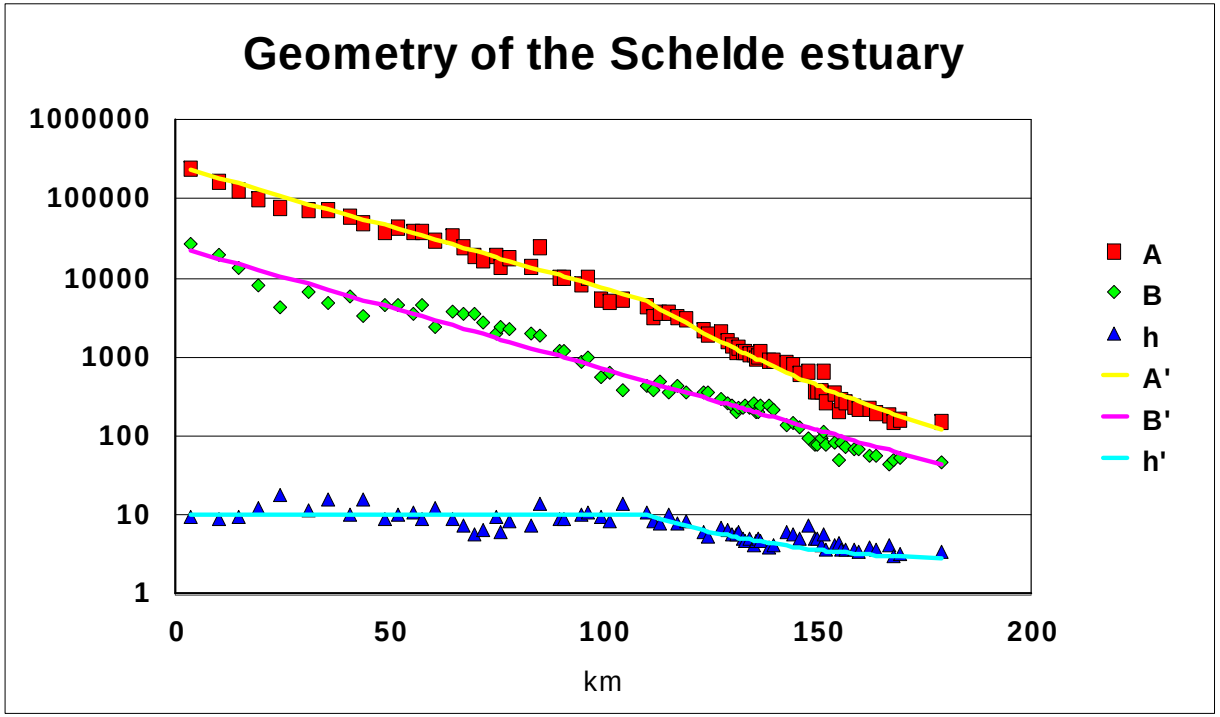
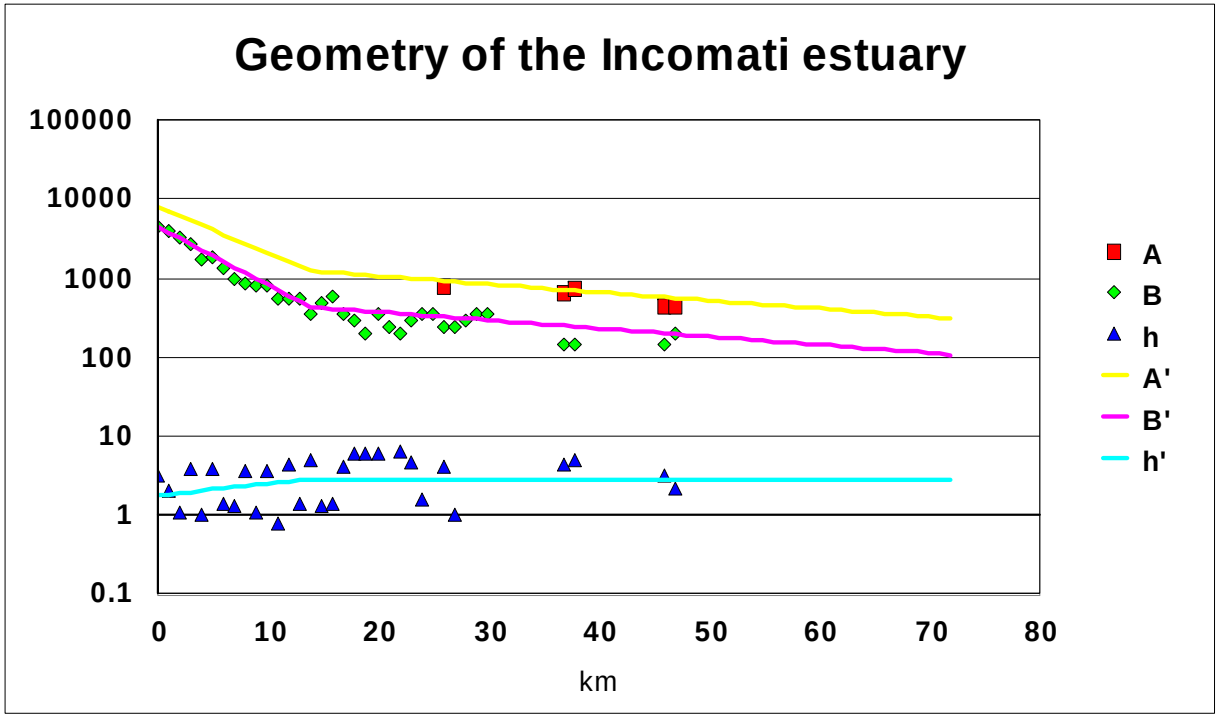
Maputo City
0 5 10 km



$$B = B_0 \exp\left(-\frac{x}{b}\right)$$



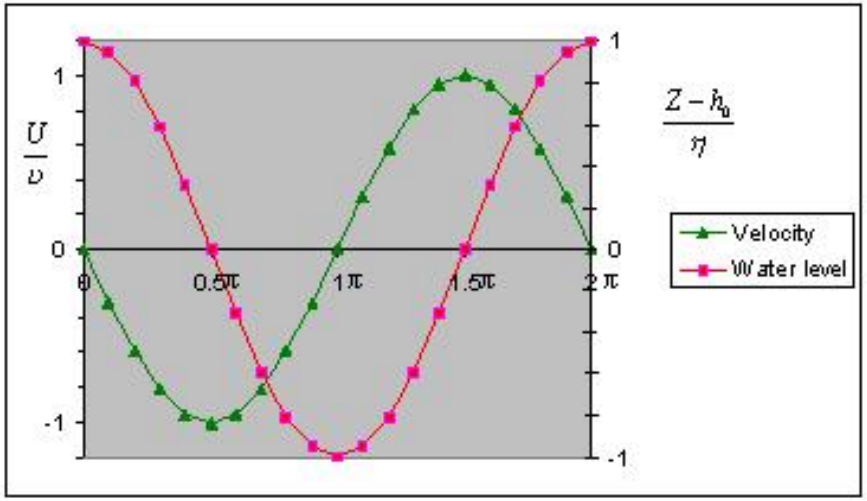
$$A = h_0 B_0 \exp\left(-\frac{x}{b}\right)$$

Equation Name	Newly derived equation	"Classical" equation
Phase Lag equation	$\tan\varepsilon = \frac{\omega b}{c(1-\delta b)} = \frac{b}{\lambda}\frac{2\pi}{(1-\delta b)}$	
Geometry-Tide relation	$\frac{H}{E} = \frac{\eta\omega}{\upsilon} = \frac{\bar{h}}{r_S b}\frac{(1-\delta b)}{\cos(\varepsilon)}$	$\frac{\eta\omega}{\upsilon} = \frac{\bar{h}}{r_S b}$
Scaling equation	$r_S\frac{\eta}{\bar{h}} = \frac{\upsilon}{c}\frac{1}{\sin(\varepsilon)}$	$r_S\frac{\eta}{\bar{h}} = \frac{\upsilon}{c}$
Damping equation Green's eq. (1937)	$\frac{\mathrm{d}H}{\mathrm{d}x}\left(1+\frac{g\eta}{c\upsilon\sin\varepsilon}\right) = H\left(\frac{1}{b}-f'\frac{\upsilon\sin\varepsilon}{\bar{h}c}\right)$	$\frac{\mathrm{d}H}{\mathrm{d}x} = H\frac{1}{2b}$
Celerity equation	$c^2 = \frac{1}{r_S}gh\Big/\left[1-\frac{\sin 2\varepsilon}{2(1+\alpha)}\left(\frac{c}{\omega b}-\frac{R'}{\omega}\right)\right]$	$c^2 = \frac{1}{r_S}gh$
Mazure's equation (1837)	$c = \frac{\omega g h\eta}{f\upsilon^2}\cos\varepsilon$	$c = \frac{\omega g h\eta}{f\upsilon^2}$

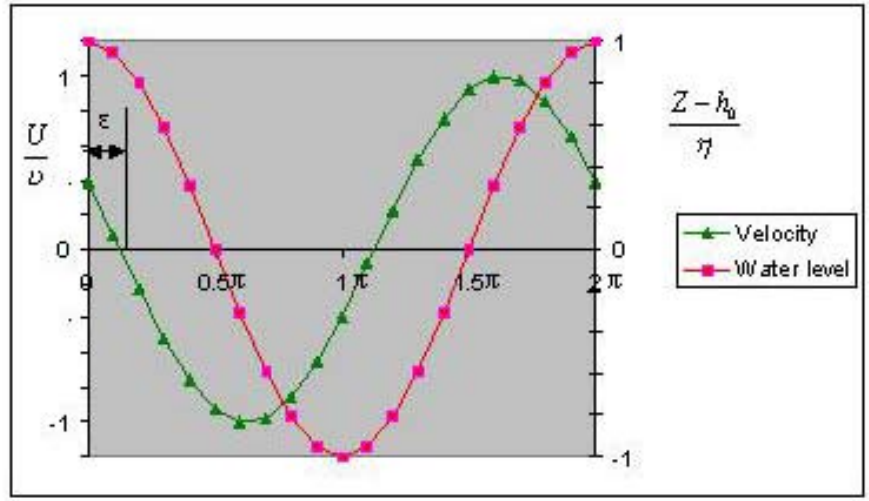
Most important differences

- Inclusion of Phase lag ε between HW and HWS
- Inclusion of tidal damping δ

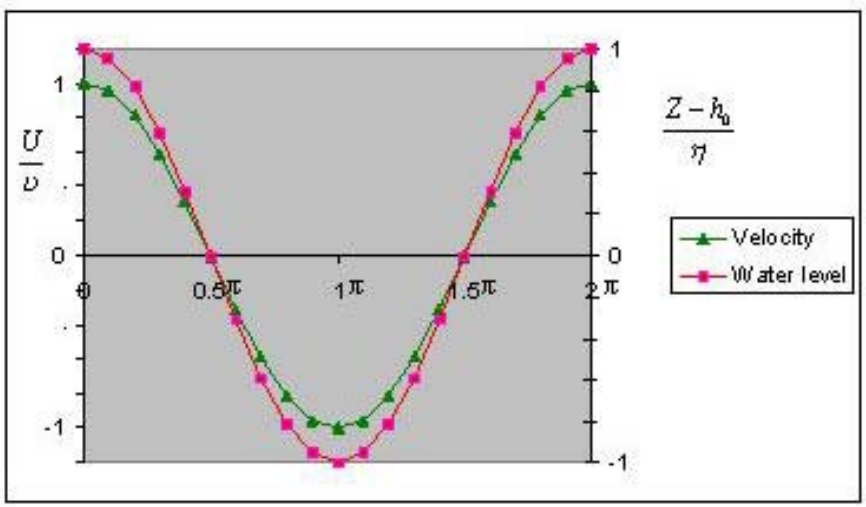
The Phase Lag between HW and HWS



Standing wave ε=0



Mixed wave 0<ε<π/2



Progressive wave ε=π/2

$$U = -\upsilon \sin(\omega t - \varepsilon)$$

$$Z = \eta \cos(\omega t) + \bar{h}$$

Phase Lag equation

Conditions at HW:
$dh/dt=0$, $V=\upsilon\sin\varepsilon$, $dV/dt=\omega\upsilon\cos\varepsilon$

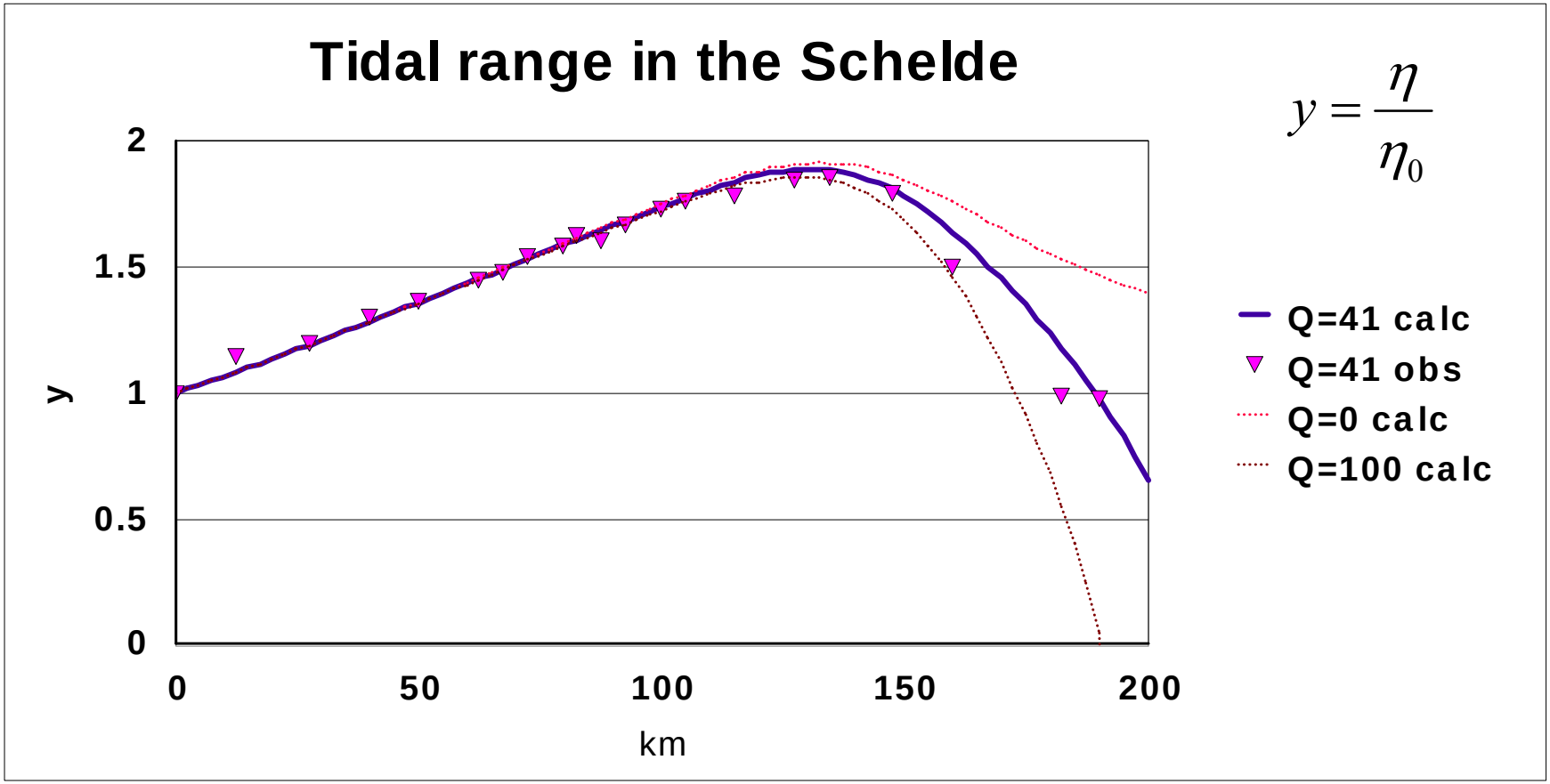
Subst. in Lagrangean Continuity Equation:

$$\tan\varepsilon = \frac{\omega b}{c(1-\delta b)} = \frac{b}{\lambda}\frac{2\pi}{(1-\delta b)}$$

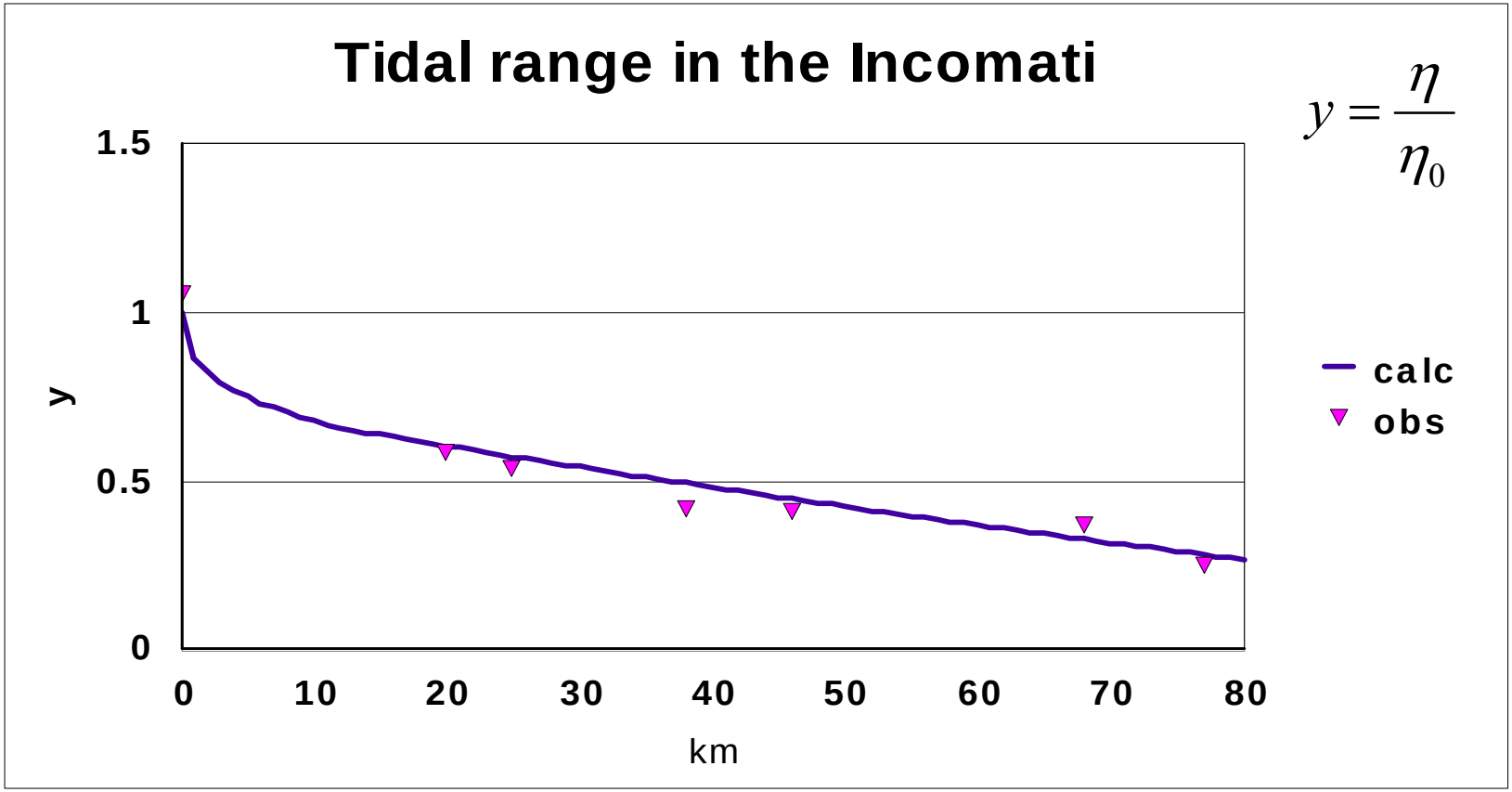
(Savenije, 1993)

Tidal Damping and Amplification

Tidal damping and amplification



Tidal damping and amplification



Tidal damping and amplification

1. Combination of continuity and momentum balance equation
2. Impose constraint for HW: $\frac{\partial h}{\partial t} = 0$ and V=υsinε
3. Impose constraint for LW: $\frac{\partial h}{\partial t} = 0$ and V=-υsinε
4. Subtract the two envelopes
5. Yields:

$$\delta = \frac{1}{\eta}\frac{\mathrm{d}\eta}{\mathrm{d}x} = \left(\frac{1}{b} - f'\frac{\upsilon \sin\varepsilon}{\bar{h}c}\right) \Big/ \left(1 + \frac{g\eta}{c\upsilon \sin\varepsilon}\right)$$

(Savenije, 1998)

Tidal damping and amplification

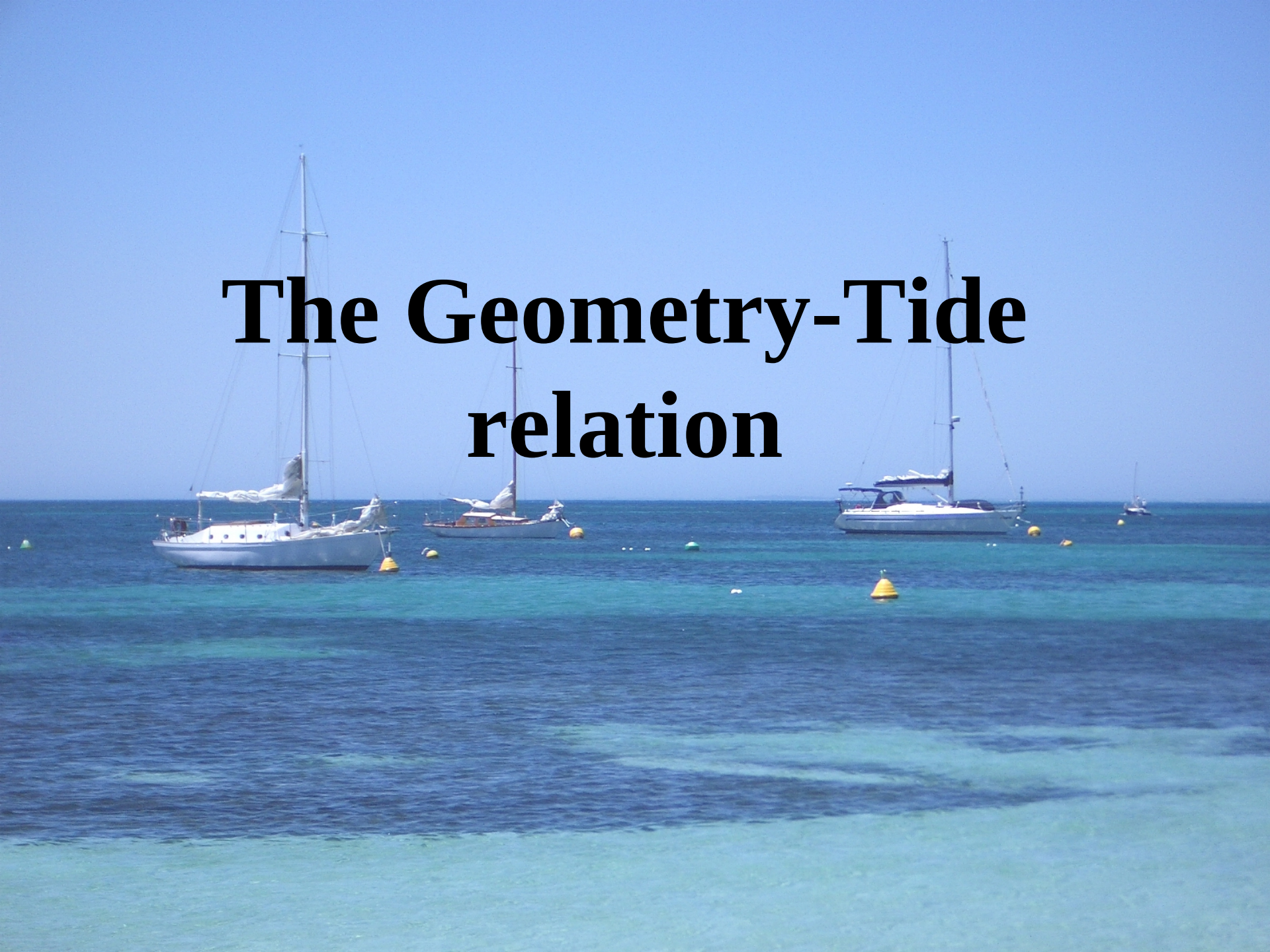
$$\delta = \frac{1}{y}\frac{\mathrm{d}\,y}{\mathrm{d}\,x} = \left(\frac{1}{b} - f'\frac{\upsilon \sin\varepsilon}{\overline{hc}}\right)\left(\frac{\alpha}{\alpha + y}\right)$$

$\boxed{\alpha = O(0.1)}$

If y>0.5 : linear amplification of damping

If y<0.1 : exponential damping

The Geometry-Tide relation



The geometry-tide relation

1. Lagrangean version of Continuity equation
2. Integration between LW and HW
3. Taylor series expansion for $E/b<1$

$$\frac{H}{E} = \frac{\eta\omega}{\upsilon} = \frac{\bar{h}}{r_S b}\frac{(1-\delta b)}{\cos(\varepsilon)}$$

(Savenije, 1993)

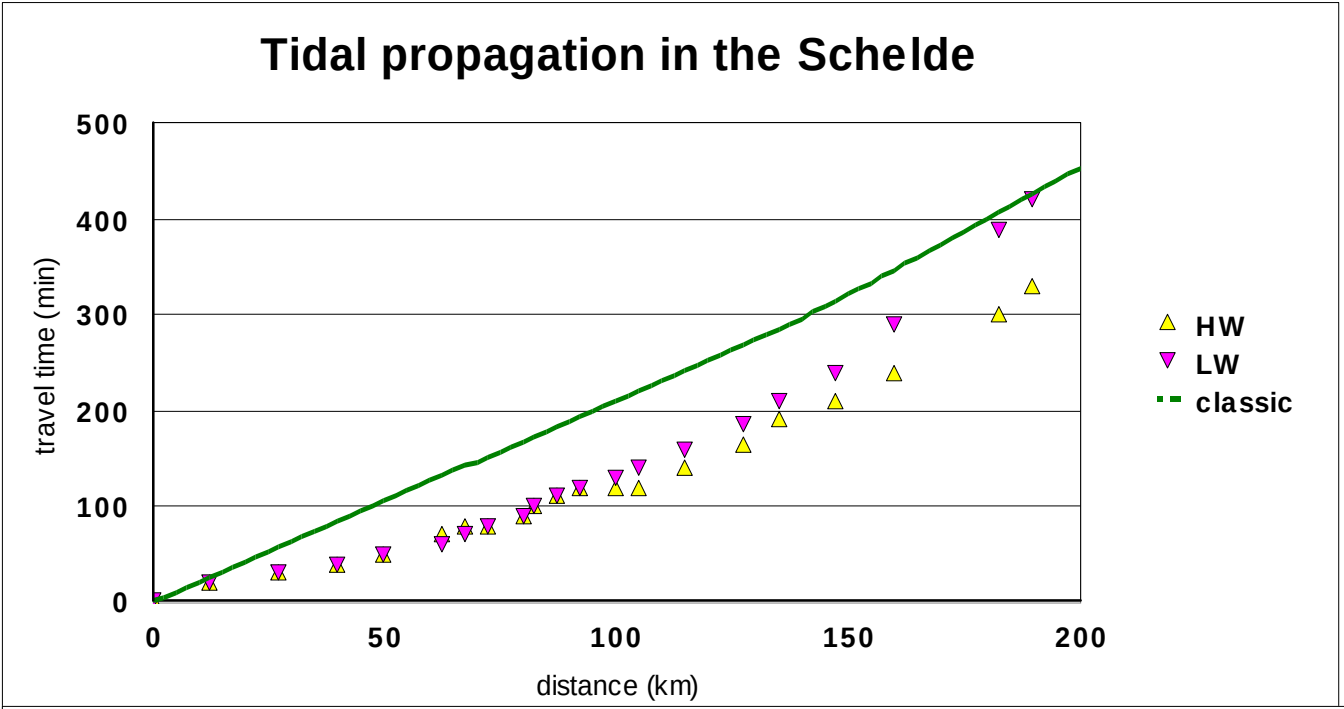
The Scaling equation

Combination of the geometry-tide relation with the Phase-Lag equation yields the Scaling Equation

$$r_S \frac{\eta}{\bar{h}} = \frac{\upsilon}{c} \frac{1}{\sin(\varepsilon)}$$

(Savenije & Veling, 2005)

Relation between wave celerity and tidal damping



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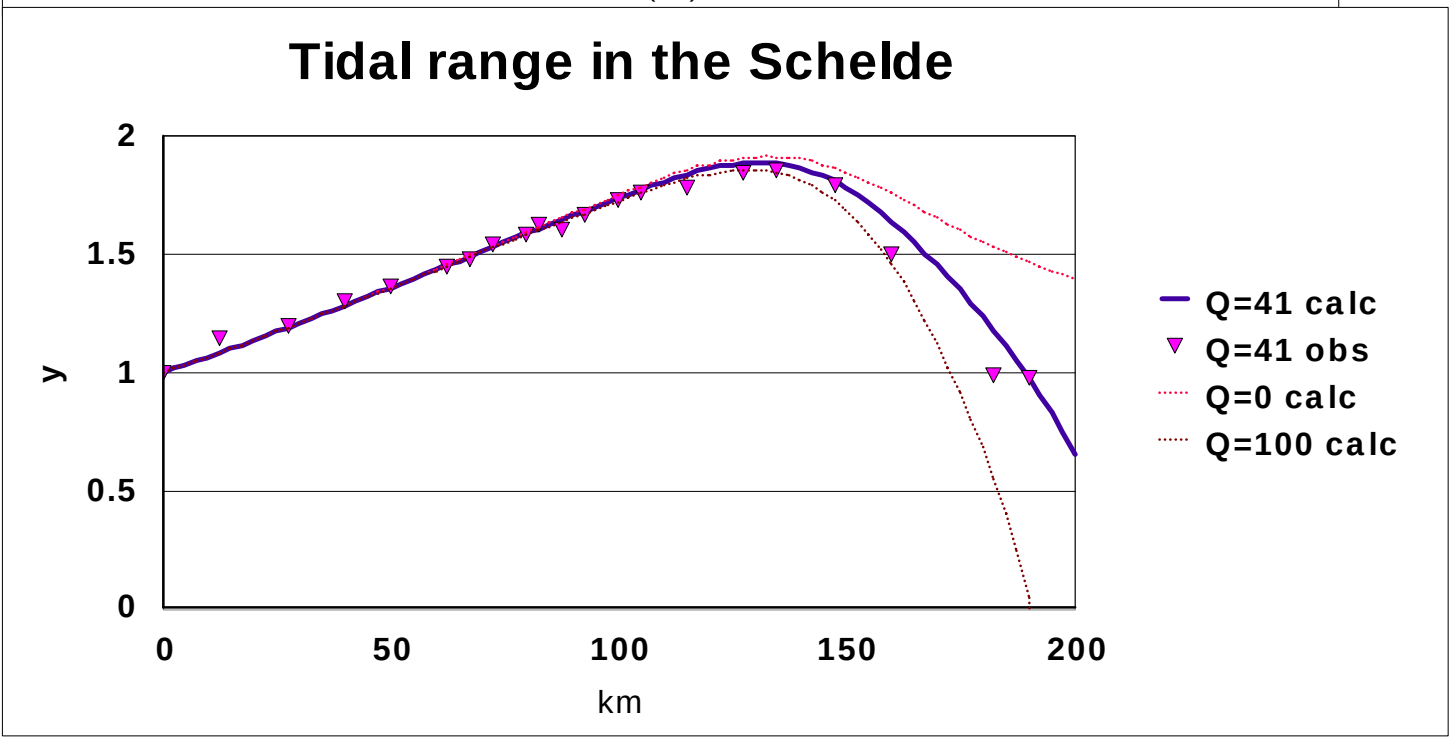
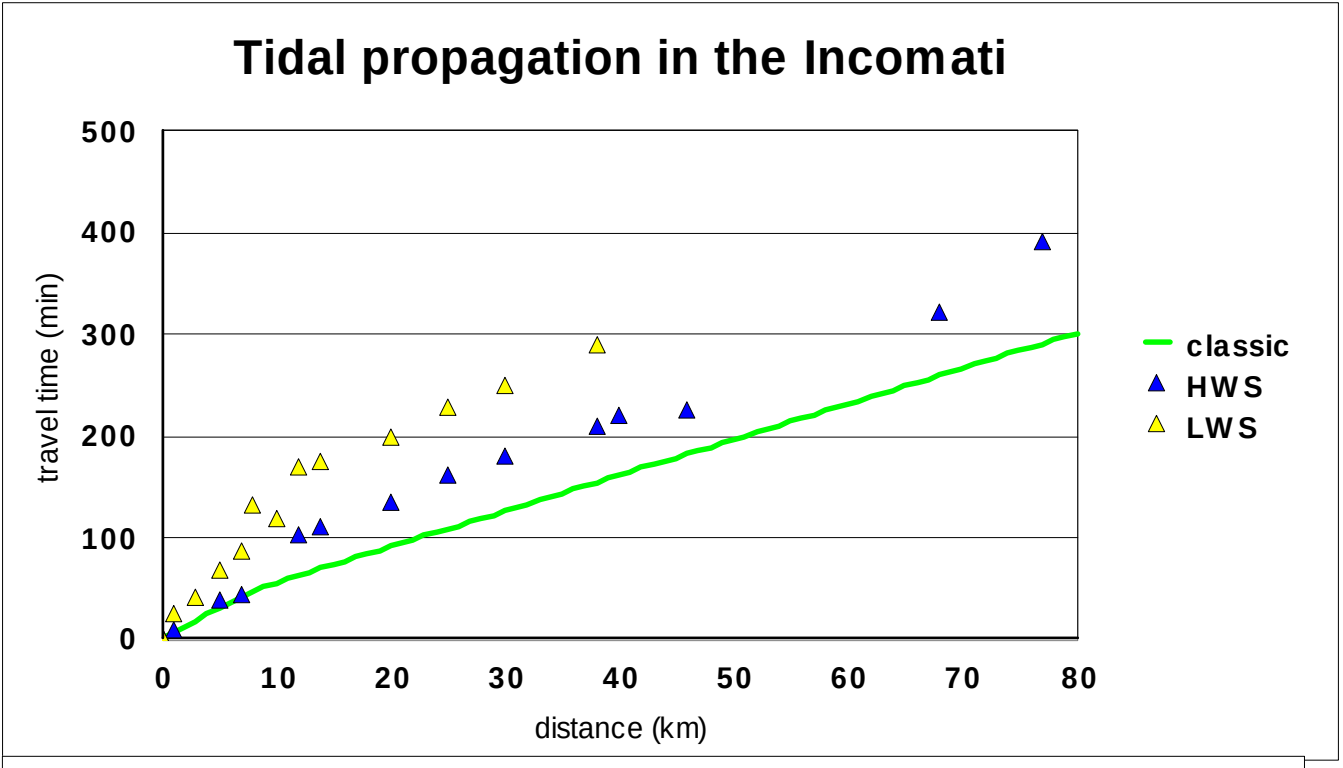
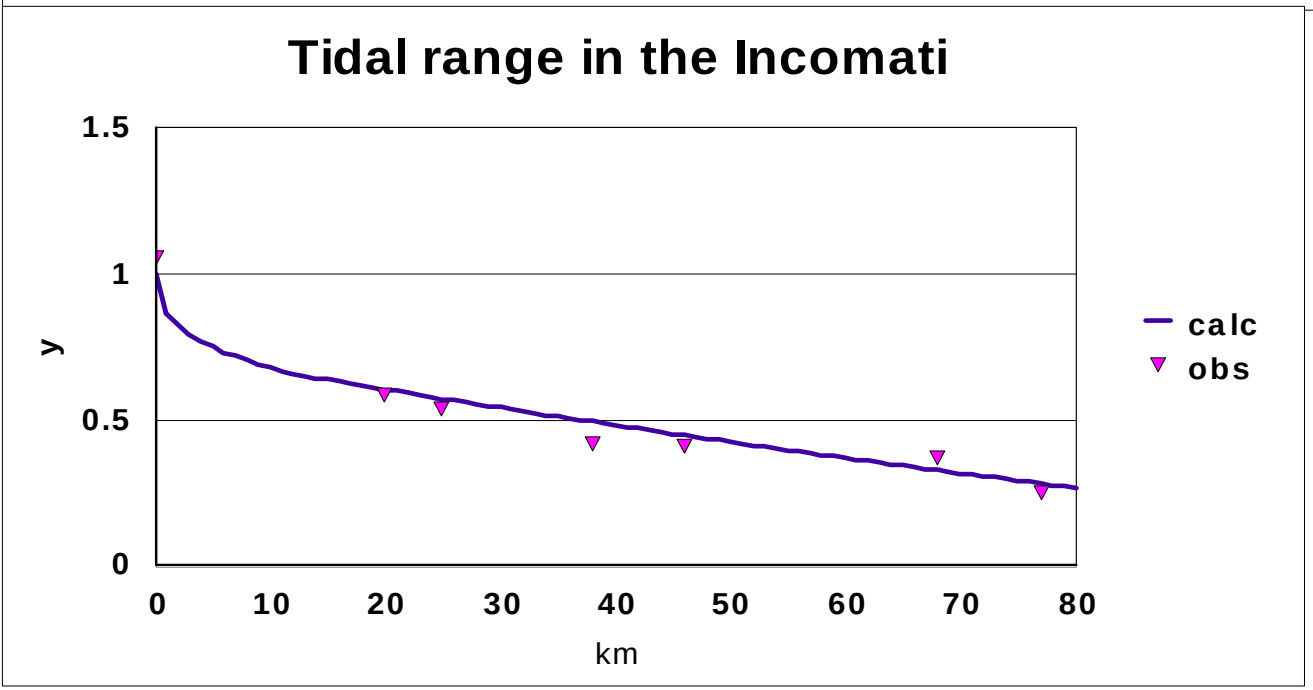
Classical equation

$$c_0 = \sqrt{\frac{1}{\beta} g h}$$



$$y = \frac{H}{H_0} = \frac{\eta}{\eta_0}$$





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Classical equation

$$c_0 = \sqrt{\frac{1}{\beta} g h}$$

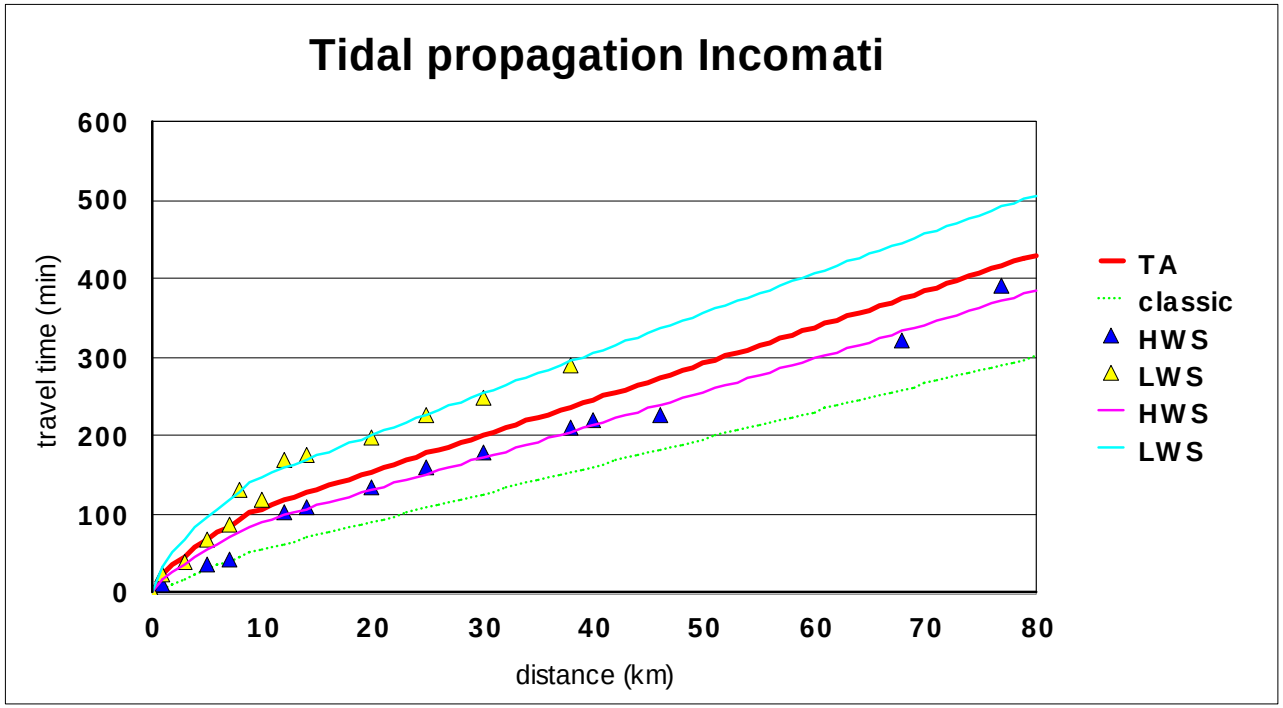
$$y = \frac{H}{H_0}$$

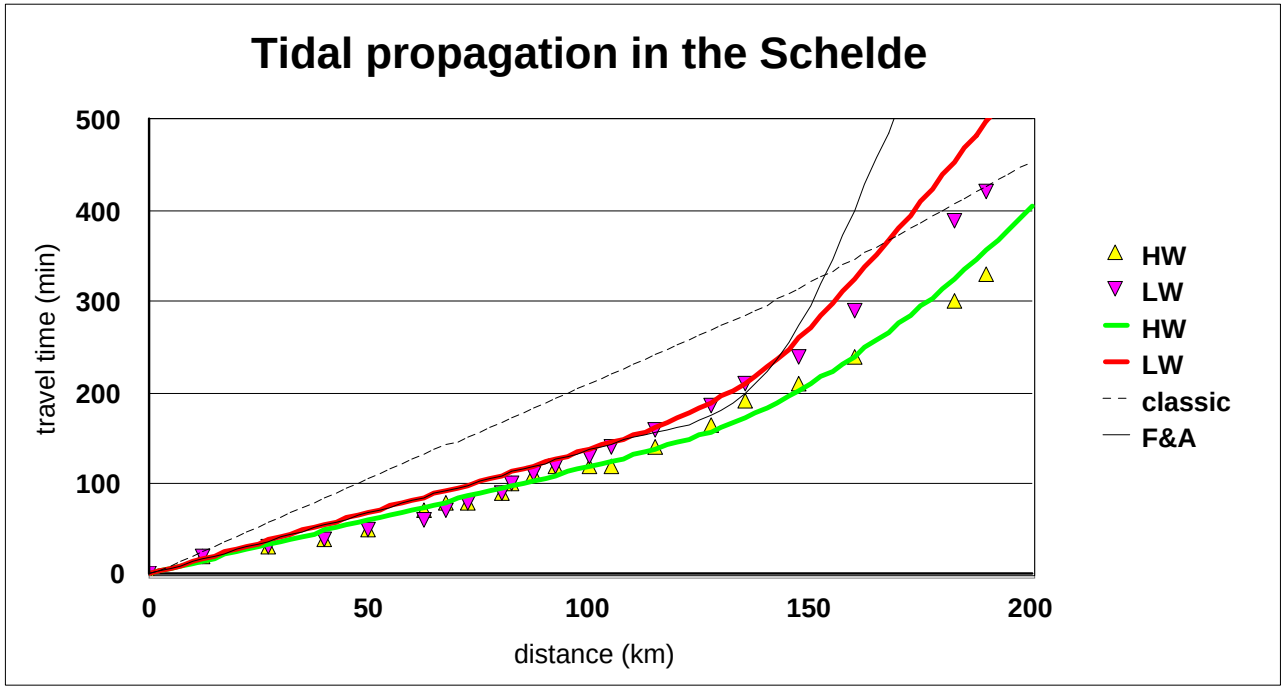
The Celerity equation

1. Allow amplification and damping of the tide
2. Assume that the scaled tidal wave propagates undeformed
3. Assume $\eta/h<1$
4. Method of Characteristics
5. Solution for HWS and LWS

$$c^2 = \frac{1}{r_S} gh \Big/ \left[1 - \frac{\sin 2\varepsilon}{2(1+\alpha)} \left(\frac{c}{\omega b} - \frac{R'}{\omega} \right) \right]$$

(Savenije & Veling, 2005)





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Comparing the new Equations with their Classical Counterparts

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Equation Name	Newly derived equation	"Classical" equation
Phase Lag equation	$\tan\varepsilon = \frac{\omega b}{c(1-\delta b)} = \frac{b}{\lambda}\frac{2\pi}{(1-\delta b)}$	
Geometry-Tide relation	$\frac{H}{E} = \frac{\eta\omega}{\upsilon} = \frac{\bar{h}}{r_S b}\frac{(1-\delta b)}{\cos(\varepsilon)}$	$\frac{\eta\omega}{\upsilon} = \frac{\bar{h}}{r_S b}$
Scaling equation	$r_S\frac{\eta}{\bar{h}} = \frac{\upsilon}{c}\frac{1}{\sin(\varepsilon)}$	$r_S\frac{\eta}{\bar{h}} = \frac{\upsilon}{c}$
Damping equation	$\frac{\mathrm{d}H}{\mathrm{d}x}\left(1+\frac{g\eta}{c\upsilon\sin\varepsilon}\right) = H\left(\frac{1}{b} - f'\frac{\upsilon\sin\varepsilon}{\bar{h}c}\right)$	$\frac{\mathrm{d}H}{\mathrm{d}x} = H\frac{1}{2b}$
Celerity equation	$c^2 = \frac{1}{r_S}gh\Big/\left[1-\frac{\sin 2\varepsilon}{2(1+\alpha)}\left(\frac{c}{\omega b}-\frac{R'}{\omega}\right)\right]$	$c^2 = \frac{1}{r_S}gh$
Mazure's equation	$c = \frac{\omega g h\eta\cos\varepsilon}{f\upsilon^2}$	$c = \frac{\omega g h\eta}{f\upsilon^2}$

Equation Name	Assumption made for "Classical" equation	"Classical" equation
Phase Lag equation	either $\varepsilon=0$ or $\varepsilon=\pi/2$	
Geometry-Tide relation	$\varepsilon=0$ and $\delta=0$	$\frac{\eta\omega}{\upsilon}=\frac{\bar{h}}{r_S b}$
Scaling equation	$\varepsilon=\pi/2$	$r_S\frac{\eta}{\bar{h}}=\frac{\upsilon}{c}$
Damping equation	$\varepsilon=\pi/2$ and $f'=0$	$\frac{\mathrm{d}H}{\mathrm{d}x}=H\frac{1}{2b}$
Celerity equation	$\varepsilon=\pi/2$ or $\delta=0$	$c^2=\frac{1}{r_S}gh$
Mazure's equation	$\varepsilon=0$	$c=\frac{\omega g h\eta}{f\upsilon^2}$

Conclusions

- The new equations demonstrate substantial differences with “classical” counterparts
- They are more general versions of the classical equations (which are special cases)
- They are consistent with each other (which the classical equations are not)
- The phase lag plays a key role and is the most important estuary parameter

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Thank You
for your attention

Some useful estuary equations

$$\beta \frac{\eta\omega}{\upsilon} = \frac{h}{b}\frac{(1-\delta b)}{\cos\varepsilon} = \beta\frac{H}{E}$$

continuity equation (Savenije, 1993)

$$\tan\varepsilon = \frac{\omega b}{(1-\delta b)c}$$

from continuity equation (Savenije, 1993)

$$\beta\frac{\eta}{h}\sin\varepsilon = \frac{\upsilon}{c} = F$$

"the Scaling Equation"
(Savenije & Veling, 2005)

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