

Chapter 1

Variables

This chapter describes the main variables which are used in traffic flow theory. Section 1.1 will show the different levels (microscopic, macroscopic and other levels) at which traffic is generally described. Section 1.2 will describe different principles (local, instantaneous and spatio-temporal) to measure the traffic flow. The last section (1.3) describes traffic flow characteristics.

1.1 Levels of description

This section will show the different levels at which traffic is generally described. Sections 1.1.1 and 1.1.2 will discuss the variables in the microscopic and macroscopic descriptions in more detail.

In a microscopic traffic description, every vehicle-driver combination is described. The smallest element in the description is the vehicle-driver combination. The other often used level of traffic flow description is the macroscopic traffic description. Different from the microscopic description, this level does not consider individual vehicles. Instead, the traffic variables are aggregated over several vehicles or, most commonly, a road stretch. Typical characteristics of the traffic flow on a road stretch are the average speed, vehicle density or flow (see section 1.1.2). Other levels of description can also be used, these are described in the last section (see section 1.1.3).

1.1.1 Microscopic

In a microscopic traffic description, the vehicle-driver combinations (often referred to as “vehicles”, which we will do from now on) are described individually. Full information of a vehicle is given in its trajectory, i.e. the specification of the position of the vehicle at all times. To have full information on these, the positions of all vehicles at all times have to be specified. A graphical representation of vehicle trajectories is given in figure 1.1

The trajectories are drawn in a space time plot, with time on the horizontal axis. Note that vehicle trajectories can never go back in time. Trajectories might move back in space if the vehicles are going in the opposite direction, for instance on a two-lane bidirectional rural road. This is not expected on motorways. The slope of the line is the speed of the vehicles. Therefore, the trajectories cannot be vertical – that would mean an infinite speed. Horizontal trajectories are possible at speed zero.

Basic variables in the microscopic representation are speed, headway, and space headway. The speed is the amount of distance a vehicle covers in a unit of time, which is indicated by v . Sometimes, the inverse of speed is a useful measure, the amount of time a vehicle needs for to cover a unit of space; this is called the pace p . Furthermore, there is the space headway or spacing (s) of the vehicle. The net space headway is the distance between the vehicle and its leader. This is also called the gap. The gross space headway of a (following) vehicle the distance *including* the length of the vehicle, so the distance from the rear bumper of the leading vehicle to the rear bumper of the following vehicle. Similarly, we can time it takes for a follower to get to reach (with its front bumper) the position of its leader’s rear bumper. This is called the net time headway. If we also add the time it cost to cover the distance of a vehicle length, we get the gross time headway. See also figure 1.2. The symbol used to indicate the headway is h .

From now on, in this reader we will use the following conventions:

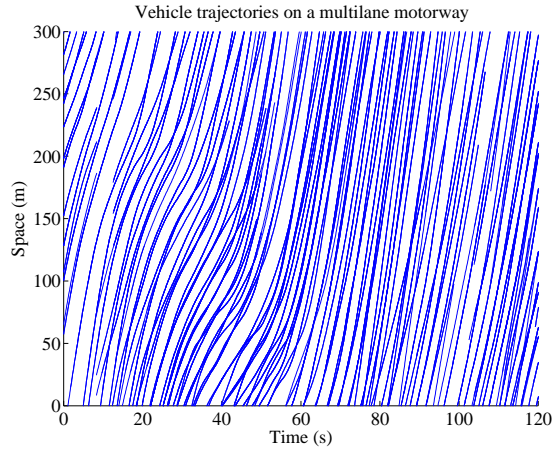


Figure 1.1: Vehicle trajectories on a multilane motorway

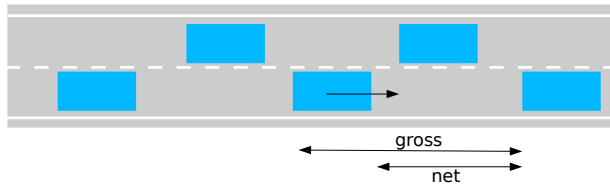


Figure 1.2: The difference between gross and net spacing (or headway)

- Unless specified otherwise, headway means time headway
- Unless specified otherwise, headways and spacing are given as gross values

Figure 1.3 shows the variables graphically. The figure shows two vehicles, a longer vehicle and a shorter vehicle. Note that the length of the vehicles remains unchanged, so the difference between the gross and net spacing is the same, namely the vehicle length. However, the difference between the gross and net time headway changes based on the vehicle speed.

In a trajectory plot, the slope of the line is the speed. If this slope changes, the vehicle accelerates or decelerates. So, the curvature of the lines in a trajectory plot shows the acceleration or deceleration of the vehicle. If the slope increases, the vehicle accelerates, if it decreases, it decelerates.

1.1.2 Marcoscopic

In a macroscopic traffic description, one does not describe individual vehicles. Rather, one describes for each road section the aggregated variables. That is, one can specify the density k , i.e. how close in space vehicles are together. Furthermore, one can specify the flow q i.e. The number of vehicles passing a reference point per unit of time. Finally, one can describe the average speed u of the vehicles on a road section. Other words for flow are throughput, volume or intensity; we will strictly adhere to the term flow to indicate this concept.

All of the mentioned macroscopic variables have their microscopic counterpart. This is summarized in table 1.1. The density is calculated as one divided by the average spacing, and is calculated over a certain road stretch. For instance, if vehicles have a spacing of 100 meters, there are $1/100$ vehicles per meter, or $1000/100=10$ veh/km. The flow is the number of vehicles that pass a point per unit of time. It can be directly calculated from the headways by dividing one over the average headway. For instance, if all vehicles have a headway of 4 seconds, there are $1/4$ vehicles per second. That means there are $3600(\text{s/h})/4(\text{s/veh})= 900$ veh/h. In table 1.1 units are provided and in the conversion from one quantity to the other, one needs to pay attention. Note that provided units are not obligatory:

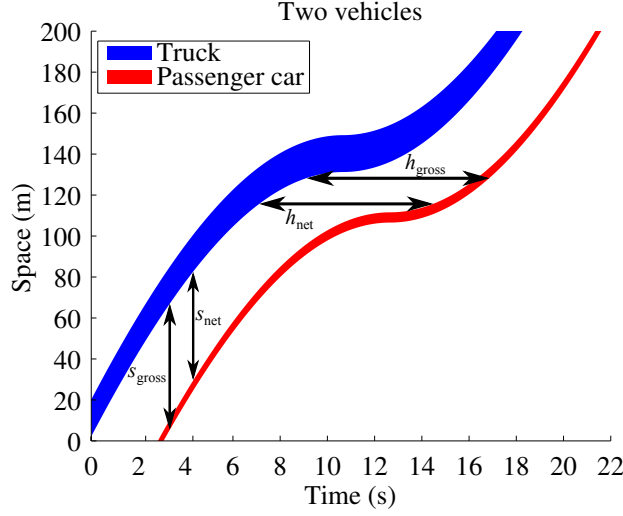


Figure 1.3: The microscopic variables explained based on two vehicles

Table 1.1: Overview of the microscopic and macroscopic variables and their relationship; the pointy brackets indicate the mean.

Microscopic	symbol	unit	Macroscopic	symbol	unit	relation
Headway	h	s	Flow	q	vtg/h	$q = \frac{3600}{\langle h \rangle}$
Spacing	s	m	Density	k	vtg/km	$k = \frac{1000}{\langle s \rangle}$
Speed	v	m/s	Average speed	u	km/u	$u = 3.6 \langle v \rangle$

one can present individual speed in km/h, or density in veh/hm. However, always pay attention to the units before converting or calculating.

Relation to the microscopic level

The average speed is calculated as an average of the speeds of vehicles at a certain road stretch. This speed differs from the average speed obtained by averaging speed of all vehicles passing a certain point. The next section explains the different measuring principles. The full explanation of the differences between the two speeds and how one can approximate the (space) average speed by speeds of vehicles passing a certain location is presented in section 4.4.

Another concept for a traffic flow, in particular in relation to a detector (see also section 1.2), is the occupancy o . This indicates which fraction of a time a detector embedded in the roadway is occupied, i.e. whether there is a vehicle on top of the detector. Suppose a detector has a length L_{det} and a vehicle a length of L_i . The occupancy is defined as the time the detector is occupied, $\tau_{occupied}$ divided by all time, i.e. the time it is occupied and time it is not occupied

$\tau_{not\ occupied}$

$$o = \frac{\tau_{occupied}}{\tau_{occupied} + \tau_{not\ occupied}} \quad (1.1)$$

The occupation time can be derived from the distances and the speed. The distance the vehicle has to cover from the moment it starts occupying the detector up to the time it leaves the detector is its own length plus the length of the detector. Hence, the occupancy time is

$$\tau_{occupied} = \frac{L_i + L_{det}}{v} \quad (1.2)$$

Once the first vehicle drives off the detector, the distance for the following vehicle to reach the detector is the gap (i.e., the spacing minus the length of the vehicle) between the vehicles minus the length of the detector. The amount of time

1.2.2 Instantaneous

Contrary to local measurements, there are instantaneous measurements. These are measurements which are taken at one moment in time, most likely over a certain road stretch. An example of such a measurement is an areal photograph. In such a measurement, one can clearly distinguish spatial characteristics, as for instance the density. However, measuring the temporal component (flow) is not possible.

1.2.3 spatio-temporal measurements

Apart from local or instantaneous measurements, one can use measurements which stretch over a period of time *and* a stretch of road. For instance, the trajectories in figure 1.1 are an example thereof. This section will introduce Edie's definitions of flow, density and speed for an area in space and time.

A combination of instantaneous measurements and local measurements can be found in remote sensing observations. These are observations which stretch in both space and time. For instance, the trajectories presented in figure 1.1 can be observed using a camera mounted on a high point or a helicopter. One can see a road stretch, and observe it for a period of time.

Measuring average speed by definition requires an observation which stretches over time and space. At one location, one cannot determine speed, nor at one moment. One needs at least two locations close by (several meters) or two time instances close by. Ignoring these short distances one can calculate a local mean speed based on speeds of the vehicles passing by location. Ignoring the short times, one can calculate the time mean speed from the speed of the vehicles currently at the road. At this moment, we suffice by mentioning these average speeds are different. Section 4.4 will show how the space mean speed can be approximated from local measurements.

Figure 1.4 shows the same trajectories as figure 1.1, but in figure 1.4 an area is selected. Trajectories within this area in space and time are coloured red. Note that an selected area is not necessarily square. It is even possible to have a convex area, or boundaries moving backwards and forwards in time. The definitions as introduced here will hold for all types of areas, regardless of their shape in space-time.

Let us consider the area X . We indicate its size by W_X , which is expressed in km-h, or any other unit of space times time. For all vehicles, we consider the distance they drive in area X , which we call $d_{X,i}$. Adding these for all vehicles i gives the total distance covered in area X , indicated by TD :

$$TD = \sum_{\text{all vehicles } i} d_{X,i} \quad (1.5)$$

For a rectangular area in space and time, the distance covered might be the distance from the upstream end to the downstream end, but the trajectory can also begin and/or end at the side of the area, at a certain time. In that case, the distance is less than the full distance.

Similarly, we can define the time a vehicle spends in area X , $t_{X,i}$, which we can sum for all vehicles i to get the total time spent in area X , indicated by TT .

$$TT = \sum_{\text{all vehicles } i} t_{X,i} \quad (1.6)$$

Obviously, both quantities grow in principle with the area size. Therefore, the traffic flow is best characterised by the quantities TD/W_X and TT/W_X . This gives the flow and the density respectively:

$$q = \frac{TD}{W_X} \quad (1.7)$$

$$k = \frac{TT}{W_X} \quad (1.8)$$

Intuitively, the relationship is best understood reasoning from the known relations of density and flow. Starting with a situation of 1000 veh/h at a cross section, and an area of 1 h and 2 km. In 1 hour, 1000 vehicles pass by, which all travel 2 kilometres in the area (There the vehicles which cannot cover the 2 km because the time runs out, but there are just as many which are in the section when the time window starts). So the total distance is the flow times the size of the area: $TD = qW_X$. This can be simply rewritten to equation 1.8.

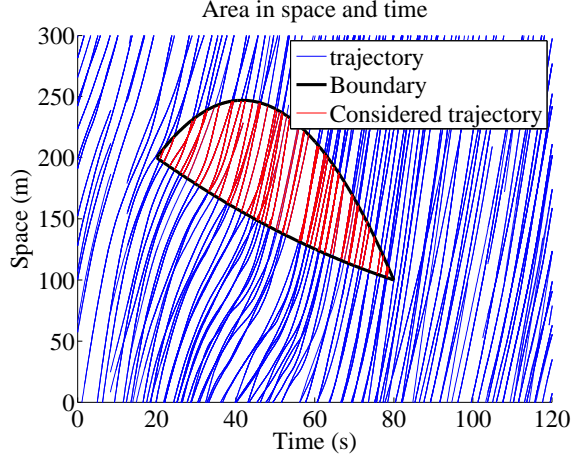


Figure 1.4: Vehicle trajectories and the selection of an area in space and time

A similar relation is constructed for the density, considering again the rectangular area of 1 hour times 2 kilometres. Starting with a density of 10 veh/km, there are 20 vehicles in the area, which we all follow for one hour. The total time spent, is hence $10 \times 2 \times 1$, or $TT = kW_X$. This can be rewritten to equation 1.8.

The average speed is defined as the total distance divided by the total time, so

$$u = \frac{TD}{TT} \quad (1.9)$$

The average travel time over a distance l can be found as the average of the time a vehicle travels over a distance l . In an equation, we find:

$$\langle tt \rangle = \left\langle \frac{l}{v} \right\rangle = l \left\langle \frac{1}{v} \right\rangle \quad (1.10)$$

In this equation, tt indicates the travel time and the pointy brackets indicate the mean. This can be measured for all vehicles passing a road stretch, for instance at a local detector. Note that the mean travel time is *not* equal to the distance divided by the mean speed:

$$\langle tt \rangle = l \left\langle \frac{1}{v} \right\rangle \neq l \frac{1}{\langle v \rangle} \quad (1.11)$$

In fact, it can be proven that in case speeds of vehicles are not the same, the average travel time is underestimated if the mean speed is used.

$$\langle tt \rangle = l \left\langle \frac{1}{v} \right\rangle \leq l \frac{1}{\langle v \rangle} \quad (1.12)$$

The harmonically averaged speed (i.e., 1 divide by the average of 1 divided by the speed) does provide a good basis for the travel time estimation. In an equation, we best first define the pace, p_i :

$$p_i = \frac{1}{v_i} \quad (1.13)$$

The harmonically averaged speed now is

$$\langle v \rangle_{\text{harmonically}} = \frac{1}{\langle p \rangle} = \frac{1}{\left\langle \frac{1}{v_i} \right\rangle} \quad (1.14)$$

The same quantity is required to find the space mean speed. Section 4.4 shows the difference qualitatively. In short, differences can be several tens of percents.

this takes is

$$\tau_{\text{not occupied}} = \frac{s - L_i - L_{\text{det}}}{v} \quad (1.3)$$

Substituting the expressions for the occupancy time and the non-occupancy time into equation 1.1 and rearranging the terms, we get

$$O = \frac{L_i + L_{\text{det}}}{s} \quad (1.4)$$

In practice, the detector length is known for a certain road configuration (usually, there are country specific standards). So assuming a vehicle length, one can calculate the spacing, and hence the density, from the occupancy.

1.1.3 Other levels

Apart from the macroscopic and microscopic traffic descriptions, there are three other levels to describe traffic. They are less common, and are therefore not discussed in detail. The levels mentioned here are mainly used in computer simulation models.

Mesoscopic

The term mesoscopic is used for any description of traffic flow which is in-between macroscopic and microscopic. It can also be a term for simulation models which calculate some elements macroscopically and some microscopically. For instance, dynasmart Dynasmart (2003), uses such a mesoscopic description.

Submicroscopic

In a submicroscopic description the total system state is determined by the sub levels of a vehicle and/or driver. Processes which influence the speed of a vehicle, like for instance mechanically throttle position and engine response, or psychologically speed perception, are explicitly modelled. This allows to explicitly model the (change in) reaction on inputs. For instance, what influence would cars with a stronger engine have on the traffic flow.

Network level

A relatively new way of describing the traffic state is the network level. This has recently gained attention after the publication by Geroliminis and Daganzo (2008). Instead of describing a part of a road as smallest element, one can take an area (e.g. a city center) and consider this as one unit. Chapter 11 is devoted to this description.

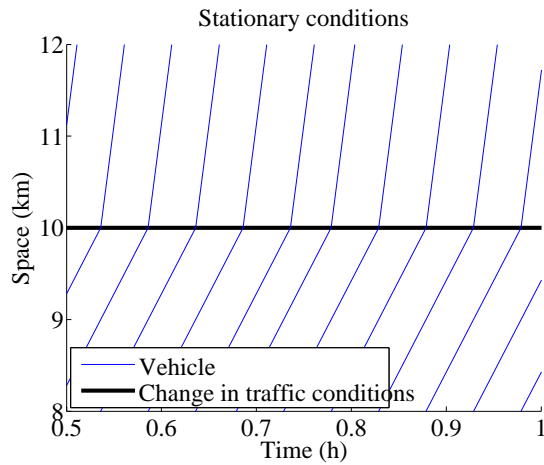
1.2 Measuring principles

Whereas the previous sections described which variables are used to describe traffic flow, this section will introduce three principles of measuring the traffic flow. These principles are local, instantaneous and spatio-temporal.

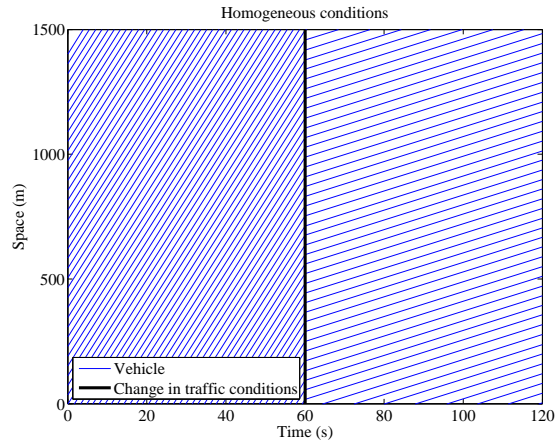
1.2.1 Local

With local measurements one observes traffic at one location. This can be for instance a position at the roadway. To measure motorway traffic, often inductive loops are used. These are coils embedded in the pavement in which a electrical current produces a (vertical) magnetic field. If a car enters or leaves this magnetic field, this can be measured in the current of the coil. Thus, one knows how long a loop is occupied. In the US, usually single loops are used, giving the occupancy of the loop. Using equation 1.4, this can be translated into density. The detectors also measure the flow. As will be explained later in section 4.1, this suffices to completely characterise the traffic flow.

This determination of density builds upon the assumption of the vehicle length being known. One can also measure the length of a vehicle for passing vehicles, using dual loop detectors. These are inductive loops which are placed a known short distance (order of 1 m) from each other. If one measures carefully the time between the moment the vehicle starts occupying the first loop and the moment it starts occupying the second loop, one can measure its speed. If its speed is known, as well as the time it occupies one loop, the length of the vehicle can also be determined.



(a) Stationary conditions



(b) Homogeneous conditions

1.3 Stationarity and homogeneity

Traffic characteristics can vary over time and/or over space. There are dedicated names for traffic if the state does not change.

Traffic is called stationary if the traffic flow does not change over time (but it can change over space). An example can be for instance two different road sections with different characteristics. An example is given in figure 1.5(a), where there first is a low speed, then the speed of the vehicles is high.

Traffic is called homogeneous if the traffic flow does not change over space (but it can change over time). An example is given in figure 1.5(b), where at time 60 the speed decreases at the whole road section. This is much less common than the stationary conditions. For this conditions to occur, externally the traffic regulations have to change. For instance, the speed limits might change at a certain moment in time (lower speeds at night).

Selected problems

For this chapter, consider problems: 5, 6, 164, 174, 175, 176, 177, 199, 220, 235

