Chapter 11

Macroscopic fundamental diagram

The idea of a Macroscopic Fundamental Diagram (MFD) or Network Fundamental Diagram (NFD) is that rather than at the level of a lint, at the level of an *area* there exists a relationship between the number of travellers on the road and the average speed of these travellers. Moreover, it could be argued that the noise in the measurements might be less. For individual detectors, one will find deviations from the fundamental diagram, (both upward and downward) which crates high scatter. Geroliminis and Daganzo (2008) showed that if one averages all detectors over a large area almost all scatter disappears (see figure 11.1). From a mathematical point, this makes sense since it simply is the law of large numbers. But although the concept seems simple, the effect might be large.

The essence of an MFD is that a high density affects the the traffic flow to even under capacity. This is in contrast to a single road with a bottleneck, where in case of a high demand, the outflow will be at capacity (or queue discharge rate). This internal congestion can only take place if the (tail of) the congestion influences the throughput. This is the case with spill back effects, e.g. a tail of the queue growing backwards and thereby influencing drivers which want to take an exit which does not pass the bottleneck. A very much simplified example of this is given by Daganzo (2007), which proposed a single ring road with entrances and exits everywhere, see fig 11.2.

In MFDs, some terminology is different (see table 11.1 for an overview). Accumulation (A) is the total number of vehicles in a network, which can be expressed as total number of vehicles or number of vehicles per roadway length. Note that since the roadway length is fixed, these two are proportional. The accumulation can be seen as an equivalent of density on a road.

Production (P) is the internal flow in the network. This can be computed by Edie's definitions for all links in the network (equation 1.8). Performance ($\mathcal{P}$)is the outflow of the network, which is the sum of the flow of the outgoing links. Geroliminis and Daganzo (2008) shows there is a strong relationship between the production and the performance. This relationship for empirical data is shown in figure 11.4. The ratio of production and performance is the length of the trip within the zone. (As a thought experiment: if one needs increases the trip length inside the zone by a factor two, the internal flow, and hence the production, needs to increase by a factor two before the same outflow, performance, is reached).

This chapter first discusses what could be a possible control application for the MFD, before continuing to conditions under which the MFD holds (section 11.2). Then, section 11.3 discusses a way to include the MFD into a simulation model, and the chapter ends with some recent developments.

Table 11.1: Variables for the MFD

Name	symbol	meaning
Accumulation	A	Number of vehicles in the network (veh, or veh/km)
Production	P	Internal flows in the network (veh/h)
Performance	$\mathcal{P}$	Outflow out of the network (veh/h)

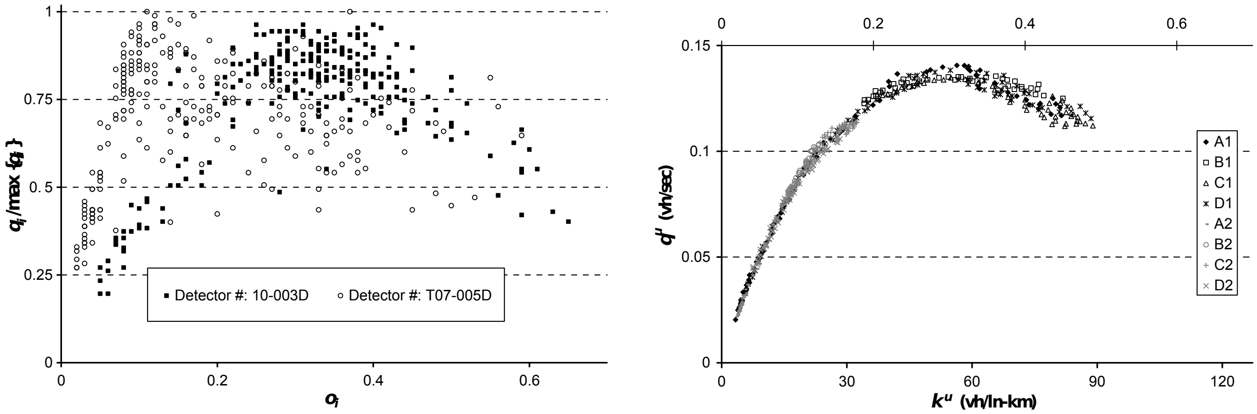


Figure 11.1: The first Macroscopic Fundamental Diagram constructed from data, from Geroliminis and Daganzo (2008).

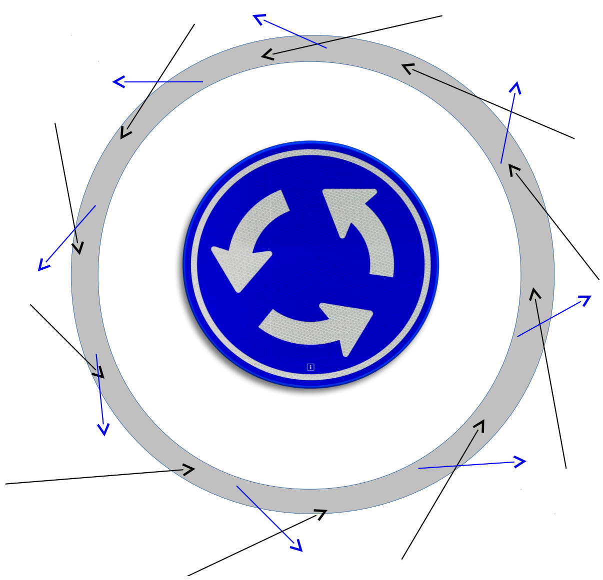


Figure 11.2: A simplified network congestion influences the outflow.

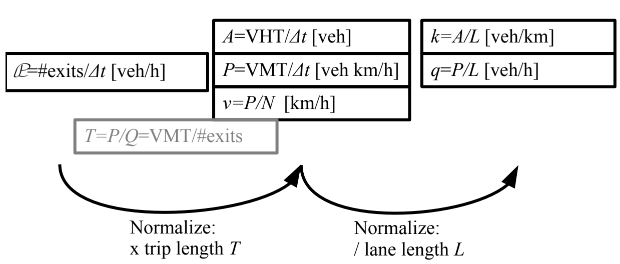
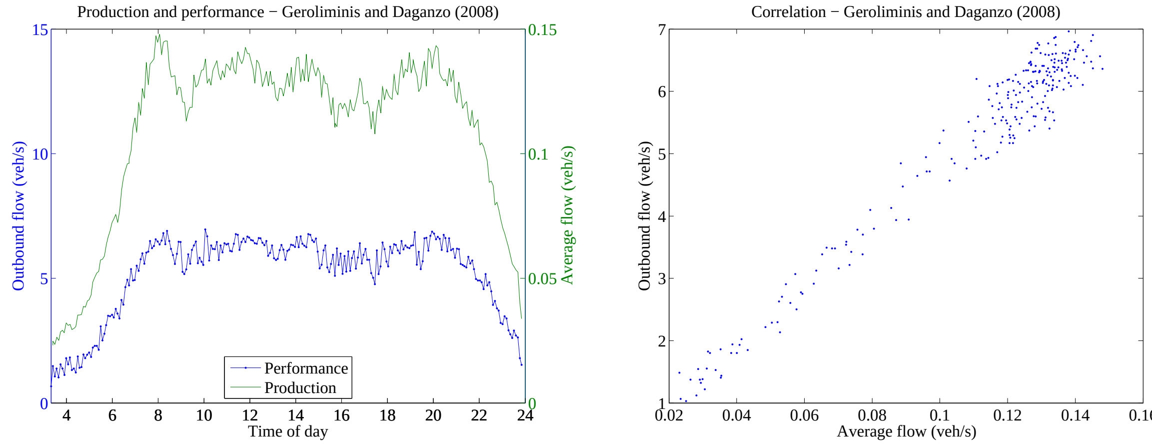


Figure 11.3: The relation between the different quantities related in the MFD



(a) timeseries (b) correlation

Figure 11.4: The relation between the production and the performance

11.1 Perimeter control

The idea behind perimeter control is as follows. As the production decreases with an increasing accumulation, it might be better to limit the accumulation in a zone, the so called protected network. Let's consider for the sake of simplicity the situation, with one zone which has this limitation, and storing vehicles outside the network will not affect traffic operations in the neigbouring zones (of course the waiting vehicles themselves are affected).

If we assume a constance trip length, for instance by assigning random exits around the ring road, the production will decrease as well. This causes an escalating effect of an further increasing accumulation and a further decreasing outflow. An example can be found in figure 11.5. This principle will now be discussed.

The top figure show the cumulative inflow curve (upper curve, indicated $A(t)$ – note that in this example, the original notation from (Daganzo, 2007) will be used), and the cumulative outflow curve without ($L(t)$) and with perimeter control ($L(t)$). The inflow curve is externally given, as is the MFD showing the production (outflow, G in this example) as function of accumulation (lower figure, n in the example, so the line is indicated $G(n)$). The accumulation in the system can be found by the difference between the inflow and outflow. Only the outflow at t=0 is specified, which equals 0, leading to the accumulation $n(0)$, which happens to be greater than the critical accumulation μ.

The dynamics of the outflow can now be predicted. In case of no perimeter control, the performance is governed by the MFD, which is the derivative of the outflow function. In the example $n(0) > \mu$, so the traffic is in a congested state, and the outflow can be read from $G(n)$. This outflow is lower than the inflow into the system, and as a consequence, the accumulation increases, leading to a decreasing outflow. Hence, the cumulative outflow curve $L(t)$ flattens. Once the accumulation increases to ω, the maximum number of vehicles in the road, the outflow reduces to 0.

Perimeter control entails limiting the inflow into the ring, and keep this at the level of the critical accumulation where the outflow is highest, i.e. not exceed μ. The inflow curve with the restriction is indicated with $A^*(t)$ in the figure. It starts with not letting any vehicles in until the accumulation is equal to μ. By consequence, the outflow increases over this time, since the accumulation becomes closer to mu, and hence the outflow closer to the maximum outflow γ. Once the accumulation is reduced to μ, the outflow is γ; to maintain this number of vehicles, the same amount of vehicles are left into the system. Once the queue is solved, i.e. no more vehicles have to wait to get into the system, the perimeter control stops. The outflow curve constructed in this way is indicated as $L^*(t)$.

The perimeter control comes at a cost, the waiting time outside the network. This can be indicated in the figure as the area between lines A^* and A. The benefit is the higher outflow, which is the shaded area between L^* and L. Note that the benefits exceed the cost.

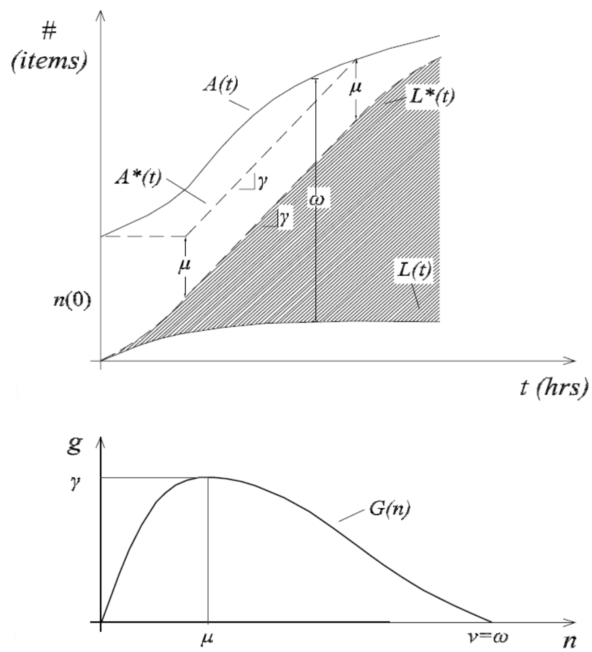


Figure 11.5: The effect of perimeter control in cumulative curves, figure from Daganzo (2007).

11.2 Traffic dynamics

A very strict claim on the application of the MFD is that it only holds if the traffic conditions are homogeneous throughout the network. In that case, all links in the area have a similar density, and the MFD simply is an average of all the same traffic states; the average then is that traffic state. In Cassidy et al. (2011) it has been shown what the averaging effects are in case a triangular fundamental diagram is assumed for the underlying links.

Figure 11.6 shows the effect of averaging two traffic states. If both states lie on the free flow branch, the average also lies on the free flow branch. If both lie on the congested branch, the average also lies on the congested branch. Only if one point lies on the congested branch and one point at the free flow branch, the average does not lie on either branch, but under the fundamental diagram. With a similar reasoning it can be shown that the average traffic state always lies *under* the fundamental diagram for one link if the fundamental diagram is concave.

One can continue this arguing and find that the larger the spread is, the further the average flow deviates from the flow which would match the average density according to the fundamental diagram. In an extreme case, some links are empty (q=0, k=0), and other links are jammed (q=0, k=k_j). All links then have a flow of 0, so the average flow also equals 0. However, the flow matching the average density, somewhere between $0 < k < k_j$ in the fundamental diagram is larger than 0.

11.2.1 Approaches to include the standard deviation

It hence makes sense to include the spread of the densities as second explanatory variable besides the average density.

One can show show the effects of the network dynamics is by performing a traffic simulation study and analysing the results (Knoop et al., 2015). One of the simplest network setups is a Manhattan grid network (one way streets in a square grid) with periodic boundary conditions. Periodic boundary conditions mean that traffic exiting at the right of the network enters at the left and traffic from exiting at the top enters at the bottom and vice versa. The network structure is shown in figure 11.7.

When the traffic starts to run, various distributed bottlenecks become active. This is shown in figure 11.8b. After some time (figure 11.8d-f), traffic problems concentrate more and more around one location. The number of vehicles

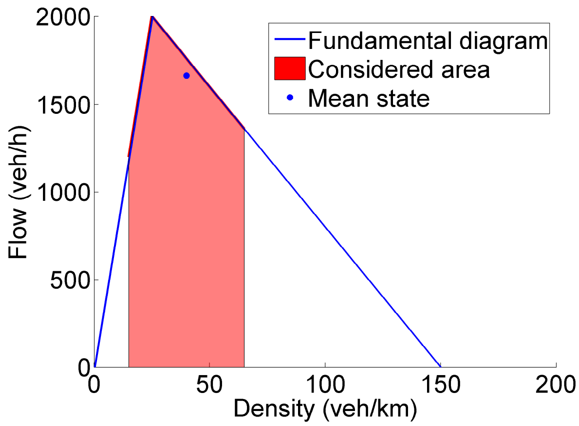


Figure 11.6: The construction of the average traffic state for two states on a triangular fundamental diagram.

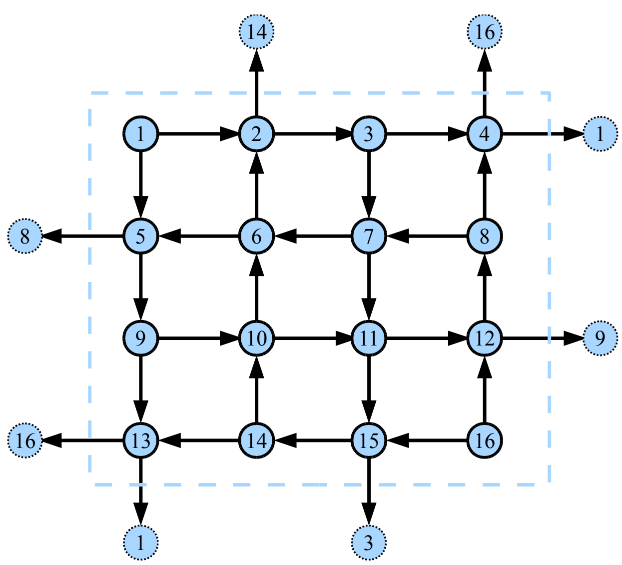
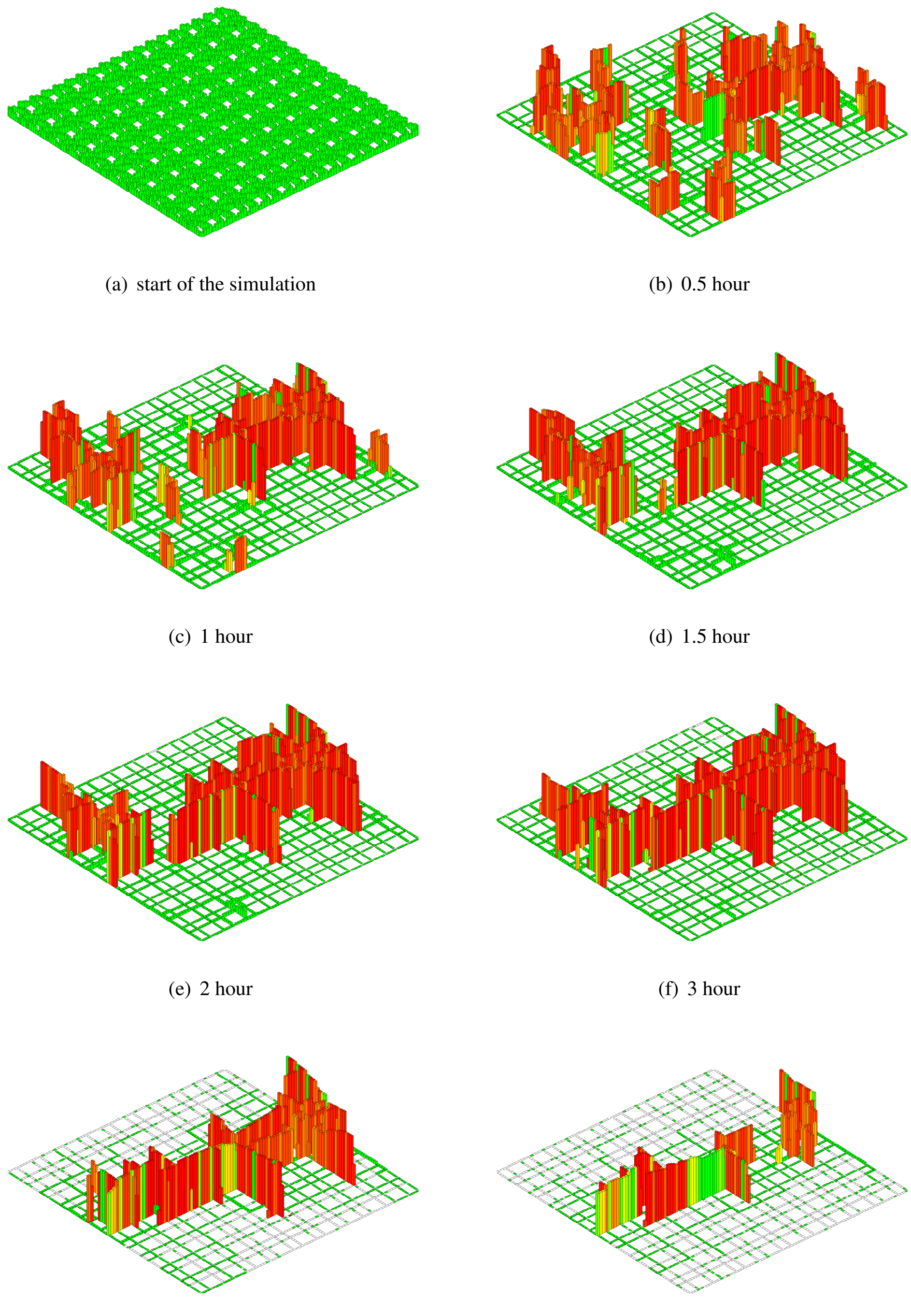


Figure 11.7: Illustration of a 4x4 grid network with periodic boundary conditions



(a) start of the simulation

(b) 0.5 hour

(c) 1 hour

(d) 1.5 hour

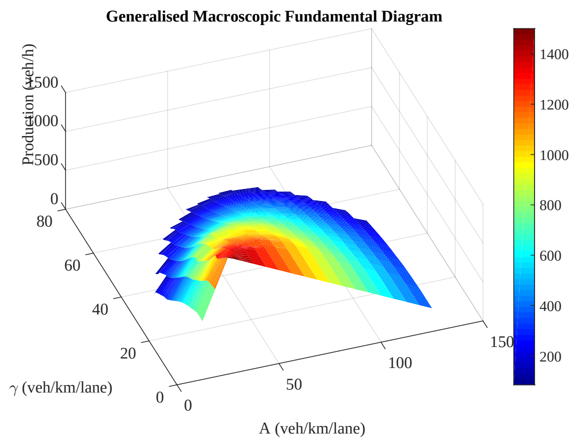
(e) 2 hour

(f) 3 hour

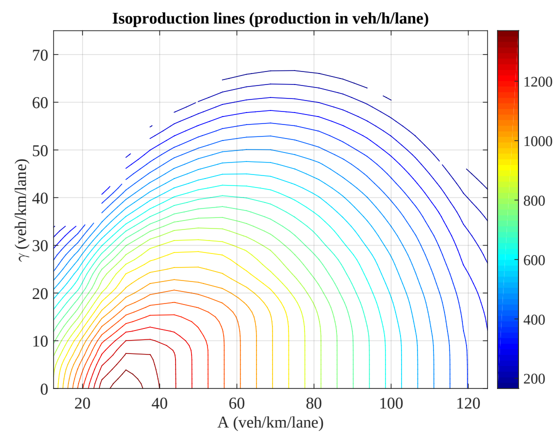
(g) 3.5 hour

(h) 4 hour

Figure 11.8: **Evolution of the densities (bar heights) and speeds (colours) in the network**



(a) The generalised macroscopic fundamental diagram



(b) Iso-production lines in the accumulation-inhomogeneity of density plane

Figure 11.9: The generalised macroscopic fundamental diagram

in the rest of the network reduces, ensuring free flow conditions there. This complete evolution can be found in figure 11.8a-f.

This indicates that with this second explanatory variable, the traffic flow can be approximated reliably. We thus define a concept, the generalised fundamental diagram (GMFD) which predicts the production as function of the accumulation and the standard deviation of density (Knoop et al., 2015)

$$P = P\left(A, \sigma_A\right). \tag{11.1}$$

Figure 11.9(a) shows the GMFD. Figure 11.9(b) shows the same surface, but now in isoproduction lines. The production decreases once the inhomogeneity increases, and that this holds for every accumulation.

11.3 Simulation

Since there is a relation between the number of vehicles in a zone and the outflow, also a dynamic simulation model can be developed. This is a recent development, see e.g. Knoop and Hoogendoorn (2015) for the Network Transmission Model. In this model, an area is divided into zones, and the traffic flow is calculated using the MFDs.

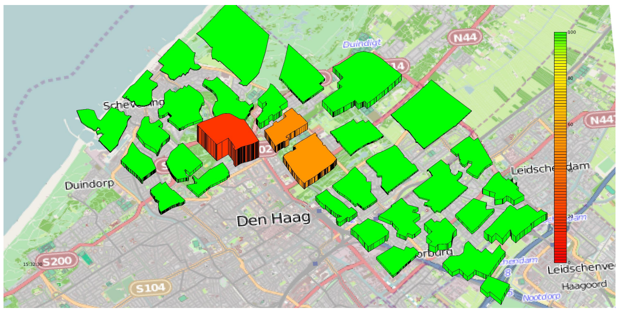
In each of the MFDs, the principle as described in the beginning of this section is implemented, where the accumulation determines the outflow. The interesting part of this concept is that, contrary to the isolated example in 11.1, the queuing outside the protected network also influences the traffic flow in that zone. Hence, there is a trade-off between letting vehicles into a protected zone, delaying the protected zone, or letting them wait outside, delaying the vehicles outside. Also, routing around the protected zone is an option, at the cost of more traffic outside (Hajiahmadi et al., 2013).

This model has been applied for the city of The Hague (Knoop et al., 2016), with a good result, showing that the rough approach works in giving the approximately right results for traffic states.

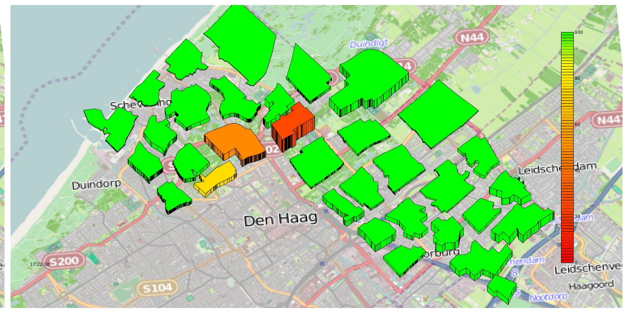
11.4 Recent insights

The field of the MFD is actively studied. The simulation models of which one example is described above in section 11.3 are actively studied, but there are other approaches as well. The division of space into different zones is actively studied, since one would like to have homogeneous areas, see Ji and Geroliminis (2012)

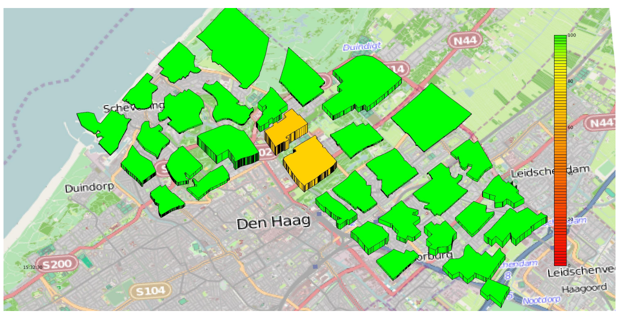
An interesting development is to estimate the MFD from scratch, without measurements. For fundamental diagrams, such estimates are possible. Laval and Castrillón (2015) uses an interesting technique to do the same for a



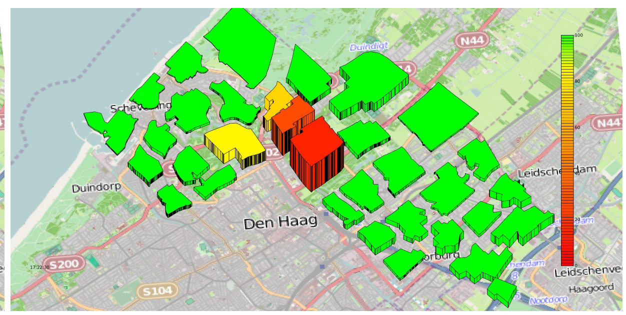
(a) Initial parameters, first part of simulation



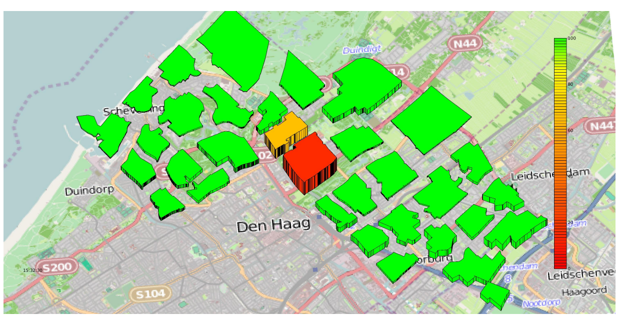
(b) Initial parameters, last part of simulation



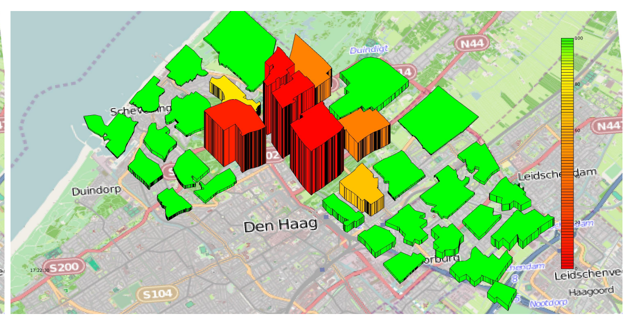
(c) Adapted parameters, first part of simulation



(d) Adapted parameters, last part of simulation



(e) Adapted parameters, accident, first part of simulation



(f) Adapted parameters, accident, last part of simulation

Figure 11.10: Screen shots of the simulation. The color indicates the speed relative to the free flow speed, and the height the accumulation, figure from Knoop et al. (2016)

zone, simplified to a ring road with random traffic lights. This paper follows other work, for instance by Leclercq and Geroliminis (2013).

Finally, the route length is not constant. Leclercq et al. (2015) indicates that the traffic patterns might lead to people using a different, non-shortest, route, thereby influencing the relationship performance-production. All in all, some aspects of the MFD still need studying, but the concept is perceived as promising.

Selected problems

For this chapter, consider problems: 26, 27, 86, 87, A.6.5, 159, 160, 161, 166, 205, 217