Chapter 12

Gap-acceptance theory and models

Summary of chapter - In traffic it happens rather frequently that a participant (a car driver, a pedestrian, a cyclist) has to 'use' a gap in an other traffic stream to carry out a manoeuvre. Examples are: crossing a street as a pedestrian; overtaking at a road with oncoming vehicles; entering a roundabout where the circulating vehicles have priority; entering a motorway from an on-ramp; a lane change on a motorway; etc.

In this chapter the process of gap-acceptance will be discussed for an overtaking manoeuvre on a two-lane road. It will be assumed that the sight distance along the road is sufficient for an overtaking. Consequently only the oncoming vehicles are impeding a desired overtaking manoeuvre. As an exercise the reader is invited to investigate which modifications are needed when the discussion is concerning a pedestrian that has to cross a road with vehicles having priority.

List of symbols

s_i	m	distance headway of vehicle i
T_r	s	reaction time
v_i	-	speed of vehicle i
τ	s	reaction time
a_i	m/s^2	acceleration of vehicle i
x_i	m	position of vehicle i
κ	$1/s$	sensitivity
u	-	mean speed

12.1 Gap acceptance at overtaking

In general terms the gap acceptance process preceding an overtaking can be described as follows: drivers that want to make an overtaking estimate the 'space' they need and estimate the available 'space'. Based on the comparison between required and available space, they decide to start the manoeuvre or to postpone it. The term space is deliberately somewhat vague; it can be expressed either in time or in distance.

Definition 57 *The* required space *is dependent on characteristics of the driver, the vehicle and the road.*

Definition 58 *The* available space *is dependent on the characteristics of the on-coming vehicles and the vehicle to be overtaken (the passive vehicle).*

Drivers have to perceive all these characteristics, process them and come to a decision. Humans differ a lot in perception capabilities, e.g. the ability to estimate distances can vary

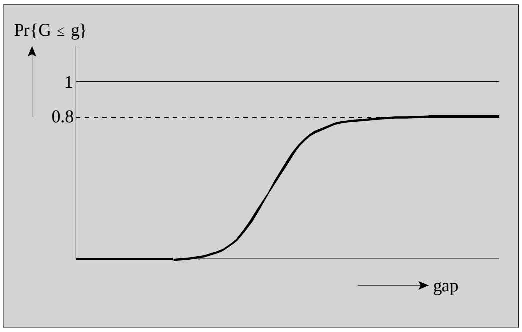

Figure 12.1: Example of a critical gap distribution with a part of the population that never carries out an overtaking, irrespective of how large the gap is.

substantially between persons, and they differ in the acceptation of risk. The total acceptance process is dependent on many factors of which only a subset is observable. This has lead to the introduction of stochastic models.

12.2 Model with a fixed critical gap for each driver

A much used model is the following: it is assumed that each driver has his/her own critical gap, i.e. gaps smaller than the critical gap are rejected and on the other hand gaps larger than the critical value are accepted. The critical gap of a driver is a constant value in a given situation and not dependent on e.g. the time a driver has already been waiting for an opportunity to carry out the overtaking (impatience is not taken into account).

Different drivers have different critical gaps and in a given situation it is assumed these critical gaps have a specific distribution. Probability densities that might be suitable should have a long tail to the right and should be limited to the left. The tail represents the careful drivers that need a long gap and the left limit is caused by more objective and mechanical factors. Suitable distributions are the *Log-normal distribution* and the *Logit distribution.* The Logit distribution of gap g is:

$$\Pr\{G \leq g\} = F(g) = \frac{1}{1 + \exp[c_1 - c_2 g]} \quad \text{with parameter } c_2 > 0 \tag{12.1}$$

Especially for overtaking on two-lane roads a distribution as sketched in Fig. 12.1 might be appropriate. In the example around 20% of the drivers have an infinite critical gap, in other words they never carry out an overtaking.From observation of rejected and accepted gaps it is possible (in principle) to derive the *gap acceptance function.* This function describes the probability that an arbitrary driver accepts an *offered gap*, i.e. starts the overtaking manoeuvre. From this gap acceptance function can be derived the distribution of critical gaps. In the special case that one uses only one observation per driver, i.e. only one accepted or rejected gap, the gap acceptance function is equal to the distribution of critical gaps.

$$\begin{aligned} Pr\{\text{acceptance gap} &= g\} && (12.2)\\ &= Pr\{\text{an arbitrary driver has a criticalgap} < g\} && (12.3)\\ &= \text{Distribution of critical gaps} && (12.4) \end{aligned}$$

If one uses more than one observation per driver, e.g. 2 rejected gaps and one accepted, then *this equality is no longer valid*. The observed gaps are far from independent and this necessitates special procedures.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
class (mean gap)	distr. crit. gaps	# offered gap	# acc. gaps	distr. acc. gaps	# rej. gaps	distr. rej. gaps
5	0.00	0	0	0.00	0	0.00
7	0.00	30	0	0.00	30	0.50
9	0.33	30	10	0.11	20	0.83
11	0.67	30	20	0.33	10	1.00
13	1.00	30	30	0.66	0	1.00
15	1.00	30	30	1.00	0	1.00
Sum		150	90		60	

Table 12.1: Main characteristics of the gap acceptance process

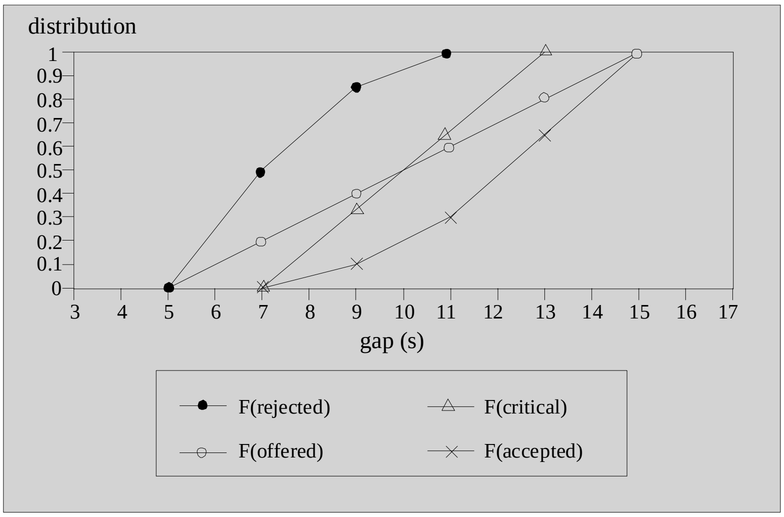

Figure 12.2: Illustration of several distribution functions that plan a role in the gap acceptance process

12.3 Example to illustrate the concepts

Table 12.1 and Fig. 12.2 illustrate the concept of the distributions of critical gaps, rejected gaps and accepted gaps. We have assumed a uniform distribution for the critical gaps; the distribution function is a straight line from 0 to 1 between 7 and 13 s. The offered gaps also have a uniform distribution, but the range is different, between 5 and 15 s.

Column (4), the number of accepted gaps, equals the product of the number of offered gaps, column (3), and the distribution function of the critical gaps, column (2). The total number of accepted gaps is 90. The distribution of the accepted gaps, column (5), is the quotient of the cumulative number of accepted gaps and the total of 90.

The number of rejected gaps, column (6), is the number of offered gaps, column (3) minus the number of accepted gaps, column (4). The distribution of the rejected gaps follows from column (6), just as column (5) from column (4).

The table and the graph show clearly there is a difference between the distribution of the accepted gaps and the distribution of the critical gaps. However, in this simple example the fraction of accepted gaps is equal to the distribution of critical gaps.

Remark 59 *In the American handbook Highway Capacity Manual (HCM) (edition 1985, Ch.*

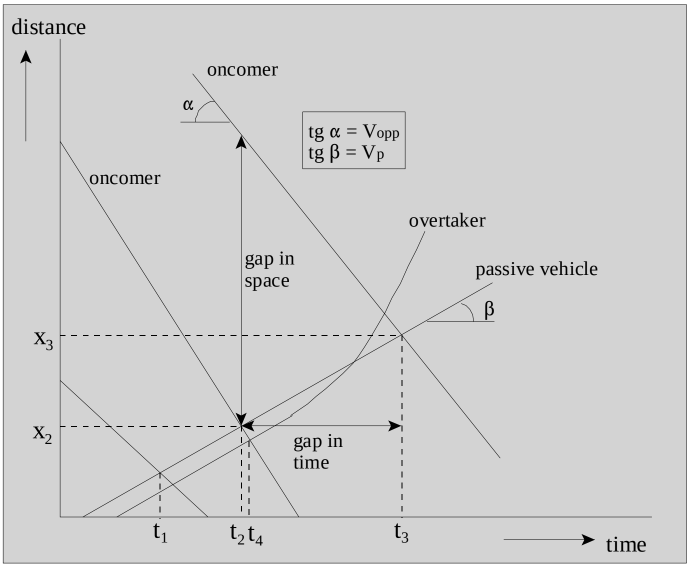

Figure 12.3: Accelerating overtaking manoeuvre in time-space plane

5 page 10; [4]) the critical gap is defined as: "the median time headway between two successive vehicles in the major street traffic stream that is accepted by drivers ...".
It is obvious that in the example presented here, this definition would lead to a biased estimate. The median (50 percentile) of the accepted gaps is around 12 s and the median of the critical gaps is 10 s. The crucial point is that characteristics of accepted gaps, depend on the gaps offered, that is on the traffic situation. A good method to determine critical gaps should always be based on both accepted and rejected gaps. In the HCM of 2000 this point has been improved.

12.4 Overtaking manoeuvre in time-space plane

After the acceptance of a gap follows the overtaking manoeuvre itself. During the manoeuvre the driver can deviate from his/her intended behaviour. He/she can for instance detect that the oncoming vehicle approaches faster than anticipated or that the acceleration of his/her car is not as good as expected. In a critical case the driver can decide to abort the manoeuvre and go back to his/her original (relative) position. Of overtakings carried out the characteristics ‘used distance’, ‘used time’, and the ‘margin’ that is left at the end of the manoeuvre are important.

Fig. 12.3 exhibits an overtaking manoeuvre in the time-space plane by using schematised vehicle trajectories. It is a so-called accelerative overtaking manoeuvre at which the overtaker has the same speed as the vehicle being overtaken at the start of the manoeuvre.

The gap expressed in time is usually defined as the duration of the interval between two successive meetings of the passive vehicle and oncomers. This interval equals the relative headway of oncomers as observed from the passive vehicle. It is the absolute value of the headway of the oncoming stream divided by $1 + v_p/v_{opp}$; index $_p$ of passive and *opp* of oncoming or opposing vehicle. The derivation of this relation is left to the reader.

In Fig. 12.3 $t_2 - t_1$ is a rejected gap and $t_3 - t_2$ an accepted gap. These gaps can not be observed directly by the overtaking driver. He/she has to estimate the distance to the oncoming vehicle and its speed and process this to get an estimation of the available gap, expressed in time.

If the driver can not estimate the speed of the oncomer, it is likely he/she bases his/her decision whether to overtake on the distance to the oncomer. Consequently this distance can also be considered to be the gap that is either rejected or accepted.

Remark 60 *Sometimes the distance covered by the passive vehicle during the gap in time, that is x_3 minus x_2, is taken as the gap in distance.*

Remark 61 *The gap in time, as defined earlier, is not completely usable for the overtaking. The nett gap starts at moment t_4, when the oncomer meets the overtaker. Moreover, the overtaker has to be back on the right lane before moment t_3 in order to prevent a collision. In reality the difference between the nett and the gross gap is not very large.*

Remark 62 *Besides the accelerative overtaking also flying overtaking manoeuvres are possible, at which the overtaking vehicle (hardly) changes its speed. Flying overtakings are difficult to investigate and to observe, e.g. the definition of the gap is not obvious.*

12.5 Studies into overtaking behaviour

Because the overtaking manoeuvre on two-lane roads is important, as well regarding traffic operation as safety, much research has been carried out. These studies can be roughly divided into:

- Experiments in laboratories concerning the perception abilities of drivers. The traffic situation is sometimes strongly schematised, e.g. two moving points on a tv screen represent the rear lights of a car at darkness.
- Experiments with vehicle simulators in a laboratory; an example is shown in Fig. 12.4.
- Experiments on laboratory roads or test tracks. In that case the situation is more realistic than in a vehicle simulator and it is still possible to control the traffic situation and offer drivers specific overtaking tasks.
- Observations in real traffic, sometimes with an instrumented vehicle and sometimes only observed from 'outside'. In those field studies one just has to wait how many interesting events do occur. Moreover the measurement of many relevant variables can be rather problematical.

The main results of the perception studies are:

- Drivers have a rather precise feeling for the speed they drive and the possibilities of their vehicles in terms of accelerations and decelerations they are prepared to apply or normally use.
- Drivers can estimate distances fairly accurate.
- In contrast the estimation of the speed of an oncoming vehicle is hardly possible for human beings. This can be understood: an acceptable time headway is around 20 s; an oncomer with a speed of 90 km/h then is at a distance of 500 m. ($20 \times 90 / 3.6$)

It is assumed that drivers estimate the distance of the oncoming vehicle and assume a value of the speed (e.g. based on the character of the road, their experience, the type of vehicle). They somehow deduce an available time from these perceived variables. Knowing this, it is not a surprise that drivers do differ so much in their gap acceptance.

The Daimler-Benz simulator in Berlin is an advanced driving simulator and uses computer-generated images. It consists of the following elements:

- A cylindrical-shaped projection dome with a diameter of 7.40 m.

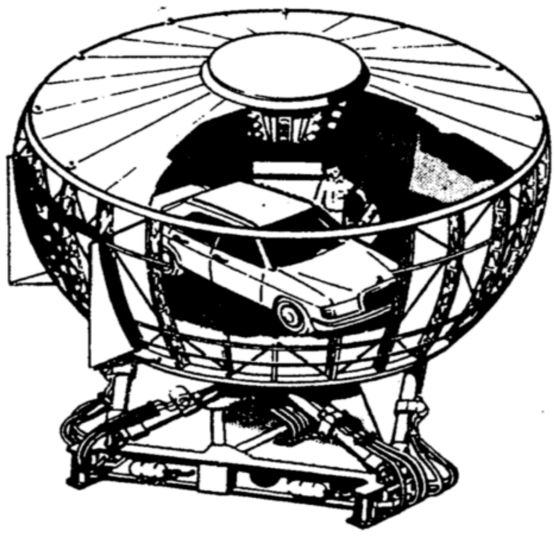

Figure 12.4: The Daimler-Benz vehicle simulator

- A complete car, a Mercedes 19O E, is positioned inside the dome in order to simulate the driver's immediate environments as realistically as possible.
- A moving base with six degrees of freedom and a maximum permissible acceleration in excess of 1g. The system allows for translatory motions of ±1.50 m and rotations of : ±33 to 45°.
- A projection system giving a visual field 33° vertically and 180° horizontally. The system comprises six video projectors.
- A digital colour picture unit (256 colours) with a library of up to 3000 basic shapes. The picture can be updated 50 times a second.
- A sound system capable of simulating noise from car engines, tires, wind, etc.
- A dynamic vehicle model that is powerful enough to be used in vehicle design.

12.5.1 Calculated required space

Based on simple mechanical rules and some empirical facts Brilon [8] has calculated the required distance to the oncoming vehicle for an accelerative overtaking manoeuvre. Important assumptions are:

- drivers use the full acceleration capabilities of their vehicles. These are dependent on the vehicle type and the speed, which in this case is the speed of the passive vehicle, $v_{p(assive)}$.
- overtakers keep accelerating until their rear side is at the same position as the front of the passive vehicle.

The following cases have been distinguished:
A: v_{opp} = 120 km/h and the overtaker respects no speed limit;
B: v_{opp} = 100 km/h and the overtaker limits its speed to 120 km/h.

It appeared that in reality most drivers use longer distances than are required according to the 'mechanical' model. This is representative for a general fact: very few drivers use the full accelerative and braking possibilities of their vehicles.

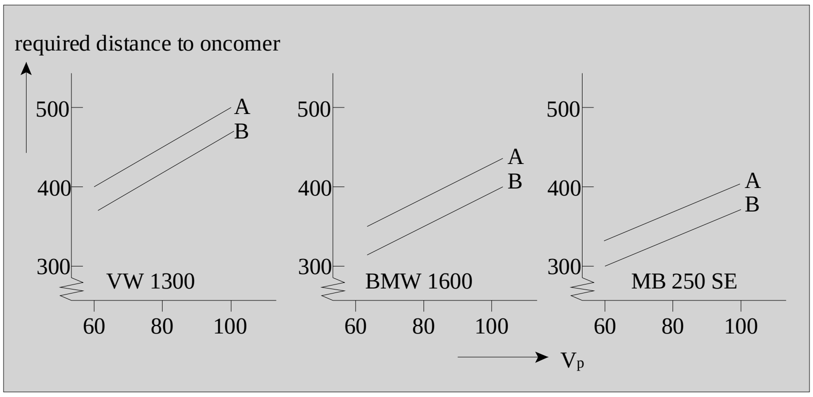

Figure 12.5: Required distance oncomer for three vehicle types and two situations

12.5.2 Studies with vehicle simulators

In the Netherlands both TNO 'Human Factors Research Institute' ('Technische Menskunde') in Soesterberg and the 'Verkeerskundig Studiecentrum' in Groningen have a modern vehicle simulator at their disposal. Advantages of studies with such a tool are:

- conditions can be controlled; e.g. one can expose different test persons to exactly the same conditions and in that way assess differences between drivers; on the other hand one can offer the same person the same conditions on different times and investigate consistency of driver behaviour.
- nearly all variables and conditions can be measured precisely.
- it is possible to investigate dangerous situations.
- it is possible to investigate new driver support systems, e.g. how will drivers react to intelligent cruise control (what distance do they prefer; when do they overrule the automatic system; etc.).

However, there are also disadvantages of using vehicle simulators and these are focussing on the 'validity' of the results. The main point always is: will test persons behave as they do in reality; see e.g. [31]. Especially when investigating dangerous situations, drivers might always be aware of the fact that the danger is not real.

The Mecedes Benz vehicle simulator has been used to investigate overtaking behaviour. The test drivers complained about:

- there were no mirrors on the vehicle to look backward;
- the scenery surrounding the car was very dreary;
- soon after each overtaking had been carried out, there appeared a new slow vehicle in front. This was done because it increased the efficiency of the experiment, i.e. it took less time. Using the simulator is very expensive.

12.6 Estimation of critical gap distributions

Many different methods to estimate the distribution of critical gaps, from observations of the gap acceptance process in reality, can be found in the literature. In a recent study [9] the most used methods have been compared using simulated data (only in this situation one knows the real outcome). It appeared that many methods with a good reputation failed the test. However, the Maximum Likelihood (ML) estimation method did not fail and will be discussed.

Suppose a driver successively rejects gaps of 3, 9, 12 and 7 s and accepts a gap of 19 s. The only thing one can conclude from these observations is that this driver has a critical gap between 12 and 19 s. Stated in other words: the critical gap can not be observed directly. Secondly it can be concluded that only the maximum of the rejected gaps is informative for the critical gap; the smaller gaps are rejected by definition (we assume a consistent driver).

The unknown distribution of the critical gaps is denoted as $F(g;\theta)$ with θ is a vector of parameters. (e.g. for a Normal distribution $\theta = [\mu, \sigma]$). Denote the maximum rejected gap by g_1 and the accepted gap by g_2. The probability that the pair (g_1, g_2) occurs, is proportional to the probability that the critical gap is between g_1 and g_2 ; that is: $F(g_2) - F(g_1)$[1]. For each observed driver i we have available a pair $[g_{1,i}, g_{2,i}]$. The combined probability of occurrence, i.e. the likelihood function, is:

$$L(g_{1,i}, g_{2,i}; \theta) = \prod_{i=1}^{n} \left(F(g_{2,i}; \theta) - F(g_{1,i}; \theta) \right) \tag{12.5}$$

The maximum likelihood procedure implies that the function L (or the logarithm of L which is usually much easier to maximize) must be maximized by varying the parameter θ; the value of θ at the maximum is the likelihood estimator.

12.7 Practical results

12.7.1 Case 1. Overtaking of long trucks

Fig. 12.6 shows the result of an Australian study [55] of overtaking behaviour of cars that overtake a long or even extra long truck. The passive vehicle (the long truck) was equipped with video camera's and other apparatus to record the overtaking behaviour and the gaps offered.

From the figure it is clear that the distribution of the critical gaps is situated more to the left than the gap acceptance function. Besides the distribution of the accepted gaps, the distribution of the overtaking times are depicted. The differences between the gaps used and the critical gaps are the margins drivers have who carry out an overtaking at their critical gap.

12.7.2 Case 2. Left turn movement at intersection, taking into account many factors

At such a manoeuvre the critical gap is much smaller (order of 5 s) than at an overtaking and the driver is able to take more factors into account in his/her decision process. A recent study in Canada [1] is a good example of such a study; see Fig. 12.7 and Table 12.2.

12.8 Conclusions and main points

- Gap acceptance is an important element of the traffic processes; it is relevant for operational aspects and safety.
- Differences between drivers are relatively large.

[1] The probability of pair (g_1, g_2) also depends on the distribution of the offered gaps, but that distribution is not dependent on the unknown parameters θ and consequently is not relevant for the estimation procedure.

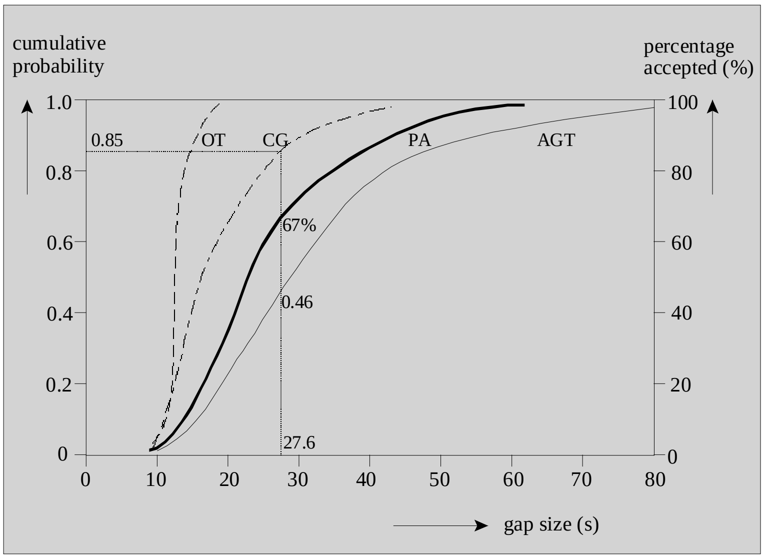

Figure 12.6: Results of overtaking study with a truck of 20 m length at a speed of 70 km/h as passive vehicle; AGT = accepted gaps; PA = percentage accepted = gap acceptance function; CG = critical gaps; OT = overtaking times

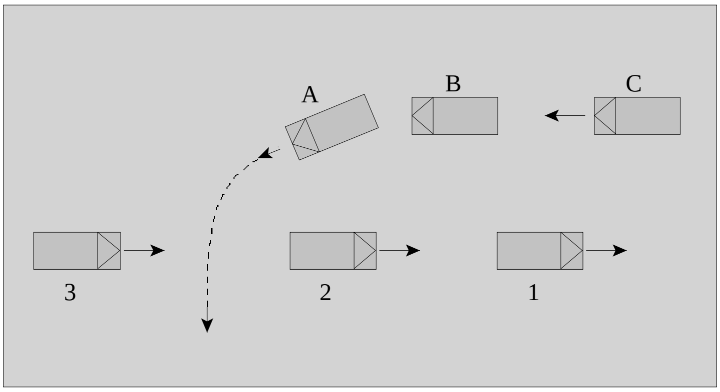

Figure 12.7: Gap acceptance process at left turn

Factor	Effect on gap acceptance
Gap in time	The most explaining factor if considering only one factor
Gap in distance and speed of oncomer	The most explaining combination of the set of explaining factors
Type of oncomer	In sequence: Motorbike-Truck-Car-Bicycle, the gap decreases
Presence vehicle behind (traffic pressure)	If present, then accepted gap $-0.2s$
Delay in queue (not being in front yet) Delay in front position Accelerative power	$10s$ more $\rightarrow$ accepted gap $-0.2s$ No monotone effect; gaps increase for delay $< 30s$ and decrease for larger delay No effect
Gender driver	If woman, then gap $+0.5s$
Age driver	No effect
Presence of passengers	If present, then gap + 0.3 s

Table 12.2: Main characteristics of the gap acceptance process

- A relatively simple assumption - each driver has his/her own constant critical gap in a given situation - implies already complex models and data analyses. Simulation models offer the opportunity to investigate more realistic assumptions and their implications.
- Data from gap acceptance in real traffic are difficult to collect. Vehicle simulators in laboratories are becoming more realistic and consequently can partially replace field studies.