

Chapter 2

Headway models

This section discusses headway models. In this chapter, the headway is always the time headway.

2.1 Relation between headway and capacity

The headway distribution describe which headways can be observed with which probability. One might argue that this is related to the flow, since the average headway is the flow. The flow is determined by the demand. However, in the bottleneck the flow is determined by the minimum headway at which drivers follow each other. Or in other words, the headway distribution determines the capacity in the bottleneck. Suppose the headway distribution is given by $P(h)$. Then, the average headway can be determined by

$$\langle h \rangle = \int_0^{\text{inf}} hP(h)dh \quad (2.1)$$

This is a mathematical way to describe the average headway once the distribution is given. The flow in the active bottleneck, hence the capacity is inverse of the mean headway, see table 1.1:

$$\cap = 1 / \langle h_{\text{bottleneck}} \rangle \quad (2.2)$$

The number of vehicles arriving in a certain period could be a useful measure. This holds for instance for traffic lights, where the number of arrivals per red period is relevant. As illustrated in figure 2.1, there could be different lanes for different directions at a traffic light. The idea is that the traffic towards one direction will not block the traffic to other directions, hence, the length should be long enough to allow the number of vehicles in the red period. The average number of vehicles in a red period can be determined from the flow. However, mostly requirements are that in $p\%$ of the red times (under a constant demand) the queue should not exceed the dedicated lane. In that case, the headway distribution can form the basis for the calculations.

2.2 Arrivals per interval

This section describes the number of arrivals per time interval. For different conditions, this distribution is different. Table 2.2 gives an overview of the distributions described in this section, and gives some characteristics. One should

Table 2.1: The different processes and the underlying assumptions

Headway dist	Process characteristic	Dist of nr of arrivals per interval
Exponential mean(h)=std(h)	Independent arrivals	Poisson
	Correlated arrivals	Binomial
	Negatively correlated arrivals	Negative binomial

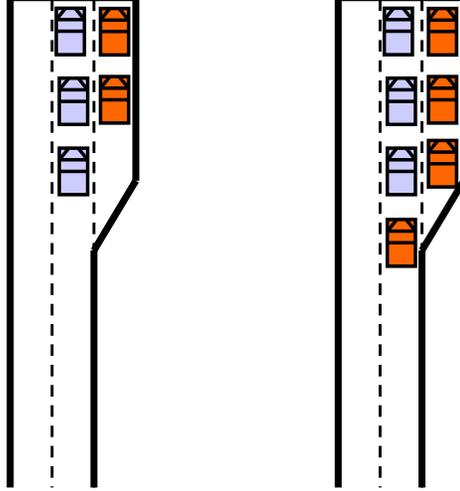


Figure 2.1: A queuing area. The orange coloured vehicles turn right, whereas the blue ones continue straight

Table 2.2: Overview of the means and variances of the different distributions. In this table, q is the flow in the observation period, p is the probability of including the observation in the period.

Distribution	Mean	Variance
Poisson	q	$q = \text{mean}$
Binomial	np	$np(1-p) < \text{mean}$
Negative binomial	$n(1p)/p$	$n(1p)/p^2 > \text{mean}$

differentiate between the probability of number of arrivals (a macroscopic characteristic, based on aggregating over a certain duration of time), described in this section, and the probability of a headway (a microscopic characteristic).

2.2.1 Poisson

The first distribution function described here is the Poisson distribution. One will observe this distribution function once the arrivals are independent (see also section 2.3.1). The resulting probability is described by a so-called Poisson distribution function. Mathematically, this function is described by:

$$P(X = k) = \frac{q^k}{k!} e^{-q}. \quad (2.3)$$

This equation gives the probability that k vehicles arrive if the average arrival rate *per period* is q . Hence note that one needs to rescale q to units of number of vehicles per aggregation period!

Figure 2.2 shows examples of the Poisson distribution. Note that for low values of the flow (expected value smaller than 1), the probability is decreasing. If the flow is higher, there is a maximum at the number of arrivals which is at a higher value than 1.

Figure 2.3 shows the best fits of this distribution on real life data. This distribution is accurate if the flow is low, and is not so good if the flow increases. This is because once the flow is low, the assumption of independent arrivals does not hold any more. Once vehicles are bound by the minimum headway, the arrivals are not independent any more. This restriction come more into play once the flow is high.

2.2.2 Binomial

The binomial distribution can be used if there are correlations between the arrivals. For instance on busy roads, one can expect that more vehicles drive at a minimum headway. Whereas in case of the Poisson distribution, the variance

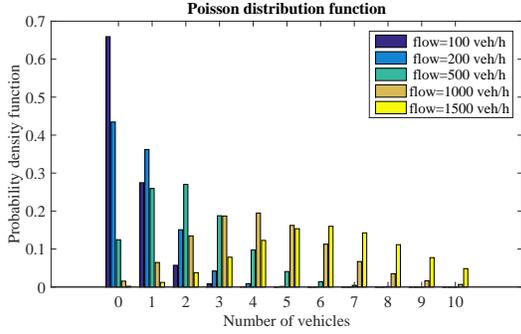


Figure 2.2: Example of the Poisson distribution for different flow values; the flow is indicated in veh/h, and these are the probabilities for arrivals in 15 seconds

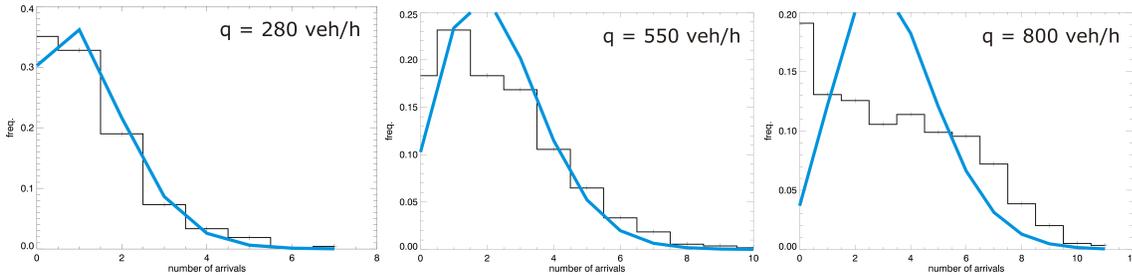


Figure 2.3: Illustration of the number of arrivals from real world data

of the distribution was equal to the mean, in this case the variance is smaller (since more drivers drive at a certain headway). The mathematical equation describing the function is:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \quad (2.4)$$

The idea behind the distribution is that one does n tries, each with an independent success rate of p . The number of successes is k . The mean of the distribution is np , see table 2.2. A certain flow specifies the mean of the arrivals, which hence determines np . This gives a freedom to choose n or p , by which one can match the spread of the function. The number of observations in the distribution can never exceed n , so an reasonable choice of n would be the interval time divided by the minimum headway. Figure 2.4 shows examples of the binomial distribution function.

Note that the variance of the binomial function is smaller than the Poisson distribution for the same flow. This can

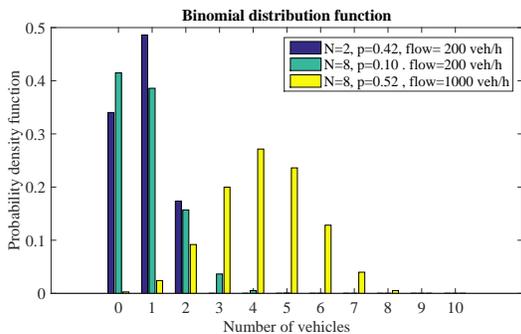


Figure 2.4: Examples of the binomial distribution function for the number of arrivals in an aggregation period

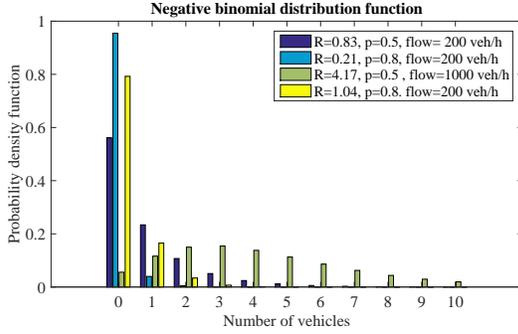


Figure 2.5: Example of the negative binomial function for the number of arrivals in an aggregation period

be a reason to choose this function.

2.2.3 Negative binomial

The negative binomial distribution can be used if there are negative correlations between the arrivals. In traffic this happens for instance downstream of a signalised intersection. If one observes several vehicles at a short headway, one gets a larger probability that next headway will be large because the traffic light will switch to red.

The probability distribution function for the number of observed vehicles in an aggregation interval is:

$$P(X = k) = \binom{k + r - 1}{k} p^k (1 - p)^r \quad (2.5)$$

This distribution describes when one observes individual and independent process with a success rate of p . One observes so until r failures are observed. X is the stochastic indicating how many successes are observed.

Figure 2.5 shows the value of this function for different parameter sets. Note that the variance (and hence standard deviation) be set independently from the mean, like in the binomial distribution function. for this distribution (see table 2.2), the mean is given by $n(1p)/p$, and the variance is given by $n(1p)/p^2$, which is the mean divided by p . Since p is a probability and has a value between 0 and 1, we can derive that the variance is larger than the mean. A larger variance is what one would intuitively expect downstream of a signalised intersection. This characteristic can be used to have an idea of the distribution to use.

2.3 Headway distributions

In this section the exponential headway distribution is described, used with independent arrivals, and composite headway models.

2.3.1 Exponential

The first distribution is the exponential distribution, shown in figure 2.6. This is defined by:

$$P(h|q) = q \exp -qh \quad (2.6)$$

Note that this equation has a *single* parameter, being the flow q . That is the inverse of the average headway. The underlying assumption of this distribution is that all drivers can choose their moment of arrival independently. Consider that each (infinitesimal small) time step a driver considers to leave with a fixed probability. One then gets a exponential distribution function for the headway.

This is a very good assumption on quiet roads, when there are no interactions between the vehicles. The interactions occur once vehicles are limited in choosing their headway, mostly indicated by the minimum headway. An illustration how this works out for a real life case is shown in figure 2.3 for the number of arrivals (see section 2.2).

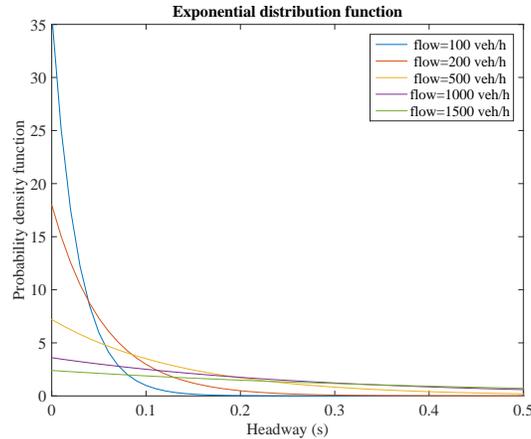


Figure 2.6: The probability density function for headways according to the exponential distribution for different flow values

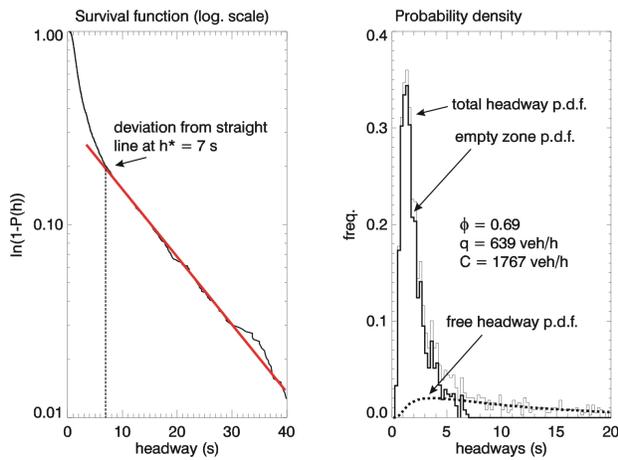


Figure 2.7: Example of the composite headway distribution and its estimation for real life data

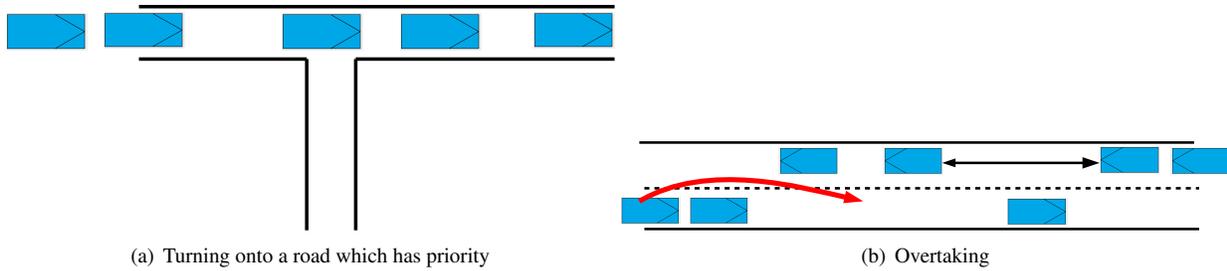
The only good way to test whether the exponential data describes the data one observes is to do a proper statistical test (e.g., a KolmogorovSmirnov test). There are also rules of thumb. A characteristic for this distribution is that the standard deviation is equal to the mean. If one has data of which one thinks the arrivals are independent and this criterion is satisfied, the arrivals are very likely to be exponentially distributed.

This distribution function for the headways matches the Poisson distribution function for the number of arrivals.

2.3.2 Composite headway models

Whereas the exponential distribution function works well for low flows, for higher flows the distribution function is not very good. For these situations, so called composite headway distributions are being used (see for more information Hoogendoorn (2005)). The basic idea is that a fraction of the traffic Φ is driving freely following a headway distribution function $P_{\text{free}}(h)$. The other fraction of the traffic $1 - \Phi$ is driving constraint, i.e., is following their leader, and have a headway distribution function $P_{\text{constraint}}(h)$.

In a composite headway distribution, these two distribution functions are combined. The combined distributino



function can hence be expressed as

$$P(h) = \Phi P_{\text{free}}(h) + (1 - \Phi) P_{\text{constraint}}(h) \quad (2.7)$$

Given the reasoning in section 2.3.1, it would make sense to choose an exponential distribution function for the free headways.

A plot of the headway distribution function (in fact, a survival function of the headway, i.e. 1- the cumulative distribution function) is shown in figure 2.7. The vertical axis is a logarithmic axis. Note that in these axis the exponential distribution function is a straight line. That is what is observed for the large headways. For the smaller headways (in the figure, for less then 7 seconds) this does not longer hold. This is due to the limitation of following distance. As the figure shows, this can be determined graphically.

2.4 Critical gap

The critical gap is the smallest gap (time gap) that a vehicle accepts to go into. Some examples are given in section 2.4.1. The relation with the inflow capacity is discussed in section 2.4.2.

2.4.1 Situations

Examples of situations where the critical gap is relevant, are turning onto a road where one has to give priority, overtaking, merging into the traffic stream or lane changing. Some of them are shown in figure 2.4.1.

The critical gap is defined in the flow. Note that the critical gap differs per person, as well as per situation. One might have a small critical gap to merge into a traffic stream moving at slow speed.

For overtaking, the critical gap (measured in headway in the opposite traffic stream) needs to be larger because the traffic is moving in the opposite direction. For instance, if a driver spends 3 seconds on lane of the opposing traffic for an overtaking manoeuvre, during that time the opposing vehicles also move in the direction of the overtaking vehicle. To find the critical (time) gap, one needs to consider the distance the overtaker uses at the other lane (overtaking time times speed) – this is the space the overtaker would need if the opposing vehicles were stationary. To this, one should *add* the distance the opposing traffic moves in the overtaking time (overtaking time times the speed of the opposing vehicles). This combined distance gap (plus arguably a safety margin) can be translated into a time gap. Since one measures the gap as headway in the opposing traffic, one needs to divide the required space gap by the speed of the opposing traffic.

2.4.2 Inflow capacity

Consider the situation as in figure 2.8(a). Consider that the main line traffic (traffic from left to right) has a headway distribution of $P(h)$, and there is an infinite line of vehicles waiting to enter the road, all with an critical gap gc . One can calculate how many vehicles can enter the road per unit of time, as will be shown below.

We know that for certain gaps, vehicles can enter. The first step is to find the number of gaps which pass per unit of time. That equals the number of vehicles (each vehicle causes one new gap). The number of vehicles per unit of time is the flow, which is the inverse of the mean of the headway, which in turn can be calculated from the headway distribution.

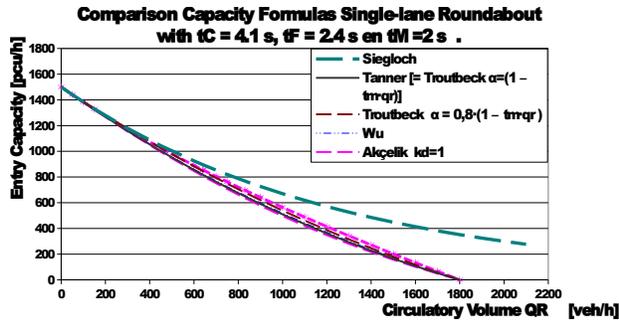


Figure 2.8: The inflow onto a road (or roundabout) as function of the main road (roundabout) traffic – figure from Fortuijn and P. (2015)

Then, for all headways larger than the critical gap, but smaller than twice the critical gap one vehicle can enter. The frequency that this happens is the flow (=gap rate) times the probability that the gap is this size, determined from integrating the probability density function of the headway over the right headways:

$$q_{in1 \text{ vehicle}} = q \int_g c^{2gc} P(h) dh \quad (2.8)$$

The probability that in one gap two vehicles can enter is the integral of the probability density function for the headways from twice the critical gap to three times the critical gap. The rate at which gaps these gaps occur is the flow q , so the rate at which these gaps occur is $q \int_{2gc}^{3gc} P(h) dh$. Per gap, two vehicles enter, so the matching inflow is

$$q_{in2 \text{ vehicles}} = 2q \int_{2gc}^{3gc} P(h) dh \quad (2.9)$$

One can continue this reasoning and find an equation for the inflow:

$$q_{in\text{total}} = \sum_n (q_{in\text{n vehicles}}) = \sum_n nq \int_{ngc}^{(n+1)gc} P(h) dh \quad (2.10)$$

This can numerically be evaluated.

Typically, one would expect the headway in the order of twice the minimum headway. Namely, after merging, one would like to have at least a minimum headway upstream (lag gap) and a minimum headway downstream (lead gap).

In the calculation of the maximum inflow, the headway distribution function plays an important role. As extreme example, suppose that all headways are equal at 99% of the critical gap. Then, the flow is at about half the capacity of the road, but the inflow is 0. For different distribution functions it can be calculated what is the maximum inflow on the main road. This is shown in figure 2.8 for different functions. It shows that the inflow capacity decreases with the main road flow. However, the precise type of distribution matters less: the graphs for the different distribution functions are quite similar.

Selected problems

For this chapter, consider problems: 38, 39, 40, 162, 189, 190, 234

