Chapter 5

Shock wave theory

This chapter describes shock wave theory to capture queuing dynamics. This differs from cumulative curves in the way that the spatial extent of the queue is considered. The section first gives examples on a fixed bottleneck (section 5.1), then gives examples of the moving bottleneck (section 5.2). Section 5.3 describes the stop-and-go waves in this modelling framework.

5.1 Fixed bottlenecks

In this section Fixed bottlenecks are discussed. At fixed bottleneck some lanes of a highway are (temporarily) blocked. Firstly the theory and derivation of equations will be discussed (section 5.1.1). Thereafter, two examples are given (Section 5.1.2 and 5.1.3)

5.1.1 Theory and derivation of equations

Let us consider a situation with two different states: state A downstream, with a matching q_A, k_A en v_A and state B upstream (q_B, k_B en v_B). The states are plotted in the space-time diagram 5.1. We choose the axis in such a way that the shockwave moves through the point t=0 at x=0. We will now derive the equation to get the speed of this wave.

In the derivation, we base the reasoning on figure 5.1. The boundary between is called a shock wave. This wave indicates where the speed of the vehicles changes. It is important to note that there are no vehicles captured in the wave itself: the wave itself does not have a physical length. Thus the assumption is that vehicles change speed instantaneously.

Because there are no vehicles in the wave, the number of vehicles entering the wave must be equal to the number of vehicles exiting exiting the wave, or we can state that rate of vehicles entering the wave must be equal to the rate of vehicles exiting the wave. This principle, in combination with the following equation (already presented in equation 4.3)is used to calculate the speed of the wave;

$$q = kv \tag{5.1}$$

The speed of the wave is indicated by w. Note that we can apply this equation to moving frames of reference as well. In that case, the flow changes, as does the speed. The density is invariant under a change of reference speed.

To determine the attachment and exit rate, we will move with the speed of the wave (in the frame of a moving observer). At the downstream end, the density is k_B. The speed in the moving frame of reference is $v_B - w$. The exit rate in the moving frame of reference is calculated using equation 5.1.

$$q_{\text{exit}} = k_B\left(v_B - w\right) \tag{5.2}$$

In the same way, the attachment rate can be determined. The upstream density is k_A. The speed of the vehicles in the frame of reference moving with the wave speed w is $v_A + w$. Using equation 5.1 again, we find the attachment rate in this moving frame of reference:

$$q_{\text{attachment}} = k_A\left(v_A - w\right) \tag{5.3}$$

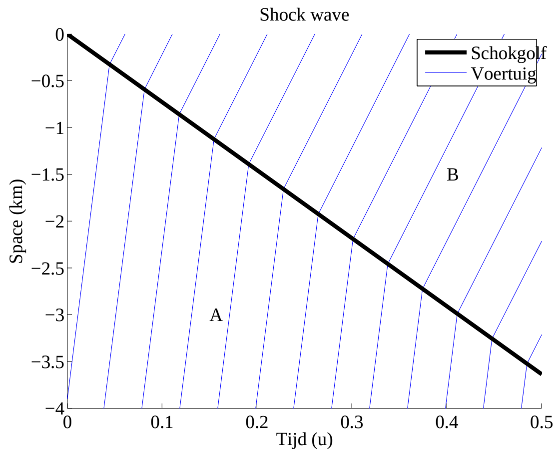


Figure 5.1: A shockwave where traffic speed changes from high to low.

Since these rates have to be equal, we find:

$$q_{\text{exit}} = q_{\text{attachment}} \quad (5.4)$$

$$k_A (v_A - w) = k_B (v_B - w) \quad (5.5)$$

This can be rewritten as:

$$k_A v_A - k_A w = k_B v_B - k_B w \quad (5.6)$$

$$q_A - k_A w = q_B - k_B w \quad (5.7)$$

In the last step, the speeds have been substituted using equation 5.1. We can solve this equation for the shock wave speed w. We find

$$q_A - q_B = (-k_B + k_A) w \quad (5.8)$$

And isolating w give the wave speed equation:

$$\boxed{w = \frac{q_A - q_B}{k_A - k_B} = \frac{\Delta q}{\Delta k}} \quad (5.9)$$

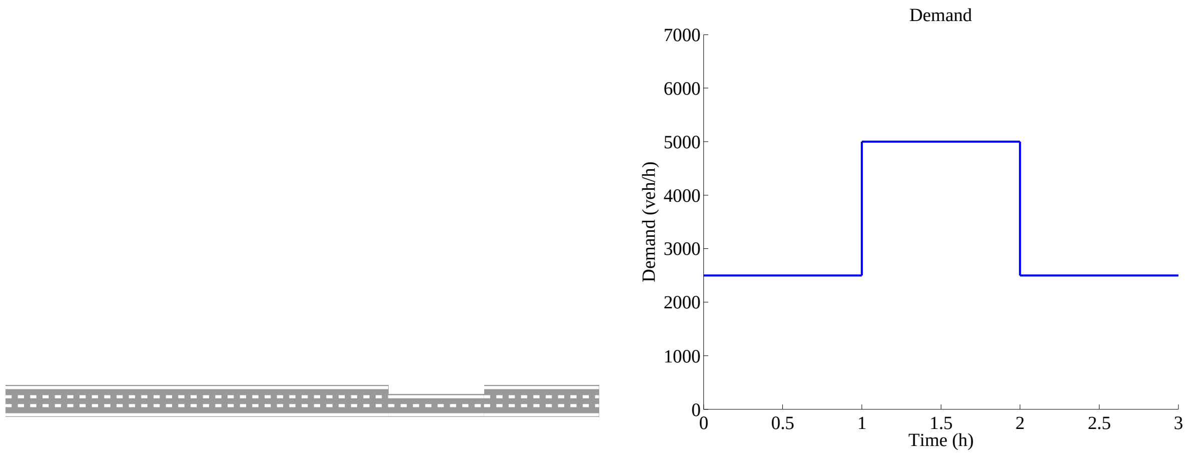
Note that in a space-time plot, the speed w is the slope of the shock wave between A and B. The right hand side is the ratio between the difference in flow and the difference in density of states A and B. This is also the slope of a line segment between A and B in the flow-density plot. This becomes very useful when constructing the traffic states.

The above reasoning holds for the speed of any shock wave, moving backward or forward. The following section shows an example for both.

5.1.2 Example: Temporal increase in demand at a road with a lane drop

Let's consider a 3 lane road with a reduction to 2 lanes over a 1 km section between x=10 and x=12.5 (see figure 5.2(b)). For the road, we assume lanes with equal characteristics, described by a triangular fundamental diagram with a free speed of 80 km/h, a capacity of 2000 veh/h/lane and a jam density of 150 veh/km/lane. At the start of the road, there is a demand of 2500 veh/h which temporarily increases to 5000 veh/h between t=1h and t=2h (see the demand profile in figure 5.2(b)). The question we will answer in this example is: *What are the resulting traffic conditions?*

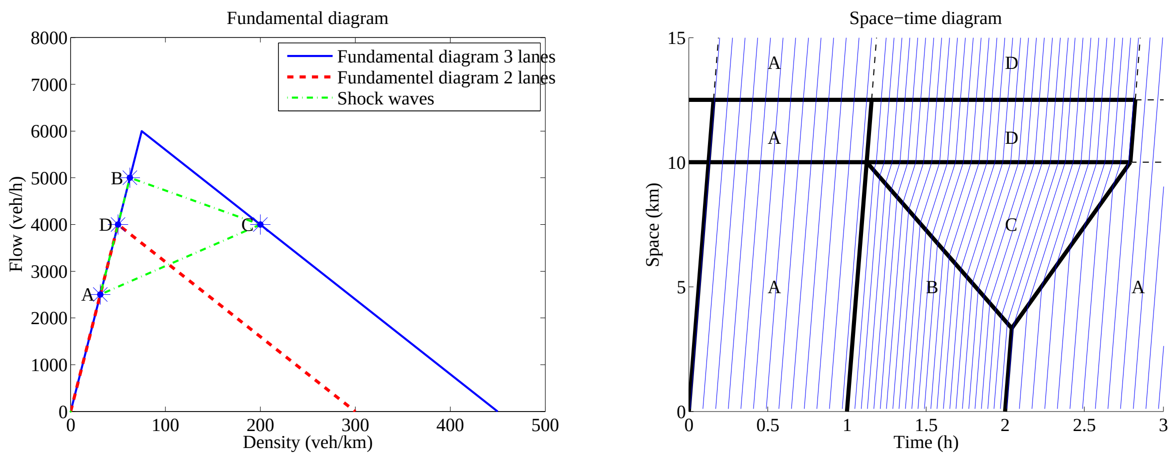
The final answer to the question are the traffic states which are shown in table 5.1, and shown on the fundamental diagram in figure 5.3(a). The speed of the shock waves is given in table 5.2, and the resulting traffic situation is shown in figure 5.3(b). We will now explain how this solution can be found.



(a) The road

(b) Demand pattern

Figure 5.2: Situation



(a) Position of traffic states at the fundamental diagram

(b) Shock waves in space-time

Figure 5.3: The situation

Table 5.1: The states on the road

Toestand	Stroom (vtg/u)	Dichtheid(vtg/km)	Snelheid (km/u)
A	2500	31.25	80
B	5000	62.5	80
C	4000	200	20
D	4000	50	80

Table 5.2: The shock waves present on the road

State 1	State 2	shock wave speed w (km/h)
A	B	80
B	C	-7.3
A	A	8.9

The inflow to the system is given, being 2500 veh/h (state A) and 5000 veh/h (state B) on a three lane road. The matching densities can be computed from the fundamental diagram for a three lane road. This gives densities of (equation 5.1) $k_A = \frac{q_A}{u_A} = \frac{2500}{80} = 31.25$veh/km and $k_B = \frac{q_B}{u_B} = \frac{5000}{80} = 62.5$veh/km. The separation with the empty road moves forward with a speed of 80 km/h, i.e. the speed of the vehicles. This can also be found by the shock wave equation, equation 5.9. The difference in flow is 2500 veh/h and the difference in density is 31.25 veh/km. The shock wave then moves with:

$$w_{0A} = \frac{q_0 - q_A}{k_0 - k_A} = \frac{-2500}{-31.25} = 80km/h \tag{5.10}$$

Note that is does not matter whether the speed of shock wave w_{0A} or shock wave w_{A0} is calculated. This remark holds for every combination of states.

Also the wave between state A and B moves forward with 80 km/h, calculated by equation 5.9:

$$w_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{2500 - 5000}{31.25 - 62.5} = 80km/h \tag{5.11}$$

Note that this equals the free flow speed. This is because we use a triangular fundamental diagram. The speed w_{ab} is the direction of the line segment AB in figure 5.3(a). This has the same slope as the slope of the free flow speed (i.e. the slope of the fundamental diagram at the origin) because its shape is triangular.

Now this wave hits the two lane section. The flow is higher then the capacity of the two lane section. That means that downstream, the road will operate at capacity, and upstream a queue will form (i.e. we have a congested state). The capacity of the downstream part is, according to the fundamental diagram, 4000 veh/h (state D). The speed follows from the (triangular) fundamental diagram, and is 80 km/h. The density hence is:

$$k_D = \frac{q_D}{u_D} = \frac{4000}{80} = 50veh/km \tag{5.12}$$

If 4000 veh/h drive onto the two lane segment, 4000 veh/h have to drive off the three lane segment (no vehicles can be lost or created at the transition from three to two lanes). That means that upstream of the transition, we have a congested state with a flow of 4000 veh/h. The density is derived from the fundamental diagram:

$$q_{\text{cong}} = q_{\text{capacity}} - q_{\text{capacity}} \frac{k - k_c}{k_j - k_c} = 4000 \tag{5.13}$$

Substituting the parameters for the fundamental diagram, and realising that the capacity is found by $q_{\text{capacity}} = l u_{\text{capacity}} k_c$ (l is the number of lanes) we calculate the density at point C, 200 veh/km. Graphically, we can find point C on the fundamental diagram by the intersection of the congested branch and a line at a constant flow value of 4000 veh/h.

The speed at which the tail of the queue moves backwards, i.e., the speed of the boundary between B and C, is calculated by the shock wave equation (equation 5.9)

$$w_{BC} = \frac{q_B - q_C}{k_B - k_C} = \frac{5000 - 4000}{62.5 - 200} = -7.3km/h \tag{5.14}$$

Note that this speed can also be derived graphically from the fundamental diagram, i.e. the slope of the line segment BC. That the slope in figure 5.3(a) and figure 5.3(b) is graphically not the same, is a consequence of different axis scales in the figures.

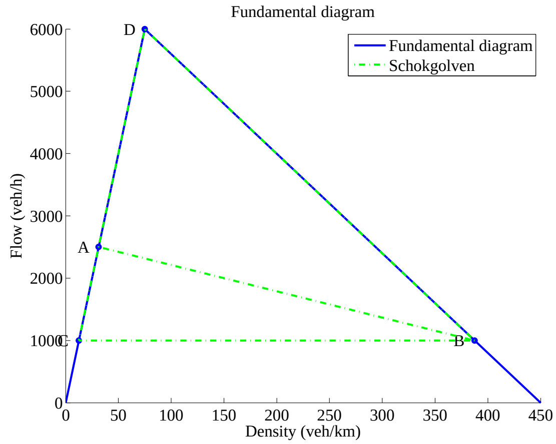
Then the demand reduces again and the inflow state returns to A. When lower demand hits the tail of the jam, the queue can solve from the tail. At the head of the queue, this change has no influence yet, since drivers are still waiting to get out onto the smaller roadway segment. The boundary between C and A moves with a speed of

$$w_{CA} = \frac{q_C - q_A}{k_C - k_A} = \frac{4000 - 2500}{200 - 31.25} = 8.9km/h \tag{5.15}$$

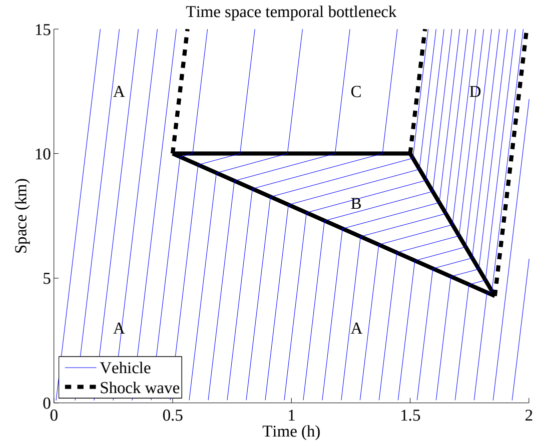
Here again, the speed could also be derived graphically, by the slope of the line segment AC.

Then, this wave arrives at the transition of the two lane road to a three lane road, and the congestion state C is dissolved. In the two lane part, the flow is now the same as the demand (the state in A, 2500 veh/h). The boundary between state D and A moves forward with a speed of

$$w_{DA} = \frac{q_D - q_A}{k_D - k_A} = \frac{4000 - 2500}{50 - 31.25} = 80km/h \tag{5.16}$$



(a) Position of traffic states at the fundamental diagram



(b) Shock waves in space-time

Figure 5.4: The situation

Table 5.3: The states on the road with a temporal bottleneck

Number	Flow	Density	Speed
A	2500	31.25	80
B	1000	387.5	
C	1000	12.5	12.5
D	6000	75	75

5.1.3 Example: Temporal capacity reduction

Another typical situation is a road with a temporal local reduction of capacity, for instance due to an accident. This section explains which traffic situation will result from that.

The case is as follows. Consider a three-lane freeway, with a triangular fundamental diagram. The free flow speed is 80 km/h, the capacity is 2000 veh/h/lane and the jam density is 150 veh/km/lane. The demand is constant at 2500 veh/h. From t=1h to t=2h, an incident occurs at x=10, limiting the capacity to 1000 veh/h. Calculate the traffic states and the shock waves, and draw them in the space-time diagram. Also draw several vehicle trajectories.

For referring to certain states, we will first show the resulting states, and then explain how these states are constructed. Figure 5.4(a) shows the fundamental diagram and the occurring states, 5.4(b) shows how the states move in space and time. The details of the states can be found in table 5.3, and the details of the shock waves can be found in table 5.4.

At the start, there are free flow conditions (state A) at an inflow of 2500 veh/h. For the assumed triangular fundamental diagram, the speed for uncongested conditions is equal to the free flow speed, so 80 km/h. The matching density can be found by applying equation 5.1: $k_A = \frac{q}{u} = \frac{2500}{80} = 31.25$ veh/km.

From t=1 to t=2, a flow limiting condition is introduced. We draw this in the space time diagram. The flow is too high to pass the bottleneck, so the moment the bottleneck occurs, a congested state (B) will form upstream.

Table 5.4: The shock waves present on the road with a temporal bottleneck

State 1	State 2	shock wave speed w (km/h)
A	C	80
B	C	0
A	B	-4.2
B	D	16

Downstream of the bottleneck we find uncongested conditions (once the vehicles have passed the bottleneck, there is no further restriction in their progress): state C. For state C, the flow equals the flow that can pass the bottleneck, which is given at 1000 veh/h. The speed is the free flow speed of 80 km/h, so the matching density can be found by applying equation 5.1: $k_C = \frac{q}{u} = \frac{1000}{80} = 12.5$ veh/km. The speed of the shock wave between state A and C can be calculated using the shock wave equation, 5.9:

$$w_{AC} = \frac{q_A - q_B}{k_A - k_B} = \frac{2500 - 1000}{31.25 - 12.5} = 80km/h \tag{5.17}$$

This equals the free flow speed. Graphically, we understand this because both states can be found at the free flow branch of the fundamental diagram (figure 5.4(a)) and the shock wave speed is the slope of the line segment connecting these states. Because the fundamental diagram is triangular, this slope is equal to the slope at the origin (i.e., the free flow speed).

Upstream of the bottleneck a congested state forms (B). The flow in this area must be the same as the flow which can pass the bottleneck. This is because at the bottleneck no new vehicles can be formed. That means state B is a congested state with a flow of 1000 veh/h. From the fundamental diagram we find the matching density in the congested branch, 387.5 veh/km. The speed at which the shock between states A and B now moves, can be calculated using equation 5.9:

$$w_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{2500 - 1000}{31.25 - 387.5} = -4.2km/h \tag{5.18}$$

The minus sign means that the shock wave moves in the opposite direction of the traffic, upstream. We could also graphically derive the speed of the shock wave by the slope of the line between point A and B in the fundamental diagram.

Once the temporal bottleneck has been removed, the vehicles can drive out of the queue: state D. Because state C is congested, the outflow will be capacity (or the queue discharge rate in case there is a capacity drop). That is for this case a flow of 3x2000 veh/h = 6000 veh/h. Realising the vehicle speed equals the free flow speed of 80 km/h, the density can be found using equation 5.1:

$$k_D = \frac{q_D}{u_D} = \frac{6000}{80} = 75veh/km \tag{5.19}$$

The shock wave between state B and D moves backward. The speed thereof can be found by applying equation 5.9

$$w_{BD} = \frac{q_B - q_D}{k_B - k_D} = \frac{1000 - 6000}{387.5 - 75} = -16km/h \tag{5.20}$$

The negative shock wave speed means the wave moves upstream. Intuitively, this is right, since the vehicles at the head of the queue can accelerate out of the queue, and thus the head moves backwards.

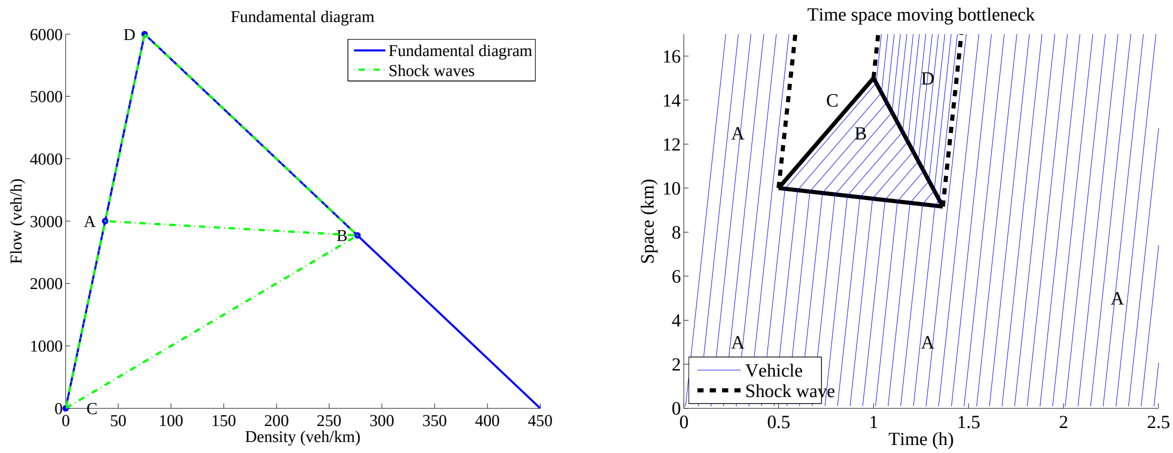
5.2 Moving bottleneck

This section describes what happens if the road is blocked, either completely or not completely, by a moving bottleneck. This can be a slow moving truck or agricultural vehicle, a funeral or wedding procession. First the theory is explained (section 5.2.1), and then three examples follow (section 5.2.2, 5.2.3 and 5.2.4).

5.2.1 Theory

For the moving bottleneck, the same theory applies as for the fixed bottleneck. The recipy is the same: check for each bottleneck weather the demand exceeds capacity. If so, there is congestion upstream and a free flow condition (or capacity) downstream. Once again, the shock wave equation applies (equation 5.9).

Different compared to the regular, fixed bottlenecks, is the position of the congested state. For fixed bottlenecks, the flow upstream of the bottleneck equals the flow downstream of the bottleneck. In case of moving bottlenecks this differs. Consider a bottleneck which moves downstream without any overtaking opportunities. The downstream flow



(a) Position of traffic states at the fundamental diagram

(b) Shock waves in space-time

Figure 5.5: The situation with a moving bottleneck without overtaking opportunities

is zero, but vehicles can accumulate in the growing area between the considered point and the moving bottleneck, so the upstream flow is not zero.

In these types of calculations, the capacity of vehicles passing the moving bottleneck is usually an input. This gives the downstream point at the fundamental diagram. To find the upstream state on the fundamental diagram, one has to realise the shock wave *is* the moving bottleneck, so the speed of the shock wave *must* equal the speed of the moving bottleneck. By constructing a shock wave moving with the speed of the moving bottleneck in the fundamental diagram from the uncongested downstream point, one finds the congested point. This is at the intersection of the line segment starting at the uncongested point moving with the bottleneck speed and the congested branch of the fundamental diagram. The following examples clarify the procedure.

5.2.2 Example 1: moving truck, no overtaking possibilities

Consider a three lane road, where at all three lanes a triangular fundamental diagram holds. The capacity is 2000 veh/h/lane, the free flow speed 80 km/h and the jam density 150 veh/km. The demand is 3000 veh/h. A truck enters the road at t=0.5h and x=10 km, and leaves the road at t=1h and x=15 km, hence driving 10 km/h. There are no overtaking opportunities. What is the traffic state at the road?

The solution of this example is given in a space-time diagram (figure 5.5(b)) and in the fundamental diagram (figure 5.5(a)). The characteristics of the states are given in table 5.5, and the characteristics of the shock waves are given in table 5.6. The explanation how these points are found follows below.

At two points states can be identified at the fundamental diagram (figure 5.5(a)), the intial state A (flow of 2500 veh/h, free flow speed 80 km/h) and the state downstream of the moving bottleneck (given “no overtaking possibilities” hence density zero and flow zero). In these examples, the states are identified by only two variables from the three flow, density and speed, since the third one can be calculated using equation 5.1.

The position upstream of the moving bottleneck can be determined by the intersection of two lines:

$$q_1 = q_C + v\,(k - k_C) \tag{5.21}$$
$$q_2 = q_D + w\,(k - k_c) \tag{5.22}$$

The first equation is a line in the fundamental diagram, figure 5.5(a) starting from point C and moving forward with the bottleneck speed v. The second line is the congested branch of the fundamental diagram, in which w is the slope of the congested branch. The intersection of these lines can be found by solving the density k from the equation $q_1 = q_2$. For this upstream density, the density of state B, we find $k_B = 277veh/km$. The matching flow is found by filling this in either equation 5.21 or 5.21, resulting in $q_B = 2769veh/km$. The speed is determined by the ratio of flow and

Table 5.5: The states on the road for a moving bottleneck without overtaking opportunities

Name	Flow	Density	Speed
A	3000	37.5	80
B	2769	277	10
C	0	0	NaN
D	6000	75	80

Table 5.6: The shock waves present on the road for a moving bottleneck without overtaking opportunities

State 1	State 2	shock wave speed w (km/h)
A	B	-0.96
B	C	10
A	C	80
D	C	80
B	D	-16
A	D	80

density: $u = \frac{q_B}{k_B} = \frac{2769}{277} = 10km/h$. Note that this is the speed of the moving bottleneck. This must be since there are no overtaking opportunities.

The speed of the shock wave between A and B can be calculated using the shock wave equation 5.9 applied to state A and B:

$$w_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{3000 - 2769}{37.5 - 277} = -0.96km/h \tag{5.23}$$

This means the shock wave moves upstream with a low speed. With a different speed of the moving bottleneck or a different demand level, the shock wave might move faster upstream (lower bottleneck speed or higher demand), or it might move downstream (higher bottleneck speed or lower demand).

The speed of the shock wave between state B and C can also be calculated with equation 5.9. It can be determined from reasoning instead of calculations as well: it must be equal to the speed of the moving bottleneck, 10 km/h.

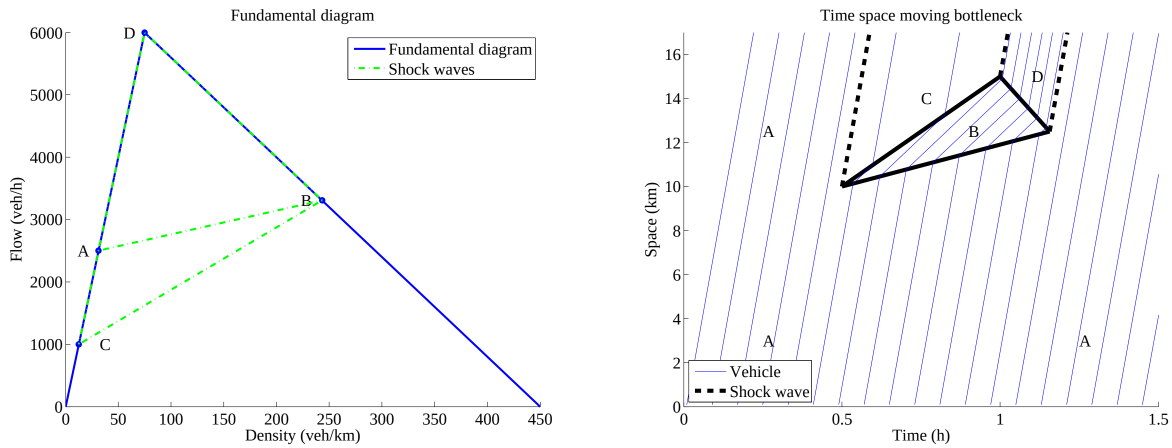
Once the moving bottleneck has left the road, vehicles will exit the jam. This means a boundary between the jam state, state B, at the capacity state, state D. For capacity the flow (6000 veh/h) and speed (80 km/h) can be derived from the road characteristics. The speed of the shock wave between state D and B can be determined by applying the shock wave equation (5.9) on state B and D. Alternatively, from the fundamental diagram in figure 5.5(a) we see that must equal the slope of the congested branch of the fundamental diagram, -16 km/h. Because any shock wave between any congested state and capacity moves with this speed, this speed is also called the wave speed of the fundamental diagram.

Speeds of shock waves between A and C, C and D, and D and A all can be calculated using the shock wave equation 5.9. Moreover, all these states lie on the free flow branch of the fundamental diagram, so the shock waves between these states move at the free flow speed of 80 km/h.

5.2.3 Example 2: moving truck with overtaking possibilities

Now, let's consider a different situation, where overtaking of the moving bottleneck is possible. We change the conditions as follows. Consider a three lane road, where at all three lanes a triangular fundamental diagram holds. The capacity is 2000 veh/h/lane, the free flow speed 80 km/h and the jam density 150 veh/km. The demand is 2500 veh/h. A truck enters the road at t=0.5h and x=10 km, and leaves the road at t=1h and x=15 km, hence driving 10 km/h. There are overtaking opportunities, such that downstream of the bottleneck the flow is 1000 veh/h. What is the traffic state at the road?

The states and fundamental diagram, as well as the resulting traffic states are show in figures 5.6(a) and 5.6(b) respectively. Tables 5.7 and 5.8 give the properties of the states and the shock waves respectively. Below, it is explained how these states and waves are found.



(a) Position of traffic states at the fundamental diagram

(b) Shock waves in space-time

Figure 5.6: The situation at a moving bottleneck with overtaking possibilities

Table 5.7: The states on the road at a moving bottleneck with overtaking possibilities

State	Flow (veh/h)	Density(veh/km)	Speed (km/h)
A	2500	31.25	80
B	3307	243	13.6
C	1000	12.5	80
D	6000	75	80

Table 5.8: The shock waves present on the road at a moving bottleneck with overtaking possibilities

State 1	State 2	shock wave speed w (km/h)
A	B	3.8
A	C	80
D	C	80
B	D	-16
A	D	80

The demand is the same as in the previous example, so state A is the same. Downstream of the moving bottleneck there is a free flow traffic state (it is downstream of the bottleneck, so it is free flow). The flow is given at 1000 veh/h, and since it is in free flow, the speed is 80 km/h. Hence, the density is $k_C = \frac{q_C}{u_C} = \frac{1000}{80} = 12.5$ veh/km.

Since the demand is higher than the capacity of the moving bottleneck, upstream of the bottleneck, a congested state occurs. Is is separated from state C by a shock wave which moves with the speed of the moving bottleneck (it *is* the moving bottleneck). This means we have to find state B in the fundamental diagram which conncects to state C with a line with a slope of 10 km/h; this line is indicated by q_1 (equation 5.24) Furthermore, state B has to lie on the congested branch of the fundamental diagram, indicated by q_2 (equation 5.25). The position upstream of the moving bottleneck can be determined by the intersection of two lines:

$$q_1 = q_C + v\,(k - k_C) \tag{5.24}$$

$$q_2 = q_D + w\,(k - k_c) \tag{5.25}$$

We find the density for state B by $q_1 = q_2$. This results in $k_B = 243$ veh/km. The matching flow can be found by subsituting this into equation 5.24 or equation 5.25, leading to q_B=3307 veh/h.

The shock wave speeds between B and C, as well as the shock wave speed between A and B can be calculated using the shock wave equation, equation 5.9:

$$w_{BC} = \frac{q_B - q_C}{k_B - k_C} = \frac{3307 - 1000}{243 - 12.5} = 10 \text{ km/h} \tag{5.26}$$

$$w_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{2500 - 3307}{31.25 - 243} = 3.8 \text{ km/h} \tag{5.27}$$

Note that the shock wave between B and C moves with the speed of the moving bottleneck.

Once the moving bottleneck leaves the road, the method is exactly the same as in the previous example. The queued vehicles exit state B at the road capacity, state D with a flow 6000 veh/h and a speed of 80 km/h. The shock wave between state D and B moves backward with a speed which can be calculated by the shock wave equation. The shock wave speed must also be equal to the wave speed of the fundamental diagram, so using the knowledge of the previous example, we find $w_{BD} = -16$ km/h.

5.2.4 Example 3: moving truck and high demand

Consider a three lane road, where at all three lanes a triangular fundamental diagram holds. The capacity is 2000 veh/h/lane, the free flow speed 80 km/h and the jam density 150 veh/km. The demand is increased to 4500 veh/h. A truck enters the road at t=0.5h and x=10 km, and leaves the road at t=1h and x=15 km, hence driving 10 km/h, limiting the flow downstream of the moving bottleneck to 1000 veh/h. What are the conditions on the road?

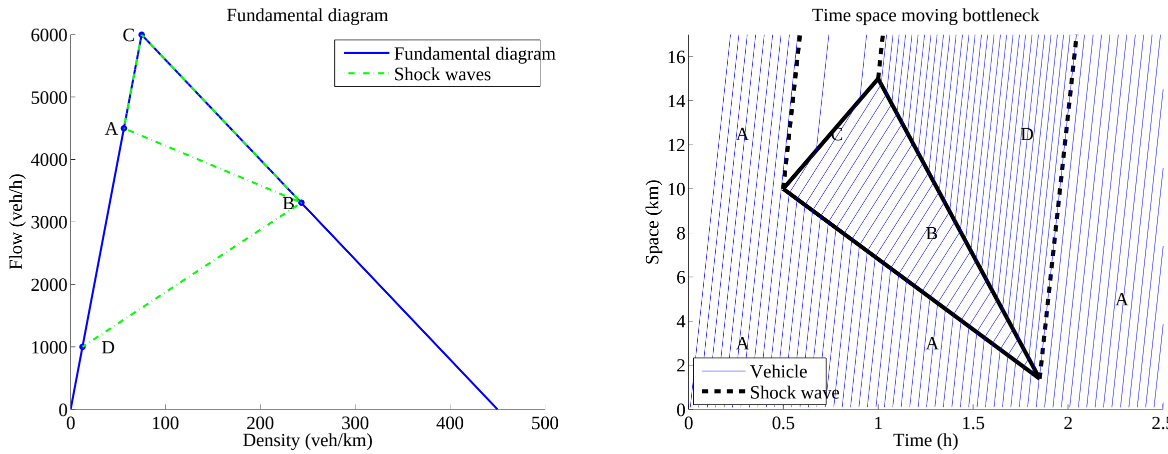
The states and fundamental diagram, as well as the resulting traffic states are show in figures 5.7(a) and 5.7(b) respectively. Tables 5.9 and 5.10 give the properties of the states and the shock waves respectively. This situation is very similar to the situation described in example 2 (section 5.2.3). Below, we will comment on the similarities and differences.

The downstream part is the same. States A and C, as well as the congested state upstream of the moving bottleneck are determined by the properties of the moving bottleneck, and are thus the same as in the previous example. The speed of the wave between state A and B differs, since state A is different. The speed can be calculated using the shock wave equation, equation 5.9:

$$w_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{4500 - 3307}{56.25 - 243} = -6.3 \text{ km/h} \tag{5.28}$$

So, the methodology to compute the shock wave speed is the same. However, the phenomenon is difference: contrary to the previous example, the shock wave moves upstream.

When the moving bottleneck leaves the road, the queue discharge (capacity) state D is also the same as in the previous example. Since states B and D are the same, the shock wave speed between the two is also the same. Also the shock wave speed between state A and D is the same, because both points are at the free flow branch of the fundamental diagram, hence the shock wave speed is the speed of the free flow branch of the (triangular) fundamental diagram.



(a) Position of traffic states at the fundamental diagram

(b) Shock waves in space-time

Figure 5.7: The situation for the moving bottleneck with overtaking opportinities and a high demand.

Table 5.9: The states on the road for the moving bottleneck with overtaking opportinities and a high demand.

State	Flow (veh/h)	Density(veh/km)	Speed (km/h)
A	4500	56.25	80
B	3077	243	13.6
C	1000	12.5	80
D	6000	75	80

Table 5.10: The shock waves present on the road for the moving bottleneck with overtaking opportinities and a high demand

State 1	State 2	shock wave speed w (km/h)
A	B	-6.3
A	C	80
D	C	80
B	D	-16
A	D	80

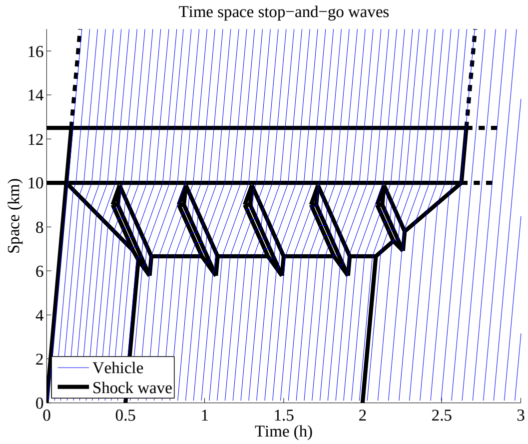


Figure 5.8: Stop and go waves in time and space

5.3 Stop and go waves

On motorways, often so called stop-and-go waves occur. These short traffic jams start from local instabilities, and the speed (end flow) in the stop-and-go waves is almost zero. For a more detailed explanation, see section 7.3. Now the speed of the boundaries of the traffic states are known, we can apply this to stop-and-go waves, and understand why this typical pattern arises (figure 5.8).

The stop-and-go waves arise in dense traffic. The traffic demand is then mostly near capacity, or at approximately the level of the queue discharge rate. The upstream boundary of such a shock moves upstream. When it is in congested conditions, both the upstream state (the congested condition) and the downstream state (the standing traffic) are congested, hence the upstream boundary moves backwards with a speed equal to the wave speed of the fundamental diagram. Once it gets out of congestion, the inflow is most likely approximately equal to the capacity of the road. That means that the upstream boundary moves with a speed which is equal to the slope of the line in the fundamental diagram connecting the capacity point with the point of jam density, which is the wave speed.

The downstream boundary separates the jam state (standing traffic) with capacity (by default: vehicles are waiting to get out of the jam, hence a capacity state occurs). The speed between these two states is found by connecting the points in the fundamental diagram. The resulting speed is the wave speed of the fundamental diagram.

After the first stop-and-go wave has moved upstream, the inflow in the second stop-and-go wave equals the outflow of the first stop-and-go wave. These are the upstream respectively the downstream state of the second wave, which thus are the same. In between, within the wave, there is another, jammed state. According to shock wave theory the shock between state A and B (i.e., the upstream state and the state within the stop-and-go wave) and the shock between B and A (i.e. the state within the stop-and-go wave and the downstream state, which equals the upstream state) is the same. This speed is approximately equal for all roads, 15-20 km/h (Schreiter et al., 2010). Hence, the length of the stop-and-go wave remains the same.

This way, stop-and-go waves can travel long distances upstream. Section 7.4 will discuss the stop-and-go waves and their characteristics in more detail.

Selected problems

For this chapter, consider problems: A.1.4, A.2.6, 67, 72, A.3.4, A.5.4, 103, 104, A.6.4, A.7.3, A.8.4, A.9.3, A.9.4, A.10.3