Chapter 5

Dynamic properties of traffic flow

Summary of the chapter. The preceding chapters have introduced the fundamental properties of traffic flow. Using the correct analytical techniques, such as queuing analysis, shock wave theory, and macroscopic traffic flow models, these fundamental flow characteristics can be provide valuable inisights into the stationary and even dynamic behaviour of traffic flow. However, insight into the dynamic (or rather, transient) properties of traffic flows is also needed for the traffic analyst.

In this chapter, we do provide some insight into these dynamic properties. The main point of interest here are the different types of congestion that been identified and there dynamic properties.

List of symbols

k	veh/m	density
u	m/s	speed
$u^e(k)$	m/s	speed as function of density
$x_i(t)$	m	position of vehicle i as function of time t
T_r	s	reaction time

5.1 Equilibrium traffic state

The different microscopic processes that constitute the characteristics of a traffic flow take time: a driver need time to accelerate when the vehicle in front drives away when the traffic signal turns green. When traffic conditions on a certain location change, for instance when the head of a queue moves upstream, it will generally take time for the flow to adapt to these changing conditions.

Generally, however, we may assume that given that the conditions remain unchanged for a sufficient period of time – say, 5 minutes – traffic conditions will converge to an *average state*. This state is ofter referred to as the *equilibrium traffic state.*

When considering a traffic flow, this equilbrium state is generally expressed in terms of the fundamental diagram. That is, when considering traffic flow under stationary conditions, the flow operations can – on average – be described by some relation between the speed, the density and the flow. This is why the speed - density relation is often referred to as the equilibrium speed $u^e(k)$.

5.2 Hysteresis and transient states

From real-life observations of traffic flow, it can be observed that many of the data points collected are not on the fundamental diagram. While some of these points can be explained by

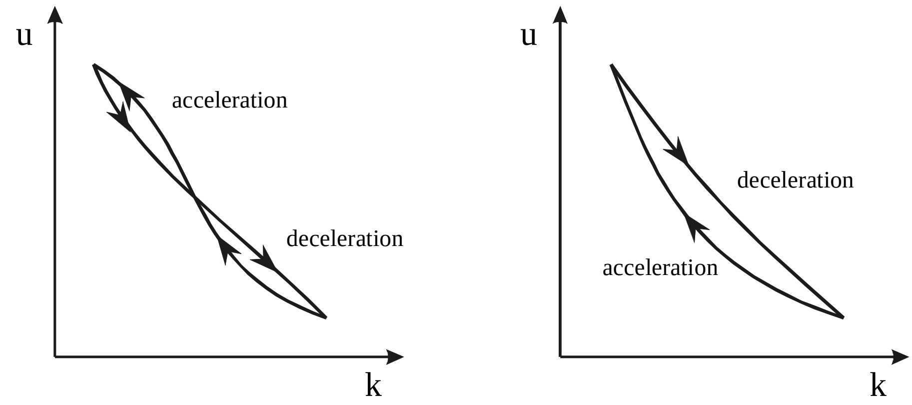

Figure 5.1: Examples of non-equilibrium curves for different data collection sites determined from [54] (left) and [35] (right)

stochastic fluctuations (e.g. vehicles have different sizes, drivers have different desired speeds and following distances), a number of researchers have suggested that these differences are structural, and stem from the dynamic properties of traffic flow. That is, they reflect so-called *transient states* (i.e. changes from congestion to free flow (*acceleration phase*) or from free flow to congestion (*deceleration phase*)) of traffic flow. In turns out that generally these changes in the traffic state are not on the fundamental diagram, that is, we have $u \neq u^e(k)$. In other words: if we consider the average behaviour of drivers (assuming stationary traffic conditions) expressed in the speed-density relation $u^e(k)$, observed mean speeds will generally not be equal to the to the 'equilibrium' speed. The term 'equilibrium' reflects the fact that the observed speeds in time will converge to the equilibrium speed, assuming that the average conditions remain the same. That is, the average speed does not adapt instantaneously to the average or equilibrium speed.

Fig. 5.1 shows examples of non-equilibrium traffic states, where we have indicated *acceleration* and *deceleration* phases. From these figures it can be concluded that no general structure can be found: acceleration phases can either lie above the fundamental diagram or below; this holds equally for deceleration curves. In general, a single acceleration or deceleration curve could not be established for generic situations.

In sum, the following could be concluded:

1. There is more than one non-equilibrium curve

2. Acceleration and deceleration are assymmetric

3. Phase trajectories form hysteresis loops

4. Both acceleration and deceleration curves are nearly smooth

5. Mixing acceleration and deceleration flow generates discontuity

In explaining the position non-equilibrium (transient) states (i.e. either above or below the speed-density curve), [59] distinguishes different transient states, namely:

1. anticipation dominant phase (phase A)

2. balances anticipation and relaxation phase (phase B)

3. relaxation dominant phase (phase C)

He asserts that drivers are usually aware of traffic conditions further downstream. This implies that their will react on downstream high density disturbances before they reach the disturbances (or the disturbance reaches them). In this case, the anticipation is stronger than the relaxation (phase A). On the contrary, under different circumstances a driver may not see that a disturbance is coming until it reaches him / her. In these cases, the response is retarded, and relaxation is dominant. Also mixes of these cases may occur.

These conjectures have been put into a microscopic model. Let $x_i(t)$ denote the position of vehicle i. For the *anticipation dominant phase A*, we assume that the the speed vehicle i is determined by the density k at position $x_i(t) + \Delta x$, i.e.

$$\frac{d}{dt}x_i(t) = v_i(t) = u^e(k(x_i(t) + \Delta x, t)) \tag{5.1}$$

Using a first-order approximation, we can easily show that

$$k(x_i(t) + \Delta x, t) = k(x_i(t), t) + \Delta x \frac{\partial k}{\partial x}(x_i(t), t) + O\left(\Delta x^2\right) \tag{5.2}$$

and thus that

$$\begin{aligned} v_i(t) &= u^e\left(k(x_i(t), t) + \Delta x \frac{\partial k}{\partial x}(x_i(t), t) + O\left(\Delta x^2\right)\right) && (5.3) \\ &= u^e(k(x_i(t), t)) + \Delta x \frac{\partial k}{\partial x}(x_i(t), t)\frac{du^e}{dk} + O\left(\Delta x^2\right) && (5.4) \end{aligned}$$

In neglecting the second-order term, we get

$$v_i(t) = u^e(k(x_i(t), t)) + \Delta x \frac{\partial k}{\partial x}(x_i(t), t)\frac{du^e}{dk} \tag{5.5}$$

Note that $\frac{du^e}{dk} \leq 0$. Now consider the following situation: a vehicle driving into a region with increasing traffic density, i.e. $\frac{\partial k}{\partial x} > 0$. In that case, the speeds $v_i(t)$ of the vehicles satisfy $v_i(t) \leq u^e(k(x_i(t), t))$ i.e. the speeds are less than the equilibrium speeds: when drivers anticipating on deteriorating traffic conditions downstream, their speed will be less than the equilibrium speed. On the contrary, in situations where drivers are in a region of decreasing traffic density, we have $\frac{\partial k}{\partial x} < 0$ and thus $v_i(t) \geq u^e(k(x_i(t), t))$. That is, drivers anticipate on the improved traffic conditions downstream causing their speeds to be higher than the equilibrium speed.

For the *relaxation dominant phase C*, the speed of vehicle i is determined by the density $k(x_i, t)$; since the speed, we have

$$\frac{d}{dt}x_i(t + T_r) = v_i(t + T_r) = u^e(k(x_i(t), t)) \tag{5.6}$$

To again show the effect of retarded reaction to the traffic conditions, we can use the Taylor approximation

$$v_i(t + T_r) = v_i(t) + T_r \frac{dv_i}{dt} + O(T_r) \tag{5.7}$$

which in turn leads to the following approximation

$$\frac{dv_i}{dt} = \frac{u^e(k(x_i(t), t)) - v_i(t)}{T_r} \tag{5.8}$$

The retarded acceleration thus causes an exponential relaxation to the equilibrium speed, rather than an instantaneous one. When a driver moves into a region with increasing densities, the speed will be higher than the equilibrium speed; on the contrary, when moving into a region of decreasing densities, the speed will generally be less than the equilibrium speed.

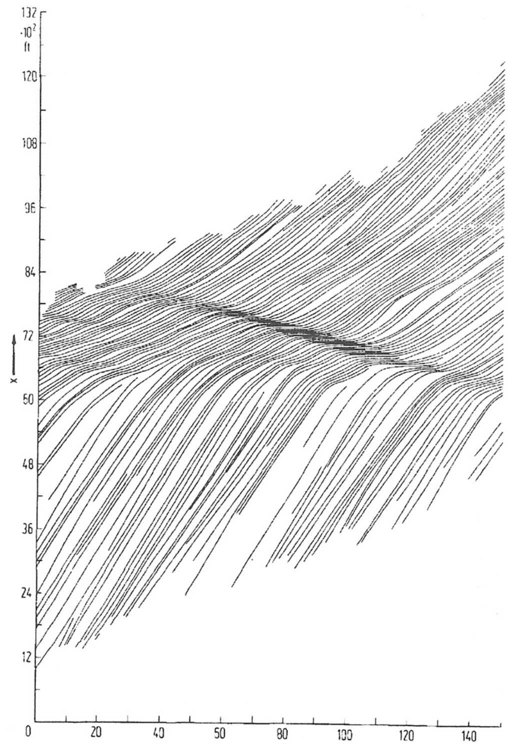

Figure 5.2: Formation of congestion in unstable flow conditions

In combining both phases A and C, we get the combined / mixed phase

$$\frac{d}{dt}x_i\left(t+T_r\right)=u^e\left(k\left(x_i\left(t\right)+\Delta t,t\right)\right) \tag{5.9}$$

It is beyond the scope of this reader to go in further detail; we refer to [59]. It will be shown in chapter 9 that these expressions form the basis for the Payne model, and thus that the emergent flow characteristics can be described using these resulting system of partial differential equations.

5.3 Metastable and unstable states

Several authors have shown that traffic flows are in fact not stable. This implies that for specific traffic flow conditions, small disturbances in the traffic flow may grow into congestion. Fig. 5.2 from [53] shows the formation of congestion in case of unstable traffic flow conditions. From the figure, we can clearly see how a small disturbance grows into a complete, upstream moving jam.

Interesting observations can be made from the San Pablo dataset described ealier. In [add reference to Daganzo here] it is shown how the flow oscillations generated by a traffic signal (the Wildcat Canyon Road) apper to damp out within half a mile of the bottleneck. However, other oscillations arise wihtin the queue farther upstream (at varied locations) which grow in amplitude as they propagate in the upstream direction. In other words, the queue appears to be stable near the bottleneck and unstable far away of the bottleneck.

In [32], transitions between the different phases of traffic is discussed and explained. Based on empirical studies of German motorways, three states of traffic flow are distinguished

1. *Free flow*, which defition is equal to the traditional definition of free flow or unconstrained traffic flow.

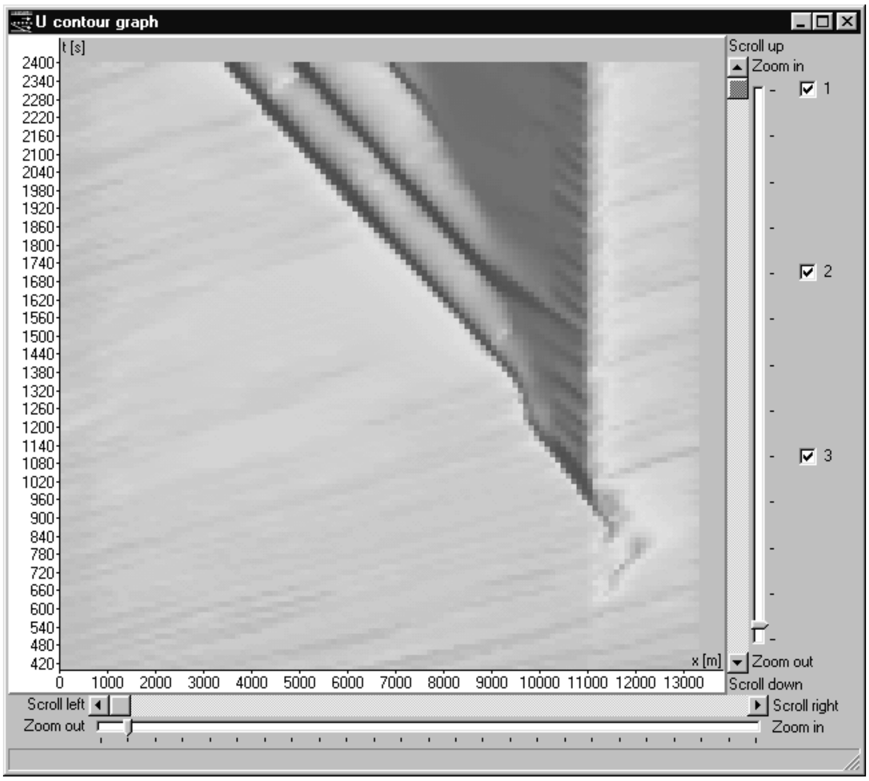

Figure 5.3: Transition from synchronised flow to wide moving jams. Results are obtained from simulations using the microscopic simulation model FOSIM.

2. *Synchronised flow*, which reflects congested (or constrained) flow operations in case traffic inside the congestion is still moving. The synchronised state is characterised by little differences in the average speeds between the lanes and the fact that it in metastable or unstable.

3. *Moving traffic jam* ('wide jam'; [32]) describes a jam in which the traffic is standing still. The traffic jam moves upstream with a fixed speed (approxmately 16 km/h). This speed is determined by the way vehicle accelerate away from the traffic jam.

Synchronised flow occurs among other things at bottle-necks (on-ramp, off-ramp, merge) when traffic demand exceeds infrastructure supply. Over time, the region of synchronised flow may spill back upstream. When the density inside the synchronised flow region is sufficiently high, small disturbances will propagate upstream (this will be explained in detail in chapter 9). The instability of the synchronised state will cause the disturbance to grow in amplitude over time when travelling upstream. In time, the disturbance will turn into a moving traffic jam, given that the region of synchronised flow extends far enough[1]. Fig. 5.3 shows the results of application of the FOSIM model for an oversaturated bottle-neck (in this case, an on-ramp). At the entry of the bottle-neck (at $x = 11150$), a region of synchronised flow forms. Here, the densities are high and the speed is low (albeit non-zero). As the synchronised flow region moves upstream (the dynamics of which can be predicted using shock wave analysis, see chapter 8), the small (stochastic) fluctuations in the flow grow into wide moving jams. FOSIM predicts that these wide moving jams travel upstream with a near constant speed.

Another issue that is address by [32] is the fact that in synchronised flow, drivers will not typically choose one particular distance headway for a certain speed. It is argued in [32] that as a result of this, measurements collected during synchronised flow will generally not lie on a

[1] This is why Kerner refers to this region as the 'pinch region'.

line in the flow-density plane, but will rather form a region. We will not discuss this issue in detail in this reader.