Contents

1 Integration in Banach spaces 1
- 1.1 Banach spaces 1
- 1.2 The Pettis measurability theorem 3
  - 1.2.1 Strong measurability 3
  - 1.2.2 Strong μ-measurability 6
- 1.3 The Bochner integral 7
  - 1.3.1 The Bochner integral 8
  - 1.3.2 The Lebesgue-Bochner spaces $L^p(A;E)$ 12
- 1.4 Exercises 13

2 Random variables in Banach spaces 15
- 2.1 Random variables 15
- 2.2 Fourier transforms 17
- 2.3 Convergence in probability 20
- 2.4 Independence 21
- 2.5 The Itô-Nisio theorem 22
- 2.6 Exercises 25

3 Sums of independent random variables 29
- 3.1 Gaussian sums 29
- 3.2 The Kahane-Khintchine inequality 35
- 3.3 Exercises 39

4 Gaussian random variables 43
- 4.1 Fernique's theorem 43
- 4.2 The covariance operator 46
- 4.3 Series representation 49
- 4.4 Convergence 51
- 4.5 Exercises 53

5 γ-Radonifying operators 57
5.1 γ-Summing operators 57
5.2 γ-Radonifying operators 61
5.3 Examples of γ-radonifying operators 66
5.4 Exercises 69

6 Stochastic integration I: the Wiener integral 73
6.1 Brownian motion 73
6.1.1 Brownian motion 73
6.1.2 Cylindrical Brownian motion 77
6.2 The stochastic Wiener integral 78
6.3 Exercises 84

7 Semigroups of linear operators 89
7.1 C_0-semigroups 89
7.2 Duality 94
7.3 The abstract Cauchy problem 96
7.4 Examples of C_0-semigroups 98
7.5 Exercises 100

8 Linear equations with additive noise I 105
8.1 Stochastic preliminaries 105
8.2 Semigroup preliminaries 108
8.3 Existence and uniqueness: cylindrical Brownian motion 110
8.4 Existence and uniqueness: Brownian motion 113
8.5 Non-existence 116
8.6 Exercises 116

9 γ-Boundedness 121
9.1 Randomised boundedness 121
9.2 Examples 127
9.3 A multiplier result 128
9.4 Exercises 132

10 Linear equations with additive noise II 135
10.1 Analytic semigroups 135
10.2 Fractional powers 139
10.3 Hölder regularity 143
10.4 Exercises 146

11 Conditional expectations and martingales 151
11.1 Conditional expectations 151
11.2 Vector-valued conditional expectations 155
11.3 Martingales 157
11.4 L^p-martingales 160
11.5 Exercises 162

12 UMD-spaces . . . 165
12.1 UMD$_p$-spaces . . . 165
12.2 p-Independence of the UMD$_p$-property . . . 168
12.2.1 Reduction to Haar martingales . . . 168
12.2.2 p-Independence for Haar martingales . . . 172
12.3 The vector-valued Stein inequality . . . 175
12.4 Exercises . . . 177

13 Stochastic integration II: the Itô integral . . . 181
13.1 Decoupling . . . 181
13.2 Stochastic integration . . . 183
13.3 Stochastic integrability of L^p-martingales . . . 189
13.4 Exercises . . . 191

14 Linear equations with multiplicative noise . . . 195
14.1 γ-Lipschitz functions . . . 196
14.2 Pisier's property . . . 197
14.3 Stochastic convolutions . . . 200
14.4 Existence and uniqueness . . . 202
14.5 Space-time regularity . . . 205
14.6 Exercises . . . 207

15 Applications to stochastic PDE . . . 211
15.1 Space-time white noise . . . 211
15.2 The stochastic wave equation . . . 212
15.2.1 Rank one multiplicative noise . . . 212
15.2.2 Additive space-time white noise . . . 214
15.3 The stochastic heat equation . . . 216
15.3.1 Rank one multiplicative noise . . . 216
15.3.2 Additive space-time white noise . . . 218
15.4 Exercises . . . 221

References . . . 223