PRODUCTION ESTIMATION BASED ON CUTTING THEORIES FOR CUTTING WATER SATURATED SAND.

Dr.ir. S.A. Miedema

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ABSTRACT.

In dredging, the excavation of soil is one of the most important processes. It is known that on the largest cutter suction dredges thousands of kW's are installed on the cutter drive.

To predict the forces on excavating elements, two-dimensional cutting theories are used. Van Os 1977 [9], Verruijt 1985 [11), Miedema 1987 [6] and 1989 [7] and van Leussen & van Os 1987 [10] described the two-dimensional cutting theory for water saturated sand and its applications extensively. The aim of developing such theories is to predict the loads on excavating elements like cutterheads, dragheads, etc. It is however also possible to predict the production, when the soil mechanics parameters, the geometry of the excavating element and the available power, are known. The soil mechanics parameters are described by van Leussen & Nieuwenhuis in 1984 [5].

The cutting process of a cutterhead is very complicated. Not only do the blades have a three-dimensional shape; also, the velocities on the blades are threedimensional, with respect to their direction, due to a combination of the swing velocity and the circumferential velocity. Other excavating elements such as dredging wheels, blades in dragheads and trenchers, may not look that complicated, but will also require the three-dimensional cutting theory to fully describe the cutting process. In 1994, Miedema [8] described the threedimensional cutting theory. To derive forces, torque, power, specific energy and production from the above-mentioned theories, requires complicated calculations, while the soil mechanics parameters of the sand have to be known. Mostly, only the SPT value of the sand is known.

This paper describes the two- and three- dimensional cutting theory in water saturated sand. A calculation model is derived on how to determine the most essential parameters, to be able to calculate the specific energy and the production if the SPT value of the sand is known. Verification for both the twoand the three dimensional cutting theories and the SPT model is given based on laboratory tests with straight blades.

The laboratory tests give a good correlation between calculated and measured cutting forces for both the two- and the three dimensional cutting theory and show that the approach for production estimation based on SPT values gives an upper limit for the specific energy and a lower limit for the production. For a more accurate prediction of the production however, more detailed soil mechanics parameters should be known.

INTRODUCTION.

In 1975 Hatamura and Chijiiwa [11] distinguished 3 failure mechanisms in soil cutting. The "shear type", the "flow type" and the "tear type". A fourth failure mechanism can be distinguished, the "curling type", as is known in metal cutting. Although it seems that the curling of the chip cut is part of the flow of the material, whether the "curling type" or the "flow type" occurs depends on several conditions. The "flow type", "tear type" and "curling type" occur in clay, while the "tear type" and, under high hydrostatic pressure, the "flow type" occur when cutting rock. The "shear type" occurs in materials with an angle of internal friction, but without cohesion like sand. Figure 1 illustrates the 4 failure mechanisms as they might occur when cutting soil.



Although the "shear type" is not a continuous cutting process, the shear planes occur so frequently, that a continuous process is considered. The Mohr-Coulomb failure criteria is used to derive the cutting forces. This derivation is made under the assumption that the stresses on the shear plane and the blade are constant and equal to the average stresses acting on the surfaces.

To estimate the production in water saturated sand, first a summary of the twodimensional cutting theory is given. Secondly, this theory is extended for angled (deviated) blades. Finally relations found in literature are used to correlate SPT values with soil mechanical parameters in order to calculate the specific energy required to cut the sand and from this an estimation of the production is made.

THE TWO-DIMENSIONAL CUTTING THEORY FOR WATER SATURATED SAND.

THE EQUILIBRIUM OF FORCES.

The forces acting on a straight blade when cutting soil, can be distinguished as (fig. 3):

1. A force normal to the blade N_2 .

2. A shear force S_2 as a result of the soil/steel friction $N_2 \tan[\delta]$.

3. A shear force A as a result of pure adhesion between the soil and the blade. This force can be calculated by multiplying the adhesive shear strength of the soil with the contact area between the soil and the blade.

4. A force W_2 as a result of water under pressure on the blade.

These forces are shown in figure 3. If the forces N_2 and S_2 are combined to a resulting force K_2 and the adhesive force and the water under pressures are known, then the resulting force K_2 is the unknown force on the blade.



Figure 2 illustrates the forces on the layer of soil cut. The forces shown are valid in general. In sand several forces can be neglected.

The forces acting on this layer are:

- 1. The forces occurring on the blade as mentioned above.
- 2. A normal force acting on the shear surface N_1 .
- 3. A shear force S_1 as a result of internal fiction $N_1 \tan[\phi]$.

4. A force W_1 as a result of water under pressure in the shear zone.

5. A shear force C as a result of pure cohesion. This force can be calculated by multiplying the cohesive shear strength with the area of the shear plane.

6. A gravity force G as a result of the weight of the layer cut.

7. An inertial force I, resulting from acceleration of the soil.



The normal force N_1 and the shear force S_1 can be combined to a resulting grain force K_1 . By taking the horizontal and vertical equilibrium of forces, an expression for the force K_2 on the blade can be derived.

The horizontal equilibrium of forces:

$$K_{1} \cdot \sin(\beta + \phi) - W_{1} \cdot \sin(\beta) + C \cdot \cos(\beta) + I \cdot \cos(\beta) - A \cdot \cos(\alpha) + W_{2} \cdot \sin(\alpha) - K_{2} \cdot \sin(\alpha + \delta) = 0$$
(1)

The vertical equilibrium of forces:

The force K_2 on the blade is now:

$$K_{2} = \frac{W_{2} \cdot \sin(\alpha + \beta + \phi) + W_{1} \cdot \sin(\phi)}{\sin(\alpha + \beta + \phi + \phi)} + \frac{G \cdot \sin(\beta + \phi) + I \cdot \cos(\phi) + C \cdot \cos(\phi) - A \cdot \cos(\alpha + \beta + \phi)}{\sin(\alpha + \beta + \phi + \delta)}$$
(3)

From this last equation, the forces on the blade can be derived. On the blade, a force component in the direction of cutting velocity F_h and a force perpendicular to this direction F_v can be distinguished.

$$F_{h} = -W_{2} \cdot \sin(\alpha) + K_{2} \cdot \sin(\alpha + \delta) + A \cdot \cos(\alpha)$$
(4)

$$F_{v} = -W_{2} \cdot \cos(\alpha) + K_{2} \cdot \cos(\alpha + \delta) - A \cdot \sin(\alpha)$$
 (5)

From literature, it is known that, during the process of cutting sand, the pore volume of the sand increases. This is caused by the phenomenon dilatancy (sec figure 4).



With a certain cutting velocity, vc there has to be a flow of water to the shear zone, the area where the pore volume increases. This causes a decrease in the pore pressure of the pore water and because the soil stress remains constant, the grain stress will increase. Van Os [9] 1977 stated: "If it is the aim of the engineer to know the average cutting farces needed to push the blade through the soil, he can take an average deformation rate $\partial e/\partial t$ to insert into the Biot equation. But it should be noted that this is purely practical reasoning and has nothing to do with Theoretical Soil Mechanics". Van Os and van Leussen published their cutting theory in 1987 [10]. Van Leussen and Nieuwenhuis [5] discussed the relevant soil mechanical parameters in 1984. Miedema [6] 1987 uses the average deformation rate as stated by van Os [9] 1977 but instead of inserting this in the Biot equation; the average deformation rate is modelled as a boundary condition in the shear zone. Although the cutting process is not solely dependent upon the phenomenon dilatancy, the above mentioned research showed that for cutting velocities in a range from 0.5 to 5 m/sec the cutting process is dominated by the phenomenon dilatancy, so the contributions of gravitational, cohesive, adhesive and inertial forces can be neglected, thus:

$$K_{2} = \frac{\mathbb{W}_{1} \cdot \sin(\phi) + \mathbb{W}_{2} \cdot \sin(\alpha + \beta + \phi)}{\sin(\alpha + \beta + \delta + \phi)}$$
(6)

This gives for the horizontal and vertical force on the blade:

$$F_{h} = -W_{2} \cdot \sin(\alpha) + K_{2} \cdot \sin(\alpha + \delta)$$
(7)

$$\mathbf{F}_{\mathbf{v}} = - \mathbf{W}_{2} \cdot \cos(\alpha) + \mathbf{K}_{2} \cdot \cos(\alpha + \delta)$$
(8)

THE WATER PORE PRESSURES.

The forces W_1 and W_2 resulting from the pore pressures are the unknowns in the equations 6, 7 and 8. Miedema [6] 1987 calculated the average pore pressures P_1 and P_2 with a Finite Element Method (FEM) program (fig. 5). With the equations 9 and 10 the forces W_1 and W_2 can be determined by substituting the results of the FEM calculations.



$$W_{1} = \frac{P_{1} \cdot \rho_{w} \cdot g \cdot v_{c} \cdot e \cdot h_{1}^{2} \cdot b}{(a_{1} \cdot k_{i} + a_{2} \cdot k_{max}) \cdot \sin(\beta)}$$
(9)
$$W_{2} = \frac{P_{2} \cdot \rho_{w} \cdot g \cdot v_{c} \cdot e \cdot h_{1} \cdot h_{b} \cdot b}{(a_{1} \cdot k_{i} + a_{2} \cdot k_{max}) \cdot \sin(\alpha)}$$
(10)

On average P_1 can be estimated by 0.15, P_2 by 0.32, a_1 by 0.5 and a_2 by 0.5. When the pore pressures reach the water vapour pressure, cavitation will occur. The pore pressures cannot decrease further with an increasing cutting velocity and remain constant. In this case the forces W_1 and W_2 can be calculated analytical, giving:

$$W_{1} = \frac{\rho_{w} \cdot g \cdot (z + 10) \cdot h_{i} \cdot b}{\sin(\beta)}$$
(11)
$$W_{2} = \frac{\rho_{w} \cdot g \cdot (z + 10) \cdot h_{b} \cdot b}{\sin(\alpha)}$$
(12)

THE SIMPLIFIED EQUATIONS FOR THE CUTTING FORCES.

Miedema [6] 1987 simplified the equations by using proportionality coefficients c_1 , c_2 , d_1 and d_2 . This leads to the first two simplified equations for the two-dimensional cutting process in water saturated sand without cavitation:

$$F_{h} = \frac{c_{1} \cdot \rho_{w} \cdot g \cdot v_{c} \cdot h_{1}^{2} \cdot b \cdot e}{k_{m}}$$
(13)
$$F_{v} = \frac{c_{2} \cdot \rho_{w} \cdot g \cdot v_{c} \cdot h_{1}^{2} \cdot b \cdot e}{k_{m}}$$
(14)
$$e = \frac{n_{max} \cdot n_{1}}{k_{m}} \qquad k \approx 0.5 \cdot k. + 0.5 \cdot k$$

With: max and max

For the cavitating cutting process the following equation are valid for the horizontal and vertical cutting force:

$$\mathbf{F}_{\mathbf{h}} = \mathbf{d}_{\mathbf{1}} \cdot \boldsymbol{\rho}_{\mathbf{w}} \cdot \mathbf{g} \cdot (\mathbf{z} + \mathbf{10}) \cdot \mathbf{h}_{\mathbf{i}} \cdot \mathbf{b}$$
(15)

$$\mathbf{F}_{\mathbf{v}} = \mathbf{d}_{2} \cdot \boldsymbol{\rho}_{\mathbf{w}} \cdot \mathbf{g} \cdot (\mathbf{z}+10) \cdot \mathbf{h}_{1} \cdot \mathbf{b}$$
(16)

The coefficients c_1 , c_2 , d_1 and d_2 are dependent upon the angle of internal friction of the sand ϕ , the soil interface friction angle δ , the blade angle α and the blade height/layerthickness ratio h_b/h_i . Detailed tables of c_1 , c_2 , d_1 and d_2 are published by Miedema [6] in 1987. Figure 6 shows the results of laboratory tests carried out by Bindt and Zwartbol [2] in 1994. The correlation between measurements and theory is satisfactory.



THE NORMAL AND FRICTION FORCES ON THE SHEAR SURFACE AND ON THE BLADE.

Although the normal and friction forces as shown in figure 2 are the basis for the calculation of the horizontal and vertical cutting forces, the approach used, requires the following equations to derive these forces by substituting 13 and 14 for the non-cavitating cutting process or 15 and 16 for the cavitating cutting process. The index 1 points to the shear surface, while the index 2 points to the blade (fig. 8):

$$F_{n1} = F_h \cdot \sin(\beta) - F_v \cdot \cos(\beta)$$
(17)

$$F_{f1} = F_h \cdot \cos(\beta) + F_v \cdot \sin(\beta)$$
(18)

$$F_{n2} = F_h \cdot \sin(\alpha) + F_v \cdot \cos(\alpha)$$
(19)

$$F_{f2} = F_h \cdot \cos(\alpha) - F_v \cdot \sin(\alpha)$$
(20)

THE THREE-DIMENSIONAL CUTTING THEORY.

The previous paragraphs summarized the two-dimensional cutting theory. On a cutterhead, the blades can be divided into small elements, at which a two dimensional cutting process is considered. However, this is correct only when the cutting edge of this element is perpendicular to the direction of the velocity of the element. For most elements this will not be the case. This means the elements can be considered to be deviated (angled) with respect to the direction of the cutting edge and a component of the cutting velocity perpendicular to the cutting edge can be distinguished. This second component results in a deviation force on the blade element, due to the friction between the soil and the blade. This force is also the cause of the transverse movement of the soil, the so called snow-plough effect.

To predict the deviation force and the direction of motion of the soil on the blade, the equilibrium equations of force will have to be solved in three directions. Since there are four unknowns, three forces and the direction of the velocity of the soil on the blade, one additional equation is required. This equation follows from an equilibrium equation of velocity between the velocity of grains in the shear zone and the velocity of grains on the blade. Since the four equations are partly non-linear and implicit, they have to be solved iteratively, Miedema [8] 1994. Figure 7 shows this phenomenon. As with snow-ploughs, the sand will flow to one side while the blade is pushed to the opposite side. This will result in a third cutting force, the deviation force F_d . To determine this force, the flow direction of the sand has to be known. Figure 8 shows a possible flow direction.



VELOCITY CONDITIONS.

For the velocity component perpendicular to the blade v_c , if the blade has a deviation angle ι and a drag velocity v_d according to figure 8, it yields:

$$\mathbf{v}_{c} = \mathbf{v}_{d} \cdot \cos(\iota) \tag{21}$$

The velocity of grains in the shear surface perpendicular to the cutting edge is now:

$$v_{rl} = v_c \cdot \frac{\sin(\alpha)}{\sin(\alpha + \beta)}$$
(22)

The relative velocity of grains with respect to the blade, perpendicular to the cutting edge is:

$$v_{r2} = v_c \cdot \frac{\sin(\beta)}{\sin(\alpha + \beta)}$$
 (23)

The grains will not only have a velocity perpendicular to the cutting edge, but also parallel to the cutting edge, the deviation velocity components v_{d1} on the shear surface and v_{d2} on the blade.

The velocity components of a grain in x, y and z direction can be determined by considering the absolute velocity of grains in the shear surface, this leads to:

$$v_{x1} = v_{r1} \cos(\beta) \cdot \cos(\iota) + v_{d1} \cdot \sin(\iota)$$
(24)

$$\mathbf{v}_{y1} = -\mathbf{v}_{r1} \cdot \cos(\beta) \cdot \sin(\iota) - \mathbf{v}_{d1} \cdot \cos(\iota)$$
(25)

$$\mathbf{v}_{z1} = \mathbf{v}_{r1} \cdot \sin(\beta) \tag{26}$$

The velocity components of a grain can also be determined by a summation of the drag velocity of the blade and the relative velocity between the grains and the blade, this gives:

$$\mathbf{v}_{\mathrm{x2}} = \mathbf{v}_{\mathrm{d}} - \mathbf{v}_{\mathrm{r2}} \cdot \cos(\alpha) \cdot \cos(\iota) - \mathbf{v}_{\mathrm{d2}} \cdot \sin(\iota)$$
(27)

$$v_{y2} = -v_{r2} \cdot \cos(\alpha) \cdot \sin(\iota) + v_{d2} \cdot \cos(\iota)$$
(28)

$$v_{z2} = v_{r1} \cdot \sin(\alpha) \tag{29}$$

Since both approaches will have to give the same resulting velocity components, the following condition for the transverse velocity components can be derived:

$$v_{x1} = v_{x2} \implies v_{d1} + v_{d2} = v_d \cdot \sin(\iota)$$
 (30)

$$v_{y1} = v_{y2} \longrightarrow v_{d1} + v_{d2} = v_d \sin(\iota)$$
 (31)

$$\mathbf{v}_{\mathbf{z}\mathbf{1}} = \mathbf{v}_{\mathbf{z}\mathbf{2}} \tag{32}$$

THE DEVIATION FORCE.

Since friction always has a direction matching the direction of the relative velocity between two bodies, the fact that a deviation velocity exists on the shear surface and on the blade, implies that also deviation forces must exist. To match the direction of the relative velocities, the following equation can be derived for the deviation force on the shear surface and on the blade (fig. 8):

$$F_{d1} = F_{f1} \cdot \frac{v_{d1}}{v_{r1}}$$
(33)
$$F_{d2} = F_{f2} \cdot \frac{v_{d2}}{v_{r2}}$$
(34)

Since perpendicular to the cutting edge, an equilibrium of forces exists, the two deviation forces must be equal in magnitude and have opposite directions.

$F_{d1} = F_{d2}$

By substituting 33 and 34 in 35 and then substituting 18 and 20 for the friction forces and 22 and 23 for the relative velocities, the following equation can be derived, giving a second relation between the two deviation velocities:

$$\frac{\mathbf{v}_{d1}}{\mathbf{v}_{d2}} = \left(\frac{\mathbf{F}_{f2}}{\mathbf{F}_{f1}}\right) \cdot \left(\frac{\mathbf{v}_{r1}}{\mathbf{v}_{r2}}\right) = \left(\frac{\mathbf{F}_{h} \cdot \cos(\alpha) - \mathbf{F}_{v} \cdot \sin(\alpha)}{\mathbf{F}_{h} \cdot \cos(\beta) + \mathbf{F}_{v} \cdot \sin(\beta)}\right) \cdot \left(\frac{\sin(\alpha)}{\sin(\beta)}\right) (36)$$

To determine F_h and F_v the angle of internal friction ϕ and the soil/interface friction angle mobilized perpendicular to the cutting edge, have to be determined by using the ratio of the transverse velocity and the relative velocity, according to:

$$\tan(\phi_{e}) = \tan(\phi) \cdot \cos\left(\operatorname{atn}\left(\frac{v_{d1}}{v_{r1}}\right)\right)$$
(37)

$$\tan(\delta_{e}) = \tan(\delta) \cdot \cos\left[\operatorname{atn}\left[\frac{v_{d2}}{v_{r2}}\right] \right]$$
(38)

THE RESULTING CUTTING FORCES.

The resulting cutting forces in x, y and z direction can be determined once the deviation velocity components are known. However, it can be seen that the second velocity condition (36) requires the horizontal and vertical cutting forces perpendicular to the cutting edge, while these forces can only be determined if the mobilized friction angles (37 and 38) are known. This creates an implicit set of equations that will have to be solved by means of an iteration process. For the cutting forces on the blade the following equation can be derived:

$$F_{x2} = F_h \cdot \cos(\iota) + F_{d2} \cdot \sin(\iota)$$
(39)

$$F_{y2} = F_h \cdot \sin(\iota) - F_{d2} \cdot \cos(\iota)$$
(40)

$$\mathbf{F}_{z2} - \mathbf{F}_{v} \tag{41}$$

The results of cutting tests with a 45 degree blade with a deviation angle of 45 degrees are shown in figure 9. The correlation between the measurements and the theory is good. It should be noted that the specific energy is considerably smaller than in the tests with a deviation angle of 0 (fig. 6).

SPECIFIC ENERGY.

To determine the production of excavating elements as a function of the SPT value of the soil, the specific energy method is used. The specific energy is the amount of energy (work) required for the excavation of 1 cubic meter of in-situ soil. The dimension of specific energy is kNm/m³ or kPa. The specific energy depends on the type of soil (soil mechanical parameters), on the geometry of the excavating element (dredging wheel, crown cutter, etc.) and on the operational parameters (haulage velocity or trail velocity, revolutions, face geometry, etc). Beside the above, the effective specific energy will be influenced by other phenomena such as spill, wear and the bull-dozer effect. The maximum production can also be limited by the hydraulic system but this will not be considered in this paper. The production can be derived from the specific energy by dividing the available power by the specific energy.

$$Q = \frac{P_a}{E_s}$$
(42)

An accurate calculation of the specific energy and thus the production can be carried out only when all off the parameters influencing the cutting process are known. If an estimate has to be made of the specific energy and the production, based on the SPT value only, a number of assumptions will have to be made and a number of approximations will have to be applied. These assumptions and approximations will maximise the specific energy and thus minimise the production. In other words, the specific energy as calculated in this paper is an upper limit, whilst the calculated production is a lower limit. Wear, spill and limitations such as the bull-dozer effect are not taken into consideration. Once the specific energy and the production per 100 kW are known, the production, giving an available power, can be calculated. This production can be either realistic, meaning that the bull-dozer effect will not occur, or not realistic meaning that this effect will occur. In the last case, the maximum production

will have to be calculated from the limitations caused by the bull-dozer effect. From the maximum production derived, the maximum swing velocity, giving a certain bank height and step size, can be determined. The type of cutting process is determined by the soil mechanical properties of the soil to be dredged, the geometry of the excavating element and the operational parameters.



SPECIFIC ENERGY AND PRODUCTION IN SAND.

As discussed previously, the cutting process in sand can be distinguished in a non-cavitating and a cavitating process, in which the cavitating process can be considered to be an upper limit to the cutting forces. Assuming that during an SPT test in water-saturated sand, the cavitating process will occur, because of the shock wise behaviour during the SPT test, the SPT test will give information about the cavitating cutting process. Whether, in practice, the cavitating cutting process will occur, depends on the soil mechanical parameters, the geometry of the cutting process and the operational parameters. The cavitating process gives an upper limit to the forces, power and thus the specific energy and a lower limit to the production and will therefore be used as a starting point for the calculations. For the specific energy of the cavitating cutting to Miedema [6, page 148]:

(43)

$$\mathbf{E}_{\mathbf{s}} = \boldsymbol{\rho}_{\mathbf{w}} \cdot \mathbf{g} \cdot (\mathbf{z} + 10) \cdot \mathbf{d}_{1}$$

The production, for an available power P_a, can be determined by:

$$Q = \frac{\frac{P_a}{E_s}}{\frac{P_a}{E_s}} = \frac{\frac{P_a}{\rho_w \cdot g \cdot (z+10) \cdot d_1}}$$
(44)

The coefficient d_1 is the only unknown in the above equation. A relation between d_1 and the SPT value of the sand and between the SPT value and the waterdepth has to be found. The dependence of d_1 on the parameters α , h_i en h_b can be estimated accurately. For normal sands there will be a relation between the angle of internal friction and the soil interface friction. Assume blade angles of 30, 45 and 60 degrees, a ratio of 3 for h_b/h_i and a soil/interface friction angle of 2/3 times the internal friction angle. For the coefficient d_1 the following equations are found by regression:

$$d_{1} = -0.185 + 0.666 \cdot e^{0.0444 \cdot \phi} \quad (\alpha = 30 \text{ degrees}) (45)$$

$$d_{1} = +0.304 + 0.333 \cdot e^{0.0597 \cdot \phi} \quad (\alpha = 45 \text{ degrees}) (46)$$

$$d_{1} = +0.894 + 0.154 \cdot e^{0.0818 \cdot \phi} \quad (\alpha = 60 \text{ degrees}) (47)$$

With: ϕ = the angle of internal friction in degrees.

Lambe & Whitman [4, page 78] (fig. 11) give the relation between the SPT value, the relative density and the hydrostatic pressure in two graphs. With some curve-fitting these graphs can be summarized with the following equation:

$$SPT = (1.82 + 0.221 \cdot (z+10)) \cdot 10^{-4} \cdot RD^{2.52}$$
(48)



Lambe & Whitman [4, page 148] (fig. 10) give the relation between the SPT value and the angle of internal friction, also in a graph. This graph is valid up to 12 m in dry soil. With respect to the internal friction, the relation given in the graph has an accuracy of 3 degrees. A load of 12 m dry soil with a density of 1.67 ton/m³ equals a hydrostatic pressure of 20 m.w.c. An absolute hydrostatic pressure of 20 m.w.c. equals 10 m of waterdepth if cavitation is considered. Measured SPT values at any depth will have to be reduced to the value that would occur at 10 m waterdepth. This can be accomplished with the following equation (see fig. 12):

$$SPT_{10} = \frac{1}{(0.646 + 0.0354 \cdot z)} \cdot SPT_{z}$$
(49)



With the aim of curve-fitting, the relation between the SPT value reduced to 10 m waterdepth and the angle of internal friction can be summarized to:

$$\phi = 54.5 - 25.9 \cdot e$$
 (+ 3 degrees value) (50)

For waterdepths of 0, 5, 10, 15, 20, 25 and 30 m and an available power of 100 kW the production is shown graphically for SPT values in the range of 0 to 100 SPT. Figure 13 shows the specific energy and figure 14 the production for a 45 degree blade angle.



THE TRANSITION NON-CAVITATING/CAVITATING.

Although the SPT value only applies to the cavitating cutting process, it is necessary to have a good understanding of the transition between the non-cavitating and the cavitating cutting process. Based on the theory in [6], an equation has been derived for this transition. If this equation is valid, the cavitating cutting process will occur.

$$\frac{\mathbf{d_1} \cdot (\mathbf{z+10})}{\mathbf{c_1} \cdot \mathbf{v_c} \cdot \mathbf{h_i} \cdot \mathbf{e/k_m}} < 1$$
(51)

The ratio d_1 / c_1 appears to have an almost constant value for a given blade angle, independent of the soil mechanical properties. For a blade angle of 30 degrees this ratio equals 11.9. For a blade angle of 45 degrees this ratio equals 7.72 and for a blade angle of 60 degrees this ratio equals 6.14. The ratio e/k_m has a value in the range of 1000 to 10000 for medium to hard packed sands. At a given layer thickness and waterdepth, the transition cutting velocity can be determined using the above equation. At a given cutting velocity and waterdepth, the transition layer thickness can be determined.



CONCLUSIONS.

To check the validity of the above derived theory, research has been carried out in the laboratory of the chair of Dredging Technology of the Delft University of Technology. The tests are carried out in a hard packed water saturated sand, with a blade of 0.3 m by 0.2 m. The blade had cutting angles of 30, 45 and 60 degrees and deviation angles of 0, 15, 30 and 45 degrees. The layer thicknesses were 2.5, 5 and 10 cm and the drag velocities 0.25, 0.5 and 1 m/s. Figure 6 shows the results with a deviation angle of 0 degrees, while figure 9 shows the results with a deviation angle of 45 degrees. The lines in this figure show the theoretical forces. As can be seen, the measured forces match the theoretical forces well. Based on two graphs from Lambe & Whitman [4] and an equation for the specific energy from Miedema [6], relations are derived for the SPT value as a function of the hydrostatic pressure and of the angle of internal friction as a function of the SPT value. With these equations also the influence of waterdepth on the production can be determined. The specific energies as measured from the tests are shown in figures 6 and 9. It can be seen that the deviated blade results in a lower specific energy. These figures also show the upper limit for the cavitating cutting process. For small velocities and/or layer thicknesses, the specific energy ranges from 0 to the cavitating value. The tests are carried out in a sand with an angle of internal friction of 40 degrees. According to figure 10 this should give an SPT value of 33. An SPT value of 33 at a waterdepth of about 0 m, gives according to figure 13, a specific energy of about 450-500 kPa. This matches the specific energy as shown in figure 6. All derivations are based on a cavitating cutting process. For small SPT values it is however not sure whether cavitation will occur. A non-cavitating cutting process will give smaller forces and power and thus a higher production. At small SPT values however the production will be limited by the bull-dozer effect or by the possible range of the operational parameters such as the cutting velocity.

The calculation method used remains a lower limit approach with respect to the production and can thus be considered conservative. For an exact prediction of the production all of the required soil mechanical properties will have to be known. As stated, limitations following from the hydraulic system are not taken into consideration.

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LIST OF SYMBOLS USED.

a ₁ , a ₂	Proportionality coefficients weighed permeability	-
А	Adhesive force on the blade	kN
b	Width of blade	m
c_1, c_2	Coefficients (no-cavitation)	-
С	Cohesive force on shear plane	kN
d_1, d_2	Coefficients (cavitation)	-
e	Volume strain	%
Es	Specific energy	kPa
F _h	Horizontal cutting force	kN
F _{f1}	Friction force on the shear surface	kN
F _{f2}	Friction force on the blade	kN
F _{n1}	Normal force on the shear surface	kN
F _{n2}	Normal force on the blade	kN
Fv	Vertical cutting force	kN
F _{d1}	Deviation force on the shear surface	kN
F _{d, d2}	Deviation force on the blade	kN
F _{x1, 2}	Cutting force in x-direction	kN
F _{y1, 2}	Cutting force in y-direction	kN
F _{z1, 2}	Cutting force in z-direction	kN
g	Gravitational constant (9.81)	m/s^2
G	Gravitational force	kN
h _i	Initial thickness of layer cut	m
h _b	Height of blade	m
k _i	Initial permeability	m/s
k _{max}	Maximum permeability	m/s
k _m	Average permeability	m/s
K_1	Grain force on the shear plane	kN
K_2	Grain force on the blade	kN

Ι	Inertial force on the shear plane	kN
n _i	Initial porosity	%
n _{max}	Maximum porosity	%
N_1	Normal grain force on shear plane	kN
N_2	Normal grain force on blade	kN
P ₁	Average pore pressure on the shear surface	kPa
P ₂	Average pore pressure on the blade	kPa
Pa	Available power for cutting	kW
Q	Production of in-situ soil	m ³ /sec
RD	Relative density	%
S_1	Shear force due to internal friction on the shear surface	kN
S_2	Shear force due to soil/steel friction on the blade	kN
Vc	Cutting velocity component perpendicular to the blade	m/s
Vd	Cutting velocity, drag velocity	m/s
v _{r1}	Velocity of grains in the shear surface	m/s
v _{r2}	Relative velocity of grains on the blade	m/s
v _{d1}	Deviation velocity of grains in the shear surface	m/s
v _{d2}	Deviation velocity of grains on the blade	m/s
v _{x1,2}	Velocity of grains in the x-direction	m/s
v _{y1,2}	Velocity of grains in the y-direction	m/s
V _{z1,2}	Velocity of grains in the z-direction	m/s
W_1	Force resulting from pore underpressure on the shear plane	kN
W_2	Force resulting from pore underpressure on the blade	kN
Ζ	Water depth	m
α	Cutting angle blade	rad
β	Shear angle	rad
φ	Angle of internal friction	rad
фe	Angle of internal friction perpendicalar to the cutting edge	rad
δ	Soil/interface friction angle	rad
δ_{e}	Soil/interface friction angle perpendicular to the cutting edge	rad
l	Deviation angle blade	rad
$\rho_{\rm w}$	Density water	ton/m ³