On the Snow-Plough Effect when Cutting Water Saturated Sand with Inclined Straight Blades.

Dr.ir. S.A. Miedema

Abstract.

To predict the forces on excavating elements, two-dimensional cutting theories are used. Miedema 1987, 1989 and 1992 described the two-dimensional cutting theory for water saturated sand and its applications extensively. On a cutterhead, the blades are divided into small elements, at which a two dimensional cutting process is considered. However, this is correct only when the cutting edge of this element is perpendicular to the direction of the velocity of the element. For most elements this will not be the case. The cutting edge and the absolute velocity of the cutting edge will not be perpendicular. This means the elements can be considered to be inclined with respect to the direction of the cutting velocity. A component of the cutting velocity perpendicular to the cutting edge and a component parallel to the cutting edge can be distinguished. This second component results in a transverse force on the blade element, due to the friction between the soil and the blade. This force is also the cause of the transverse movement of the soil, the snow-plough effect.

To predict the transverse force and the direction of motion of the soil on the blade, the equilibrium equations of force will have to be solved in three directions. Since there are four unknowns, three forces and the direction of the velocity of the soil on the blade, one additional equation is required. This equation follows from an equilibrium equation of velocity between the velocity of grains in the shear zone and the velocity of grains on the blade. Since the four equations are partly non-linear and implicit, they have to be solved iteratively. The results of solving these equations have been compared with the results of laboratory tests. The correlation between the two was very satisfactory, with respect to the magnitude of the forces and with respect to the direction of the forces and the flow of the soil on the blade.

Introduction

In dredging the excavation of soil is one of the most important processes. It is known, that on the largest cutter suction dredges thousands of kW's are installed on the cutter drive. The cutting process of a cutter head is very complicated. Not only do the blades have a three-dimensional shape, also the velocities on the blades are three-dimensional, with respect to their direction, due to a combination of the swing velocity and the circumferential velocity. Other excavating elements such as dredging wheels, blades in dragheads and trenchers, may not look that complicated, but will also require the three-dimensional cutting theory to fully describe the cutting process.

The Two-Dimensional Cutting Theory for Water Saturated Sand.

From literature it is known that, during the process of cutting sand, the pore volume of the sand increases. This is caused by the phenomenon dilatancy (see figure 1). With a certain cutting velocity v_c there has to be a flow of water to the shear zone, the area where the pore volume increases. This causes a decrease in the pore pressure of the pore water and because the soil stress remains constant the grain stress will increase. Van Os 1977 stated: "If it is the aim of the engineer to know the average cutting forces needed to push the blade through the soil, he can take an average deformation rate $\partial \mathbf{e}/\partial \mathbf{t}$ to insert into the Biot equation. But it should be noted that this is purely practical reasoning and has nothing to do with Theoretical Soil Mechanics". Van Os and van Leussen published their cutting theory in 1987. Van Leussen and Nieuwenhuis discussed the relevant soil mechanical parameters in 1984. Miedema 1987 uses the average deformation rate as stated by van Os 1977 but instead of inserting this in the Biot equation, the average deformation rate is modeled as a boundary condition in the shear zone. Although the cutting process is not solely dependent upon the phenomenon dilatancy, the above mentioned research showed that for cutting velocities in a range from 0.5 to 5 m/sec the cutting process is dominated by the phenomenon dilatancy, so the contributions of gravitational, cohesive, adhesive and inertial forces can be neglected.

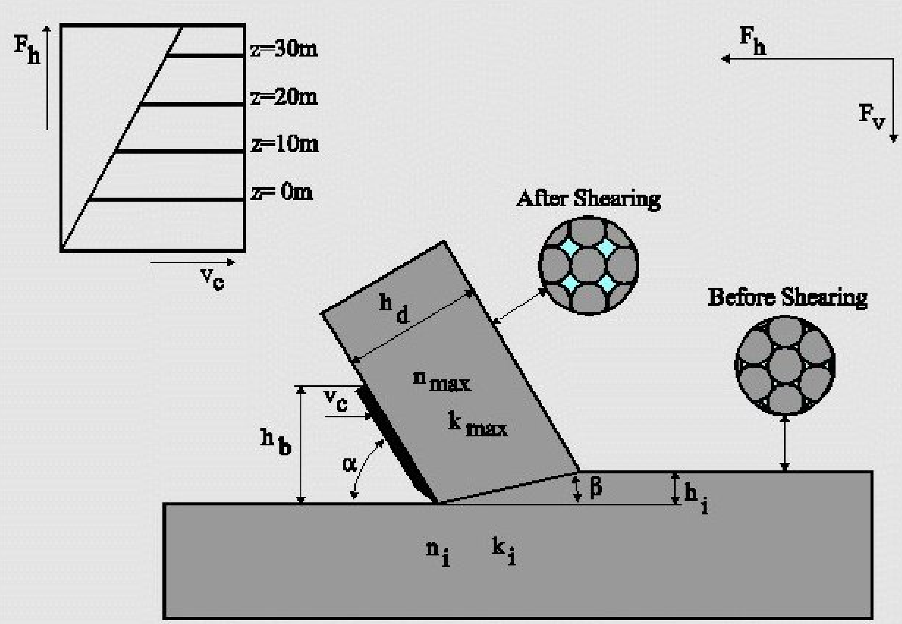

Fig. 1: Dilatation during the cutting of water saturated sand.

The Equilibrium of Forces on the Layer Cut.

As stated above, the cutting process is governed by the pore pressures and by the grain stresses. Figure 2 shows the pore pressures and the grain stresses acting on the layer of sand cut. If the cutting velocity (drag velocity) is considered to be horizontal, the unknown forces K_1 and K_2 can be determined by taking the horizontal and vertical equilibrium of forces acting on the layer cut, according to the equations 1 and 2.

The horizontal equilibrium of forces:

$$K_1 \cdot \sin(\beta+\phi) - W_1 \cdot \sin(\beta) + C \cdot \cos(\beta) + I \cdot \cos(\beta) - A \cdot \cos(\alpha) + W_2 \cdot \sin(\alpha) - K_2 \cdot \sin(\alpha+\delta) = 0 \quad (1)$$

The vertical equilibrium of forces:

$$- K_1 \cdot \cos(\beta+\phi) + W_1 \cdot \cos(\beta) + C \cdot \sin(\beta) + I \cdot \sin(\beta) + G + A \cdot \sin(\alpha) + W_2 \cdot \cos(\alpha) - K_2 \cdot \cos(\alpha+\delta) = 0 \quad (2)$$

The force K_2 on the blade is now:

$$K_2 = \frac{W_1 \cdot \sin(\phi) + W_2 \cdot \sin(\alpha+\beta+\phi)}{\sin(\alpha+\beta+\delta+\phi)} \quad (3)$$

From this last equation, the forces on the blade can be derived. On the blade, a force component in the direction of cutting velocity F_h and a force perpendicular to this direction F_v can be distinguished.

$$F_h = - W_2 \cdot \sin(\alpha) + K_2 \cdot \sin(\alpha+\delta) \quad (4)$$

$$F_v = - W_2 \cdot \cos(\alpha) + K_2 \cdot \cos(\alpha+\delta) \quad (5)$$

The Water Pore Pressures.

The forces W_1 and W_2 resulting from the pore pressures are the unknowns in the equations 4 and 5. Miedema 1987 calculated the average pore pressures P_1 and P_2 with a FEM program. With the equations 6 and 7 the forces W_1 and W_2 can be determined by substituting the results of the FEM calculations.

$$W_1 = \frac{P_1 \cdot \rho_w \cdot g \cdot v_c \cdot e \cdot h_i^2 \cdot b}{(a_1 \cdot k_i + a_2 \cdot k_{max}) \cdot \sin(\beta)} \tag{6}$$

$$W_2 = \frac{P_2 \cdot \rho_w \cdot g \cdot v_c \cdot e \cdot h_i \cdot h_b \cdot b}{(a_1 \cdot k_i + a_2 \cdot k_{max}) \cdot \sin(\alpha)} \tag{7}$$

When the pore pressures reaches the water vapor pressure, cavitation will occur. The pore pressures cannot decrease further with an increasing cutting velocity and remain constant. In this case the forces W_1 and W_2 can be calculated analytical, giving:

$$W_1 = \frac{\rho_w \cdot g \cdot (z + 10) \cdot h_i \cdot b}{\sin(\beta)} \tag{8}$$

$$W_2 = \frac{\rho_w \cdot g \cdot (z + 10) \cdot h_b \cdot b}{\sin(\alpha)} \tag{9}$$

The Simplified Equations for the Cutting Forces.

Miedema [6] 1987 simplified the equations by using proportionality coefficients c_1 , c_2 , d_1 and d_2 . This leads to the first two simplified equations for the two-dimensional cutting process in water saturated sand without cavitation:

$$F_h = \frac{c_1 \cdot \rho_w \cdot g \cdot v_c \cdot h_i^2 \cdot b \cdot e}{k_m} \tag{10}$$

$$F_v = \frac{c_2 \cdot \rho_w \cdot g \cdot v_c \cdot h_i^2 \cdot b \cdot e}{k_m} \tag{11}$$

With: $e = \frac{n_{max} - n_i}{1 - n_{max}}$ and $k_m \approx 0.5 \cdot k_i + 0.5 \cdot k_{max}$

For the cavitating cutting process the following equation are valid for the horizontal and vertical cutting force:

$$F_h = d_1 \cdot \rho_w \cdot g \cdot (z+10) \cdot h_i \cdot b \tag{12}$$

$$F_v = d_2 \cdot \rho_w \cdot g \cdot (z+10) \cdot h_i \cdot b \tag{13}$$

The coefficients c_1 , c_2 , d_1 and d_2 are dependent upon the angle of internal friction of the sand ϕ, the soil interface friction angle δ, the blade angle α and the blade height/layer thickness ratio h_b /h_i. Detailed tables of c_1 , c_2 , d_1 and d_2 are published by Miedema [6] in 1987.

The Normal and Friction Forces on the Shear Surface and on the Blade.

Although the normal and friction forces as shown in figure 2 are the basis for the calculation of the horizontal and vertical cutting forces, the approach used, requires the following equations to derive these forces by substituting 10 and 11 for the non-cavitating cutting process or 12 and 13 for the cavitating cutting process. The index 1 points to the shear surface, while the index 2 points to the blade.

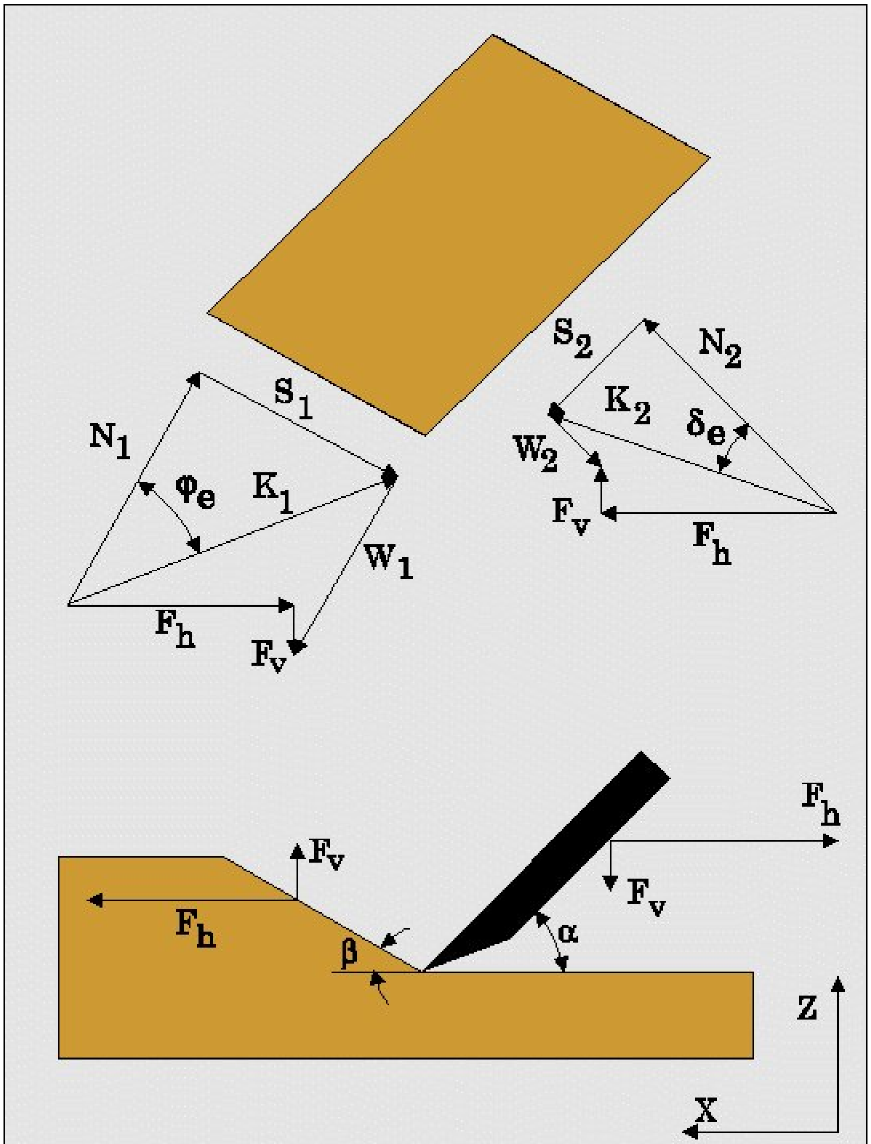

Fig. 2: The forces on the layer cut.

$$F_{n1} = F_h \cdot \sin(\beta) - F_v \cdot \cos(\beta) \tag{14}$$

$$F_{f1} = F_h \cdot \cos(\beta) + F_v \cdot \sin(\beta) \tag{15}$$

$$F_{n2} = F_h \cdot \sin(\alpha) + F_v \cdot \cos(\alpha) \tag{16}$$

$$F_{f2} = F_h \cdot \cos(\alpha) - F_v \cdot \sin(\alpha) \tag{17}$$

The Three-Dimensional Cutting Theory.

The previous paragraphs summarized the two-dimensional cutting theory. However, as stated in the introduction, in most cases the cutting process is not two-dimensional, because the drag velocity is not perpendicular to the cutting edge of the blade. Figure 3 shows this phenomenon. As with snow-ploughs, the sand will flow to one side while the blade is pushed to the opposite side. This will result in a third cutting force, the transverse force Ft . To determine this force, the flow direction of the sand has to be known. Figure 4 shows a possible flow direction.

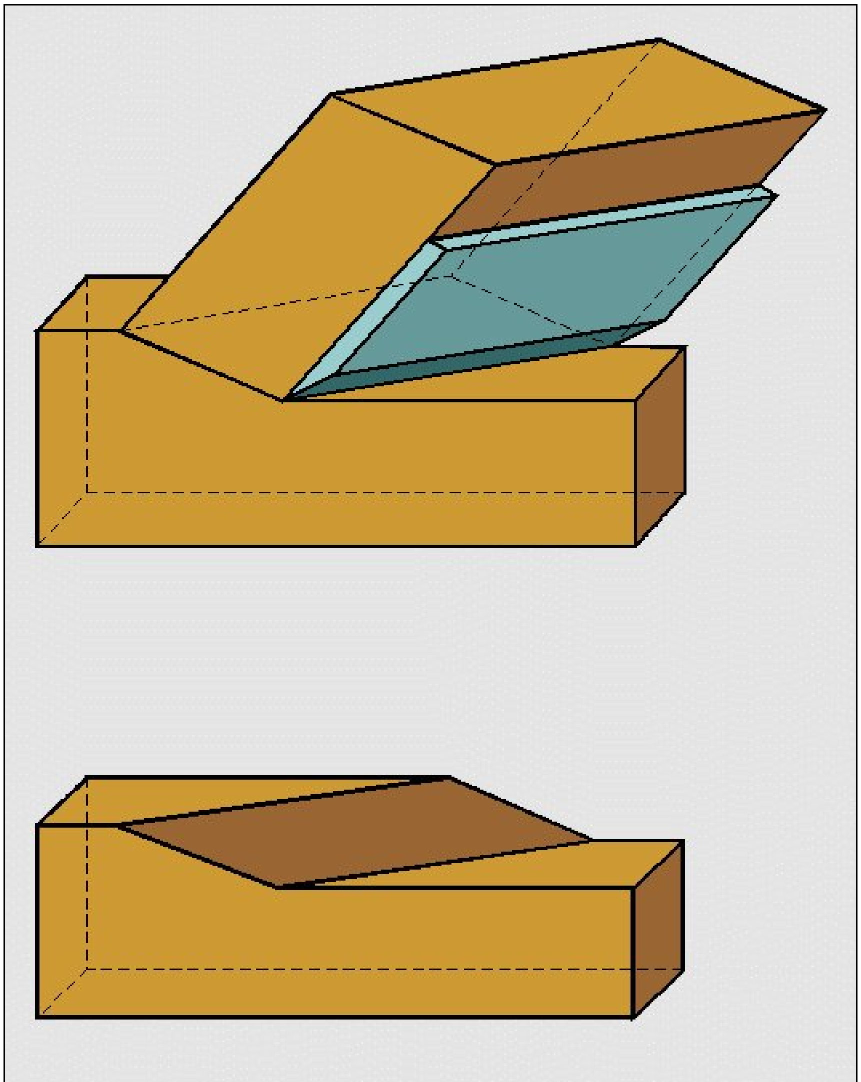

Fig. 3: The 3D cutting process

Velocity Conditions.

For the velocity component perpendicular to the blade v_c, if the blade has a deviation angle ι and a drag velocity v_d according to figure 4, it yields:

$$v_c = v_d \cdot \cos(\iota) \tag{18}$$

The velocity of grains in the shear surface perpendicular to the cutting edge is now:

$$v_{r1} = v_c \cdot \frac{\sin(\alpha)}{\sin(\alpha+\beta)} \tag{19}$$

The relative velocity of grains with respect to the blade, perpendicular to the cutting edge is:

$$v_{r2} = v_c \cdot \frac{\sin(\beta)}{\sin(\alpha+\beta)} \tag{20}$$

The grains will not only have a velocity perpendicular to the cutting edge, but also parallel to the cutting edge, the deviation velocity components v_{d1} on the shear surface and v_{d2} on the blade. The velocity components of a grain in x, y and z direction can be determined by considering the absolute velocity of grains in the shear surface, this leads to:

$$v_{x1} = v_{r1} \cdot \cos(\beta) \cdot \cos(\iota) + v_{d1} \cdot \sin(\iota) \tag{21}$$

$$v_{y1} = - v_{r1} \cdot \cos(\beta) \cdot \sin(\iota) - v_{d1} \cdot \cos(\iota) \tag{22}$$

$$v_{z1} = v_{r1} \cdot \sin(\beta) \tag{23}$$

The velocity components of a grain can also be determined by a summation of the drag velocity of the blade and the relative velocity between the grains and the blade, this gives:

$$v_{x2} = v_d - v_{r2} \cdot \cos(\alpha) \cdot \cos(\iota) - v_{d2} \cdot \sin(\iota) \tag{24}$$

$$v_{y2} = - v_{r2} \cdot \cos(\alpha) \cdot \sin(\iota) + v_{d2} \cdot \cos(\iota) \tag{25}$$

$$v_{z2} = v_{r1} \cdot \sin(\alpha) \tag{26}$$

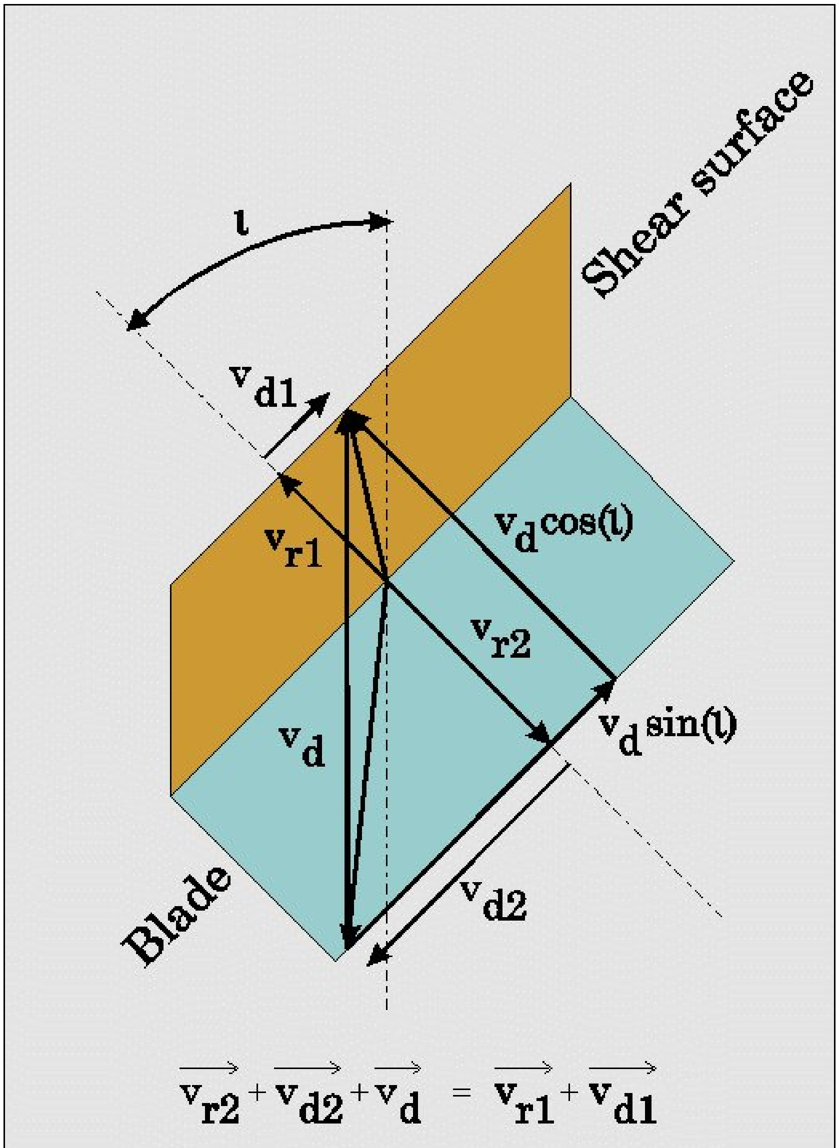

Fig. 4b: Velocity conditions.

Since both approaches will have to give the same resulting velocity components, the following condition for the transverse velocity components can be derived:

$$v_{x1} = v_{x2} \Longrightarrow v_{d1} + v_{d2} = v_d \cdot \sin(\iota) \tag{27}$$

$$v_{y1} = v_{y2} \Longrightarrow v_{d1} + v_{d2} = v_d \cdot \sin(\iota) \tag{28}$$

$$v_{z1} = v_{z2} \tag{29}$$

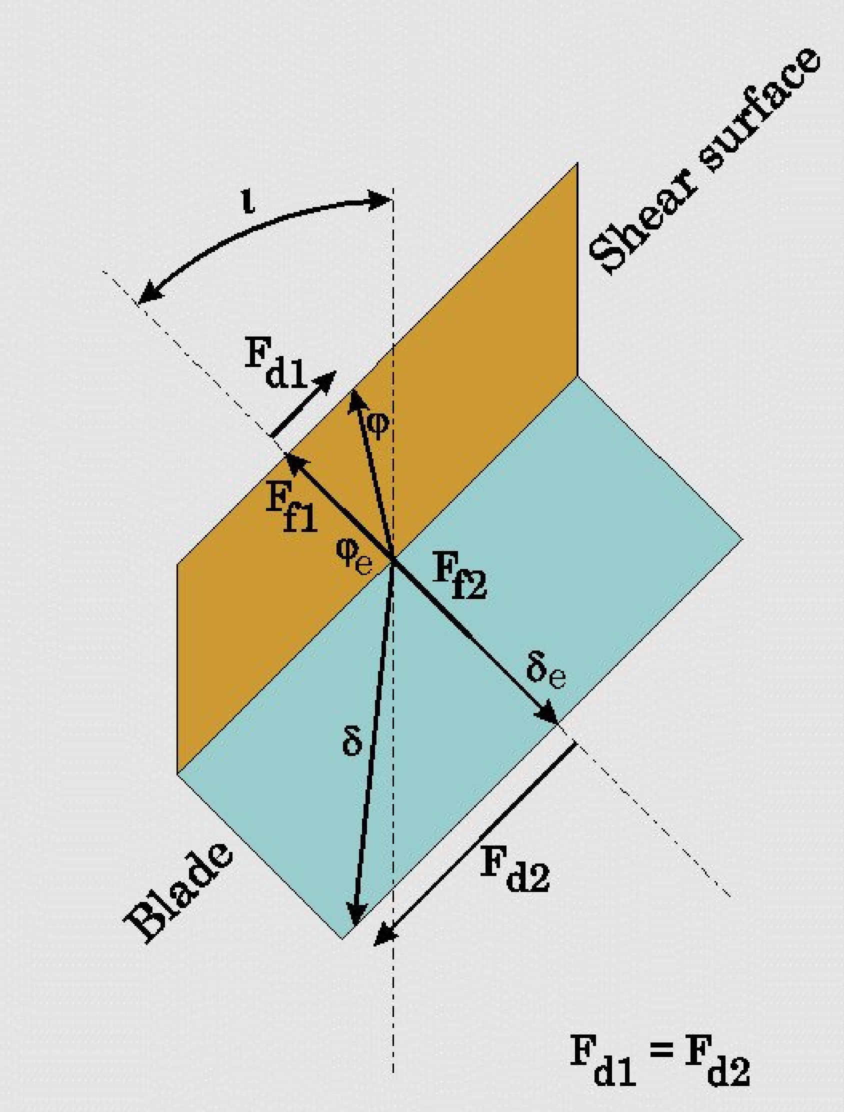

Fig. 4a: Force directions.

The Deviation Force.

Since friction always has a direction matching the direction of the relative velocity between two bodies, the fact that a deviation velocity exists on the shear surface and on the blade, implies that also deviation forces must exist. To match the direction of the relative velocities, the following equation can be derived for the deviation force on the shear surface and on the blade (fig. 4):

$$F_{d1} = F_{f1} \cdot \frac{v_{d1}}{v_{r1}} \quad (30)$$

$$F_{d2} = F_{f2} \cdot \frac{v_{d2}}{v_{r2}} \quad (31)$$

Since perpendicular to the cutting edge, an equilibrium of forces exists, the two deviation forces must be equal in magnitude and have opposite directions.

$$F_{d1} = \left|F_{d2}\right| \quad (32)$$

By substituting 30 and 31 in 32 and then substituting 15 and 17 for the friction forces and 19 and 20 for the relative velocities, the following equation can be derived, giving a second relation between the two deviation velocities:

$$\frac{v_{d1}}{v_{d2}} = \left(\frac{F_{f2}}{F_{f1}}\right) \cdot \left(\frac{v_{r1}}{v_{r2}}\right) = \left(\frac{F_h \cdot \cos(\alpha) - F_v \cdot \sin(\alpha)}{F_h \cdot \cos(\beta) + F_v \cdot \sin(\beta)}\right) \cdot \left(\frac{\sin(\alpha)}{\sin(\beta)}\right) \quad (33)$$

To determine F_h and F_v the angle of internal friction ϕ and the soil/interface friction angle mobilized perpendicular to the cutting edge, have to be determined by using the ratio of the transverse velocity and the relative velocity, according to:

$$\tan(\phi_e) = \tan(\phi) \cdot \cos\left(\operatorname{atn}\left(\frac{v_{d1}}{v_{r1}}\right)\right) \quad (34)$$

$$\tan(\delta_e) = \tan(\delta) \cdot \cos\left(\operatorname{atn}\left(\frac{v_{d2}}{v_{r2}}\right)\right) \quad (35)$$

The Resulting Cutting Forces.

The resulting cutting forces in x, y and z direction can be determined once the deviation velocity components are known. However, it can be seen that the second velocity condition (33) requires the horizontal and vertical cutting forces perpendicular to the cutting edge, while these forces can only be determined if the mobilized friction angles (34 and 35) are known. This creates an implicit set of equations that will have to be solved by means of an iteration process. For the cutting forces on the blade the following equation can be derived:

$$F_{x2} = F_h \cdot \cos(\iota) + F_{d2} \cdot \sin(\iota) \quad (36)$$

$$F_{y2} = F_h \cdot \sin(\iota) - F_{d2} \cdot \cos(\iota) \quad (37)$$

$$F_{z2} = F_v \quad (38)$$

Conclusions.

To check the validity of the above derived theory, research has been carried out in the laboratory of the chair of Dredging Technology of the Delft University of Technology. The tests are carried out in a hard packed water saturated sand, with a blade of 0.3 m by 0.2 m. The blade had a cutting angle of 45 degrees and inclination angles of 0, 15, 30 and 45 degrees. The layer thicknesses were 2.5, 5 and 10 cm and the drag velocities 0.25, 0.5 and 1 m/s. Figure 5 shows the results with an inclination angle of 45 degrees. The lines in this figure show the theoretical forces. As can be seen, the measured forces match the theoretical forces well. Since the research is still in progress, further publications on this subject will follow.

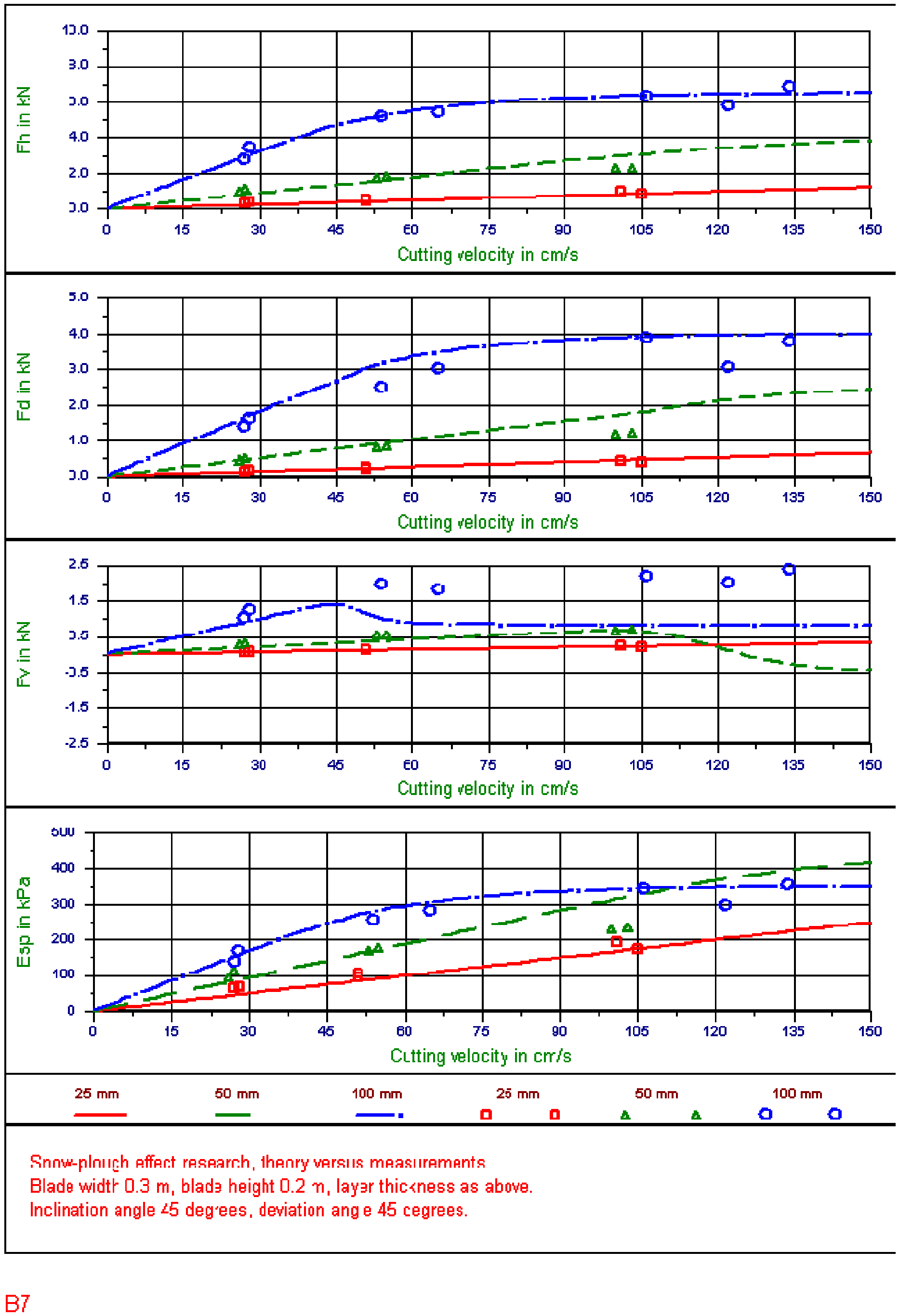

Fig. 5: Results of cutting tests with a deviation angle of 45 degrees.

Acknowledgements.

The author wishes to thank Mr. Zwartbol and Mr. Keuzekamp for the work they carried out on this research. He also thanks the Delft University for giving him the opportunity to carry out this research.

Bibliography.

[1] Leussen, W. van & Nieuwenhuis J.D., "Soil Mechanics Aspects of Dredging". Geotechnique 34 No.3, pp. 359-381, 1984.

[2] Miedema, S.A., "The Calculation of the Cutting Forces when Cutting Water Saturated Sand, Basic Theory and Applications for 3-Dimensional Blade Movements with Periodically Varying Velocities for in Dredging Usual Excavating Elements" (in Dutch). Ph.D. thesis, Delft, 1987, the Netherlands.

[3] Miedema, S.A., "On the Cutting Forces in Saturated Sand of a Seagoing Cutter Suction Dredge". Proc. WODCON XII, Orlando, Florida, USA, April 1989. And Terra et Aqua No. 41, December 1989, Elseviers Scientific Publishers.

[4] Becker, S. & Miedema, S.A. & Jong, P.S. de & Wittekoek, S., "On the Closing Process of Clamshell Dredges in Water Saturated Sand". Proc. WODCON XIII, Bombay, India, 1992. And Terra et Aqua No. 49, September 1992, IADC, The Hague.

[5] Os, A.G. van, "Behavior of Soil when Excavated Underwater". International Course Modern Dredging. June 1977, The Hague, The Netherlands.

[6] Os, A.G. van & Leussen, W. van, "Basic Research on Cutting Forces in Saturated Sand". Journal of Geotechnical Engineering, Vol. 113, No.12, December 1987.

List of Symbols Used.

b	Width of blade	m
c_1, c_2	Coefficients (no-cavitation)	-
d_1, d_2	Coefficients (cavitation)	-
e	Volume strain	%
F_h	Horizontal cutting force	kN
F_{f1}	Friction force on the shear surface	kN
F_{f2}	Friction force on the blade	kN
F_{n1}	Normal force on the shear surface	kN
F_{n2}	Normal force on the blade	kN
F_v	Vertical cutting force	kN
F_{d1}	Deviation force on the shear surface	kN
$F_{d, d2}$	Deviation force on the blade	kN
$F_{x1, 2}$	Cutting force in x-direction	kN
$F_{y1, 2}$	Cutting force in y-direction	kN
$F_{z1, 2}$	Cutting force in z-direction	kN
g	Gravitational constant (9.81)	m/s^2
h_i	Initial thickness of layer cut	m
h_b	Height of blade	m
k_i	Initial permeability	m/s
k_{max}	Maximum permeability	m/s
k_m	Average permeability	m/s
K_1	Grain force on the shear plane	kN
K_2	Grain force on the blade	kN
n_i	Initial porosity	%
n_{max}	Maximum porosity	%
P_1	Average pore pressure on the shear surface	kPa
P_2	Average pore pressure on the blade	kPa
v_c	Cutting velocity component perpendicular to the blade	m/s
v_d	Cutting velocity, drag velocity	m/s
v_{r1}	Velocity of grains in the shear surface	m/s
v_{r2}	Relative velocity of grains on the blade	m/s
v_{d1}	Deviation velocity of grains in the shear surface	m/s
v_{d2}	Deviation velocity of grains on the blade	m/s

$v_{x1,2}$	Velocity of grains in the x-direction	m/s
$v_{y1,2}$	Velocity of grains in the y-direction	m/s
$v_{z1,2}$	Velocity of grains in the z-direction	m/s
W_1	Force resulting from pore underpressure on the shear plane	kN
W_2	Force resulting from pore underpressure on the blade	kN
z	Water depth	m
α	Cutting angle blade	rad
β	Shear angle	rad
ϕ	Angle of internal friction	rad
ϕ_e	Angle of internal friction perpendicular to the cutting edge	rad
δ	Soil/interface friction angle	rad
δ_e	Soil/interface friction angle perpendicular to the cutting edge	rad
ι	Deviation angle blade	rad
ρ_w	Density water	ton/m^3