## SOIL-WATER MIXTURE AND ITS PHASES

### 2.1 SOIL PROPERTIES

Different sorts of soil are subject to dredging. At different locations dredged solids may differ strongly in density, size, shape and consistency. It is a very different experience to dredge the iron ore of density round $4700 \mathrm{~kg} / \mathrm{m}^{3}$ or the sand of density $2650 \mathrm{~kg} / \mathrm{m}^{3}$. If clay (a typical particle size is in an order of microns) is dredged it is necessary to know whether it is cohesive or non-cohesive. The non-cohesive clay will be transported as a pseudo-homogeneous mixture in a pipeline while the cohesive clay, cuttered in slices from the bottom of a waterway, will form clay balls in a pipeline. The balls might be of hundreds of millimetres in a diameter. Such balls will definitely not be suspended in a carrying liquid. They will roll and slide at the bottom of a dredging pipeline, giving a totally different flow pattern in a pipeline than cohensionless clay. As a result the flow resistance will be very different. However, a majority of dredged materials are just different sorts of sand or gravel that are non-cohesive and of almost identical density.

The properties of a transported soil are of great importance for a determination of flow conditions in dredging pipelines. A determination of properties of a transported soil requires:
a. taking samples of soil

- in situ
- in a pipeline
b. analysis of samples using suitable methods.

Particle size distribution, soil density and in situ concentration of soil are the most important soil properties influencing behaviour of solid particles in a mixture transported in a dredging installation. If clay is transported the cohesion (described by liquid limit and plastic limit) is a further important parameter. In the following text, only the particle size distribution and solids density will be discussed. A determination of other properties is described e.g. in van den Berg (1998).

### 2.1.1. Particle size distribution (PSD)

Several systems are defined for identification and a classification of solids according to their particle size (e.g. American norm ASTM Standard D288). For the dredging purposes the following system is used (Table 2.1)

Table 2.1. Identification and classification of soil for dredging purposes

| Main type of soil |  | Particle size |  |
| :---: | :---: | :---: | :---: |
|  |  | Identification | size in [mm] |
| Boulders | Granular <br> Non-cohesive | - | $>200$ |
| Cobbles |  | - | $200-60$ |
| Gravel |  |  | $60-20$ |
|  |  | Coarse | $20-6$ |
|  |  | Medium | $6-2$ |
|  |  | Fine | $2-0.6$ |
| Silt | Coarse | $0.6-0.2$ |  |
|  |  | Medium | $0.2-0.06$ |
|  | Cohesive | Fine | $0.06-0.02$ |
| Clay |  | Coarse | Medium |
|  |  | Fine | $0.02-0.006-0.002$ |

Dredged soil is seldom uniformly graded. Two techniques are used to determine particle size distribution (PSD) in samples of dredged sand or gravel - the sieving and the sedimentation tests, finer solids as silt and clay are tested using the hydrometric method.

### 2.1.1.1 Methods for PSD determination

A choice of the PSD method is dependent on the range of particle sizes in the tested sample of soil. Samples containing particles of sand- or gravel size are usually tested using the screen method (the sieving) or the sedimentation method (the sedimentation in the water column). For finer particles ( $\mathrm{d}<0.074 \mathrm{~mm}$ ) the distribution is determined by a sedimentation process in a hydrometer.
A. Sieving

A weighted sample of dry solids is sieved through a series of sieves with standard sieve meshes. The sample fraction remaining in each sieve is weighted. The fractions by mass are recalculated to obtain a percentage of the mass of the entire sample, $p_{i}$. The fraction $p_{i}$ is considered to contain particles of characteristic diameter $d_{i}$ represented by the size of opening of the sieve at which the fraction $\mathrm{p}_{\mathrm{i}}$ remained.

Table 2.2. PSD for the narrow-graded medium sand (determined by a sieving analysis of a one-kilogram dry sample).

| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| sieve <br> opening <br> $[\mathrm{mm}]$ | weight <br> fraction <br> $[\mathrm{g}]$ | percentage of <br> total weight, <br> $\mathrm{p}_{\mathrm{i}}[\%]$ | cumulative <br> $\%$ <br> mass | characteristic <br> particle size, <br> $\mathrm{d}_{\mathrm{j}}[\mathrm{mm}]$ |
| 0.85 | 0 | 0 | 100 | 0.85 |
| 0.6 | 6.7 | 0.67 | 99.33 | 0.6 |
| 0.5 | 37.7 | 3.77 | 95.56 | 0.5 |
| 0.42 | 447.8 | 44.78 | 50.78 | 0.42 |
| 0.355 | 366.4 | 36.64 | 14.14 | 0.355 |
| 0.3 | 74.2 | 7.42 | 6.72 | 0.3 |
| 0.21 | 55.1 | 5.51 | 1.21 | 0.21 |
| 0.15 | 10.6 | 1.06 | 0.15 | 0.15 |
| 0.00 | 1.5 | 0.15 | 0 | 0.00 |
|  | $\Sigma 1000$ | $\Sigma 100$ |  |  |

## Legend:

column 1: sieve opening of the sieves used to test the sample
column 2: weight of soil fraction remaining at the sieve of the opening in col. 1
column 3: weight in col. 2 as a percentage of the weight of the total sample ( 1 kg )
column 4: cumulative percentage, $\Sigma \mathrm{p}_{\mathrm{i}}$
column 5: particle diameter representing the soil fraction remaining at the sieve

Table 2.3. PSD for the broad-graded medium sand (determined by a sieving analysis of a one-kilogram dry sample).

| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| sieve <br> opening <br> $[\mathrm{mm}]$ | weight <br> fraction <br> $[\mathrm{g}]$ | percentage of <br> total weight, <br> $\mathrm{p}_{\mathrm{i}}[\%]$ | cumulative <br> $\%$ <br> mass | Characteristic <br> Particle size, <br> $\mathrm{d}_{\mathrm{j}}[\mathrm{mm}]$ |
| 0.85 | 5 | 0.5 | 99.50 | 0.85 |
| 0.6 | 162.4 | 16.24 | 83.26 | 0.6 |
| 0.5 | 162.4 | 16.24 | 67.02 | 0.5 |
| 0.42 | 162.4 | 16.24 | 50.78 | 0.42 |
| 0.355 | 125.7 | 12.57 | 38.21 | 0.355 |
| 0.3 | 125.7 | 12.57 | 25.64 | 0.3 |
| 0.21 | 125.7 | 12.57 | 13.07 | 0.21 |
| 0.15 | 125.7 | 12.57 | 0.5 | 0.15 |
| 0.00 | 5 | 0.5 | 0 | 0.00 |
|  | $\Sigma 1000$ | $\Sigma 100$ |  |  |

The columns 4 and 5 of the Tables 2.2 and 2.3 are plotted to the cumulative Particle Size Distribution curve.

Samples of relatively narrow graded fine sand (Sand a) and medium sand (Sand c) are processed together with a broad graded sand (Sand b) in Table 2.4 and plotted to the cumulative PSD curve on Fig. 2.1.

Table 2.4. Processed PSD for three sorts of sand.

| $(\%)$ | zand a <br> $(\mu \mathrm{m})$ | zand b <br> $(\mu \mathrm{m})$ | zand c <br> $(\mu \mathrm{m})$ |
| :---: | :---: | :---: | :---: |
| 10 | 250 | 1300 | 850 |
| 20 | 180 | 500 | 620 |
| 30 | 150 | 270 | 500 |
| 40 | 140 | 170 | 400 |
| 50 | 120 | 120 | 350 |
| 60 | 90 | 75 | 290 |
| 70 | 75 | 55 | 240 |
| 80 | 60 | 40 | 180 |
| 90 | 40 | 25 | 130 |
|  |  |  |  |
| Totaal | 1105 | 2555 | 3560 |
| dmf | 123 | 284 | 396 |



Figure 2.1. Cumulative curves of particle size distribution for three sorts of sand according to Table 2.4.

## b. Sedimentation column

The method of sedimentation a long sedimentation column is used as alternative to the sieving method for solid particles of sand (and gravel) size. This method has the advantage of direct provision of the settling velocity of the particles, which is the parameter characterising the solids impact on the slurry flow behaviour rather than the
particle size. The particle size distribution can be obtained from the settling velocity distribution using appropriate settling velocity equation.
A weighted sample of solids is collected in a cup at the top of a sedimentation column filled with water. The water level in the sedimentation column touches the bottom of the cup. The cup is opened and the time is measured which solid particles of the sample need to reach a plate at the bottom of the sedimentation column. The distance between the cup and plate is known. The measuring principle is the sensing of the progression of the deflection in time of a thin metal membrane connected with both the cup and the plate. The membrane senses an impulse determining the time at which the cup is opened and the time-dependent increase in the weight of the plate as the particles settle on its surface. This signal is converted and plotted as a summation curve of $\%$ solids mass passed versus particle settling velocity.

The effect of the concentration of solids in a settling cloud on the particle settling velocity in a sedimentation column is negligible because of small solids samples and thus the low concentration of solids in the settling cloud. The disadvantage of this method is that the length of the sedimentation column is too short to test very coarse particles, which need only a few seconds to reach the plate at the bottom of the sedimentation column.

## c. Hydrometer tests

This method determines a particle size distribution for soils too fine to be sieved, i.e. finer than 0.074 mm . A hydrometer is a small glass bottle with a calibrated volume. In a hydrometer a small sample of soil is suspended in distilled water. At certain position density of suspension is measured. Due to slow sedimentation of particles in quiescent water in a hydrometer the suspension density at the position gradually decreases. The change of density with time is sensed. This can be interpreted as a development of settling velocity of particles in a soil sample and thus according to Stokes law of settling as a particle size distribution. As the sieving analysis the hydrometer analysis provides the cumulative PSD curve.

### 2.1.1.2. Characteristic diameters of particle

The mass-median diameter $\mathrm{d}_{50}$ is the diameter for which $50 \%$ (by mass) of the particles in a soil sample are finer. In other words one half of all particles in a sample is larger and one half of all particles is smaller than is the size $\mathrm{d}_{50}$. The $\mathrm{d}_{85}$ size gives the diameter for which $85 \%$ (by mass) of the particles in a soil sample are finer.

A comparison of PSD for these two sand samples demonstrates clearly that the mass-median diameter $\mathrm{d}_{50}$ of particles does not represent fully the particle size in a soil sample. The $\mathrm{d}_{50}$ value is identical for both samples $\left(\mathrm{d}_{50}=0.42 \mathrm{~mm}\right)$ but the deviation from the median size is much larger in a broadly graded sample. A deviation from the median size has to be incorporated in the parameter(s) representing the solids size in mixture flow models. Usually such a parameter is sufficiently represented by mean diameter or decisive particle diameter defined below, but for specific cases as is that simulated in Table 2.3 is this not enough. Therefore the flow models employing at least two characteristic particle diameters (e.g. $\mathrm{d}_{50}$ and d85) are more suitable.

The mean diameter of particles in a type of solid with a certain degree of grading is determined as

$$
\begin{equation*}
\mathrm{d}_{\mathrm{s}}=\frac{\sum_{\mathrm{i}} \mathrm{~d}_{\mathrm{i}} \mathrm{p}_{\mathrm{i}}}{\sum_{\mathrm{i}} \mathrm{p}_{\mathrm{i}}} \tag{2.1}
\end{equation*}
$$

where $\sum_{i} p_{i}=1.00$.

In a dredging practice the Fuhrboter classification of the characteristic particle size is often used. This so-called decisive particle diameter (in Dutch "maatgevende korreldiameter") is defined as

$$
\begin{equation*}
\mathrm{d}_{\mathrm{mf}}=\frac{\mathrm{d}_{10}+\mathrm{d}_{20}+\ldots \ldots \ldots \ldots \ldots . .+\mathrm{d}_{80}+\mathrm{d}_{90}}{9} \tag{2.2}
\end{equation*}
$$

Table 2.5. Comparison of characteristic particle sizes for medium sand samples in Table 2.2 and Table 2.3.

|  | Narrow-graded <br> sand <br> (Table 2.1) | Broadly <br> graded sand <br> (Table 2.2) |
| :---: | :---: | :---: |
| $\mathrm{d}_{50}$ | 0.42 | 0.42 |
| $\mathrm{~d}_{85}$ | 0.47 | 0.62 |
| $\mathrm{~d}_{15}$ | 0.36 | 0.22 |
| $\mathrm{~d}_{\mathrm{S}}$ | 0.38 | 0.38 |
| $\mathrm{~d}_{\mathrm{mf}}$ | 0.42 | 0.42 |

### 2.1.1.3. Form factor of a particle

The form (shape) of the particle influences the settling process of a particle in a carrying liquid, but also the wear of components in contact with flowing mixture and packing of solids in granular beds. The shape factor is determined experimentally by measuring the settling velocity of non-spherical particles. The shape-factor for non-spherical particles is $\xi=\frac{\mathrm{v}_{\mathrm{t}}}{\mathrm{v}_{\mathrm{ts}}}$ (as already discussed in Intermezzo I) and its value drops from one for spherical particles to the lowest values for the particles that have one dimension much smaller than the other two.

### 2.1.2. Density of solid grains and granular body

Density of solids is a property independent of temperature and pressure of the surrounding matter or other parameters that might vary during the solids transportation in a mixture.

Density is determined by a pycnometric method. A pycnometer is a glass bottle with a calibrated volume gauge. This is used to determine a volume of dry solids sample of known weight. The dry sample is dropped to the pycnometer containing a known volume of water. The change in a volume is read out as the change in the position of the water level at the pycnometer gauge. This gives a total volume of particles of a sample. The ratio of the weight of a sample and its volume gives density of solid grains, $\rho_{\mathrm{S}}$, in a sample.

For dredging purposes, however, an another parameter associated with the mass and the volume of transported soil is important. The amount of cubic meters of transported material decisive for dredging work payments is not based on the density of solid particles, $\rho_{\mathrm{S}}$, but rather on the in situ density of the soil body, $\rho_{\mathrm{si}}$. If a known volume of a soil (i.e. a granular body) submerged in water is weighted, the ratio of the weight and body volume gives the in situ density of a soil, $\rho_{\mathrm{si}}$. This is lower than the density of solids occupying a granular body since it includes the volume of water (lighter than solids) present in the pores between particles in the granular body.

A measure of the difference between the density of solid particles and density of soil occupied by the solid particles is given by the porosity of a soil, $n$, that is obtained as

$$
\begin{equation*}
\mathrm{n}=\frac{\rho_{\mathrm{S}}-\rho_{\mathrm{si}}}{\rho_{\mathrm{S}}-\rho_{\mathrm{f}}} \tag{2.3}
\end{equation*}
$$

| n | porosity of soil |
| :--- | :--- |
| $\rho_{\mathrm{S}}$ | density of solid grains |
| $\rho_{\mathrm{si}}$ | density of soil in situ |
| $\rho_{\mathrm{f}}$ | density of liquid |

$\rho_{\mathrm{S}} \quad$ density of solid grains $\quad\left[\mathrm{kg} / \mathrm{m}^{3}\right]$
$\rho_{\mathrm{f}}$ density of liquid $\quad\left[\mathrm{kg} / \mathrm{m}^{3}\right]$

The typical value of the in situ density is $2000 \mathrm{~kg} / \mathrm{m}^{3}$ for a submerged sand bed. This indicates that $60 \%$ of the sand-bed volume is occupied by sand particles having density $2650 \mathrm{~kg} / \mathrm{m}^{3}$ and the interstitial water of density $1000 \mathrm{~kg} / \mathrm{m}$ occupies $40 \%$ of the total volume ${ }^{3}$. The porosity $\mathrm{n}=0.40$.

Table 2.6. Typical values of density and porosity of various soils

|  | Density of <br> solids <br> $\rho_{\mathrm{S}}\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ | Density of soil <br> in situ (wet) <br> $\rho_{\mathrm{Si}}\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ | Porosity |
| :---: | :---: | :---: | :---: |
| $\mathrm{n}[\%]$ |  |  |  |
| silt | 2650 | $1100-1400$ | $80-90$ |
| loose clay | 2650 | $1400-1600$ | $60-80$ |
| packed clay | 2650 | $1800-2000$ | $35-50$ |
| sand with clay | 2650 | $1800-2000$ | $40-50$ |
| sand | 2650 | $1900-2000$ | $35-45$ |
| coarse sand <br> with gravel | 2650 | $2050-2200$ | $28-36$ |
| clay boulders | 2650 | 2320 | 20 |

The ratio $\rho_{\mathrm{S}} / \rho_{\mathrm{W}}$ gives the relative density (also "specific gravity" of solid particle), $\mathrm{S}_{\mathrm{S}}$. The parameter $\rho_{\mathrm{W}}$ is the density of water with the typical value $1000 \mathrm{~kg} / \mathrm{m}^{3}$. More precise values are discussed in the following paragraph.

### 2.2 LIQUID PROPERTIES

Density and viscosity are the most important properties of a carrying liquid from the point of view of the mixture flow in a dredging pipeline. Both parameters are sensitive to temperature. Sensitivity to pressure can be neglected over the range of conditions encountered in the hydraulic transport, the liquids are considered incompressible.

Viscosity that relates the shear stress and the shear rate according to Newton's law of viscosity (see Chapter 1) is called dynamic viscosity, $\mu$, and it has a unit Pa.s. The ratio of the dynamic viscosity of liquid and density of liquid is known as kinematic viscosity, $v=\mu / \rho$, having a unit $\mathrm{m}^{2} / \mathrm{s}$.

Table 2.7. Properties of water

| Temperature $\mathrm{T}\left[{ }^{\circ} \mathrm{C}\right]$ | Density, $\rho_{\mathrm{W}}\left[\mathrm{~kg} / \mathrm{m}^{3}\right]$ | Dynamic viscosity, $\mu_{\mathrm{W}} \text { [Pa.s] }$ | Kinematic viscosity, $v_{\mathrm{W}}\left[\mathrm{~m}^{2} / \mathrm{s}\right]$ | Vapour <br> pressure, $\mathrm{p}_{\mathrm{V}}[\mathrm{~Pa}]$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 999.8 | $1.781 \times 10^{-3}$ | $1.785 \times 10^{-6}$ | $0.61 \times 10^{3}$ |
| 5 | 1000.0 | $1.518 \times 10^{-3}$ | $1.519 \times 10^{-6}$ | $0.87 \times 10^{3}$ |
| 10 | 999.7 | $1.307 \times 10^{-3}$ | $1.306 \times 10^{-6}$ | $1.23 \times 10^{3}$ |
| 15 | 999.1 | $1.139 \times 10^{-3}$ | $1.139 \times 10^{-6}$ | $1.70 \times 10^{3}$ |
| 20 | 998.2 | $1.002 \times 10^{-3}$ | $1.003 \times 10^{-6}$ | $2.34 \times 10^{3}$ |
| 25 | 997.0 | $0.890 \times 10^{-3}$ | $0.893 \times 10^{-6}$ | $3.17 \times 10^{3}$ |
| 30 | 995.7 | $0.798 \times 10^{-3}$ | $0.800 \times 10^{-6}$ | $4.24 \times 10^{3}$ |

Density of water, $\rho_{\mathrm{W}}\left[\mathrm{kg} / \mathrm{m}^{3}\right]$, can be approximated within a temperature range $5<\mathrm{T}$ $<100{ }^{\circ} \mathrm{C}$ using

$$
\begin{equation*}
\rho_{\mathrm{W}}=999.7-0.10512(\mathrm{~T}-10)-0.005121(\mathrm{~T}-10)^{2}+0.00001329(\mathrm{~T}-10)^{3} \tag{2.4}
\end{equation*}
$$

Dynamic viscosity of water, $\mu_{\mathrm{W}}$ [Pa.s], can be approximated using

$$
\begin{equation*}
\mu_{\mathrm{W}}=\frac{0.10}{2.1482\left([\mathrm{~T}-8.435]+\sqrt{8078.4+[\mathrm{T}-8.435]^{2}}\right)-120} \tag{2.5}
\end{equation*}
$$

in which T is temperature in ${ }^{0} \mathrm{C}$.
Density of seawater is slightly higher than that of river water due to the contents of dissolved salt. The typical value is $\rho_{\mathrm{f}}=1025 \mathrm{~kg} / \mathrm{m}^{3}$, i.e. relative density $\mathrm{S}_{\mathrm{f}}=\rho_{\mathrm{f}} / \rho_{\mathrm{W}}=$ 1.03 .

### 2.3 MIXTURE PROPERTIES

### 2.3.1 Mixture density and concentration of solids

Mixture is composed of two phases: solids and liquid. The density of the mixture, $\rho_{\mathrm{m}}$, is influenced by the fraction of solids present in a carrying liquid. The fraction is determined by the parameter called concentration. The volumetric concentration, $\mathrm{C}_{\mathrm{v}}$, determines the fraction of the mixture volume that is occupied by solids. The $\mathrm{C}_{\mathrm{V}}=$ 0.60 means that $60 \%$ of the total volume of mixture is occupied by solid particles. The fraction of solids in mixture can be also expressed in weight giving the weight concentration of solids, $\mathrm{C}_{\mathrm{w}}$. The $\mathrm{C}_{\mathrm{w}}=0.60$ means that $60 \%$ of the total weight of mixture is exerted by solid particles present in the mixture.
A relationship between the mixture density and the volumetric concentration of solids in a mixture proportion is derived from the general formula

$$
\begin{equation*}
\text { MASS }_{\text {mixture }}=\text { MASS }_{\text {liquid }}+\text { MASS }_{\text {solids }} \tag{2.6}
\end{equation*}
$$

considering MASS $=$ density $(\rho) \times$ volume $(U)$ and $U_{m}=U_{f}+U_{S}$

$$
\begin{equation*}
\rho_{\mathrm{m}} \mathrm{U}_{\mathrm{m}}=\rho_{\mathrm{f}} \mathrm{U}_{\mathrm{f}}+\rho_{\mathrm{S}} \mathrm{U}_{\mathrm{S}}=\rho_{\mathrm{f}}\left(\mathrm{U}_{\mathrm{m}}-\mathrm{U}_{\mathrm{S}}\right)+\rho_{\mathrm{S}} \mathrm{U}_{\mathrm{S}} \tag{2.7}
\end{equation*}
$$

dividing by $\mathrm{U}_{\mathrm{m}}$ and considering $\mathrm{C}_{\mathrm{V}}=\mathrm{U}_{\mathrm{S}} / \mathrm{U}_{\mathrm{m}}$ gives finally

$$
\begin{equation*}
\rho_{\mathrm{m}}=\rho_{\mathrm{f}}\left(1-\mathrm{C}_{\mathrm{V}}\right)+\rho_{\mathrm{s}} \mathrm{C}_{\mathrm{v}} \tag{2.8}
\end{equation*}
$$

Rearranging gives the equation for the volumetric concentration

$$
\begin{equation*}
\mathrm{C}_{\mathrm{V}}=\frac{\rho_{\mathrm{m}}-\rho_{\mathrm{f}}}{\rho_{\mathrm{s}}-\rho_{\mathrm{f}}}=\frac{\mathrm{S}_{\mathrm{m}}-\mathrm{S}_{\mathrm{f}}}{\mathrm{~S}_{\mathrm{s}}-\mathrm{S}_{\mathrm{f}}} \tag{2.9}
\end{equation*}
$$

The corresponding weight concentration

$$
\begin{array}{cl}
\mathrm{C}_{\mathrm{W}}=\frac{\mathrm{m}_{\mathrm{s}}}{\mathrm{~m}_{\mathrm{m}}}=\frac{\rho_{\mathrm{S}} \mathrm{U}_{\mathrm{S}}}{\rho_{\mathrm{m}} \mathrm{U}_{\mathrm{m}}}=\frac{\rho_{\mathrm{S}}}{\rho_{\mathrm{m}}} \mathrm{C}_{\mathrm{V}} & (2.10) \\
\mathrm{C}_{\mathrm{V}} & \text { volumetric concentration of solid grains } \\
\mathrm{C}_{\mathrm{w}} & \text { weight concentration of solid grains } \\
\rho_{\mathrm{S}} & \text { density of solid grains } \\
\rho_{\mathrm{m}} & \text { density of mixture } \\
\rho_{\mathrm{f}} & \text { density of liquid } \\
\mathrm{S}_{\mathrm{S}} & \text { relative density of solid grains } \\
\mathrm{S}_{\mathrm{m}} & \text { relative density of mixture } \\
\mathrm{S}_{\mathrm{f}} & \text { relative density of liquid } \\
\mathrm{m}_{\mathrm{s}} & \text { mass of solid grains } \\
\mathrm{m}_{\mathrm{m}} & \text { mass of mixture } \\
\mathrm{U}_{\mathrm{S}} & \text { volume of solid grains } \\
\mathrm{U}_{\mathrm{m}} & \text { volume of mixture } \\
{\left[\mathrm{kg} / \mathrm{m}^{3}\right]} \\
\left.\mathrm{m}^{3}\right] \\
{\left[\mathrm{kg} / \mathrm{m}^{3}\right]} \\
{[-]} \\
{[-]} \\
& {[-]} \\
& {[\mathrm{kg}]} \\
& {[\mathrm{kg}]} \\
{\left[\mathrm{m}^{3}\right]} \\
& {\left[\mathrm{m}^{3}\right]}
\end{array}
$$

Volumetric concentration is a parameter more often used than weight concentration in a dredging practice. This is because the payment of dredging works is based on the amount of cubic meters of material either dredged from the bottom of a waterway or dumped at the deposit site. The prize is determined for $1 \mathrm{~m}^{3}$ of the in situ soil dredged.

The volumetric concentration of in situ soil is determined as

$$
\begin{equation*}
\mathrm{C}_{\mathrm{vsi}}=\frac{\rho_{\mathrm{m}}-\rho_{\mathrm{f}}}{\rho_{\mathrm{si}}-\rho_{\mathrm{f}}} \tag{2.11}
\end{equation*}
$$

The volume of in situ soil body, $\mathrm{U}_{\mathrm{si}}$, dredged is composed of the volume of grains, $\mathrm{U}_{\mathrm{s}}$, and the volume of water, $\mathrm{U}_{\mathrm{f}}$, captured in voids (pores) between grains in a submerged granular body, i.e. $\mathrm{U}_{\mathrm{Si}}=\mathrm{U}_{\mathrm{S}}+\mathrm{U}_{\mathrm{f}}$. The volume of water in the in situ granular body is expressed by the parameter called porosity $n=U_{f} / U_{S i}$. Then $U_{\text {Si }}(1-n)$ $=U_{S}$ and the volumetric concentration of in situ solids is

$$
\begin{equation*}
\mathrm{C}_{\mathrm{vsi}}=\frac{\mathrm{U}_{\mathrm{si}}}{\mathrm{U}_{\mathrm{m}}}=\frac{\mathrm{U}_{\mathrm{S}}}{(1-\mathrm{n}) \mathrm{U}_{\mathrm{m}}}=\frac{\mathrm{C}_{\mathrm{v}}}{1-\mathrm{n}} \tag{2.12}
\end{equation*}
$$

The typical value of porosity for loose-poured bed of sand grains is $n=0.4$. For this bed the concentration $\mathrm{C}_{\mathrm{v}}=0.60$. Remember that $\mathrm{C}_{\mathrm{V}}=0.60$ means that $60 \%$ of the total volume of mixture is occupied by solid particles and $40 \%$ by water. If the porosity $\mathrm{n}=0.4$ (i.e. $40 \%$ ) the $\mathrm{C}_{\mathrm{Vsi}}=1.00$. This says that for a dredging operation a creation of the loose-poured bed in a deposit site is a reference level considered as unity.

The volumetric concentration of solids, $\mathrm{C}_{\mathrm{V}}$, is an important parameter in mixture flow modeling, the volumetric concentration of in situ soil, $\mathrm{C}_{\mathrm{Vsi}}$, is an important parameter in calculating production and thus economical costs of a dredging operation.

For the FLOWING MIXTURES the physical meaning of the solids concentration must be further specified. A different fraction of solids is determined in a mixture flowing in a pipeline when it is measured at two different places:

- in a horizontal length section of a pipeline (e.g. by weighting of the pipe section) and
- at the outlet of a pipeline (e.g. by collecting the discharged mixture to the tank).

The volume fraction of solids resident in an isolated length section of a horizontal pipe somewhere within the pipeline determines the spatial (resident) volumetric concentration of solids, $\mathrm{C}_{\mathrm{vi}}$, in a pipe. The volume fraction of solids in the mixture discharged to the collecting tank, i.e. delivered from a pipeline, determines the delivered volumetric concentration of solids, $\mathrm{C}_{\mathrm{Vd}}$ -

The spatial volumetric concentration gives the fraction of solids actually resident in a slurry pipeline and it is calculated as the ratio between solids and slurry volumes in a pipeline section

$$
\begin{equation*}
\mathrm{C}_{\mathrm{vi}}=\frac{\mathrm{U}_{\mathrm{S}}}{\mathrm{U}_{\mathrm{m}}} \tag{2.13}
\end{equation*}
$$

The delivered volumetric concentration gives a fraction of solids delivered from a slurry pipeline (Fig. 2.2) and it is calculated as the ratio between solids and slurry flow rates

$$
\begin{equation*}
\mathrm{C}_{\mathrm{vd}}=\frac{\mathrm{Q}_{\mathrm{S}}}{\mathrm{Q}_{\mathrm{m}}}=\frac{\Delta \mathrm{U}_{\mathrm{S}}}{\Delta \mathrm{t}} \frac{\Delta \mathrm{t}}{\Delta \mathrm{U}_{\mathrm{m}}} \tag{2.14}
\end{equation*}
$$

| $\mathrm{C}_{\mathrm{Vi}}$ | spatial volumetric concentration of solids | $[-]$ |
| :--- | :--- | :--- |
| $\mathrm{C}_{\mathrm{Vd}}$ | delivered volumetric concentration of solids | $[-]$ |
| $\mathrm{U}_{\mathrm{s}}$ | volume fraction of solids in mixture | $\left[\mathrm{m}^{3}\right]$ |
| $\mathrm{U}_{\mathrm{m}}$ | total volume of mixture, $\mathrm{U}_{\mathrm{m}}=\mathrm{U}_{\mathrm{s}}+\mathrm{U}_{\mathrm{f}}$ | $\left[\mathrm{m}^{3}\right]$ |
| $\mathrm{Q}_{\mathrm{s}}$ | volumetric flow rate of solids | $\left[\mathrm{m}^{3} / \mathrm{s}\right]$ |
| $\mathrm{Q}_{\mathrm{m}}$ | volumetric flow rate of mixture, $\mathrm{Q}_{\mathrm{m}}=\mathrm{Q}_{\mathrm{S}}+\mathrm{Q}_{\mathrm{f}}$ | $\left[\mathrm{m}^{3} / \mathrm{s}\right]$ |
| $\Delta \mathrm{U}_{\mathrm{S}}$ | volume of solids delivered by a pipe during <br> time period $\Delta \mathrm{t}$ |  |
|  |  | $\left[\mathrm{m}^{3}\right]$ |
| $\Delta \mathrm{U}_{\mathrm{m}}$ | volume of mixture delivered by a pipe during <br> time period $\Delta \mathrm{t}$ |  |
|  |  | $\left[\mathrm{m}^{3}\right]$. |



Figure 2.2. Definition of volumetric delivered concentration.

Table 2.8. Maximum average attainable concentration $\mathrm{C}_{\mathrm{vdsi}}$ by various dredges (typical values of $\mathrm{C}_{\mathrm{vdsi}}$ according van den Berg, 1998)

|  | $\mathrm{C}_{\mathrm{vdsi}}$ <br> in situ <br> $[-]$ | Density of <br> mixture <br> $\rho_{\mathrm{m}}\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ | $\mathrm{C}_{\mathrm{Vd}}$ <br> solid grains <br> $[-]$ |
| :---: | :---: | :---: | :---: |
| CSD without SP | 0.25 | 1250 | 0.15 |
| CSD with SP | 0.30 | 1300 | 0.18 |
| Bucket wheel dredge <br> with SP | 0.50 | 1500 | 0.30 |
| Plain suction dredge | $0.40-0.60$ | $1400-1600$ | $0.24-0.36$ |
| Modern THSD during <br> hopper loading <br> (suction process) | $0.30-0.40$ | $1300-1600$ | $0.18-0.36$ |
| Modern THSD during <br> hopper unloading <br> (pumping to shore) | $0.70-0.80$ | $1700-1800$ | $0.42-0.48$ |

Legend:
CSD cutter suction dredge
SP submerged pump
THSD trailing hopper suction dredge
The difference between the spatial concentration and the delivered concentration can be illustrated on a hypothetical case in a dredging pipeline delivering the sand-water mixture. Imagine that the carrying liquid velocity in a dredging pipeline drops suddenly to the value that is not sufficient to carry the sand particles. All sand particles will settle down to the bottom of a pipeline and form a stationary bed. The carrying water will flow above this stationary bed. Measurements of concentration in a horizontal pipe section will indicate a fraction of solids in a pipe section but measurements at the pipeline outlet will not register any solids in discharging carrier (Fig. 2.3). The spatial concentration will reach a reasonably high value but the delivered concentration will be just zero.


Figure 2.3. Difference between spatial and delivered concentrations.

This extreme case is unlikely to occur during a dredging operation but an operation during which a certain portion of transported sand forms a slowly sliding bed or even a stationary bed is not quite unusual. During such operation the different values of $\mathrm{C}_{\mathrm{vi}}$ and $\mathrm{C}_{\mathrm{vd}}$ also occur in a pipeline.

The difference between the spatial and the delivered concentration indicates slip (hold-up) within the mixture flow caused by the different velocities of the carrying liquid and that of the solid phase within a mixture stream. Govier \& Aziz (1972) described this phenomenon as follows:
"When the phases of two-phase flow differ in density and/or viscosity, one of them - usually the less dense phase - tends to flow at a higher in situ average velocity than does the other. This gives rise to an all-important characteristic of two-phase flow, the existence of "slip" of one phase past the other, or "holdup" of one phase relative to the other."
Govier and Aziz also summarised the factors influencing slip:

- the existence of a velocity profile across the pipeline cross section
- the existence of a concentration profile across the pipeline cross section
- the local relative velocity between phases ( $\mathrm{v}_{\mathrm{S}}-\mathrm{v}_{\mathrm{f}}$ ) caused by gravitational effects.

The slip between two phases in a cross section of slurry pipeline can be quantified by the mean slip velocity in a pipeline cross section, $\mathrm{V}_{\mathrm{S}}-\mathrm{V}_{\mathrm{f}}$, or by the ratio of the mean velocities of solids and mixture in a pipeline cross section, $\mathrm{V}_{\mathrm{S}} / \mathrm{V}_{\mathrm{m}}$. This latter parameter, called the slip ratio (or "transport factor" in a dredging practice), is exceptionally suitable for the evaluation of slip in a pipeline. This ratio is also equal to the ratio of mean concentrations in pipeline cross sections $\mathrm{C}_{\mathrm{Vd}} / \mathrm{C}_{\mathrm{vi}}$ since

$$
\begin{equation*}
\mathrm{C}_{\mathrm{vd}}=\frac{\mathrm{Q}_{\mathrm{S}}}{\mathrm{Q}_{\mathrm{m}}}=\frac{\mathrm{V}_{\mathrm{S}} \mathrm{~A}_{\mathrm{s}}}{\mathrm{~V}_{\mathrm{m}} \mathrm{~A}}=\frac{\mathrm{V}_{\mathrm{s}} \mathrm{C}_{\mathrm{vi}} \mathrm{~A}}{\mathrm{~V}_{\mathrm{m}} \mathrm{~A}} \tag{2.15}
\end{equation*}
$$

in which $\mathrm{A}_{\mathrm{S}}$ is the part of the cross sectional area of the pipeline occupied by solids, thus

$$
\begin{equation*}
\frac{\mathrm{V}_{\mathrm{s}}}{\mathrm{~V}_{\mathrm{m}}}=\frac{\mathrm{C}_{\mathrm{vd}}}{\mathrm{C}_{\mathrm{vi}}} \tag{2.16}
\end{equation*}
$$

$\mathrm{C}_{\mathrm{vd}} \quad$ delivered volumetric concentration of solids $\quad[-]$
$\mathrm{C}_{\mathrm{Vi}} \quad$ spatial volumetric concentration of solids $\quad[-]$
$\mathrm{V}_{\mathrm{S}} \quad$ mean velocity of solids in pipeline cross section $[\mathrm{m} / \mathrm{s}]$
$\mathrm{V}_{\mathrm{m}} \quad$ mean velocity of mixture in pipeline cross section $[\mathrm{m} / \mathrm{s}$ ]

The slip phenomenon may influence the accuracy of the determination of the solids flow rate from the measurements on a dredging pipeline. Solids flow rate through a slurry pipeline connected with a dredge is often determined from measurements of the mean solids concentration and the mean slurry velocity in a horizontal pipeline section. The radiometric measurement of the concentration in a horizontal pipeline gives the value of the spatial concentration (not the delivered concentration that might be lower) so that $\mathrm{Q}_{\mathrm{S}}$ calculated as $\mathrm{C}_{\mathrm{Vi}} \mathrm{V}_{\mathrm{m}} \mathrm{A}$ overestimates the real solids flow rate
unless the slip in a pipeline is negligible. Generally, it is desirable to take the slip into account during slurry flow calculations.

Table 2.9. Transport factor $\mathrm{C}_{\mathrm{Vd}} / \mathrm{C}_{\mathrm{vi}}$ for various solids and solids concentrations in a mixture flow of velocity range $\mathrm{V}_{\mathrm{dl}}<\mathrm{V}_{\mathrm{m}}<2 \mathrm{~V}_{\mathrm{dl}}$ (typical $\mathrm{C}_{\mathrm{vd}} / \mathrm{C}_{\mathrm{vi}}$ values according to Matousek, 1997).

|  | At low <br> $\mathrm{C}_{\mathrm{Vd}}$ | At high <br> $\mathrm{C}_{\mathrm{Vd}}$ |
| :---: | :---: | :---: |
| Silt and finer solids | 1.00 | 1.00 |
| Fine to medium sand | $0.80-1.00$ | $0.90-1.00$ |
| Medium to coarse sand | $0.70-0.90$ | $0.85-1.00$ |
| Coarse sand | $0.65-0.85$ | $0.75-0.95$ |
| Fine gravel | $0.65-0.85$ | $0.75-0.90$ |
| Boulders | $0.40-0.65$ | $0.40-0.65$ |

### 2.4 REFERENCES

van den Berg, C.H. (1998). Pipelines as Transportation Systems. European Mining Course Proceedings, MTI.
Govier, G.W. \& Aziz, K. (1972). The Flow of Complex Mixtures in Pipes. Van Nostrand Reinhold Company.
Matousek, V. (1997). Flow Mechanism of Sand-Water Mixtures in Pipelines. Delft University Press.

## CASE STUDY 2

Consider that sand particles occupy 27 per cent of the total volume of a dredging pipeline. The rest is occupied by carrying water. The sand-water mixture is discharged from a dredging pipeline at a deposit site. The porosity of sand in a deposition $\mathbf{n}=\mathbf{0 . 4}$.

Determine the density $\rho_{m}$ of sand-water mixture in the pipeline and the weight concentration $C_{W}$ of solids in the mixture. Further, determine the in situ density $\rho_{S i}$ and the spatial volumetric concentration of in situ sand, $C_{\nu s i}$, in a deposition and in a pipeline.

## Inputs:

The spatial volumetric concentration of solids in mixture flow: $\quad \mathbf{C}_{\mathbf{V}}=\mathbf{0 . 2 7}$
The density of sand particles:
$\rho_{\mathrm{S}}=2650 \mathrm{~kg} / \mathrm{m}^{3}$
The density of carrying water:
$\rho_{f}=1000 \mathrm{~kg} / \mathrm{m}^{3}$
Porosity of sand body in a deposition:
$\mathrm{n}=0.4$.

## Solution:

## a. Density of mixture $\left(\rho_{m}\right) \&$ weight concentration of solids in mixture ( $C_{w}$ )

Eq. 2.9 determines the density of mixture

$$
\rho_{\mathrm{m}}=\rho_{\mathrm{f}}\left(1-\mathrm{C}_{\mathrm{V}}\right)+\rho_{\mathrm{S}} \mathrm{C}_{\mathrm{V}}=1000(1-0.27)+2650 \times 0.27=1445.5 \mathrm{~kg} / \boldsymbol{m}^{3}
$$

Eq. 2.10 determines the weight concentration

$$
\mathrm{C}_{\mathrm{W}}=\frac{\rho_{\mathrm{S}}}{\rho_{\mathrm{m}}} \mathrm{C}_{\mathrm{V}}=\frac{2650}{1445.5} 0.27=0.495, \text { i.e. } 49.5 \%
$$

b. In-situ density ( $\rho_{s i}$ ) \& in-situ volumetric concentration of soil $\left(C_{v s i}\right)$ in a deposition

According to Eq. 2.3 the porosity $\mathrm{n}=\frac{\rho_{\mathrm{S}}-\rho_{\mathrm{Si}}}{\rho_{\mathrm{S}}-\rho_{\mathrm{f}}}$ and thus

$$
\rho_{\mathrm{si}}=\rho_{\mathrm{S}}-\left(\rho_{\mathrm{S}}-\rho_{\mathrm{f}}\right) \mathrm{n}=2650-1650 \times 0.4=1990 \mathrm{~kg} / \boldsymbol{m}^{3} .
$$

In a deposition the density of the sand-water mixture is equal to $\rho_{\mathrm{si}}$ and thus $\mathrm{C}_{\mathrm{vsi}}=1$ (see Eq. 2.11).

## c. The in-situ volumetric concentration in a pipeline

Eq. 2.12 determines the volumetric concentration of in situ material in a pipeline as

$$
\mathrm{C}_{\mathrm{Vsi}}=\frac{\mathrm{C}_{\mathrm{V}}}{1-\mathrm{n}}=\frac{0.27}{1-0.4}=0.45 \text {, i.e. } 45 \% .
$$

