Bending Deflection – Method of Superposition

AE1108-II: Aerospace Mechanics of Materials

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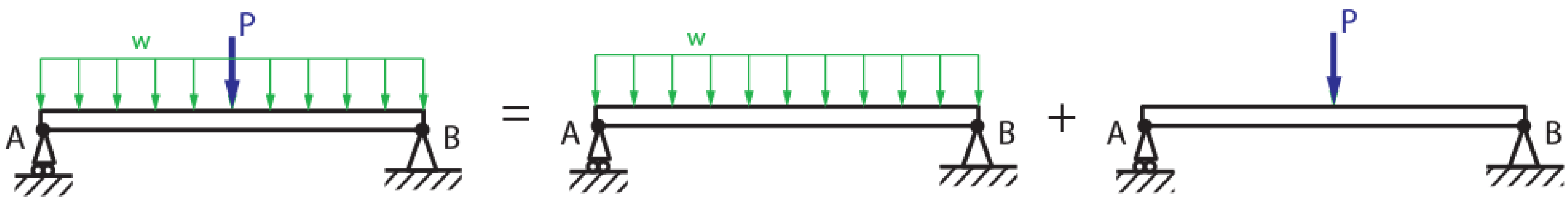
Deflection by Superposition

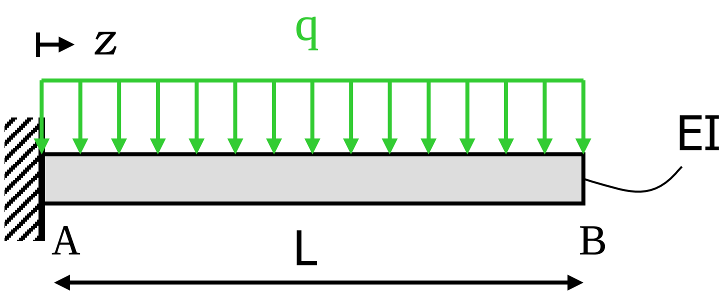
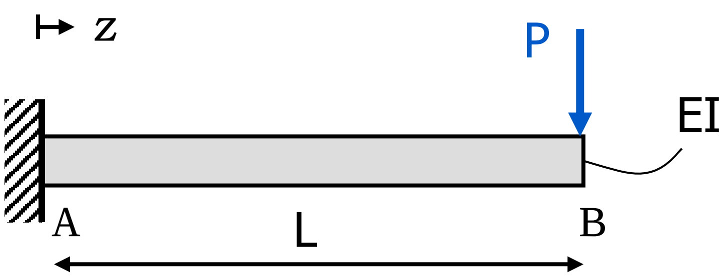
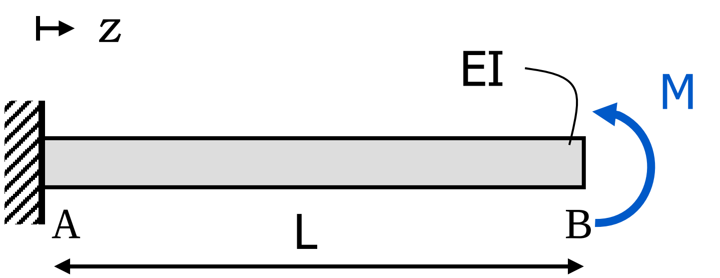
- If stress-strain behaviour of the beam material remains *linear elastic*, principle of superposition applies



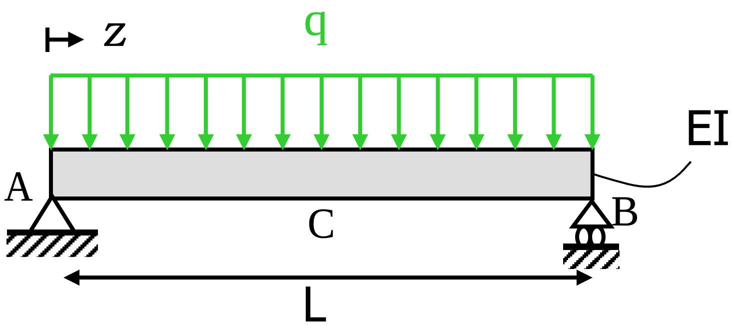
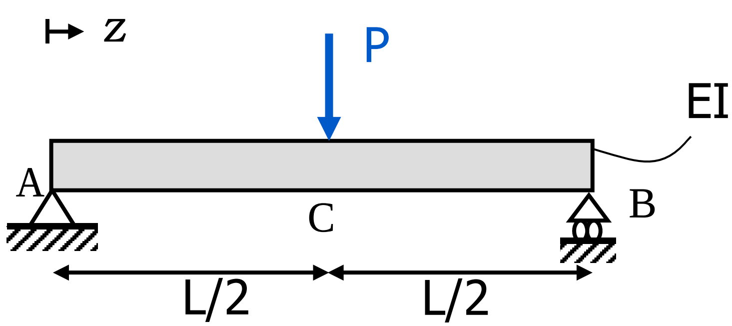
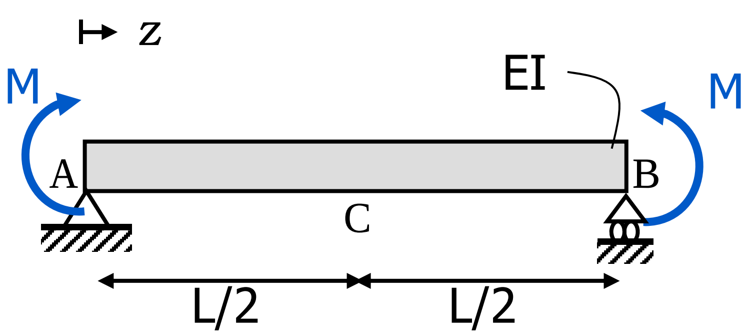
- Problem can be broken down into simple cases for which solutions may be easily found, or obtained from data handbooks (see Appendix C of the textbook)

The following slides show all the standard solutions you will be permitted to use on the exam!

Cantilever Beam Standard Solutions

	Deflection (at B)	Slope (at B)
	$v_B = \frac{qL^4}{8EI}$	$\theta_B = \frac{qL^3}{6EI}$
	$v_B = \frac{PL^3}{3EI}$	$\theta_B = \frac{PL^2}{2EI}$
	$v_B = -\frac{ML^2}{2EI}$	$\theta_B = -\frac{ML}{EI}$

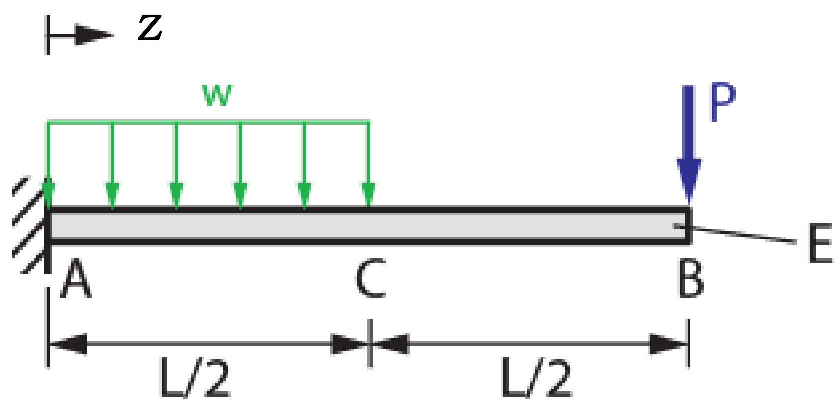
Simply Supported Beam Standard Solutions

	Deflection	Slope
	$$v_C = \frac{5qL^4}{384EI}$$	$$\theta_{A,B} = \frac{qL^3}{24EI}$$
	$$v_C = \frac{PL^3}{48EI}$$	$$\theta_{A,B} = \pm\frac{PL^2}{16EI}$$
	$$v_C = \frac{ML^2}{8EI}$$	$$\theta_C = 0$$

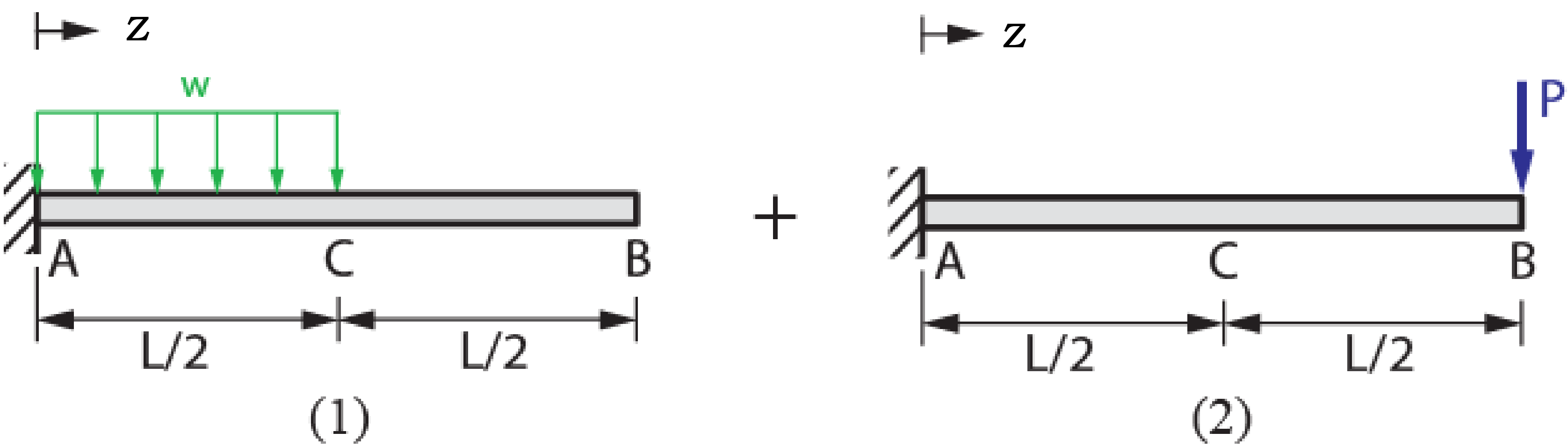
Example

Example Problem

Find the deflection at B



We can break into two sub problems



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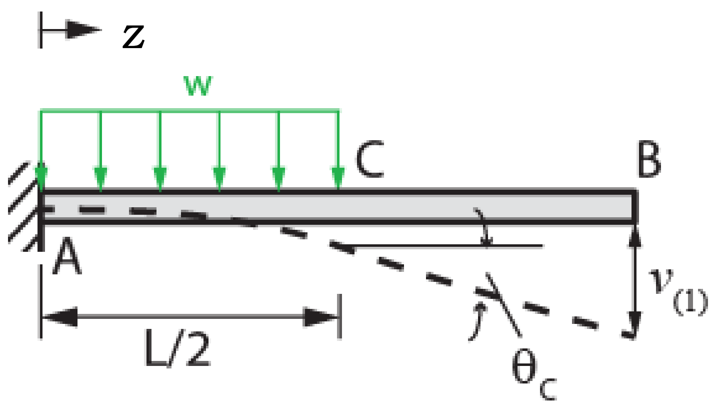
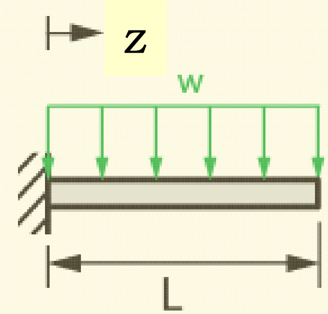
Example Problem (cont)

Sub Problem (1)



Standard Solution 1:



$$v = \frac{wL^4}{8EI}$$

$$\theta = \frac{wL^3}{6EI}$$

$$v_C = \frac{w\left(L/2\right)^4}{8EI} = \frac{wL^4}{128EI}$$

$$\theta_C = \frac{w\left(L/2\right)^3}{6EI} = \frac{wL^3}{48EI}$$

$$v_{(1)} = v_C + \theta_C\left(L/2\right)$$

$$v_{(1)} = \frac{7wL^4}{384EI}$$

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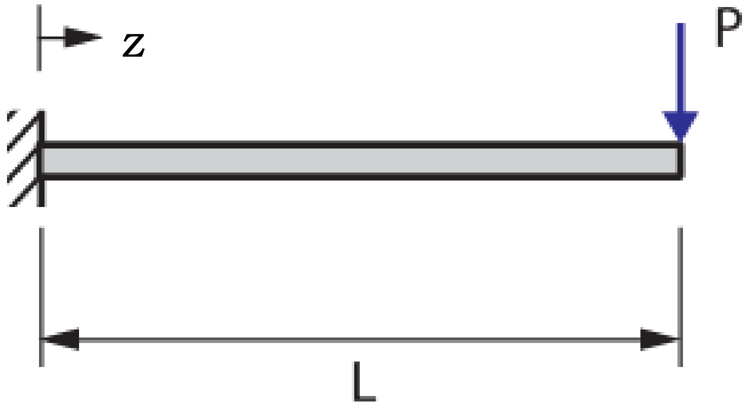
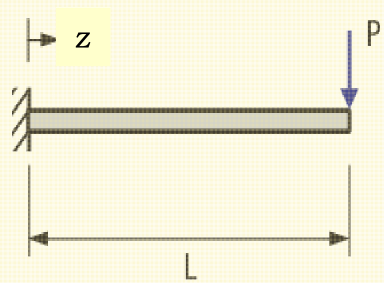
Example Problem (cont)

Sub Problem (2)



Standard Solution 2:



$$v = \frac{PL^3}{3EI}$$

$$\theta = \frac{PL^2}{2EI}$$

$$\therefore v_{(2)} = \frac{PL^3}{3EI}$$

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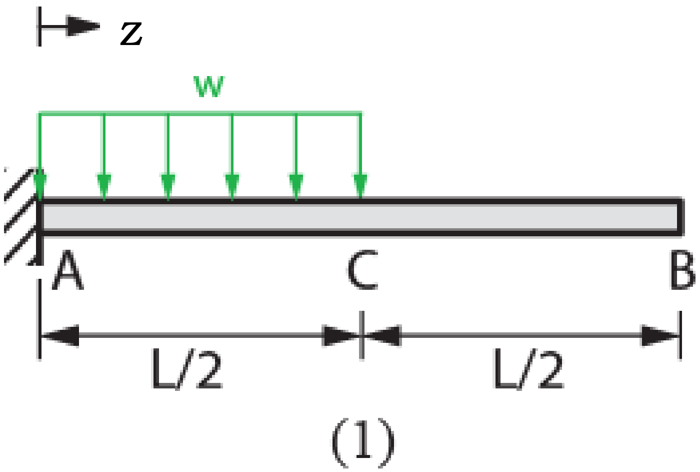
Example Problem (cont)

Superposition:



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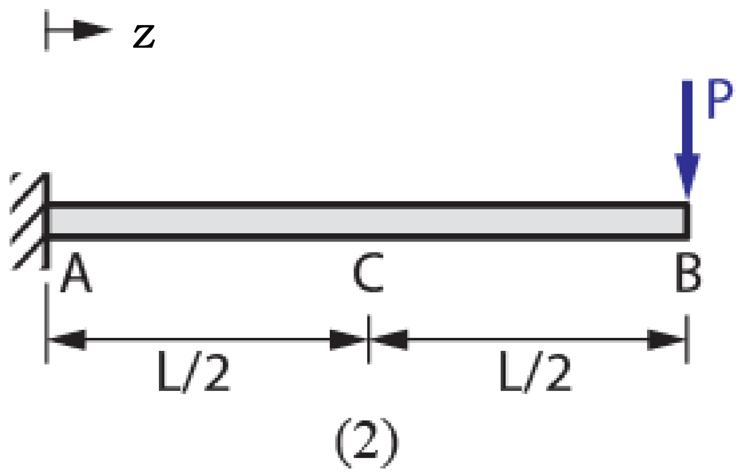

$$v_{(1)} = \frac{7wL^4}{384EI}$$

$$v_{(2)} = \frac{PL^3}{3EI}$$

$$\therefore v_B = \frac{7wL^4}{384EI} + \frac{PL^3}{3EI}$$

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