



Prof. Ir. W.J. Vlasblom

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1. INTRODUCTION

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Centrifugal pumps are particular suitable for pumping solids due to a small number of moveable part.

More advantages of this pump type are:

- a continuous pump capacity
- the possibility of a direct drive
- relatively cheap and maintenance friendly

Centrifugal pumps (dredge pumps) as used in the dredging industry are to distinguished by "ordinary water pumps" by:

• a large bore in the impeller as well as in the pump casing, without any restriction in the direction of the flow.

at the impeller inlet the bore is most small

• a small number (3, 4 or 5) and short vanes in the impeller as a compromise between a large bore and an efficient pump action

• a large clearance between the cutwater (Dutch puntstuk) and the impeller (10 to 20% of the impeller diameter

• An easy replacement of wear parts

• the use of gland water for flushing the space between the impeller shrouds and the wearing plates on the pump cover, in order to prevent particles to enter the shaft seals





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3D VIEW OF DREDGE PUMP



PUMPROOM VIEW

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Pumps and Systems

2. **DEFINITIONS**

CAPACITY Q:

The volume liquid pumped per second; dimension [m³/s]

MANOMETRIC PRESSURE p_m :

The total pressure which can be delivered by the pump, dimension $[N/m^2]$, is defined as:



EFFICIENCY: $\eta = \frac{PUMPPOWER}{ENGINEPOWER} \times 100\%$ or $\eta = \frac{Qp_m}{P} \times 100\%$

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3. SET OF PUMP CHARACTERISTICS



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4. EULERS EQUATION FOR CENTRIFUGAL PUMPS

The changes of momentum for rotating bodies is:

$$T = \frac{d(mvr)}{dt}$$

m = mass [kg] v = rotational velocity [m/s] r = radius [m] T = torque [Nm] t= time [s]



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$$T = \frac{a}{dt} \left[mr_2 c_2 \cos \alpha_2' - mr_1 c_1 \cos \alpha_1' \right]$$

For a stationary flow the mass: $m = Q\rho$

Q = capacity [m³/s] $\rho = density [kg/m³]$

 c_1 and c_2 are the absolute velocities and α_1 and α_2 true directions of the liquid particles.

If all losses in the pump are disregarded, the required power equals delivered power

$$P = T\omega = Q\rho\omega[r_2c_2\cos\alpha_2 - r_1c_1\cos\alpha_1] = Qp_{th}$$

 p_{th} the theoretical delivered pump pressure $[n/m^2]$;

Resulting in:

$$p_{ih} = \rho \omega [r_2 c_2 \cos \alpha'_2 - r_1 c_1 \cos \alpha'_1] = \rho [u_2 c_{u2} - u_1 c_{u1}]$$

With $\omega r = u$ the peripheral velocity of the impeller and $c \cdot \cos \alpha' = c_u$ the component of the absolute velocity on the peripheral velocity.

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When expressed in the vane angles β_1 and β_2 the equation becomes.

$$p_{e} = \rho \left[u_{2}^{2} - u_{1}^{2} - \left(\frac{u_{2}c_{r2}}{\tan \beta_{2}} - \frac{u_{1}c_{r1}}{\tan \beta_{1}} \right) \right]$$

and because
$$c_{r1} = \frac{Q}{2\pi r_1 b}$$
 and $c_{r2} = \frac{Q}{2\pi r_2 b}$
 $p_e = \rho \left[u_2^2 - u_1^2 - \frac{Q}{2\pi b} \left(\frac{u_2}{r_2 \tan \beta_2} - \frac{u_1}{r_1 \tan \beta_1} \right) \right]$

This is Euler's pump equation

If the liquid enters impeller without a tangential component thus radial for centrifugal pumps then $C_{u_1} = 0$ and Euler's equation becomes

$$p_e = \rho \left[u_2^2 - \frac{u_2 c_{r_2}}{\tan \beta_2} \right]$$

Note:

 p_{th} is based on actual velocities and directions. Unfortunately those are in practice unknown. therefore p_e is based on the known velocities and vane angles. For constant speed (u=constant) the equation reduced to: $p_e = A - B \cdot Q$

with
$$A = \rho \left[u_2^2 - u_1^2 \right]$$
 and $B = \left[\frac{\rho}{2\pi b} \cdot \left(\frac{u_2}{r_2 \tan \beta_2} - \frac{u_1}{r_1 \tan \beta_1} \right) \right]$

Which is an equation of a strait line.

Comform $p_{e_1} p_{th}$ can be written as: $p_{th} = A' - B' \cdot Q$

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Because the power can be written as: $P = p_e Q = AQ - B \cdot Q^2$

Which is the equation of a parabola.



When the liquid has prerotation before approaching the impeller eye the $c_{u1} > 0$ and Eulers had will be lower by the term $\rho u_1 c_{u1}$



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4.1. VELOCITY DISTRIBUTION BETWEEN THE BLADES

Already stated true velocity angles β_1 and β_2 may not be the same as the blade angles β_1 and β_2 . However the last ones are used in impeller design because it is easier to calculate flow velocities based on those angles than the actual flow velocities.

Derivation of the fluid from the vane direction reduces the peripheral component of the absolute velocity c_{u_0} . This causes a reduction in head. This phenomenon is called slip and is a

consequence of the non-uniform velocity distribution across the impeller channel. The input power keeps roughly the same because the capacity doesn't change. Note:

 $\beta_1 = \beta_1'$ no-shock condition at entry

$$\beta_2 = \beta'_2$$
 no fluid slip at exit

The difference in head between those angles is called the head reduction factor μ .

$$\mu = \frac{p_{th}}{p_e} = \frac{c_{u_2}}{c_{u_{2\infty}}}$$



Slip velocity Δc_{u_2} is defined as: $\Delta c_{u_2} = c_{u_{2\infty}} - c_{u_2}$

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With
$$\mu = \frac{c_{u_2}}{c_{u_{2\infty}}}$$
 this gives $\mu = \frac{c_{u_{2\infty}} - \Delta c_{u_2}}{c_{u_{2\infty}}} = 1 - \frac{\Delta c_{u_2}}{c_{u_{2\infty}}}$

4.2. THE EXISTENCE OF SLIP.

To transmit power to the liquid the pressure on the leading front of the vane should be higher than on the back.

For any force exerted by the vane to the fluid has an equal and opposite reaction.

this means that the relative velocities at the back of the vane are higher than at the front.

This velocity profile in the impeller can be regarded as the through flow on which a relative eddy is superimposed

Such a relative circulation can also be explained by the orientation of fluid particles through the impeller



RELATIVE MOTION OF FLUID PARTICLES IN AN IMPELLER

Fluid particles moving through the impeller fails to turn around their axes. So the eddy has the same but opposite angular velocity as the impeller

These two flows cause that the direction of the flow at the outlet is inclined

Stodola has estimated $\Delta c_{u_2} = \frac{\omega \cdot e}{2}$ (mean velocity in the channel) e = channel width at outlet and is: $e = \frac{2\pi r_2 \sin \beta_2}{z}$

z = number of vanes of thickness zero So

$$\Delta c_{u_{2\infty}} = \frac{\pi\omega \cdot r_2 \sin\beta_2}{z} = \frac{\pi \cdot u_2 \sin\beta_2}{z}$$

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FURTHER



The relative eddy between impeller blades [Jonker 1995]

 C_{r_2} = is the component of the absolute velocity normal to the peripheral velocity This results in

$$\mu = 1 - \frac{\pi \cdot u_2 \sin \beta_2}{z \left(u_2 - \frac{c_{r_2}}{\tan \beta_2} \right)}$$

with
$$p_e = \rho \left[u_2^2 - \frac{u_2 c_{r_2}}{\tan \beta_2} \right]$$

gives
$$\mu = 1 - \frac{\pi \cdot u_2 \sin \beta_2}{z \frac{p_e}{\rho \cdot u_2}} = \frac{p_{th}}{p_e}$$

$$\mu = 1 - \frac{\rho \pi \cdot u_2^2 \sin \beta_2}{z p_e} = \frac{p_{th}}{p_e}$$

or

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$\frac{\text{Wb 3413}}{1 - \frac{p_{th}}{p_e}} = \frac{\rho \pi \cdot u_2^2 \sin \beta_2}{z p_e}$

$$p_e - p_{th} = \frac{\rho \pi \cdot u_2^2 \sin \beta_2}{z}$$

Being a line parallel with p_e

Tests with 3 and 4 vane impeller do show a shift in the pressure curve.



Input power was the same for both impellers.

Another formula to calculate the slipfactor is proposed by Pfleiderer:

 $\mu = \frac{1}{1 + a \left(1 + \frac{\beta_2}{60}\right) \frac{2}{1 - \left(r_1 / r_2\right)^2}}$ with a between 0.65 and 0.85 for volute type pumps and r_1 and r_2

respectively the radius at entrance and discharge.

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5. CORRECTION ON THE THEORETICAL CHARACTERISTICS.



1. FRICTION LOSSES:

In pump and impeller friction loss is can be written as: $\Delta p_h = c_1 Q^2$

2. SHOCK LOSSES:

Impact losses at the impeller blades because direction of flow differs from the blade angles. At best efficiency point these losses are zero; so

 $\Delta p_s = c_2 (Q - Q_s)^2$

3. SECONDARY LOSSES

Leakage, recirculation in pump casing

ACTUAL OR MANOMETRIC PRESSURE:

$$p_{m} = p_{i} - c_{1}Q^{2} - c_{2}(Q - Q_{s})^{2}$$
$$p_{m} = A' - B'Q - c_{1}Q^{2} - c_{2}(Q - Q_{s})^{2}$$

More general: $p_m = A_0 + A_1Q + A_2Q^2$

The equation is only valid for centrifugal pumps and not for axial flow pumps!

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Wb 3413 EFFICIENCIES



HYDRAULIC EFFICIENCY:

 $\eta_{h} = \frac{p_{hydraulic}}{p_{hydraulic} + p_{losses}} = \frac{\text{ACTUAL PUMP PRESSURE}}{\text{THEORETICAL PUMP PRESSURE}}$

VOLUMETRIC EFFICIENCY:

 $\eta_{\underline{Q}} = \frac{Q}{Q + Q_{loss}} = \frac{Q}{Q_{imp}} = \frac{\text{FLOW RATE TROUGH PUMP}}{\text{FLOW RATE TROUGH IMPELLER}}$ MECHANICAL EFFICIENCY:

 $\eta_m = \frac{Q_i p_{th}}{P} = \frac{\text{POWER SUPPLIED TO IMPELLER}}{\text{POWER INPUT TO SHAFT}}$

OVERALL EFFICIENCY: $\eta = \frac{Qp}{p} = \frac{\text{FLUID POWER DEVELOPED BY PUMP}}{\text{SHAFT POWER INPUT}}$

 $\eta = \eta_h \cdot \eta_v \cdot \eta_m$

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6. DIMENSIONLESS PUMP CONSTANTS (SIMILARITY CONSIDERATIONS)

Flows conditions in two geometrically similar systems are called similar if all fluid velocities change with a constant ratio.

So in hydraulic machines similarity of flow requires a constant ration between fluid velocities and peripheral velocities.

 $\frac{c}{u} = \text{constant}$ So $c_{m_2} = \frac{Q}{2\pi r b} \text{ and } u_2 = \omega r \Rightarrow \frac{c_{m_2}}{u_2} = \frac{Q}{2\pi r b u_2} = \frac{Q}{2\pi r b \cdot \omega r} = \Phi$

Or

$$\Phi = \frac{Q}{(\pi Db)\left(\frac{\pi nD}{60}\right)}$$

Full similarity is only obtained if the width b changes with the same ratio as D, so:

$$\Phi \! \Rightarrow \! \frac{Q}{nD^3}$$

For similarity of the pressure $\frac{p}{\rho} = const \cdot u^2$

When p is devided by ρu_2^2 the term

$$\frac{p}{\rho \cdot u_2^2} = \frac{p}{\rho \cdot \omega^2 r_2^2} = \Psi = \frac{p}{\rho \left(\frac{\pi nD}{60}\right)^2} = const \frac{p}{\rho n^2 D^2}$$

Becomes dimensionless and is called the dimensionless pressure.

Dimensionless power can be defined as:

$$\Pi = \frac{\Psi\Phi}{\eta} = \frac{P}{\rho \left(\frac{n\pi D}{60}\right)^3 \cdot (\pi Db)} = const \frac{P}{\rho n^3 D^5}$$

From the momentum follows that full similarity is only got when viscous effects do not change.

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this is only the case for high Reynolds numbers!

EULERS EQUATION:

$$p_e = \rho \left[u_2^2 - u_1^2 - \frac{Q}{2\pi b} \left(\frac{u_2}{r_2 \tan \beta_2} - \frac{u_1}{r_1 \tan \beta_1} \right) \right]$$

can now be rewritten in:

$$\Psi = 1 - \frac{u_1^2}{u_2^2} - \frac{Q}{2\pi b \cdot u_2} \cdot \left(\frac{1}{r_2 \tan \beta_2} - \frac{u_1}{u_2} \cdot \frac{1}{r_1 \tan \beta_1}\right)$$

because
$$\frac{u_1}{u_2} = \frac{u_1 \cdot v_1}{\omega \cdot r_2}$$

 $\Psi = 1 - \frac{r_1^2}{r_2^2} - \frac{Q}{2\pi b \cdot u_2 r_2} \cdot \left(\frac{1}{\tan \beta_2} - \frac{1}{\tan \beta_1}\right)$

$$\Psi = 1 - \frac{r_1^2}{r_2^2} - \boldsymbol{\Phi} \cdot \left(\frac{1}{\tan\beta_2} - \frac{1}{\tan\beta_1}\right)$$

In case of radial flow into the impeller, this equation reduce to

$$\Psi = 1 - \Phi \frac{1}{\tan \beta_2}$$

So the dimensionless Euler equation is only determined by the discharge angle β_2

7. AFFINITY LAWS:





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This condition is strictly only true for the impeller action and for the location of the best efficiency point.

However it can be stated:

If pump tests of centrifugal pumps do not fulfil these laws, check the results or the measuring devices.

If there is prerotation the affinity law regarded to the diameter is less applicable . (see: dimensionless Euler's equation)

For variable speed and constant impeller diameter, lines of constant efficiencies are parabolas going through the origin.

The condition $\eta = \frac{\eta_1}{\eta_2} = 1$ is strictly only true for the impeller action.

The influence of the impeller casing results in an optimum speed with the highest efficiency, however the best efficiency point at different speed are still located at a parabola through the origin.



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Due to the flow in the volute there is a small deviation of this theory. instead of parabola of constant efficiency it appeared to be more or less ellipses



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8. DIMENSIONLESS PUMP CHARACTERISTICS

For centrifugal pumps and to a lesser extend for half-axial flow pumps as well, the dimensionless pump characteristics can be written as a power serie of the second degree So for the pressure: $W_{n} = \sigma_{n+1} + \sigma_{n+2} + \sigma_$

 $\Psi = \alpha_0 + \alpha_1 \Phi + \alpha_2 \Phi^2$ and for the power: $\Pi = \beta_0 + \beta_1 \Phi + \beta_2 \Phi^2$



DIMENSIONLESS PUMP CHARACTERISTICS

Calculating the actual pump characteristics from the dimensionless gives for the pressure:

$$\frac{p}{\rho\left(\frac{n\pi D}{60}\right)^2} = \alpha_0 + \alpha_1 \left[\frac{\frac{1}{\pi Db}}{\frac{n\pi D}{60}}\right] Q + \alpha_2 \left(\frac{1}{\frac{n\pi D}{60}\pi Db}\right)^2 Q^2$$

or

$$p = \rho \left\{ \alpha_0 \left[\rho \left(\frac{n \pi D}{60} \right)^2 \right] + \alpha_1 \left[\left(\frac{n \pi D}{60} \right) \cdot \left(\frac{1}{\pi D b} \right) \right] Q + \alpha_2 \left[\left(\frac{1}{\pi D b} \right)^2 \right] Q^2 \right\}$$

FOR THE POWER CHARACTERISTIC:

$$P = \rho \left\{ \beta_0 \left[(\pi Db) \left(\frac{n\pi D}{60} \right)^3 \right] + \beta_1 \left(\frac{n\pi D}{60} \right)^2 Q + \beta_2 \left[\left(\frac{n\pi D}{60} \right) (\pi Db)^{-1} \right] Q^2 \right\}$$

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9. SPECIFIC SPEED

In the selection of pumps the discharge Q, the pressure p and the pump speed n are usually known.

A dimensionless combination of these variables at the best efficiency point is known as the specific speed:

$$n_{s} = \frac{\omega\sqrt{Q}}{\left(gH\right)^{\frac{3}{4}}} = \frac{\rho^{\frac{3}{4}}\omega\sqrt{Q}}{\left(p\right)^{\frac{3}{4}}} \Big|_{--}$$

Comment: Page: 23 In literature $n_s = \frac{n\sqrt{Q}}{(H)^{\frac{3}{4}}}$ is frequently use, however this number is not dimensionless!

The specific speed is used as a "type" number and to compare different impeller designs and dimensions such as b/D and inlet over outlet diameters $\frac{D_1}{D_2}$

By defining $Q = 2\pi b \omega r_2^2 \Phi$ and $p = \rho \omega^2 r_2^2 \Psi'$ in which Φ and Ψ are based on the best efficiency point.

$$n_{s} = \frac{\rho^{\frac{3}{4}} \omega r_{2} \sqrt{2\pi b \omega \Phi}}{\rho^{\frac{3}{4}} \Psi^{\frac{3}{4}} (\omega r)^{1.5}} = \frac{\Phi^{\frac{1}{2}}}{\Psi^{\frac{3}{4}}} \cdot 2\sqrt{\pi \left(\frac{b}{D}\right)} = 2n_{s}^{\prime} \sqrt{\pi \left(\frac{b}{D}\right)}$$
or

Because for simualar impellers the ratio b/D is constant the ratio $\frac{\Phi^{\frac{1}{2}}}{\Psi^{\frac{3}{4}}}$ can be used as a type

number or another form of specific speed: $n_s' = \frac{\Phi^{\frac{1}{2}}}{\Psi^{\frac{3}{4}}} = \frac{n_s}{2\sqrt{\pi(\frac{b}{D})}} = \frac{1}{2}n_s\sqrt{\frac{D}{\pi b}}$

An increase in specific speed requires a wider impeller and/or a smaller impeller. A change in the diameter results in a shift of the specific speed.

Figure below shows typical impeller shapes with their specific speeds

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HYDRO DYNAMIC ROTORS OF DIFFERENT SPECIFIC SPEEDS [JONKER 1995]

The pump types have different characteristics in a well-defined region of head en flow as shown in the next graph.



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EFFECT OF SPECIFIC SPEED ON IMPELLER SHAPE [JONKER 1995]

Experiments have shown that for each type of impeller shape the maximum efficiency is in a narrow range.

In dredging practice only centrifugal and half-axial flow (mixed) pumps are used.

The first in all type of dredgers and the latter mainly as additional "submerged" pumps on board of trailing suction hopper dredgers when equipped for dredging over the 50 m depth. In that case low head and low head and high capacity is required.

Submerged pumps used on cutter dredgers or plain suction dredgers are mainly from the centrifugal type. Because there head is mostly much more than required to pump the mixture to the inboard pump. The additional head is used for overcome the pipeline resistance of the discharge line.

Figure below shows specific head and capacity as function of specific speed of pumps used in the dredging industry.

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On basis of figure ??? the dimensions of the impeller and the pumpspeed can be determined.

Example:

Assume $Q=2 \text{ m}^3/\text{s}$ and p=750 kPa; Determine pumpspeed and diameter.

For N_s= 0.3, ϕ and ψ can be estimated from the graph above; ϕ =0.042 and ψ =0.6. The rotational speed ωr can be calculated from ψ and impeller internal width from ϕ .

1		
$N'_{s} = \frac{\phi^{\overline{2}}}{\psi^{\overline{4}}}$	$\phi = \frac{Q}{\omega r 2\pi b}$	$\psi = \frac{p}{\rho(wr)^2}$
0.3	0.042	0.6
	$b = \frac{Q}{\omega r \cdot 2\pi\phi}$ $= \frac{2}{2525 \cdot 2\pi^{0.042}} = 0.214 \text{ [m]}$	$\omega r = \sqrt{\frac{p}{\rho \psi}} = \sqrt{\frac{750}{1*0.6}} = \sqrt{1250} = 35.35 [\text{m/s}]$
	55.55 2/10.042	

With the figures ???? the ratio b/D can be estimated. Note that in these figure the specific speed is n_s while in figure ??? this is n'_s

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10. INFLUENCE OF ENGINE CHARACTERISTIC ON THE PUMP CHARACTERISTICS

At characteristic of electrical engines types one can distinct:

- constant speed
- constant power

variable torque



Note:

Constant power condition is also possible with diesel engines with special gearboxes (f.i. hydrodynamic)

For diesel engines this

- constant speed
- constant torque

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10.1. EQUATION OF THE CONSTANT POWER LINE.

The equation of the actual power can be rewritten as:

$$P = A_0 n^3 D^4 + A_1 n^2 D^2 Q + A_2 n Q^2$$

with:

$$A_0 = \rho \frac{\pi^4 b}{60^3} \beta_0$$
, $A_1 = \rho \frac{\pi^2}{60^2} \beta_1$ AND $A_2 = \rho \frac{1}{60b} \beta_2$

Are for a certain pump β_0 , β_1 , β_2 , D and b given, then the pump speed can be determined as function of the capacity q. (the solution of a cubic equation or numerical solution by Newton Raphson)

Substituting the results in the pressure equation gives the so-called constant power line. (see enclosure a)

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CHARACTERISTICS FOR CONSTANT POWER



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For a for a given pump speed and capacity, optimum impeller diameter can be determined by solving the equation:

P

$$A_0 n^3 D^4 + A_1 n^2 D^2 Q + A_2 n Q^2 - P = 0$$

this gives:
$$D_{opt} = \sqrt{\frac{-A_1 n^2 Q + \sqrt{(A_1 n^2 Q)^2 - 4A_0 n^3 (A_2 n Q^2 - 2A_0 n^3)}}{2A_0 n^3}}$$

10.2. CONSTANT TORQUE LINE.

The same technique can be applied for the case of constant torque. The torque can be written as:

$$T = \frac{P}{\omega} = \frac{\beta_0}{\omega} \left[\rho(\pi Db) \left(\frac{n\pi D}{60}\right)^3 \right] + \beta_1 \left[\rho\left(\frac{n\pi D}{60}\right)^2 \right] Q + \beta_2 \left[\rho \cdot \left(\frac{n\pi D}{60}\right) (\pi Db)^{-1} \right] Q^2$$

with $\omega = \frac{2\pi n}{60}$
or with the simplified equation as:
$$T = \frac{1}{\frac{2\pi n}{60}} \left[A_0 n^3 D^4 + A_1 n^2 D^2 Q + A_2 n Q^2 \right]$$

or:

$$T = B_0 n^2 D^4 + B_1 n D^2 Q + B_2 Q^2$$

here in is:

$$B_n = A_n \cdot \frac{30}{\pi}$$

The line of constant torque can be found by solving the equation:

$$B_0 n^2 D^4 + B_1 n D^2 Q + B_2 Q^2 - T = 0$$

giving:
$$n = \frac{-B_1 D^2 Q + \sqrt{(B_1 D^2 Q)^2 - 4B_0 D^4 (B_2 Q^2 - T)}}{2B_0 D^4}$$

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10.3. VARIABLE TORQUE LINE.

Is the available torque a function of the speed, such as in the case of electric motors, then $T = C_0 - C_1 n$. In that case the solution is:

$$n = \frac{-(B_1 D^2 Q - C_1) + \sqrt{(B_1 D^2 Q - C_1)^2 - 4B_0 D^4 (B_2 Q^2 - C_0)}}{2B_0 D^4}$$

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11.CAVITATION

Cavitation is a condition in a liquid in which the local pressure has dropped below the vapour pressure corresponding to the temperature of the water. (boiling)

Cavitation can occur at:

- high points in a pipeline f.i. siphons
- high velocities (Bernoulli)
- large suction heights or long suction lines
- high fluid densities.
- high altitudes (reservoirs) or low atmospheric pressure

Results:

- 1. Collapse of the vapour bubbles when they enter the high-pressure zone
- 2. Drop of the manometric pressure- and efficiency curves
- 3. Pitting and corrosion

In dredge pumps low pressure is on the entrance side and cavitation start between the vanes



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11.1. NET POSITIVE SUCTION HEAD (NPSH)

The NPSH is defined as total (energy) head available to the pump above the vapour pressure in front of the pump.

$$(NPSH)_a = \frac{p_s}{\rho g} - \frac{p_v}{\rho g} + \frac{v^2}{2g} \text{ [m]}$$

or
$$(NPSH)_a = p_s - p_s + \frac{1}{2} \text{ov}^2 \text{ [Pa]}$$

$$\left(NPSH\right)_{a} = p_{s} - p_{v} + \frac{1}{2}\rho v^{2} \text{ [Pa]}$$

 p_S = absolute pressure in front of pump $p_V =$ vapour pressure of liquid v = velocity

This can be written as:

 $(NPSH)_a = p_a - p_v - \rho g h_s - \Sigma L$ [Pa] $p_a = Atmospheric pressure$ [Pa] $h_s =$ suction height $\rho =$ fluid density [Pa] $[kg/m^3]$ Σ L= all pipeline losses [Pa]





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11.2. THE DELIVERED (or produced) NPSH OF A PUMP

The minimum NPSH delivered by a pump is a function of the capacity at which the pressure drop due to cavitation with a certain value f.i. 5 % . It can only determined by testing the pressure drop by trottling progressively the pump inlet.



The pressure- and efficiency drop are measured as function of net positive suction head.

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No cavitation if $(NPSH)_d < (NPSH)_a$

To estimate (NPSH)_d around the best efficiency point use can be made of Specific NPSH number:

$$S_{\omega} = \frac{\omega \sqrt{Q}}{\left(g \cdot NPSH\right)^{\frac{3}{4}}}$$

For dredge pumps $S_{\omega} = 3 - 3.5$

Because (NPSH)_d is proportional with liquid velocity squared, it also means that $NPSH \div u^2$ and so with n²

So affinity law: $\frac{(NPSH)_1}{(NPSH)_2} = \frac{n_1^2}{n_2^2} \text{ and } \frac{Q_1}{Q_2} = \frac{n_1}{n_2}$

11.3. RELATION BETWEEN (NPSH)d AND DECISIVE VACUUM

$$(NPSH)_d = \frac{p_a}{\rho g} - \frac{p_v}{\rho g} + \frac{v^2}{2g}$$

$$p_a = p_b - (Vac)_d$$

$$(NPSH)_{d} = \frac{p_{b}}{\rho g} - \frac{p_{v}}{\rho g} + \frac{v^{2}}{2g} - (Vac)_{d}$$

or
$$\therefore (Vac)_{d} = -(NPSH)_{d} + \frac{p_{b}}{\rho g} - \frac{p_{v}}{\rho g} + \frac{v^{2}}{2g}$$



Relation vaccum and NPSH

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12. INFLUENCE OF DENSITY AND VISCOSITY ON THE PUMP CHARACTERISTICS FOR NEWTONIAN FLUIDS

12.1. Fluids having the same viscosity but another density than water.

The manometric pressure for a fluid other than water relates to that of water by:

$$p_{fluid} = p_{water} \cdot \frac{\rho_{fluid}}{\rho_{water}}$$

and for the power

$$P_{fluid} = P_{water} \cdot \frac{\rho_{fluid}}{\rho_{water}}$$

12.2. Fluids having the same density but another viscosity than water. (Stepanoff 1967)

Due to the viscous effects affinity laws hold with less accuracy than for water,

capacity varies with speed. Because efficiency is mostly higher at higher specific speeds, power increases less than the cube of the speed and the pressure more than the square of the speed

When speed varies specific speed at the bep-points remains the same.

3

$$n_{s} = \frac{\omega\sqrt{Q}}{p^{\frac{3}{4}}} \implies \frac{n_{s_{1}}}{n_{s_{2}}} = \frac{n_{1}}{n_{2}} \frac{\sqrt{Q_{1}}}{\sqrt{Q_{2}}} \frac{p^{\frac{3}{4}}}{p^{\frac{3}{4}}_{1}} = \frac{n_{1}}{n_{2}} \frac{n^{\frac{1}{2}}_{1}}{n^{\frac{1}{2}}_{2}} \frac{n^{\frac{3}{2}}_{2}}{n^{\frac{3}{2}}_{1}} = 1$$

This relation stands irrespective of the deviation of the affinity laws.

At constant speed pressure curve decreases as viscosity increases in such a way that the specific speed at "bep" remains constant

$$n_{s} = \frac{\omega\sqrt{Q_{1}}}{p_{1}^{\frac{3}{4}}} = \frac{\omega\sqrt{Q_{2}}}{p_{2}^{\frac{3}{4}}}$$

so for the same speed at different viscocities the relationship

$$\frac{\underline{Q}_1}{\underline{Q}_2} = \left(\frac{\underline{p}_1}{\underline{p}_2}\right)^{\frac{3}{2}}$$

Is valid.

At constant speed pressure curve decreases as viscosity increases, but head at zero capacity remains the same. However the influence of the pump casing on the characteristics is higher more than when pumping water.

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For constant viscosity and variable speed. Efficiency at "bep" increases at higher speeds. (higher Reynolds numbers give less resistance's so higher efficiencies.



Influence viscosity on pump performance (Stepanoff, 1957)

A change in Reynolds number due to a change in viscosity causes a change in the hydraulic **losses** are estimated for water the hydraulic losses for another viscosity can be calculated according:

$$1 - \left(\eta_{hydr.}\right)_{fluid} = 1 - \left(\eta_{hydr.}\right)_{water} \cdot \frac{\lambda_{fluid}}{\lambda_{water}}$$

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In which λ is the Darcy-Weisbach resistance coefficient?



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Wb 3413Pumps and Systems13.INFLUENCE OF SOLIDS ON THE PUMP CHARACTERISTICS.

Solids in suspension cannot posses or transmit any pressure energy. Solids can only acquire kinetic energy. When a particle is accelerated the required energy is taken from the liquid phase. When a particle is de-accelerated by the fluid, the kinetic energy is transformed to turbulent energy from which only a part is transformed to pressure energy.

13.1. PUMP CHARACTERISTICS FOR MIXTURES

For homogeneous flows the required power is proportional with the density of the fluid. (see page 8 $P = T\omega = Q\rho\omega[r_2c_2\cos\alpha_2 - r_1c_1\cos\alpha_1] = Qp_{th}$)

$$P_{mixture} = P_{water} * \frac{\rho_{mixture}}{\rho_{water}}$$

Solids transform their kinetic energy partially to pressure energy(potential) According to Stepanoff:

$$p_m = p_w \frac{\rho_m}{\rho_w} f_c$$

and because $P_m = P_w * \rho_m$ it follows that:

$$\frac{\eta_m}{\eta_w} = f_c$$

$$f_c = \left\{ 1 - C_{cd} \left[.8 + .6 \log(d_{50}) \right] \right\}$$

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Influence of particle size on pump performance (Stepanoff)



Research in the laboratory of Dredging Technology TUD have shown the following: For fine and medium sand efficiency is less than according Stepanoff but increase more than linear at high concentrations

For course sand efficiency is lower than according Stepanoff

For fine and medium sand power is proportional with the density but for coarse sand the required power increases strongly with delivered concentration.

A more general solution can be obtained with a distinction between the different effects.:

$$\frac{\eta_m}{\eta_w} = f_\eta \text{ and } \frac{p_m}{p_w} \frac{\rho_w}{\rho_m} = f_p \implies \frac{P_m}{P_w} \frac{\rho_w}{\rho_m} = \frac{f_p}{f_\eta}$$

Wilson has published a more generalised solids-effect diagram for slurry pumps. He concludes that the sloids effect on pressure, efficiency, and power may be strongly influenced by the size of the dredge pump (Scale effects).

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GENERALISED SOLIDS EFFECT DIAGRAM BY WILSON

Note:

Wilson's consideration it is only based on limited experimental data.

14.INFLUENCE OF SOLIDS ON CAVITATION

In principal a negative influence.

The presents of solids in the flow will incept cavitation earlier.

Silt and clay can cause a higher vapour pressure.

However the most important aspect of pumping solids is the higher-pressure drop in vertical lines due to the higher density.

As a consequence the decisive vacuum is reached earlier.

In order to avoid cavitation in suction lines there are in principle three possibilities:

- 1. Reduce the concentration of the mixture.
- 2. Put the pump (further) below the water level.
- 3. Reduce the velocity

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This can easily proved by the so-called vacuum formulae for homogeneous transport

$$\rho_{water}gH + Vac = \rho_{mengsel}gh_z + \xi \frac{1}{2}\rho_{mengsel}v^2 = \rho_{mengsel}g(H-k) + \xi \frac{1}{2}\rho_{mengsel}v^2$$
$$Vac = -\rho_{water}gH + \rho_{mengsel}g(H-k) + \xi \frac{1}{2}\rho_{mengsel}v^2$$

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15.PUMP PIPELINE COMBINATION

When pumping water under a constant boundary conditions, there is only one operating point, butten the operating conditions are variable, there is a operating area.







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When the operating point shifts to the constant power or constant torque line the engine speed will decrease in order to avoid overloading of the motor.



For diesel engines this speed reduction is limited by the smoke limit. This is the point where insufficient air is available for a complete combustion. At lower speed the available torque will drop sharply and heavily polluted gasses are emitted resulting in higher wear.

The position of the smoke limit depends mainly on the degree of supercharging. Rule of thumb 90% of the nominal speed.

In case of normally aspirated engines speed drops of 60-70% of nominal speed are possible.

The allowable torque at speeds lower than at the smoke limit depends on the type of engine. When the allowable torque results in a decreasing capacity with decreasing head the operating point can easily come below the critical capacity resulting in a blockage of the pipe. Installing an impeller with a smaller diameter is now the only solution to get a normal operating condition.

As already said cavitation causes a drop of the manometric head. Working under high cavitation condition can reduce the available pump pressure remarkable.

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15.1. PUMPING AT CONSTANT SPEED



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For a pump-pipeline combination with a short suction line compared by the discharge line $(L_{SUCTION LINE} \ll L_{DISCHARGE LINE})$ the operating points are:

- 1. When the complete line (suction and discharge line) are filled with water
- 2. Suction line filed with mixture and pump and discharge line filled with water.
- 3. The complete system filled with mixture
- 4. Suction line and pump filled with water, discharge line with mixture.

15.2. PUMPING AT CONSTANT TORQUE OR POWER

The numbering is now clockwise

Pressure [kPa]



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Flow [m3/s]

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16.RELATION BETWEEN PRODUCTION PUMPING DISTANCE

In case of pump speed is maximum, the maximum output of solids per unit of time (production) depends on the pumping distance.

Using the expression for empirical correlation for the pressure gradient between mixture and

water:
$$\phi = \frac{I_m - I_f}{C_{vd}I_f} \Longrightarrow I_m = I_f (1 + \varphi C_{vd})$$

The pressure loss can be written as:

$$\Delta p_l = AQ^2 \left(1 + \frac{B}{Q^3} C_{vd} \right)$$

with $A = \lambda \frac{\rho_w}{2} \frac{L}{D} \frac{1}{\left(\frac{\pi}{4} D^2\right)^2}$ and b depending on the particle size and pipe diameter.

Because Δp_l and q vary only slowly with C_{vd} so, 1 can increase if C_{vd} decreases.



PUMP-PIPELINE CHARACTERISTICS

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Section I : Production is determined by other factors than pump or engine Section II : Operating point at constant torque or constant power line Section III: Operating pony at constant speed line

PRODUCTION-PIPELINE LENGTH DIAGRAM

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Wb 3413 17.SERIES OPERATION:

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Purpose of serie operation is:

• dredging at greater depth.

• pumping over greater distance.

From operation point of view there is hardly any difference between pumping with one pump or with more than one pump. However the pumps should be designed for the same operation area.

For dredgers having more than one pump the first pump is in general a suction pump. (relative low pressure and a high decisive vacuum)

The pump characteristics of pumps in series can easily be determined by super position of the manometric pressure and the required power at a given capacity.

So:

$$p_t = \sum_{n=1}^{N} p_n(Q) \text{ AND } P_t = \sum_{n=1}^{N} P_n(Q)$$

The total efficiency is defined as:

$$\eta_s = \frac{Q \cdot \sum_{n=1}^{N} p_n(Q)}{\sum_{n=1}^{N} P_n(Q)} \times 100\%$$

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17.1. THE LOCATION OF THE BOOSTER

As long as the incoming pressure at the booster is sufficient positive and out coming pressure is not too high for the pump and its component, then the location does not matter.



FIGURE: PRESSURELINES ALONG PIPELINE

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Wb 3413Pumps and Systems18.PARALLEL OPERATION OF PUMPS AND PIPES

Parallel operation is used when a higher capacity is required. Examples:

• Trailing suction hopper dredgers.



• Special purpose vessel "Cardium" used during the delta works



SUCTION MOUTH "CARDIUM"

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• Jet pumps systems.

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Jetpump systems on board of trailing suction hopper dredgers have often the possibility to work in serial and parallel operation.

• Drink water works (variable demand)

Due to the variable demand parallel operation is normal in drinkwater supply

Parallel operation with to dredge pumps on one line is some times to be seen on board of trailing suction hopper dredgers. The two dredgepumps deliver the mixture via one shute or discharge pipe into the hopper.

18.1. PUMP CHARACTERISTICS OF PARALLEL OPERATION

The combined characteristics can be determined by super position of the capacities at a given pressure.

This implies that the capacity is expressed as function of the pressure.

$$Q_t = \sum_{n=1}^{N} Q_n(p)$$
 AND $P_t = \sum_{n=1}^{N} P_n(Q) = f(Q_t)$

The total efficiency is:

$$\eta_s = \frac{p \cdot \sum_{n=1}^{N} Q_n(p)}{P_t(Q_t)} \times 100\%$$

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18.2. PARALLEL PIPELINES

Parallel operation of dredge pumps on one line is only done in the dredging field when reclamation areas have small fill heights. In that case the main pipeline is devided in two smaller lines with an equal cross section.

Comform with parallel operating pumps the pipeline characteristic can be determined by super position of the capacities at a given pressure.

When different pipeline length are used beware of the critical velocity in the long line!



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If parallel or serie operation is useful depends on pipeline characteristic as shown below.



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19.INFLUENCE OF WEAR ON THE PERFORMANCE OF PUMPS.

Wear is mainly determined, except from the mineral composition of the grains, by the speed of the mixture.

For pumps it is assumed that the wear is proportional with the third power of the peripheral speed.

Therefore the peripheral speed is limited to 35- 40 m/s.

The performance of a pump changes as the sizes and shapes differ from the original ones.

19.1. WEAR AT THE SUCTION INLET

Wear at the inlet occurs when the pump is working at a capacity, which differs substantial from the design capacity.

(shock losses)

The inlet geometry is decisive for the cavitation performance of centrifugal pumps. So wear at the inlet results mostly in a reduction of the decisive vacuum.

19.2. WEAR AT THE OUTLET.

The manometric pressure is mainly determined by the geometry at the outlet.

Reduction of the impeller diameter due to wear will result in a decrease of the manometric pressure.

Because the wear is proportional with the third power of the peripheral speed, more wear can be expected when pumping over long distances.

19.3. WEAR AT THE LINING PLATES

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Recirculation between impeller and pumphouse

19.4. WEAR AT THE CUTWATER

Wear at the cutwater does increase the quantity of recirculation water in the pump casing. However, compared to water pumps, dredge pumps do have a large cap between the impeller and the pump casing at the cutwater.

So the influence of wear at the cutwater will decrease the efficiency slightly.

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Wb 3413 ENCLOSURE A

THE CUBIC EQUATION FOR THE SOLUTION OF THE CONSTANT POWER LINE

GIVEN
$$z^3 + a_2 z^2 + a_1 z + a_0 = 0$$

LET $q = \frac{1}{3}a_1 - \frac{1}{9}a_2^2$; $r = \frac{1}{6}(a_1a_2 - 3a_0) - \frac{1}{27}a_2^3$
AND $s_1 = \sqrt[3]{r + \sqrt{q^3 + r^2}}$; $s_2 = \sqrt[3]{r - \sqrt{q^3 + r^2}}$

THEN IS IF:

 $q^{3} + r^{2} = c^{2} > 0;$

ONE REAL ROOT AND A PAIR OF COMPLEX CONJUGATE ROOTS.

 $s_1 = \sqrt[3]{r+c}; s_2 = \sqrt[3]{r-c}$ AND BOTH REAL THEN:

- z_1 IS REAL AND z_2, z_3 ARE COMPLEX
- $q^3 + r^2 = c^2 = 0$ ALL REAL ROOTS AND AT LEAST TWO ARE EQUAL.

$$s_1 = s_2 = \sqrt[3]{r} \implies s_1 - s_2 = 0$$

 $q^3 + r^2 = c^2 < 0$ ALL ROOTS REAL

 $s_1 = \sqrt[3]{r+ci}; \ s_2 = \sqrt[3]{r-ci}$ AND ARE COMPLEX $(2-2)^{\frac{1}{2}} \left[(\theta + 2k\pi) ... (\theta + 2k\pi) \right]$

$$s_{1} = (r^{2} + c^{2})^{6} \cdot \left[\cos\left(\frac{-\theta}{3}\right) + i\sin\left(\frac{-\theta}{3}\right) \right]$$
$$s_{2} = (r^{2} + c^{2})^{\frac{1}{6}} \cdot \left[\cos\left(\frac{-\theta + 2k\pi}{3}\right) + i\sin\left(\frac{-\theta + 2k\pi}{3}\right) \right]$$

SO

$$s_{1} + s_{2} = 2(r^{2} + c^{2})^{\frac{1}{6}} \cdot \cos\left(\frac{\theta + 2k\pi}{3}\right)$$
$$s_{1} - s_{2} = 2(r^{2} + c^{2})^{\frac{1}{6}} \cdot i\sin\left(\frac{\theta + 2k\pi}{3}\right)$$

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 $\frac{\text{Wb 3413}}{\text{WITH } \theta = \arctan\left(\frac{c}{r}\right)}$

RESULTING IN THE REAL ROOTS $\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3$

FOR ALL CONDITIONS THE ROOTS z_1, z_2, z_3 ARE:

$$z_{1} = (s_{1} + s_{2}) - \frac{a_{2}}{3}$$

$$z_{2} = -\frac{1}{2}(s_{1} + s_{2}) - \frac{a_{2}}{3} + \frac{i\sqrt{3}}{2}(s_{1} - s_{2})$$

$$z_{3} = -\frac{1}{2}(s_{1} + s_{2}) - \frac{a_{2}}{3} - \frac{i\sqrt{3}}{2}(s_{1} - s_{2})$$

AND

$$z_1 + z_2 + z_3 = -a_2$$

$$z_1 z_2 + z_1 z_3 + z_2 z_3 = a_1$$

$$z_1 z_2 z_3 = -a_0$$

APPLIED TO THE CONSTANT POWER EQUATION $z^3 + a_2 z^2 + a_1 z + a_0 = 0$

$$P = A_0 n^3 D^4 + A_1 n^2 D^2 Q + A_2 n Q^2 \implies n^3 + \frac{A_1 Q}{A_0 D^2} n^2 + \frac{A_2 Q^2}{A_0 D^4} n - P = 0$$

WITH

$$a_{2} = \frac{A_{1}Q}{A_{0}D^{2}}, a_{1} = \frac{A_{2}Q^{2}}{A_{0}D^{4}}, a_{0} = P$$
SO
$$q = \frac{A_{2}Q^{2}}{3A_{0}D^{4}} - \frac{1}{9} \left(\frac{A_{1}Q}{A_{0}D^{2}}\right)^{2}, r = \frac{1}{6} \left(\frac{A_{2}Q^{2}}{A_{0}D^{4}} \cdot \frac{A_{1}Q}{A_{0}D^{2}} - 3P\right) - \frac{1}{27} \left(\frac{A_{1}Q}{A_{0}D^{2}}\right)^{2}$$
AND
$$q^{3} + r^{2} = \left[\frac{A_{2}Q^{2}}{3A_{0}D^{4}} - \frac{1}{9} \left(\frac{A_{1}Q}{A_{0}D^{2}}\right)^{2}\right]^{3} + \left[\frac{1}{6} \left(\frac{A_{2}Q^{2}}{A_{0}D^{4}} \cdot \frac{A_{1}Q}{A_{0}D^{2}} - 3P\right) - \frac{1}{27} \left(\frac{A_{1}Q}{A_{0}D^{2}}\right)^{2}\right]^{2}$$

$$s_{1} = \sqrt[3]{\frac{1}{6} \left(\frac{A_{2}Q^{2}}{A_{0}D^{4}} \cdot \frac{A_{1}Q}{A_{0}D^{2}} - 3P\right) - \frac{1}{27} \left(\frac{A_{1}Q}{A_{0}D^{2}}\right)^{2} + \sqrt{\left[\frac{A_{2}Q^{2}}{3A_{0}D^{4}} - \frac{1}{9} \left(\frac{A_{1}Q}{A_{0}D^{2}}\right)^{2}\right]^{3} + \left[\frac{1}{6} \left(\frac{A_{2}Q^{2}}{A_{0}D^{4}} \cdot \frac{A_{1}Q}{A_{0}D^{2}} - 3P\right) - \frac{1}{27} \left(\frac{A_{1}Q}{A_{0}D^{2}}\right)^{2}\right]^{2}}$$

$$s_{2} = \sqrt[3]{\frac{1}{6} \left(\frac{A_{2}Q^{2}}{A_{0}D^{4}} \cdot \frac{A_{1}Q}{A_{0}D^{2}} - 3P\right) - \frac{1}{27} \left(\frac{A_{1}Q}{A_{0}D^{2}}\right)^{2} - \sqrt{\left[\frac{A_{2}Q^{2}}{3A_{0}D^{4}} - \frac{1}{9} \left(\frac{A_{1}Q}{A_{0}D^{2}}\right)^{2}\right]^{3} + \left[\frac{1}{6} \left(\frac{A_{2}Q^{2}}{A_{0}D^{4}} \cdot \frac{A_{1}Q}{A_{0}D^{2}} - 3P\right) - \frac{1}{27} \left(\frac{A_{1}Q}{A_{0}D^{2}}\right)^{2}\right]^{2}}$$

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Wb 3413SUBSTITUTED IN z_1, z_2, z_3 GIVES THE REQUIRED ROOTS.

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