Time Measurements and Isochronism

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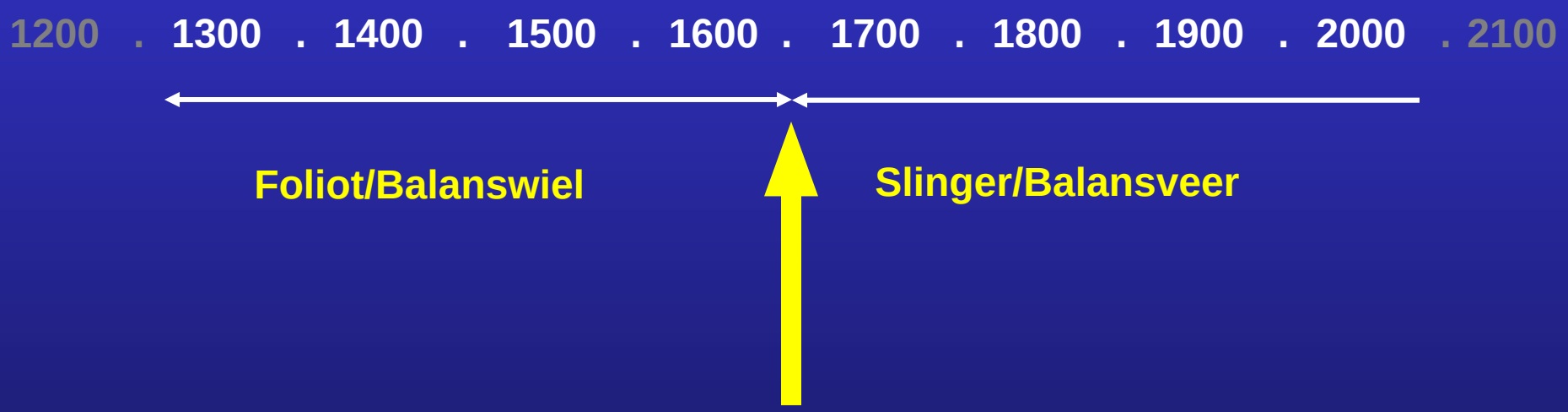
Museum of the Dutch Clock, Zaandam, The Netherlands

Academic Medical Center

Titan: From Discovery to Encounter, Noordwijk 16 April, 2004

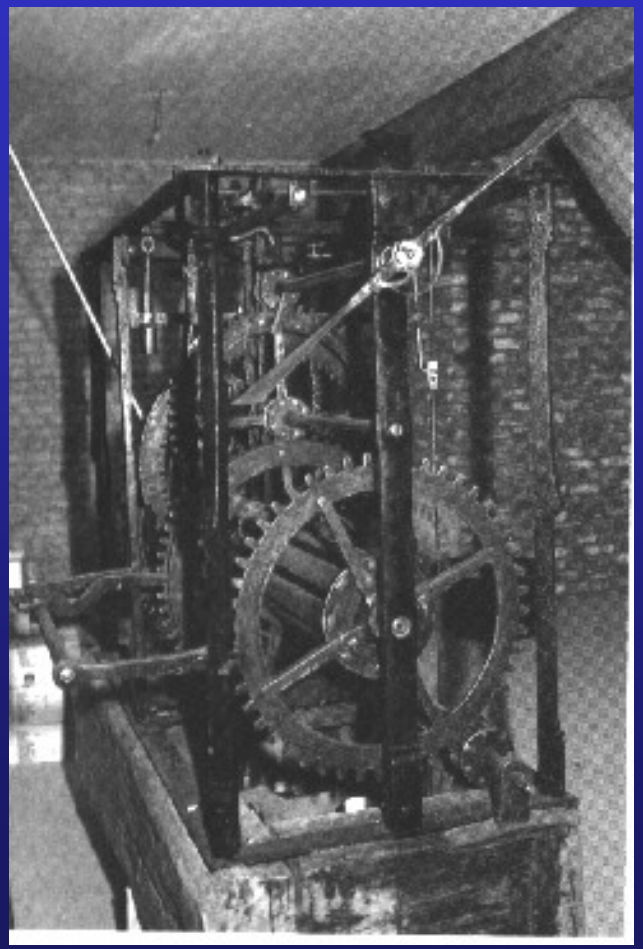
MNU

Mechanische Uurwerken (> 1300)

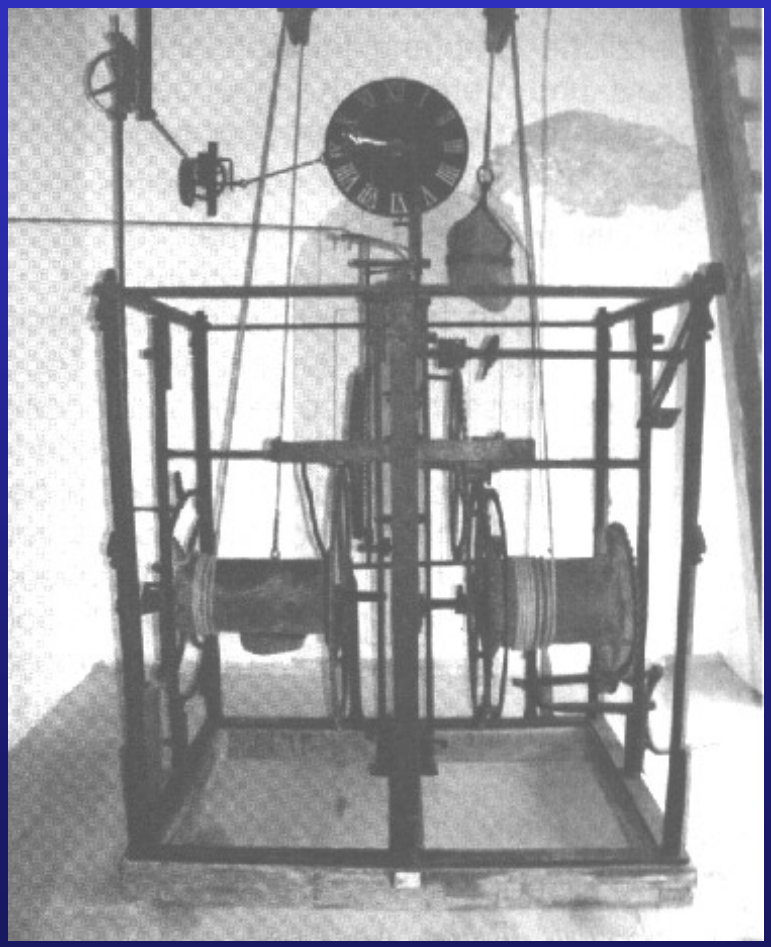


Early Turret Clocks

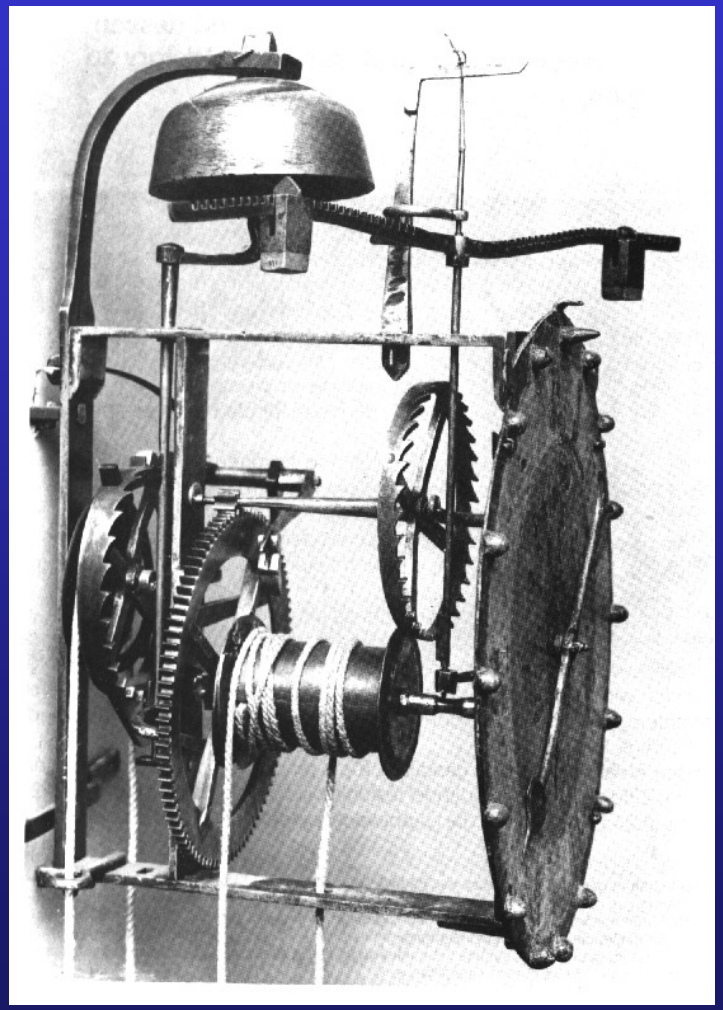
Maastricht 1400?;



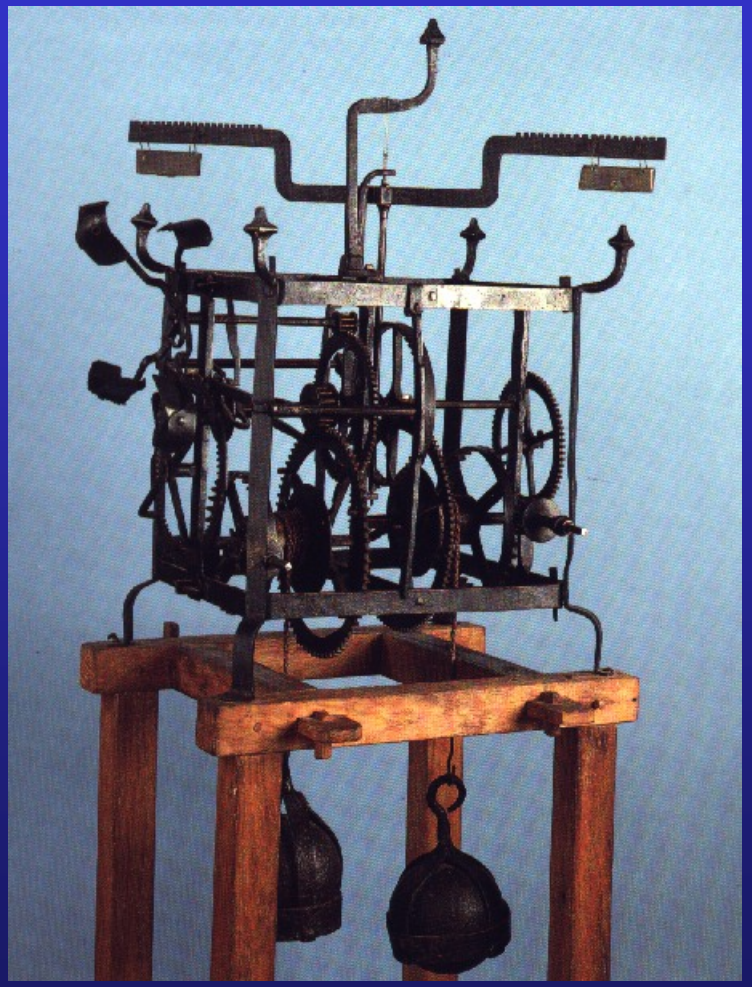
Winkel (N-H) ~1460



Early Mechanical Clocks



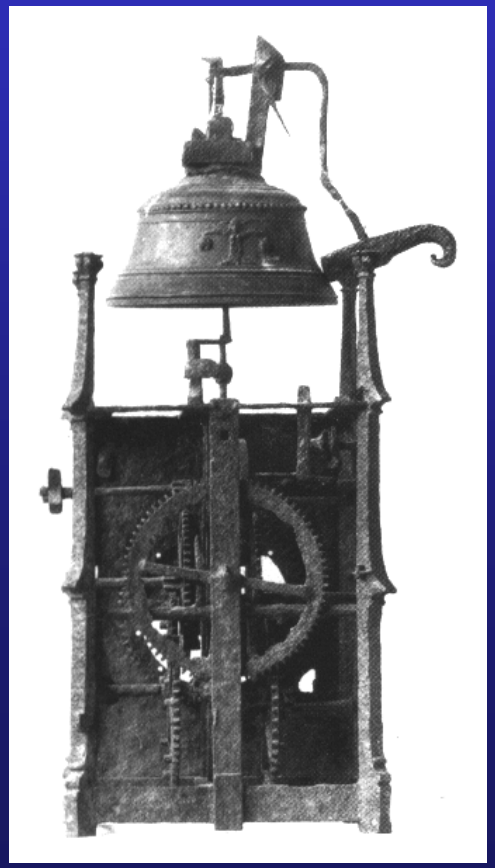
Nürnberg ca 1450



Nederland ca 1520 (MNU)

Museum of the Dutch Clock

Exhibition Willem Barents and his Clock; 1596-1996

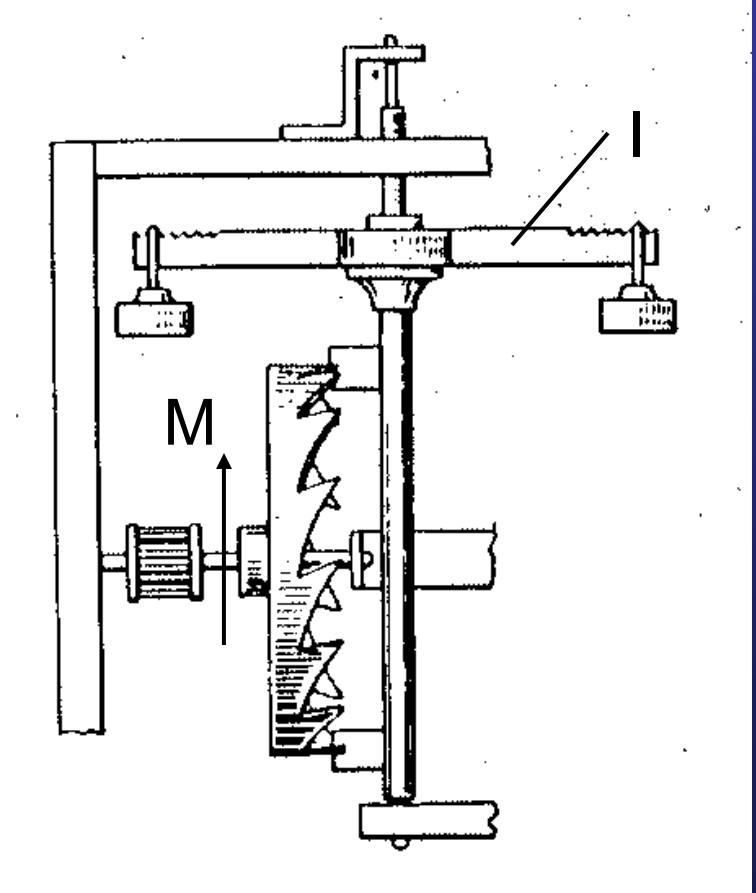


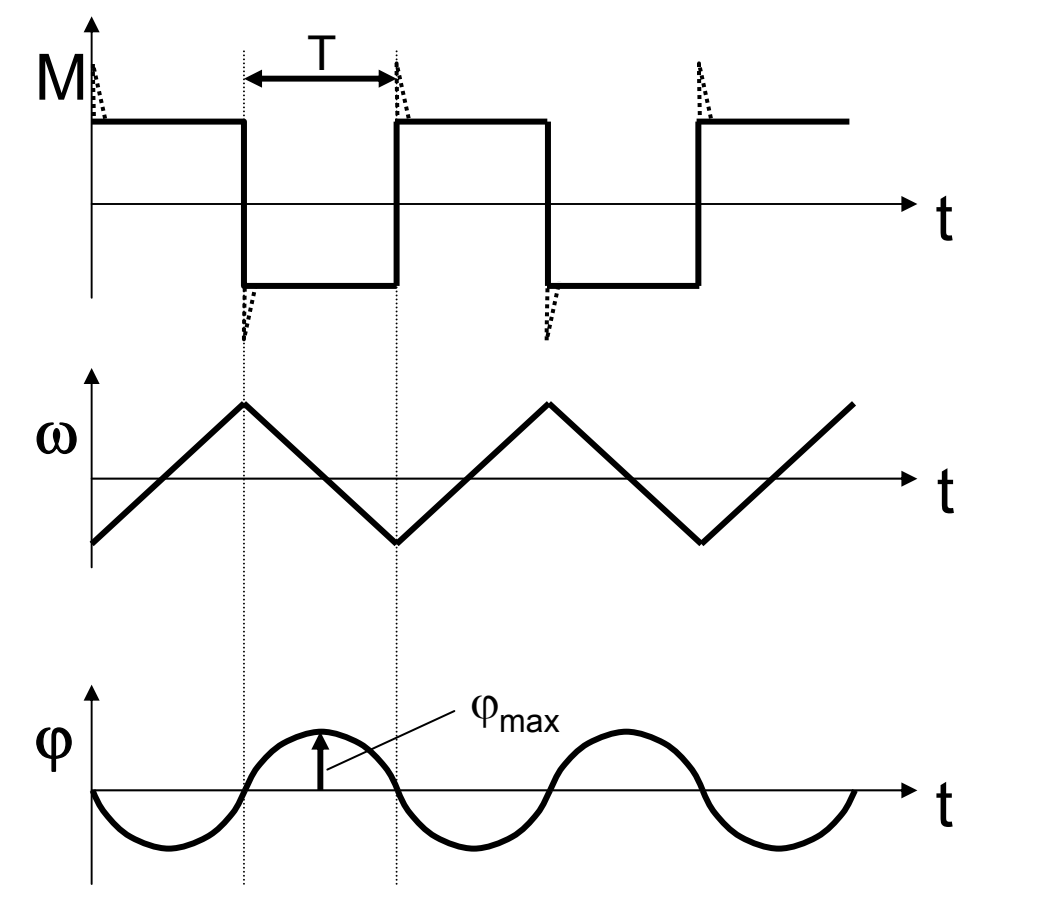
- Model from ca 1450
- Not touched since 1596
- Collection Rijksmuseum

Mechanical Clocks; 1300 - 1656

Foliot/Vertical Verge-Escapement

$M = I\,\partial\omega/\partial t = I\,\partial^2\phi/\partial t^2$





Mechanical Clocks; 1300 - 1656

Foliot/Vertical Verge-Escapement

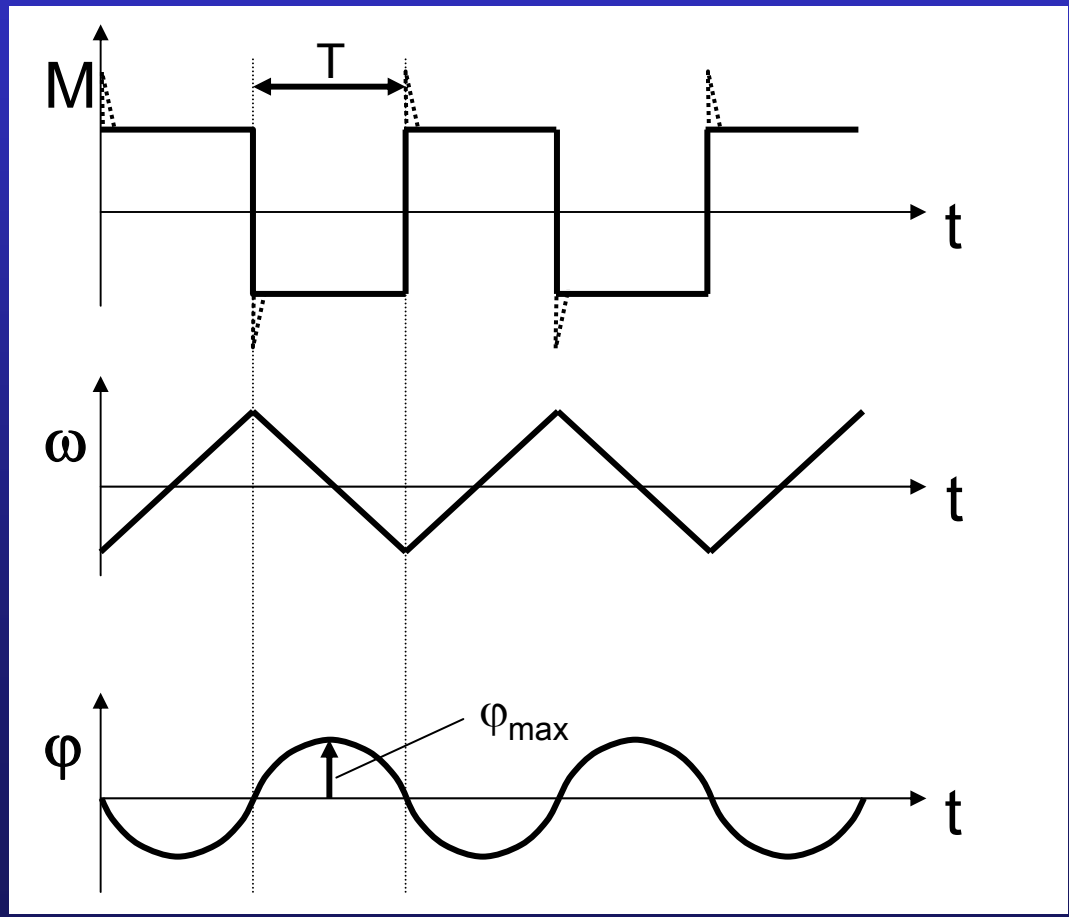
$M = I\,\partial\omega/\partial t = I\,\partial^2\phi/\partial t^2$

$\omega = (M/I).t$

$\phi = \frac{1}{2}\,(M/I).t^2$

$\phi_{max} = \frac{1}{2}\,(M/I).(T/2)^2$

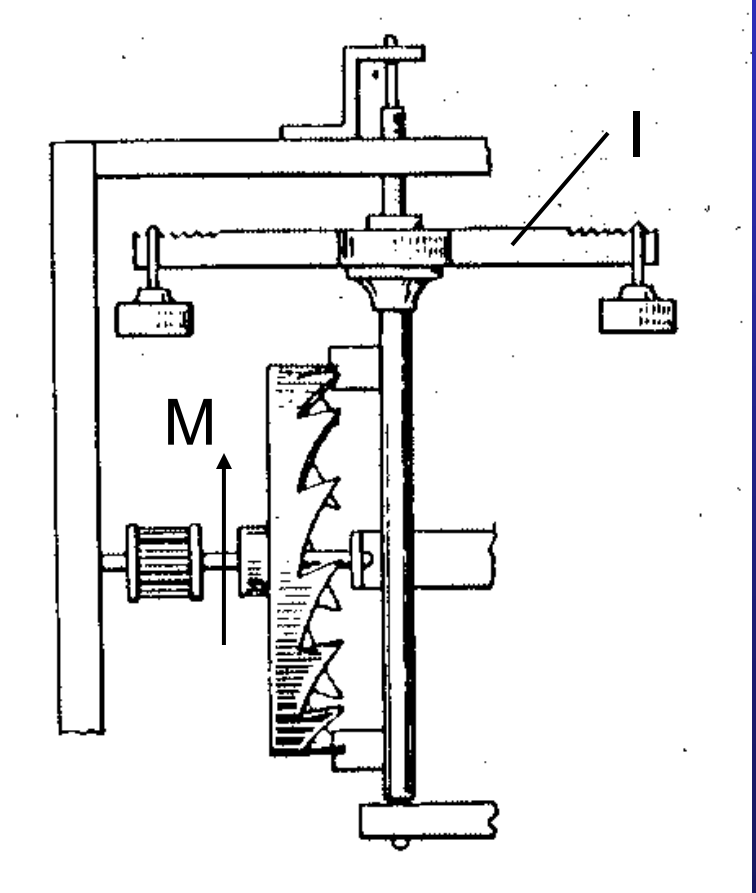
$T/2 = \sqrt{2\,\phi max(I/M)}$

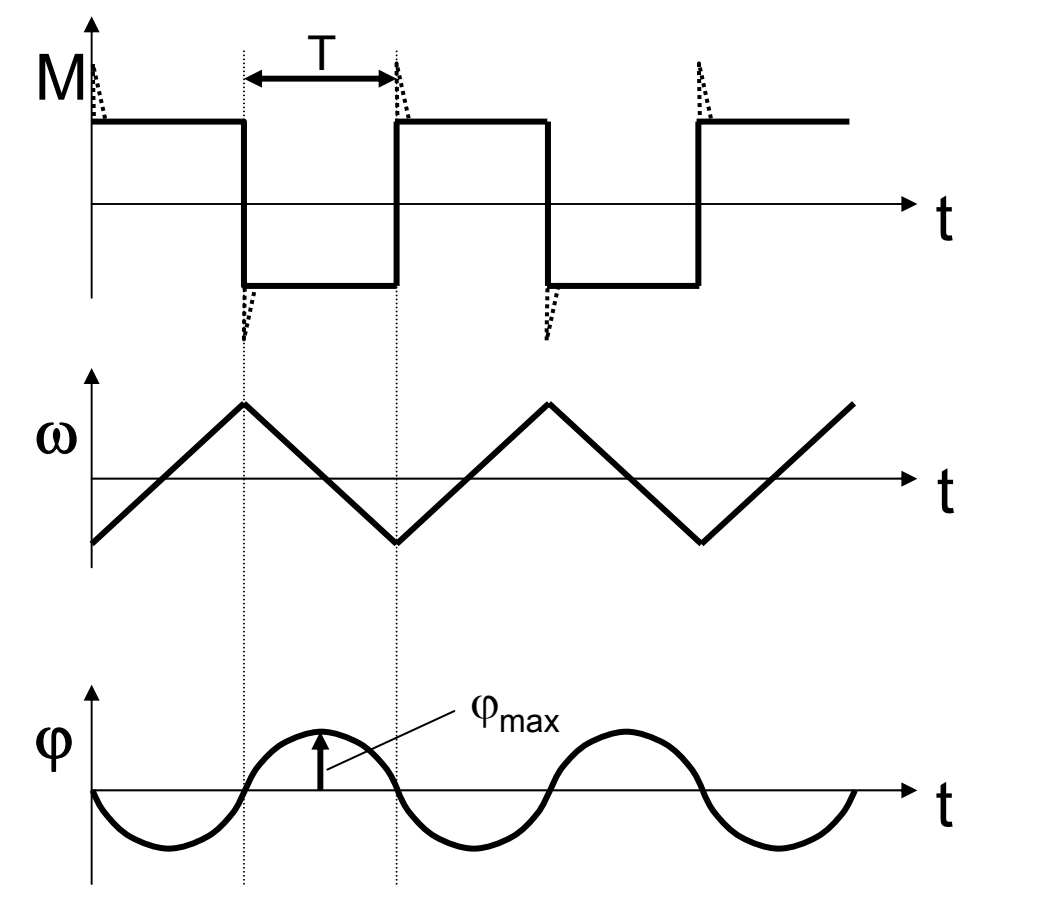


Mechanical Clocks; 1300 - 1656

Foliot/Vertical Verge-Escapement

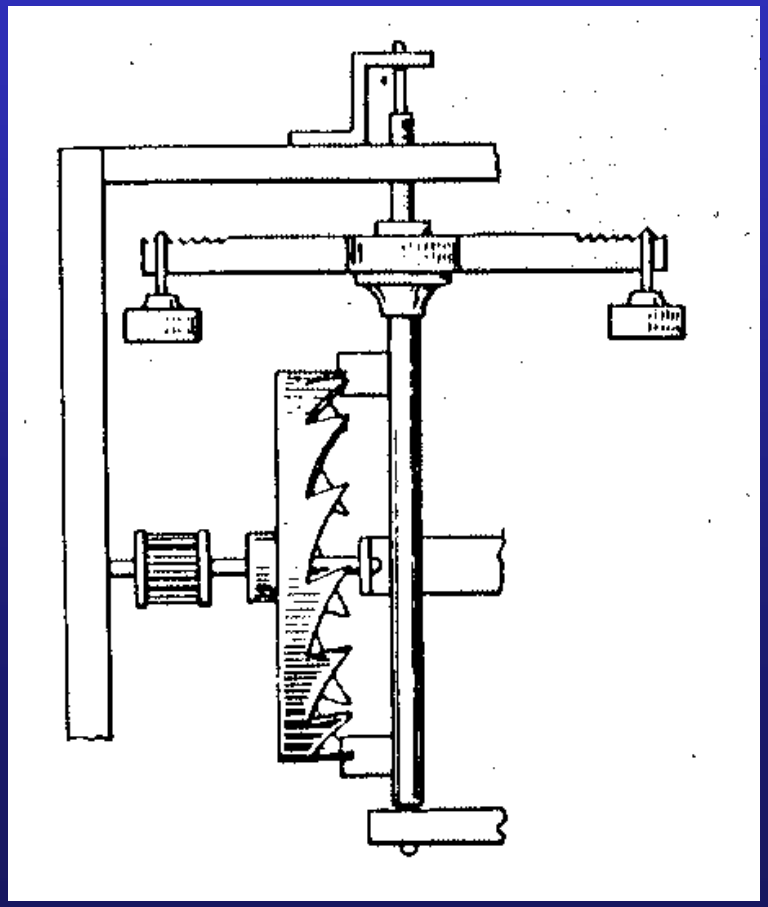
$T = 2\sqrt{2\,\phi_{max}(I/M)}$





Mechanical Clocks; 1300 - 1656

Foliot/Vertical Verge-Escapement



+ Balanced (independent of gravity)

+ Self starting

$$T = 2\sqrt{2\,\phi_{max}(I/M)}$$

- Dependent on driving moment

- Dependent on friction

- Dependent on temperature

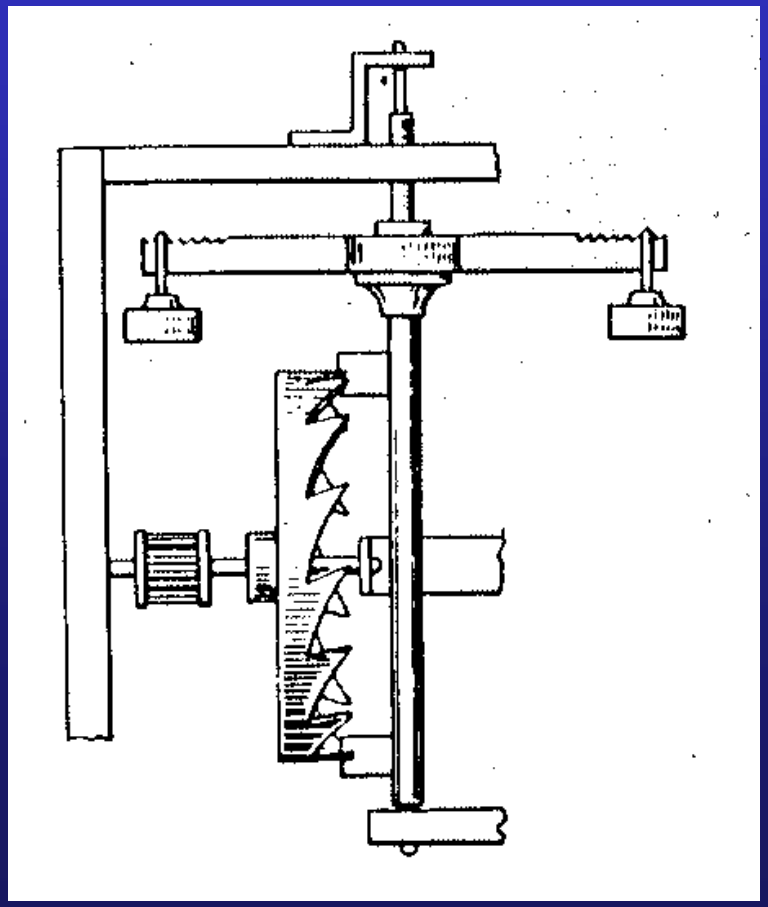
-Dependent on ϕ_{max}:

$$dT/T = \tfrac{1}{2}\; d\phi_{max}/\phi_{max}$$

➡ NO INTRINSIC FREQUENCY

Mechanical Clocks; 1300 - 1656

Foliot/Vertical Verge-Escapement



- Dependent on ϕ_{max}:

$$T = 2\sqrt{2\,\phi_{max}(I/M)}$$

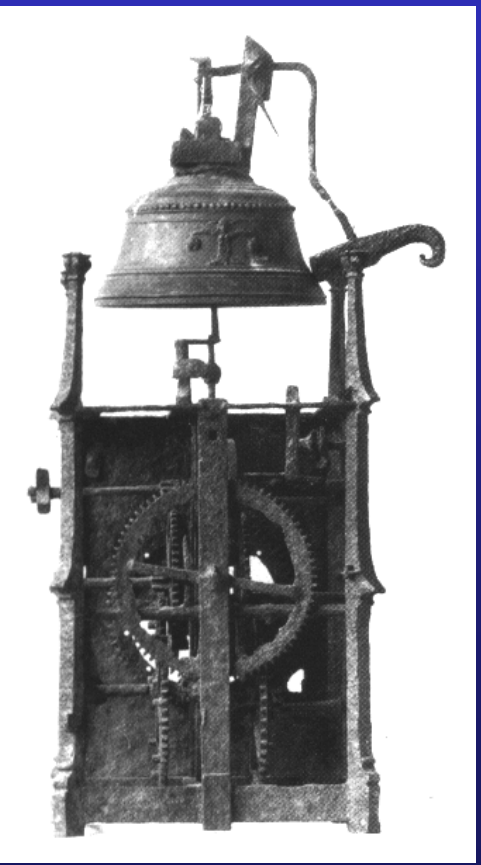
➡ NO INTRINSIC FREQUENCY

$$dT/T = \tfrac{1}{2}\; d\phi_{max}/\phi_{max}$$

Voorbeeld: $d\phi_{max}/\phi_{max}$ = 1%

dT/T = ½ % = 18 s/uur = 7.2 min/dag

Clocks; Developments 1200-1650



- Iron Clocks → Brass bearings/Iron axles (ca 1550)

  Identical materials: Wear (lubrication!)

  Combining: Soft bearing/Hard axle: Less wear

- Fixed by wedges → Rivets/Screws (1500 –)

  Much more design freedom

  From cage to plate construction

  (blacksmith) (locksmith)

- Spring drive → Table clocks/Watches

  Fusee

Clocks; Developments 1200-1650

- Iron Clocks → Brass bearings/Iron axles (ca 1550)

  Identical materials: Wear (lubrication!)

  Combining: Soft bearing/Hard axle: Less wear

Germany 1545 - 1550

Clocks; Developments 1200-1650

- Iron Clocks → Brass bearings/Iron axles (ca 1550)

  Identical materials: Wear (lubrication!)

  Combining: Soft bearing/Hard axle: Less wear

London ca 1585

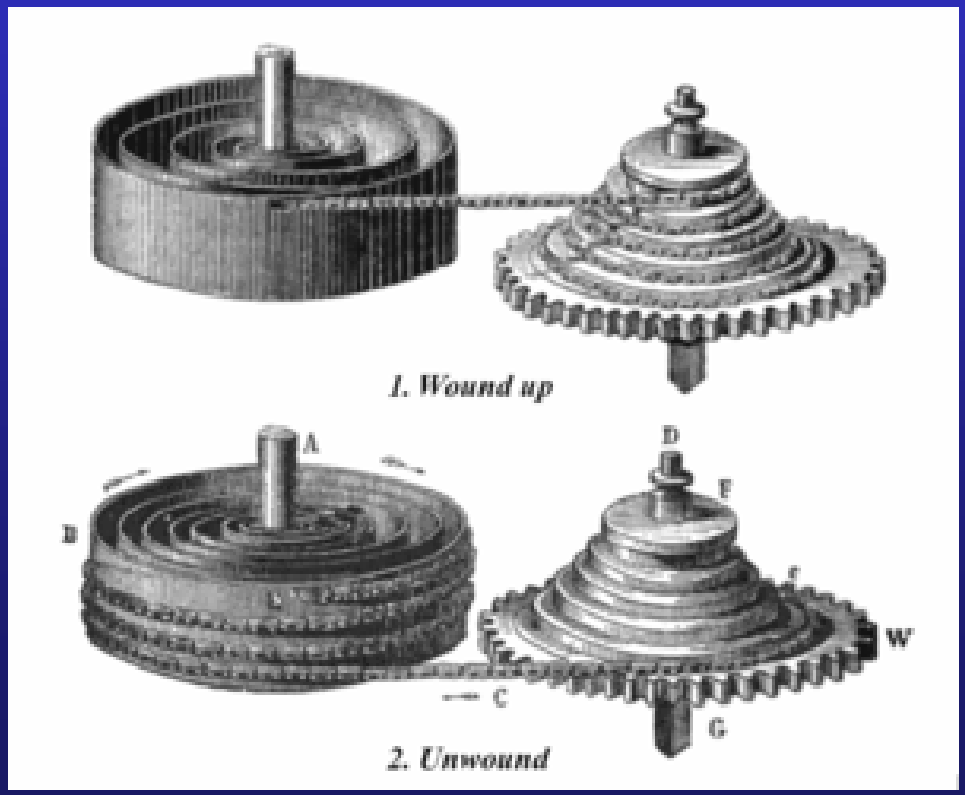
Clocks; Developments 1200-1650

- Spring drive → Table clocks/Watches:

Fusee

Burgundy ca 1450

Model movement



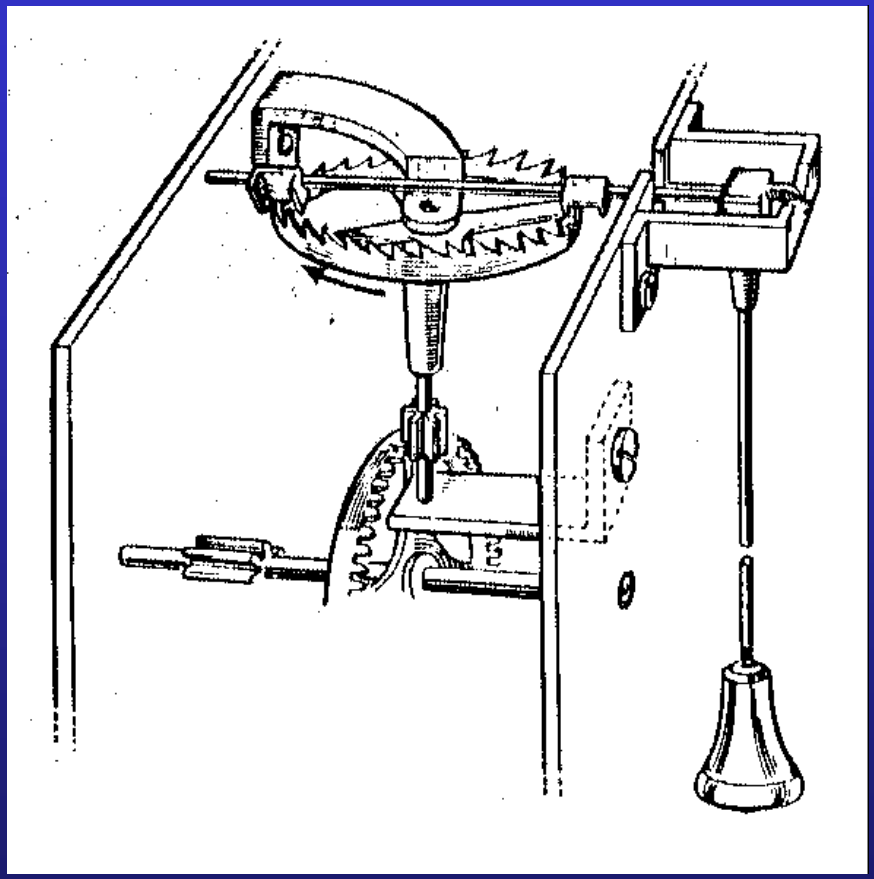
M = F x R → Constant drive moment

Christiaan Huygens

14 april 1629 - 8 juli 1695

P. Bourguignon ca 1688; coll KNAW

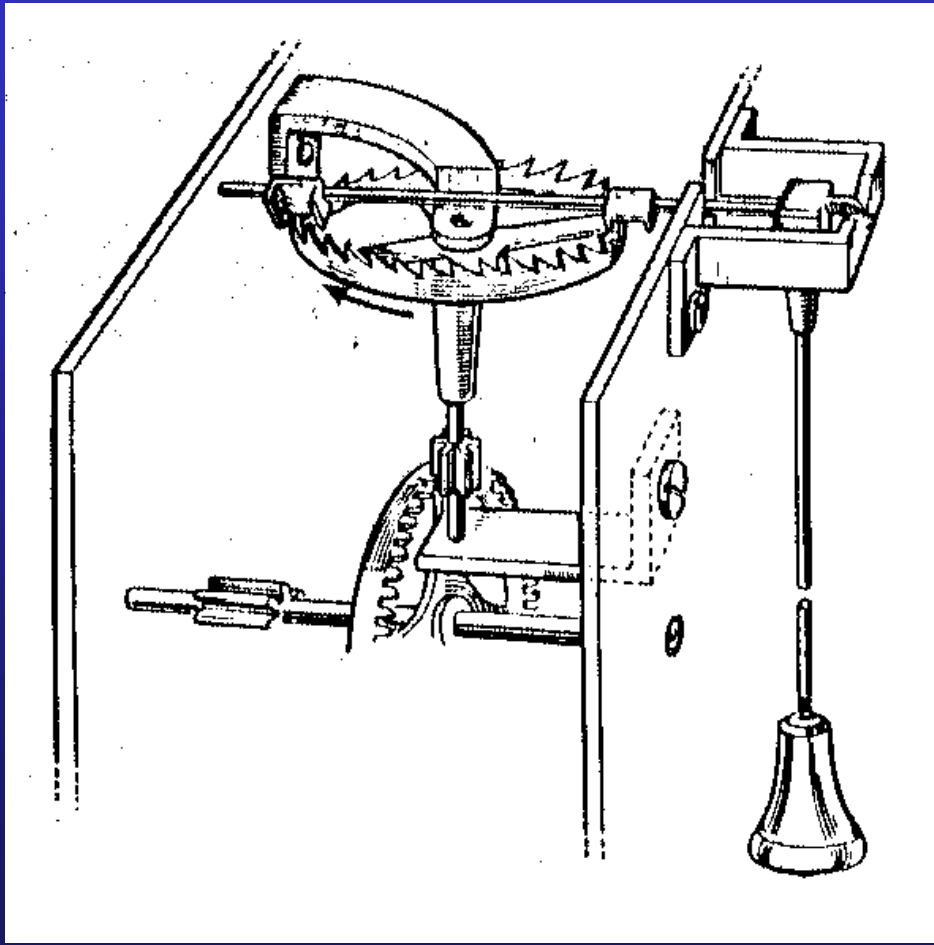
Christiaan Huygens 1629 - 1695



1657: Pendulum/horizontal verge escapement

Mechanical Clocks; > 1656

Pendulum/Horizontal Verge-Escapement $T = 2\pi\sqrt{l/g}$



Slinger / *Pendulum*

$$ml\,d^2\varphi/dt^2 = -mg\sin\varphi$$
$$d^2\varphi/dt^2 = -(g/l)\sin\varphi$$
$$\approx -(g/l)\varphi$$

Oplossing / *Solution* $\varphi = \varphi_{max}\sin(2\pi t/T)$

Periode / *Period* $T = 2\pi\sqrt{(l/g)}$
$$\approx 2.0\sqrt{l}$$

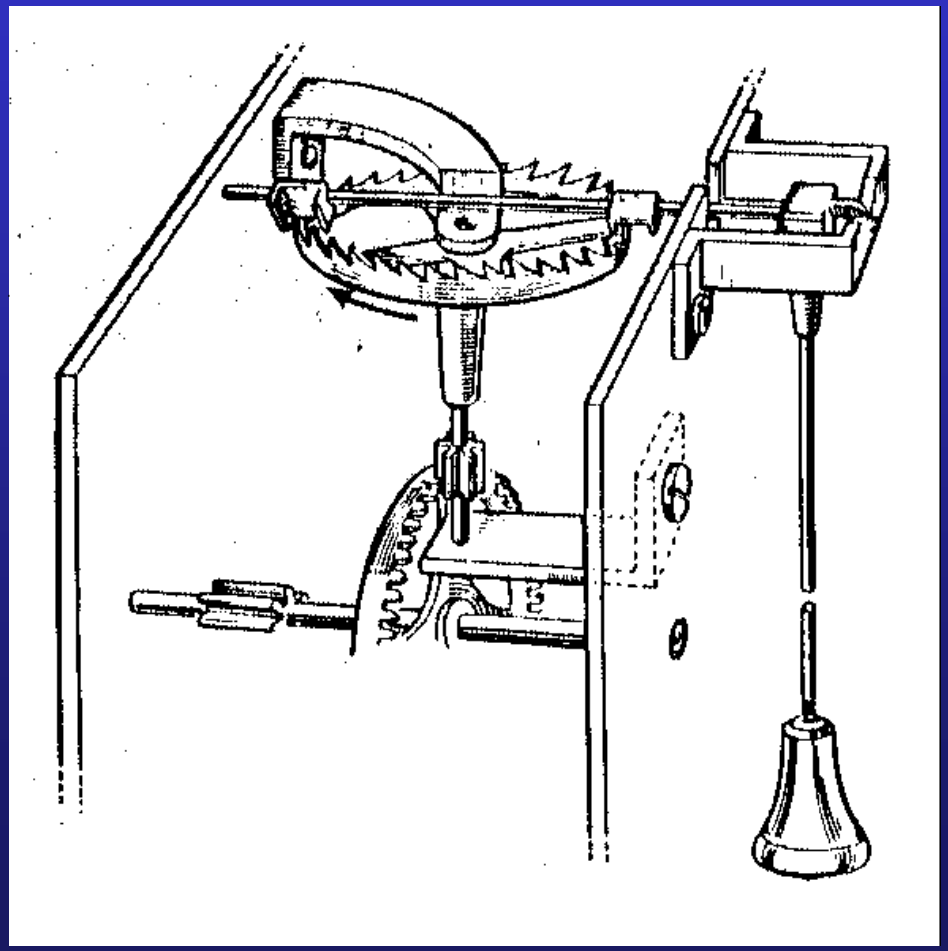
l = slingerlengte / *length of pendulum* [m]

g = versnelling zwaartekracht *gravitation constant* [m/s^2]

The evolution of the Dutch Clock, MNU 1991

Mechanical Clocks; > 1656

Pendulum/Horizontal Verge-Escapement $T = 2\pi\sqrt{l/g}$



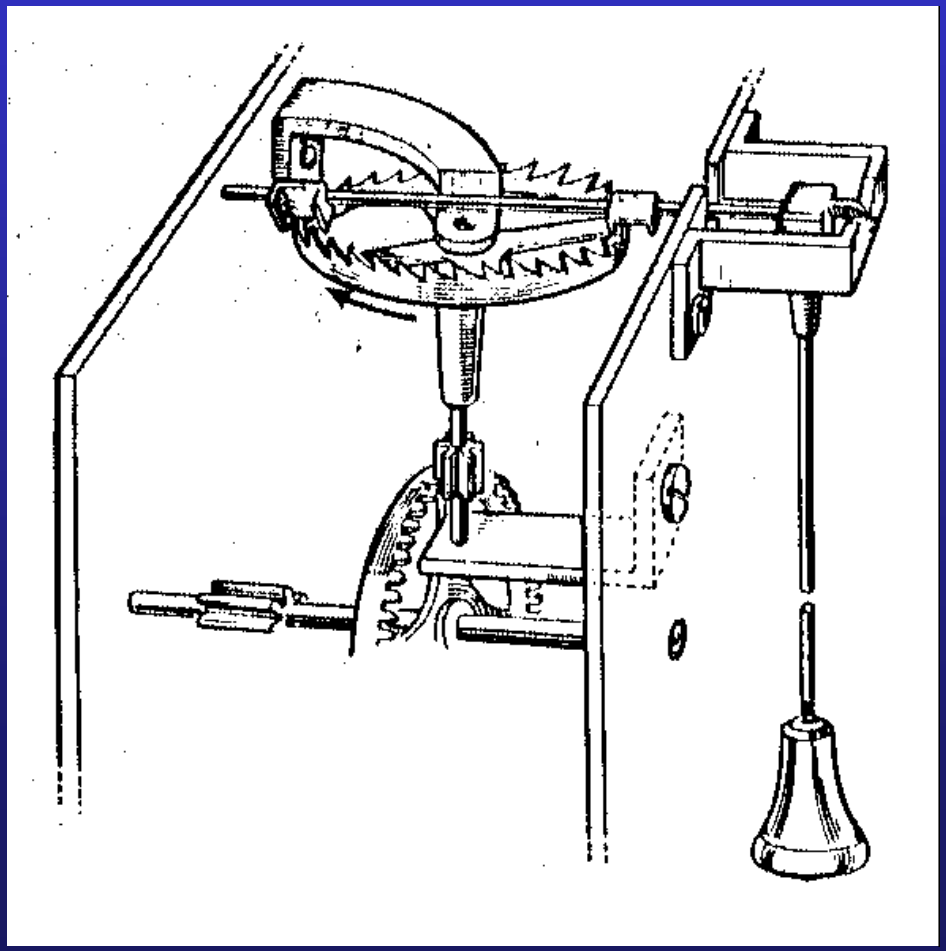
+ Dependent on length pendulum
+ Independent of friction
+ Amplitude dependence can be compensated (cycloidal cheeks)

→ INTRINSIC FREQUENCY

- Dependent on gravity (g-tables)
- Can stand still
- Dependent on driving moment
- Dependent on temperature

Mechanical Clocks; > 1656

Pendulum/Horizontal Verge-Escapement $T = 2\pi\sqrt{(l/g)}$



ISOCHRONISME

Amplitude dependance

$$T = 2\pi\sqrt{(l/g)}\ (1 + (\tfrac{1}{2})^2\sin^2\varphi_{max}/2 + (1\cdot 3/2\cdot 4)^2\sin^4\varphi_{max}/2 + \dots\ )$$

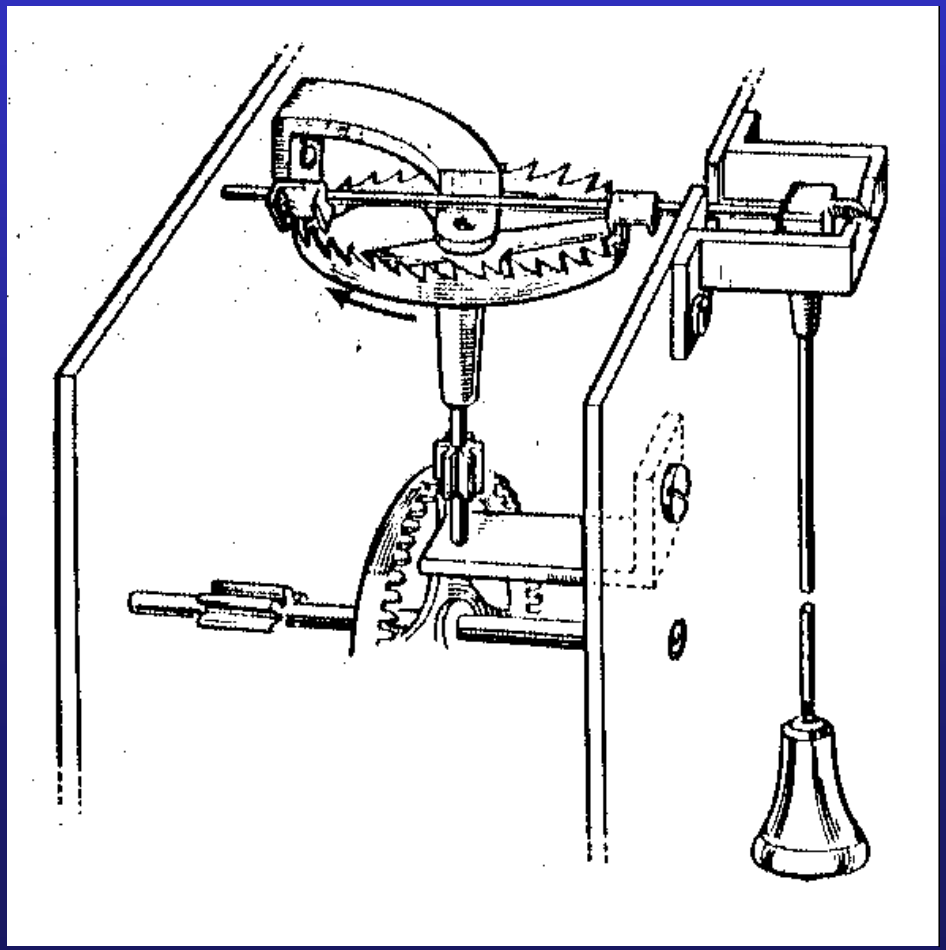
$$T \approx 2\pi\sqrt{l/g}\ \{1 + (1/16)\ \varphi_{max}^{\,2}\}$$

$$dT/T = 1/8\ (\varphi_{max})^2\ d\varphi_{max}/\varphi_{max}$$

$$dT/T = 3.8\ 10^{-5}\ (\varphi_{max}(^0))^2\ d\varphi_{max}/\varphi_{max}$$

Mechanical Clocks; > 1656

Pendulum/Horizontal Verge-Escapement $T = 2\pi\sqrt{(l/g)}$



ISOCHRONISME

Amplitude dependance

$$dT/T = 1/8\ (\varphi_{max})^2\ d\varphi_{max}/\varphi_{max}$$

$$dT/T = 3.8\ 10^{-5}\ (\varphi_{max}(^0))^2\ d\varphi_{max}/\varphi_{max}$$

Voorbeeld: $d\phi_{max}/\phi_{max} = 1\%$

$dT/T = 1/800\ \phi_{max}^2$

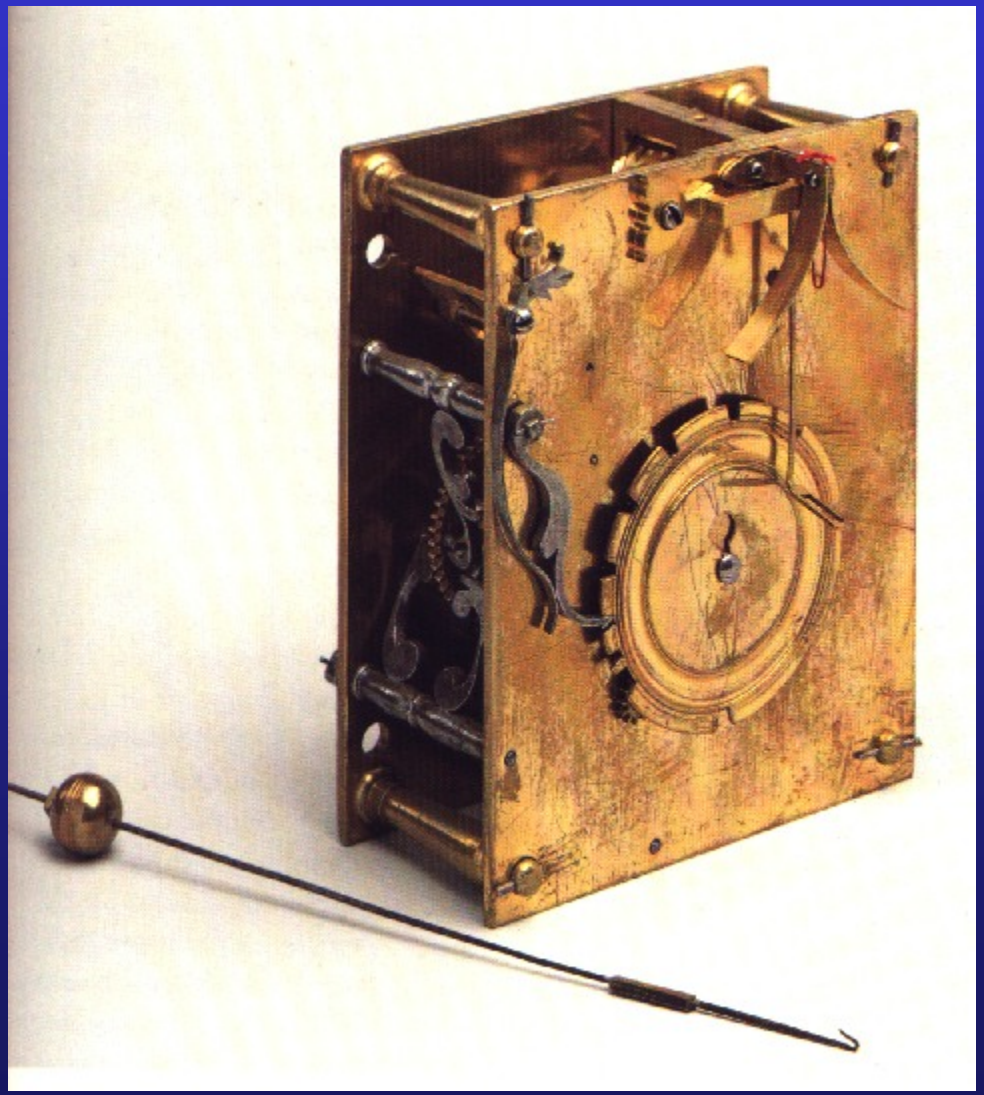
ϕ_{max}= 1 rad (57^o) dT/T= (1/8)% = 1.8 min/dag

ϕ_{max}= 3^o dT/T= 3.4 10^{-6} = 0.3 s/dag

Salomon Coster with ‘privilege’ Christiaan Huygens;

Collection Museum van het Nederlandse Uurwerk, ca 1658

Coster Clocks 1657-1659



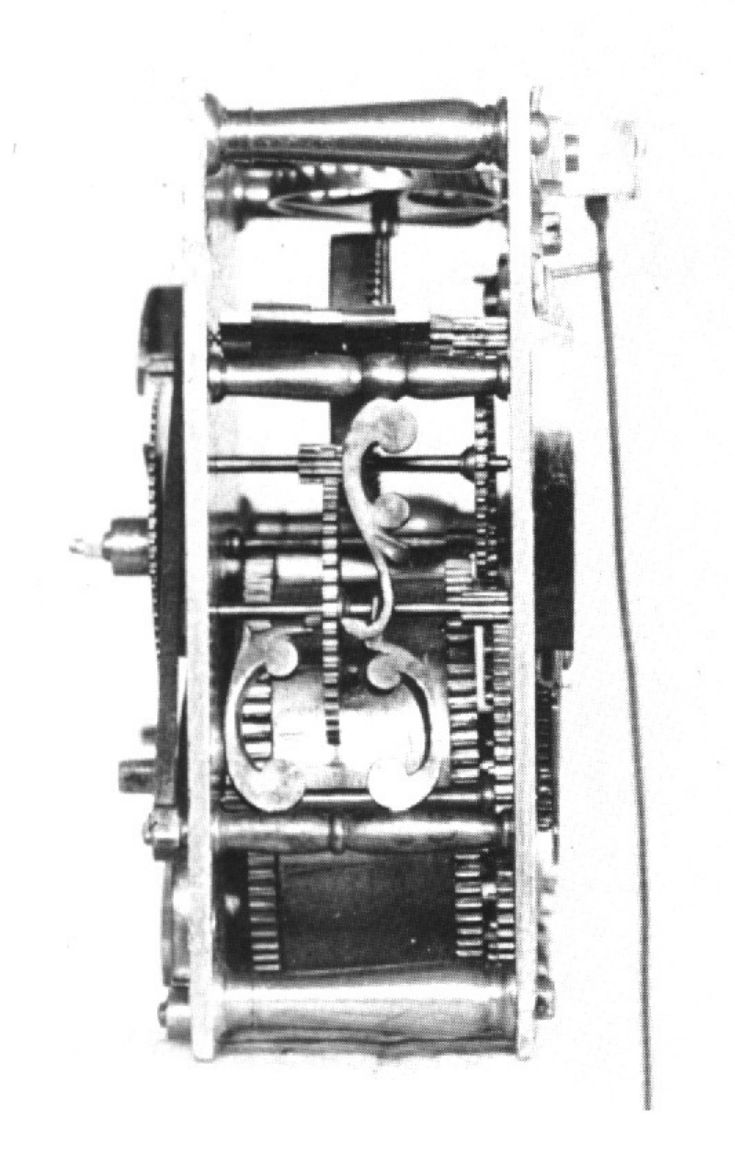
Salomon Coster

ca 1658

Collection MNU

- Double barrel
- Horizontal verge escapement
- Cycloidal cheeks/silk suspended pendulum
- Striking train

Coster Clocks 1657-1659



Salomon Coster

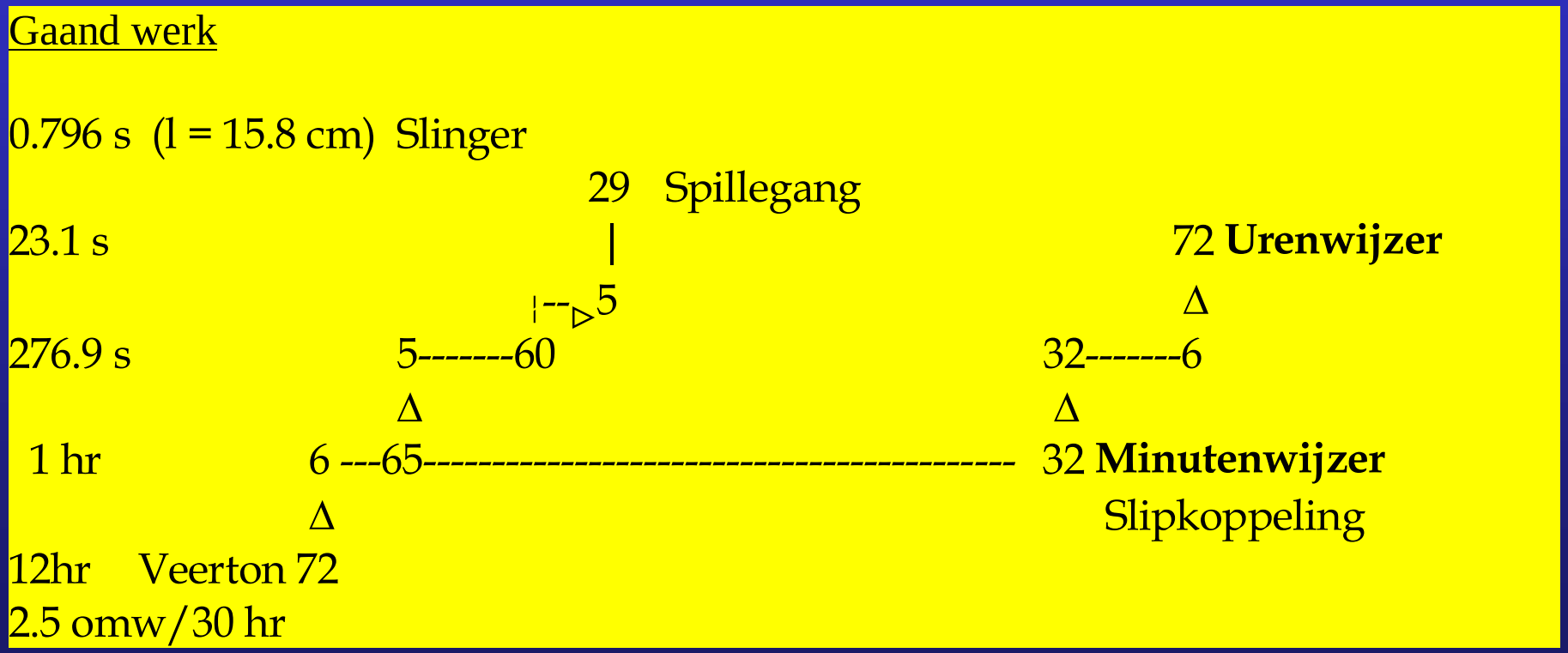
ca 1658

Collection MNU

- Double barrel
- Horizontal verge escapement
- Cycloidal cheeks/silk suspended pendulum
- Striking train

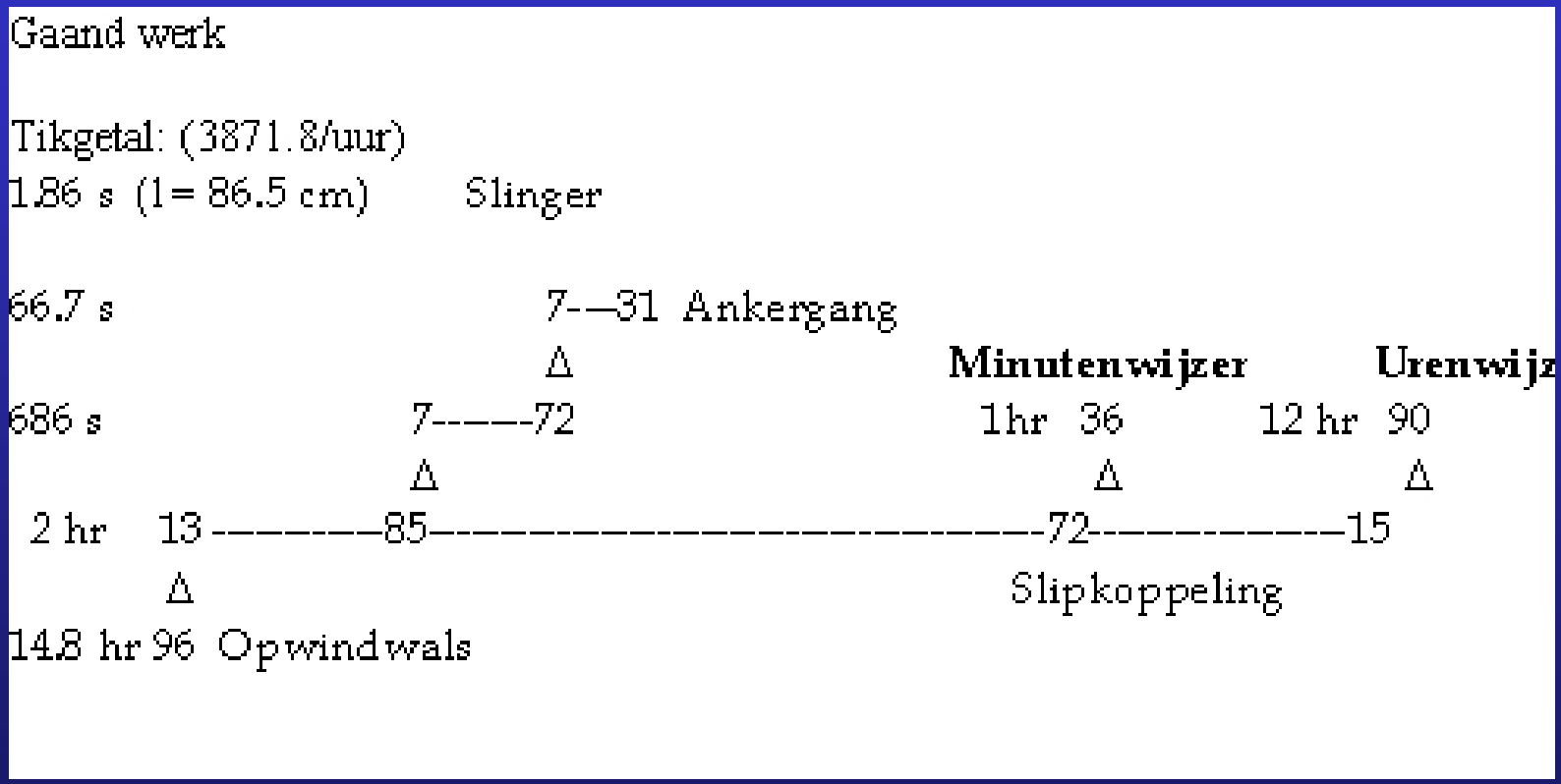
Museum van het Nederlandse Uurwerk

Opbouw “Coster-klok”



Museum van het Nederlandse Uurwerk

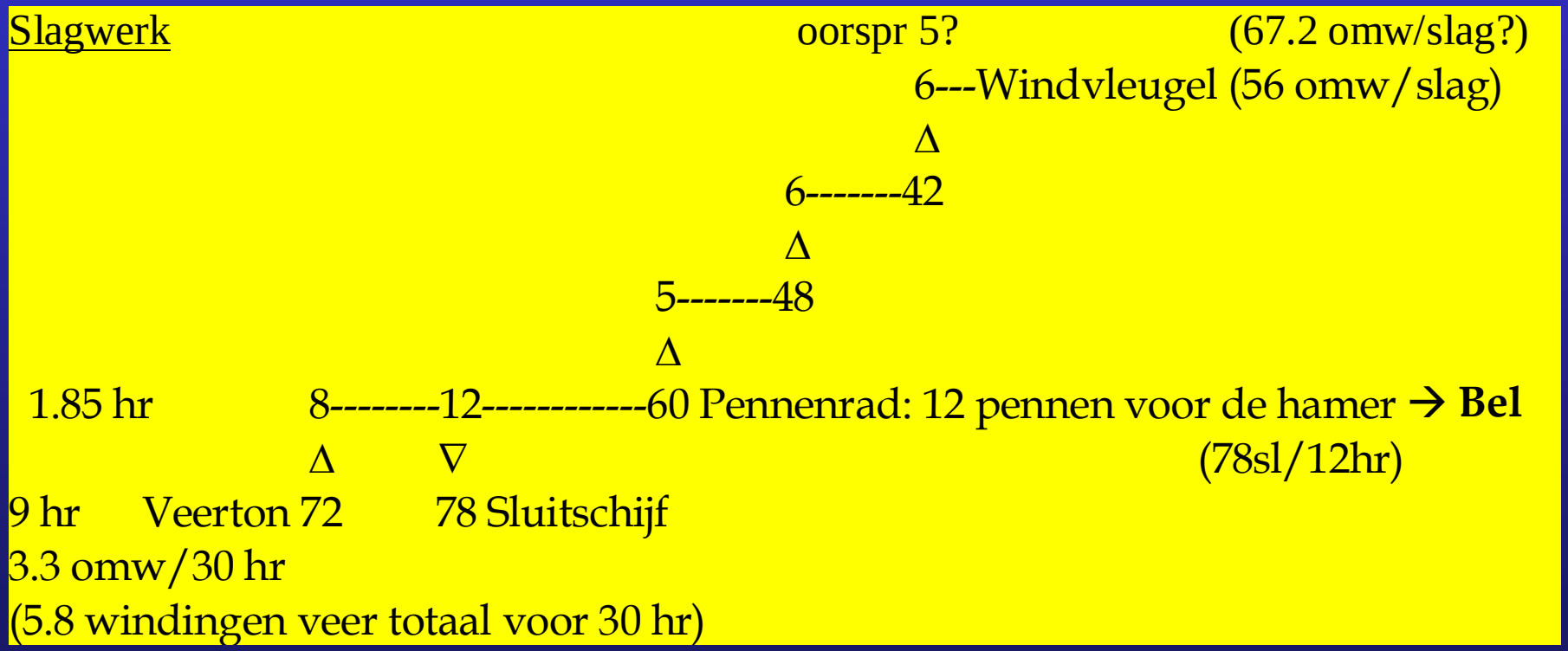
Opbouw Comtoise-klok Elzas ca 1850



Rondsels priemgetallen/Overbrenging geen geheel getal: →
Regelmatige slijtage/montage niet standgevoelig

Museum van het Nederlandse Uurwerk

Opbouw slagwerk “Costerklok”: Sluitschijf (va 1300)



Museum van het Nederlandse Uurwerk

Opbouw Comtoise-klok Elzas ca 1850

Slagwerk

7----Windvleugel (14 omw/slag)

Δ

7-----98 Scheprad

Δ

1 hr 7----98-----14 Sterrad → **Bel** (168/12 hr → gem 1 omw/uur)

Δ

13.7 hr 96 Opwindwals

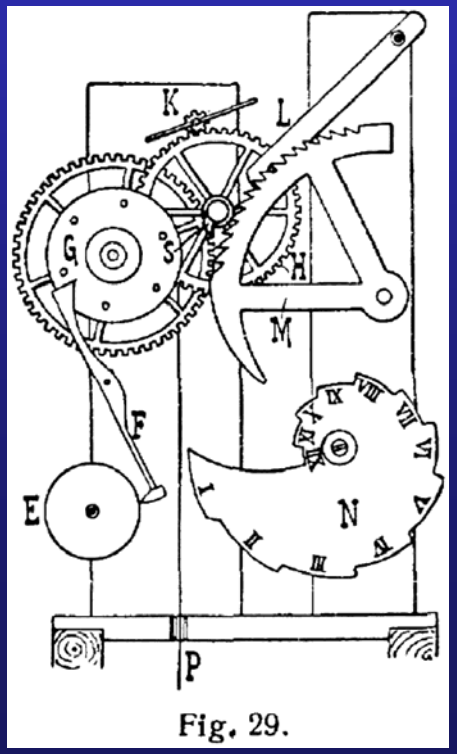


Fig. 29.

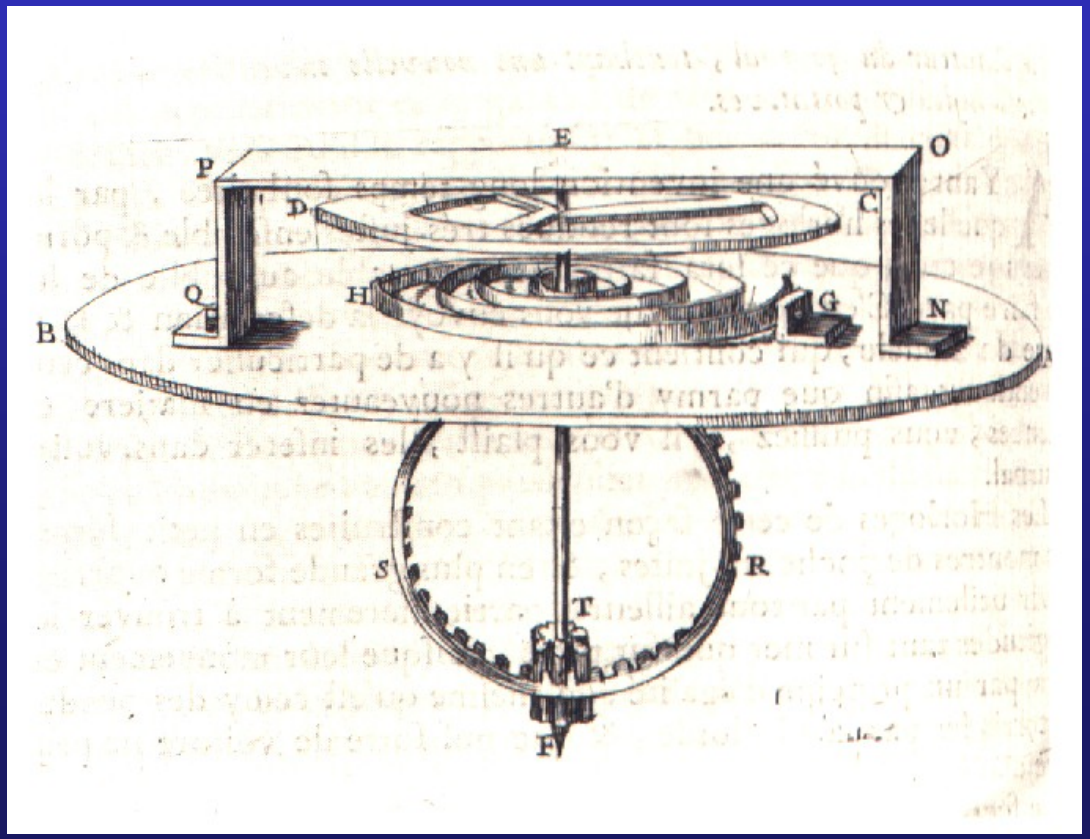
Repetitieslagwerk: Aftasting op uurwijzer (Slakkenhuis): →

Klok kan niet van slag raken

Barlow 1676

Balance Wheel with Spring

Journal des Sçavans, January 20, 1675



Balanswiel met spiraalveer
Balance wheel with spring

$$Id^2\varphi/dt^2 = -b\varphi$$
$$d^2\varphi/dt^2 = -(b/I)\varphi$$

Oplossing / *Solution* $\varphi = \varphi_{max}\sin(2\pi t/T)$

Periode / *Period* $T = 2\pi\sqrt{(I/b)}$
$= 2\pi\sqrt{(mr^2/2b)}$

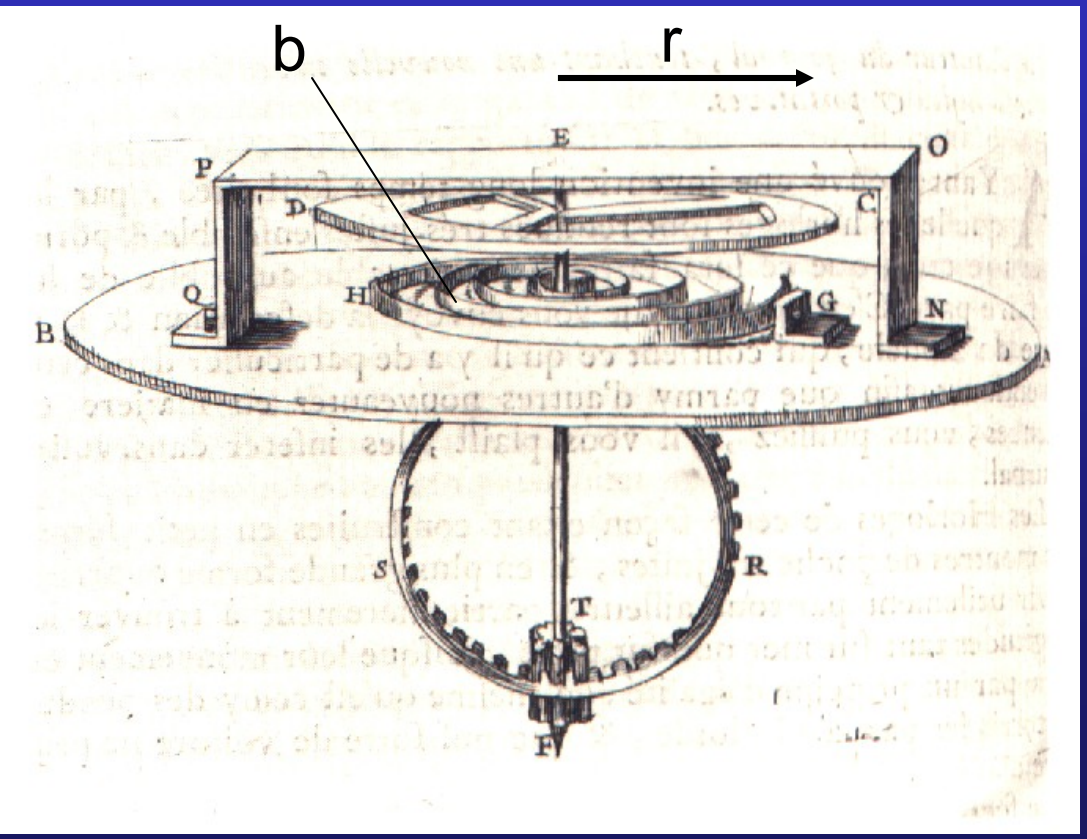
b = veerconstante / *spring constant* [Nm/rad]

I = traagheidsmoment / *moment of inertia* [kgm²]

Balance Wheel with Spring

Journal des Sçavans, January 20, 1675

$T = 2\pi\sqrt{mr^2/2b}$

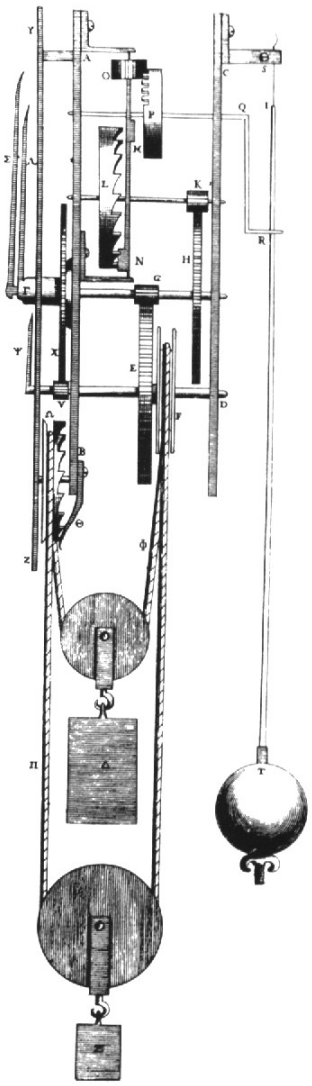


+ Dependent on diameter wheel, mass and spring constant
+ Independent of friction
+ Independent of gravity (balanced)

➡ INTRINSIC FREQUENCY

- Can stand still (less easily)
- Dependent on driving moment
- Dependent on temperature

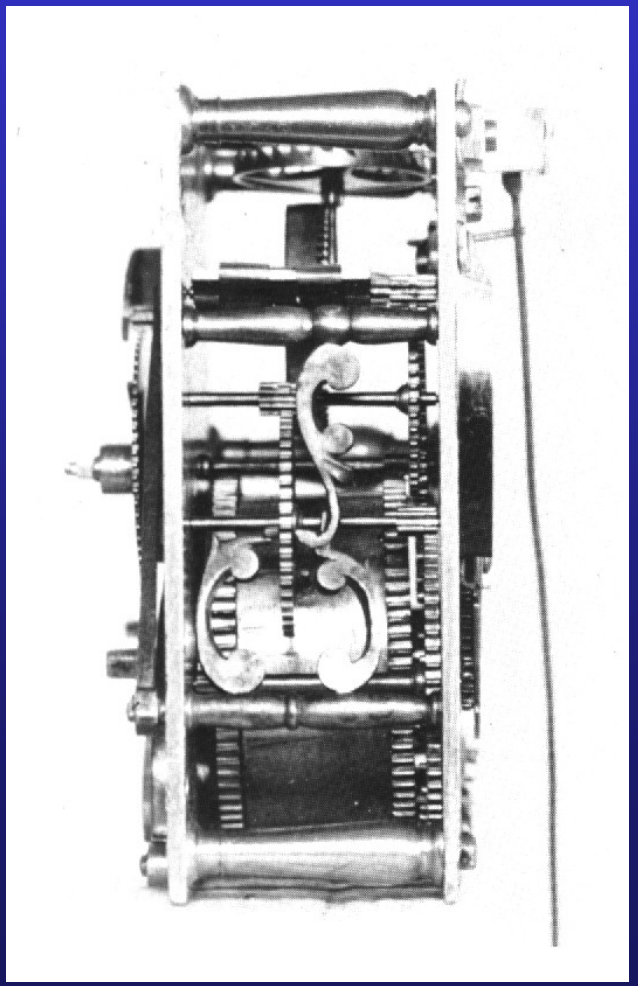
Limitations; driving moment, maintaining power



Horologium 1658

Double barrel

Endless cord



Salomon Coster ca 1658
Collection MNU

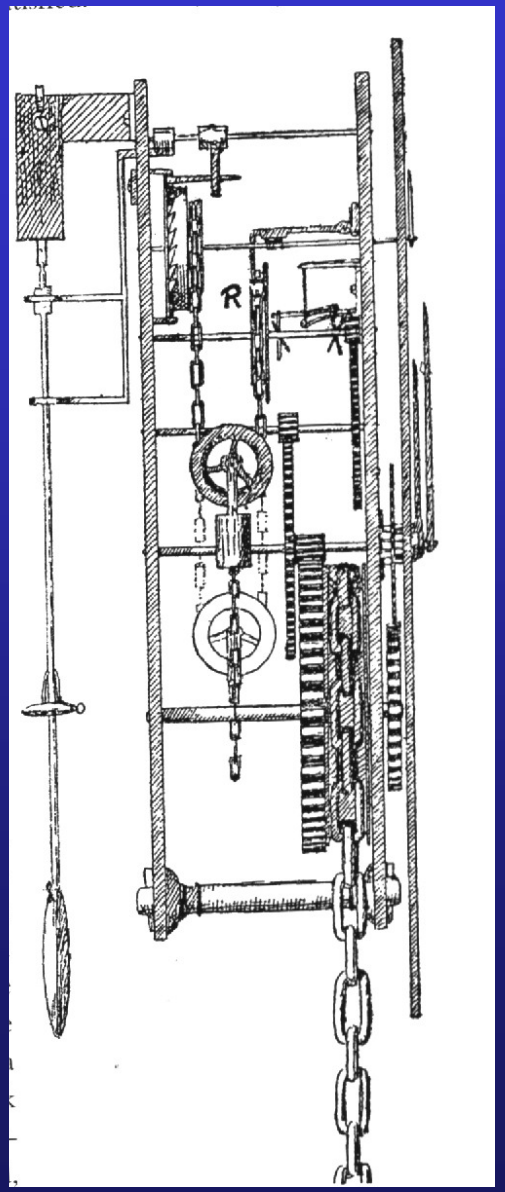
Limitations; driving moment,

Remontoir

Weight Remontoir,

period of half a minute

Design drawing ca 1664



Marine Timekeepers

Determining longitude at sea

Most important problem to be solved by time measurements!

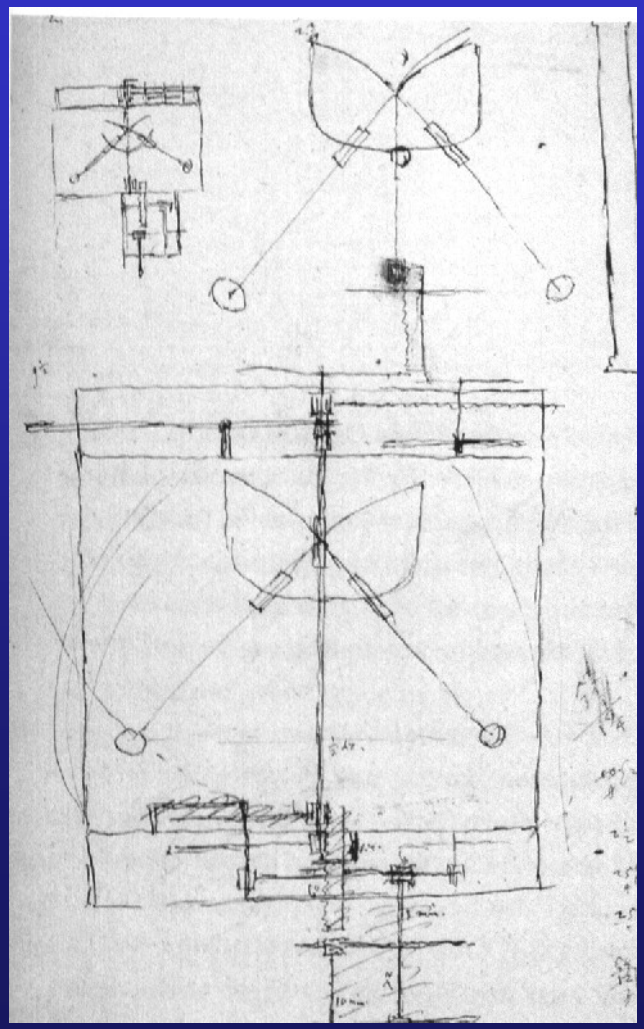
Golden Age of the Netherlands!

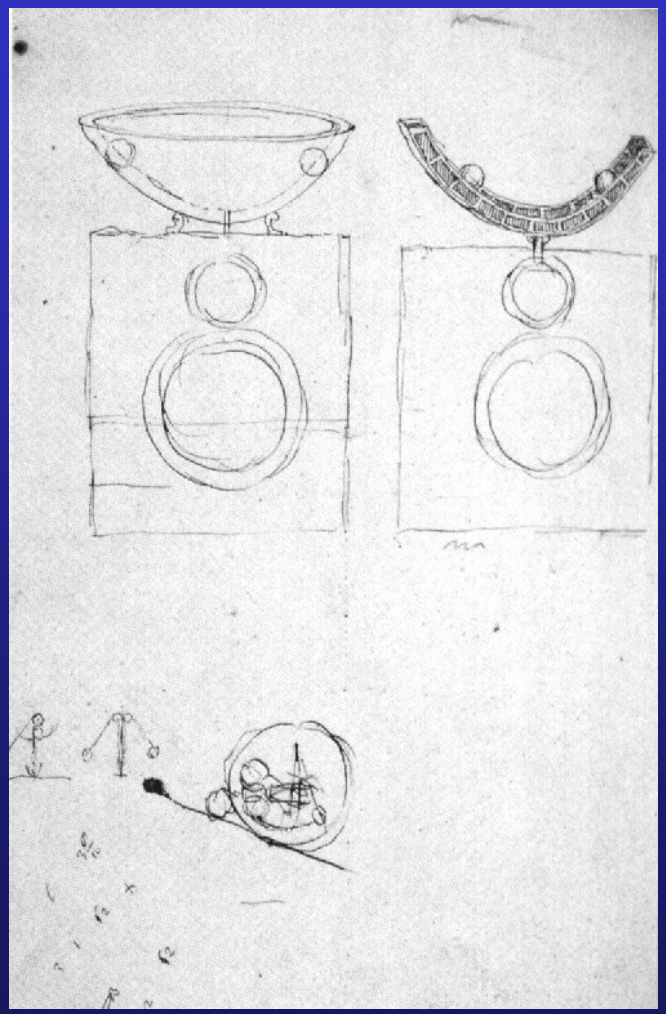
Verenigde Oost-Indische Compagnie (VOC)!

Huygens from 1658 on

- Philips III, Spain 1598, 6000 ducats
- Proposal Galilei Dutch States General 1636
- England 1714 £ 20.000,- ← !!
- France 1715 10.000 Livres

Marine Timekeepers (I)



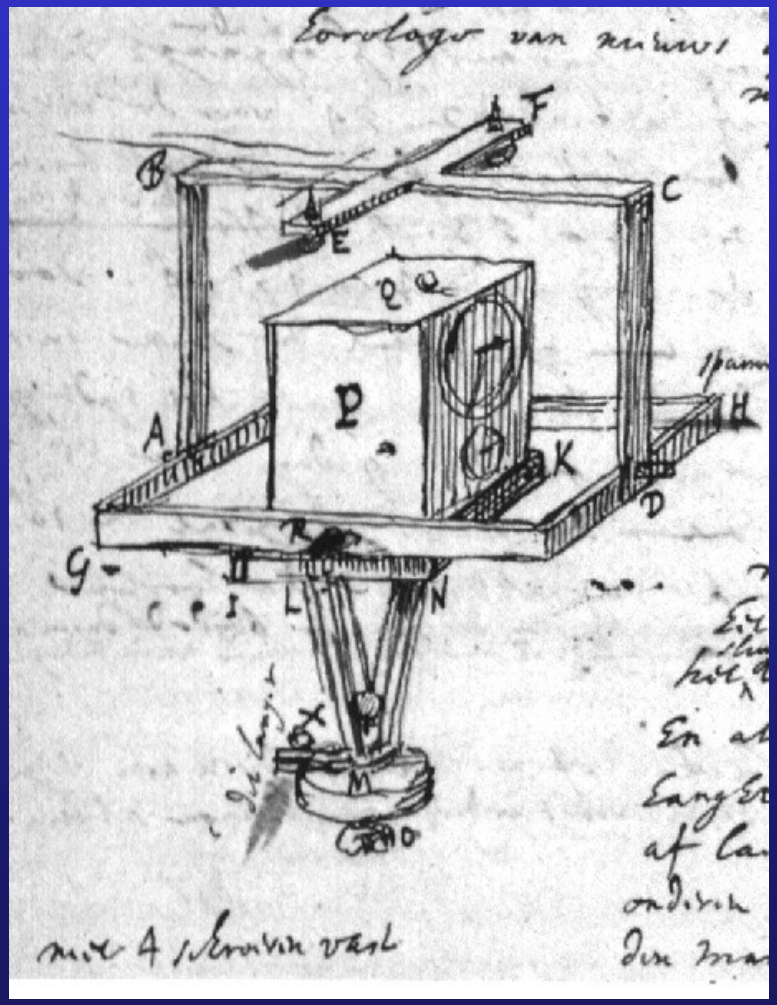


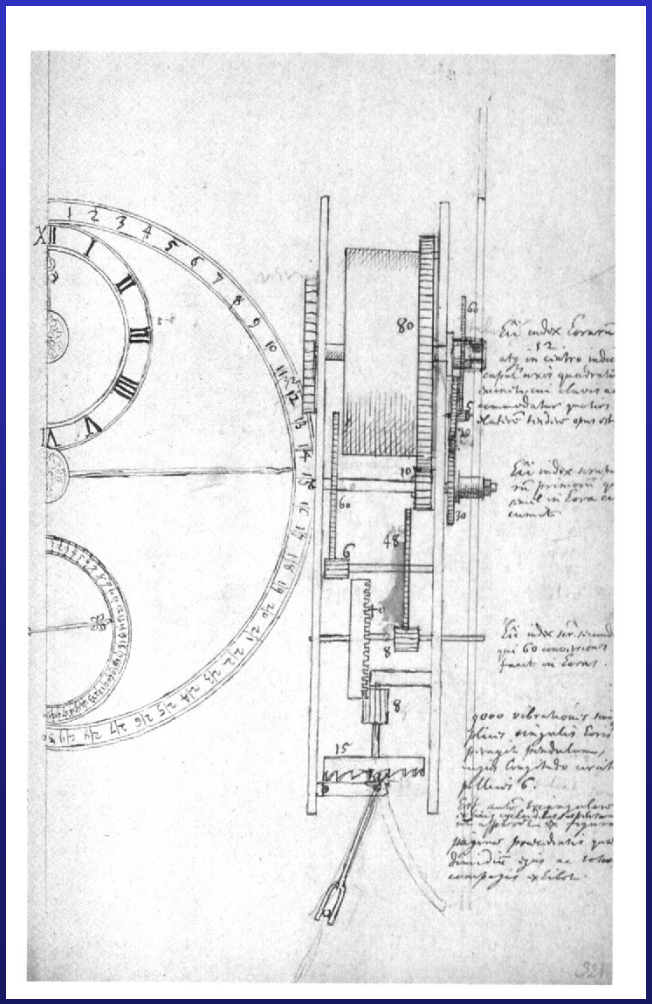
Design drawings

- Clock with two parabolic-conical pendulums 1666
- Clock with a parabolic track 1666 (right)

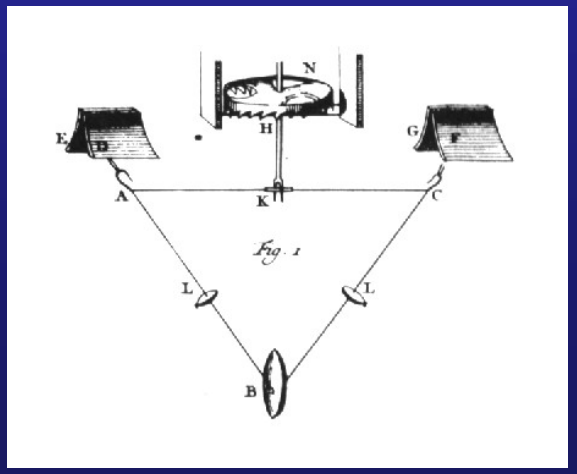
Marine Timekeepers (II)

Triangular pendulum



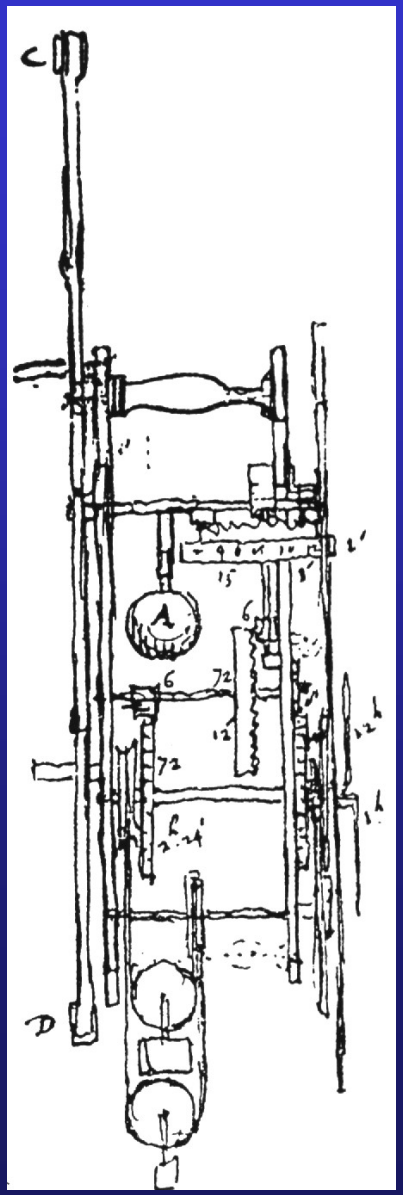


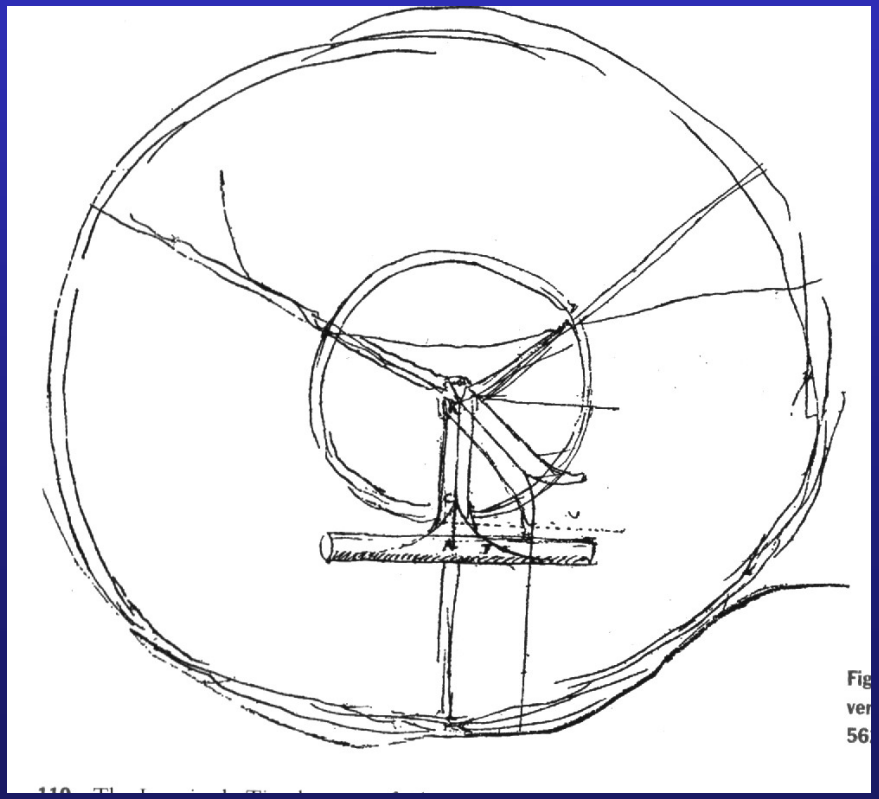
- Sea clock in gimbals with triangular pendulum 1671
- Design drawing for a clock with a triangular pendulum 1672 (right)



Marine Timekeepers (III)

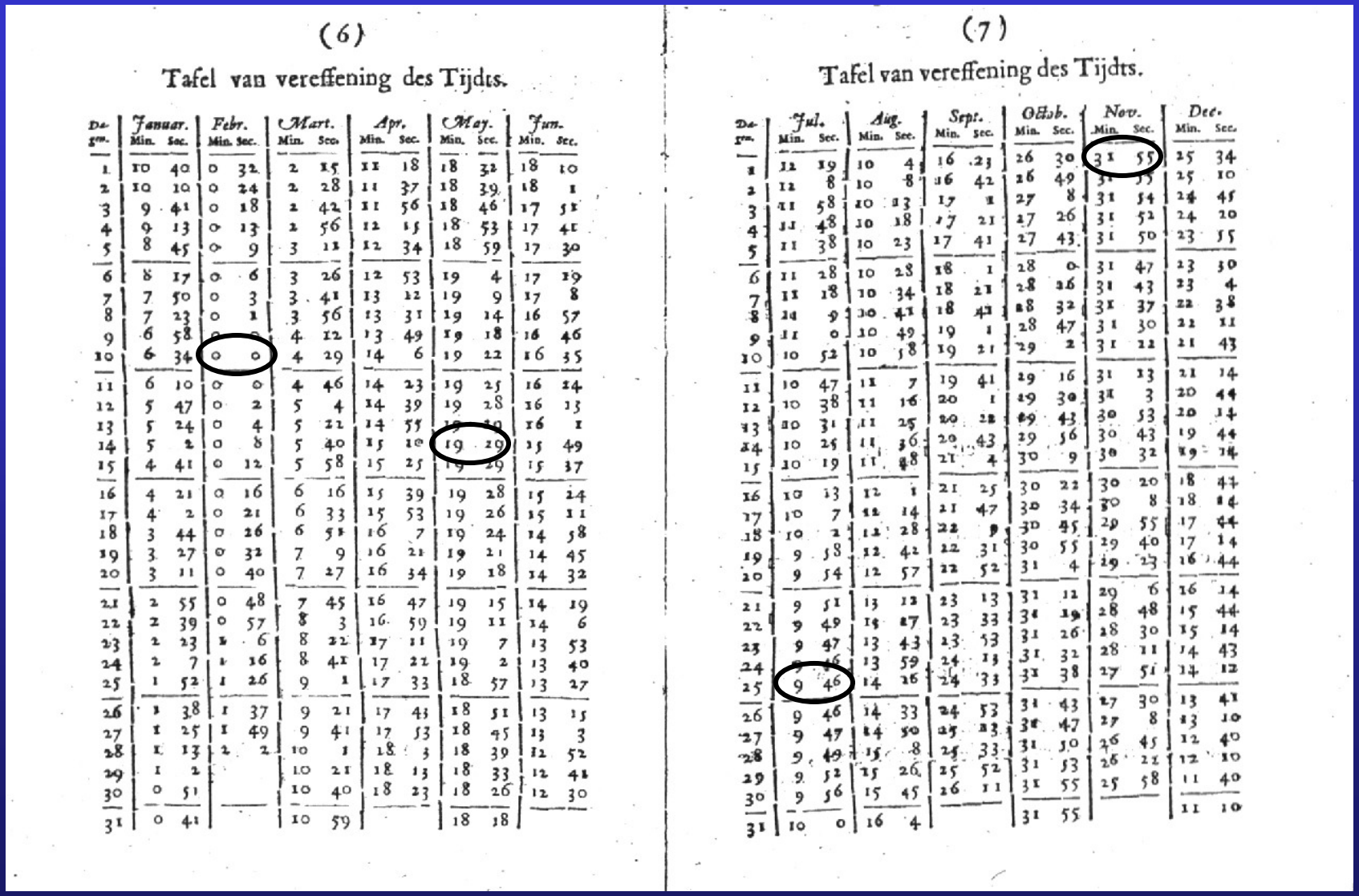
‘Perfect’ Marine Balance





- The Perfect Marine Balance, version of March 1693
- Sea clock with Perfect Marine Balance, March 1693 (left)

Table Equation of time, Kort Onderwys, 1665



(6)

Tafel van vereffening des Tijdts.

Dag.	Januar. Min.	Sec.	Febr. Min.	Sec.	Mart. Min.	Sec.	Apr. Min.	Sec.	May. Min.	Sec.	Jun. Min.	Sec.
1	10	40	0	32	2	15	11	18	18	32	18	10
2	10	10	0	24	2	28	11	37	18	39	18	1
3	9	41	0	18	2	42	11	56	18	46	17	51
4	9	13	0	13	2	56	12	15	18	53	17	41
5	8	45	0	9	3	11	12	34	18	59	17	30
6	8	17	0	6	3	26	12	53	19	4	17	19
7	7	50	0	3	3	41	13	12	19	9	17	8
8	7	23	0	1	3	56	13	31	19	14	16	57
9	6	58	0	0	4	12	13	49	19	18	16	46
10	6	34	0	0	4	29	14	6	19	22	16	35
11	6	10	0	0	4	46	14	23	19	25	16	24
12	5	47	0	2	5	4	14	39	19	28	16	13
13	5	24	0	4	5	22	14	55	19	29	16	1
14	5	2	0	8	5	40	15	10	19	29	15	49
15	4	41	0	12	5	58	15	25	19	29	15	37
16	4	21	0	16	6	16	15	39	19	28	15	24
17	4	2	0	21	6	33	15	53	19	26	15	11
18	3	44	0	26	6	51	16	7	19	24	14	58
19	3	27	0	32	7	9	16	21	19	21	14	45
20	3	11	0	40	7	27	16	34	19	18	14	32
21	2	55	0	48	7	45	16	47	19	15	14	19
22	2	39	0	57	8	3	16	59	19	11	14	6
23	2	23	1	6	8	22	17	11	19	7	13	53
24	2	7	1	16	8	41	17	22	19	2	13	40
25	1	52	1	26	9	1	17	33	18	57	13	27
26	1	38	1	37	9	21	17	43	18	51	13	15
27	1	25	1	49	9	41	17	53	18	45	13	3
28	1	13	2	2	10	1	18	3	18	39	12	52
29	1	2			10	21	18	13	18	33	12	41
30	0	51			10	40	18	23	18	26	12	30
31	0	41			10	59			18	18		

(7)

Tafel van vereffening des Tijdts.

Dag.	Jul. Min.	Sec.	Aug. Min.	Sec.	Sept. Min.	Sec.	Octob. Min.	Sec.	Nov. Min.	Sec.	Dec. Min.	Sec.
1	12	19	10	4	16	23	26	30	31	55	25	34
2	12	8	10	8	16	42	26	49	31	55	25	10
3	11	58	10	13	17	1	27	8	31	54	24	45
4	11	48	10	18	17	21	27	26	31	52	24	20
5	11	38	10	23	17	41	27	43	31	50	23	55
6	11	28	10	28	18	1	28	0	31	47	23	30
7	11	18	10	34	18	21	28	26	31	43	23	4
8	11	9	10	41	18	43	28	32	31	37	22	38
9	11	0	10	49	19	1	28	47	31	30	22	11
10	10	52	10	58	19	21	29	2	31	22	21	43
11	10	47	11	7	19	41	29	16	31	13	21	14
12	10	38	11	16	20	1	29	30	31	3	20	44
13	10	31	11	25	20	22	29	43	30	53	20	14
14	10	25	11	36	20	43	29	56	30	43	19	44
15	10	19	11	48	21	4	30	9	30	32	19	14
16	10	13	12	1	21	25	30	22	30	20	18	47
17	10	7	12	14	21	47	30	34	30	8	18	24
18	10	2	12	28	22	9	30	45	29	55	17	44
19	9	58	12	42	22	31	30	55	29	40	17	14
20	9	54	12	57	22	52	31	4	29	23	16	44
21	9	51	13	12	23	13	31	12	29	6	16	14
22	9	49	13	27	23	33	31	19	28	48	15	44
23	9	47	13	43	23	53	31	26	28	30	15	14
24	9	46	13	59	24	13	31	32	28	11	14	43
25	9	46	14	16	24	33	31	38	27	51	14	12
26	9	46	14	33	24	53	31	43	27	30	13	41
27	9	47	14	50	25	13	31	47	27	8	13	10
28	9	49	15	8	25	33	31	50	26	45	12	40
29	9	52	15	26	25	52	31	53	26	22	12	10
30	9	56	15	45	26	11	31	55	25	58	11	40
31	10	0	16	4			31	55			11	10

Short education for the use of clocks for the determination of longitude from East to West

1 nov
31m55s

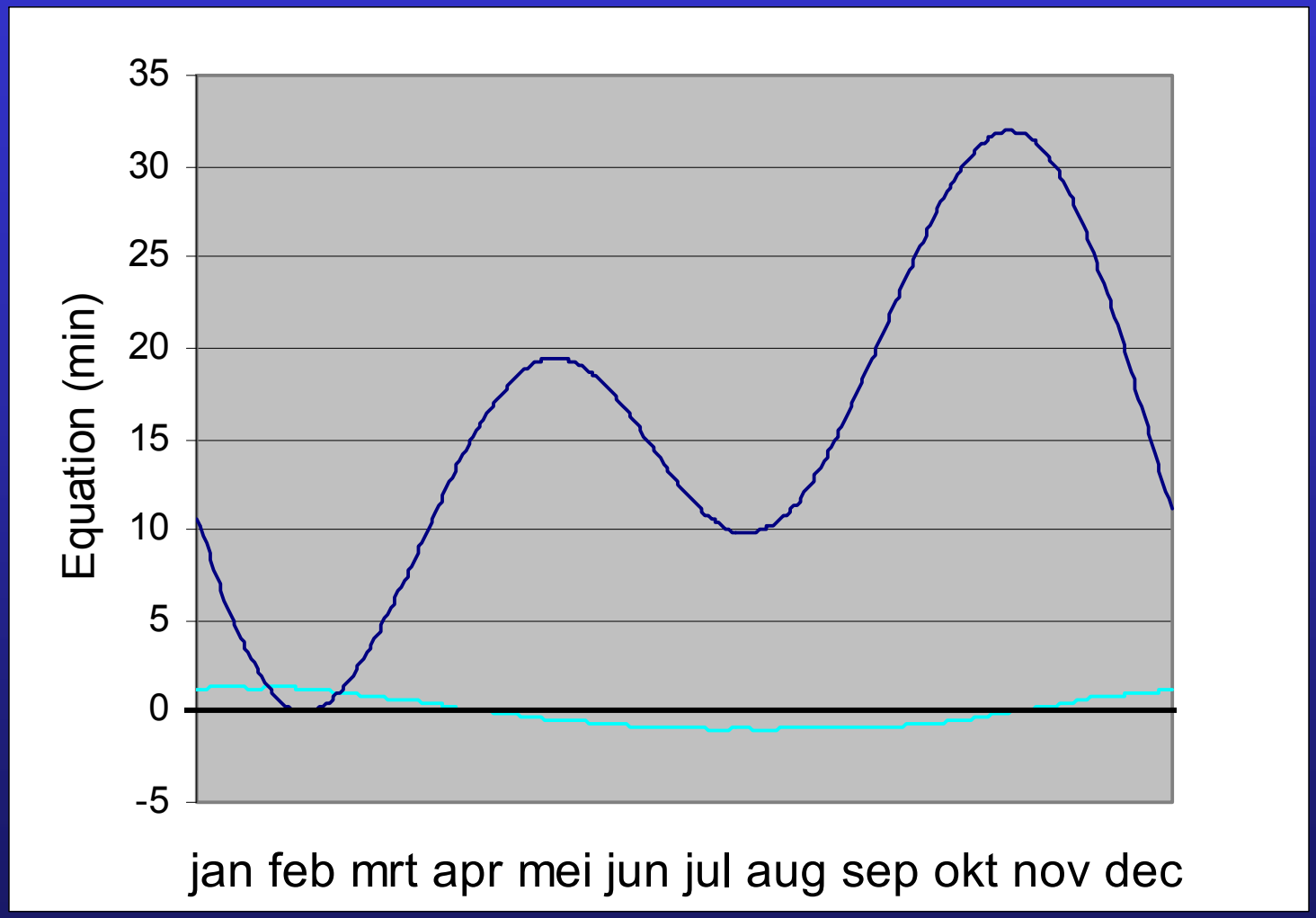
10 feb
0m0s

14 mei
19m29s

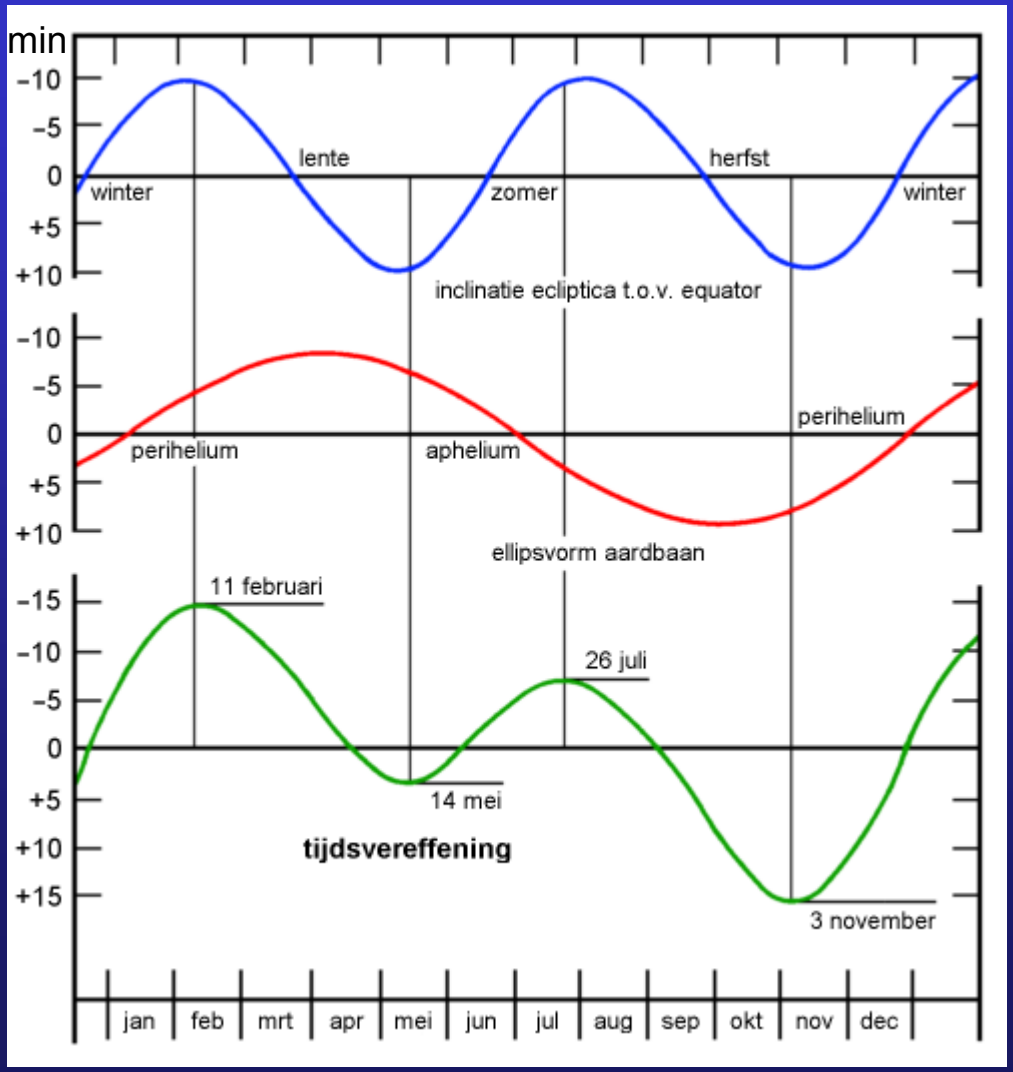
25 jul
9m46s

OC XVIII, p 110: “In computanda tabula hac duplicem causam adhibui, utramque astronomis notam, Eclipticae nimirum obliquitatem & Solaris motus anomaliam”

Huygens: Equation Table Hor. Oscill. 1673



Equation of time (min)

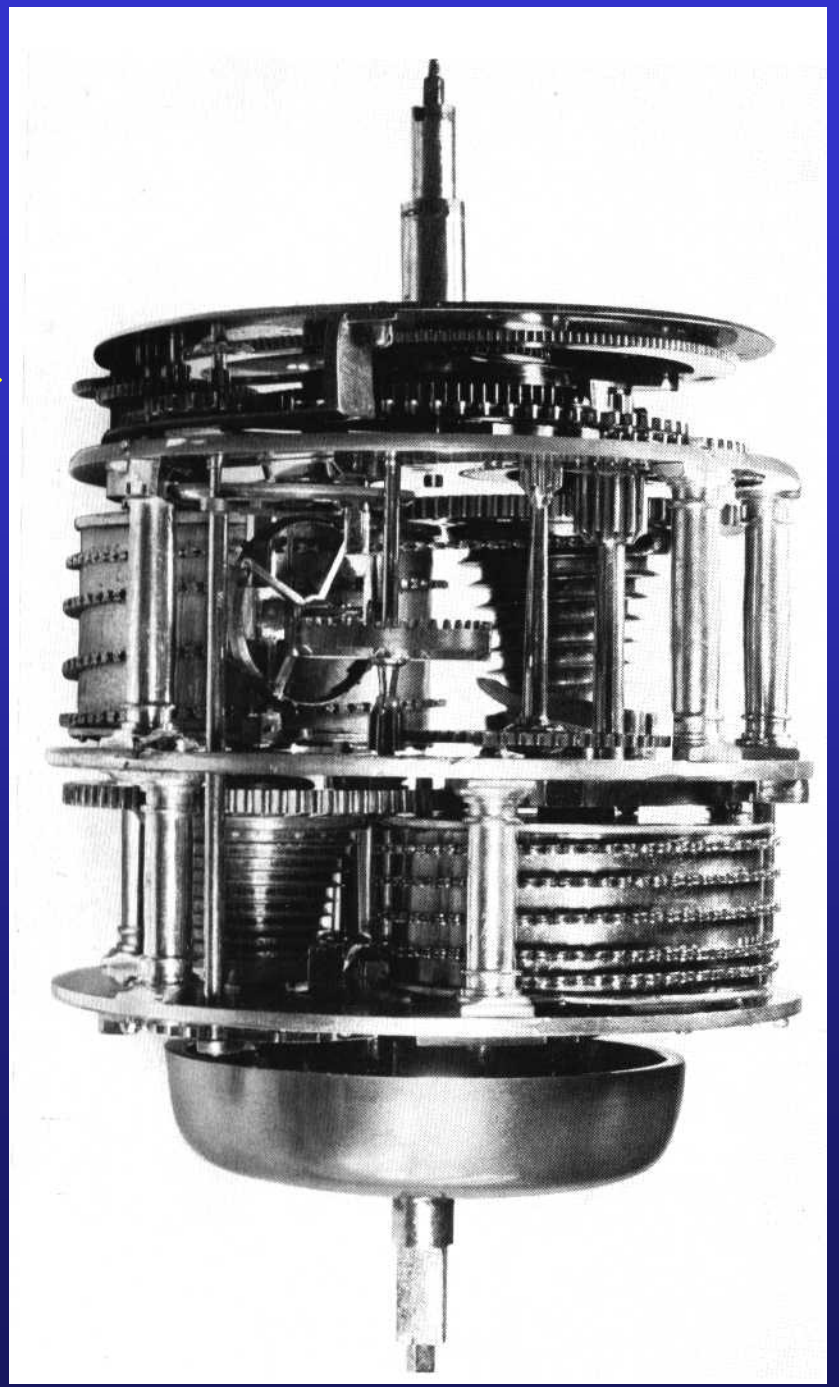


4 min ≡ 1^0

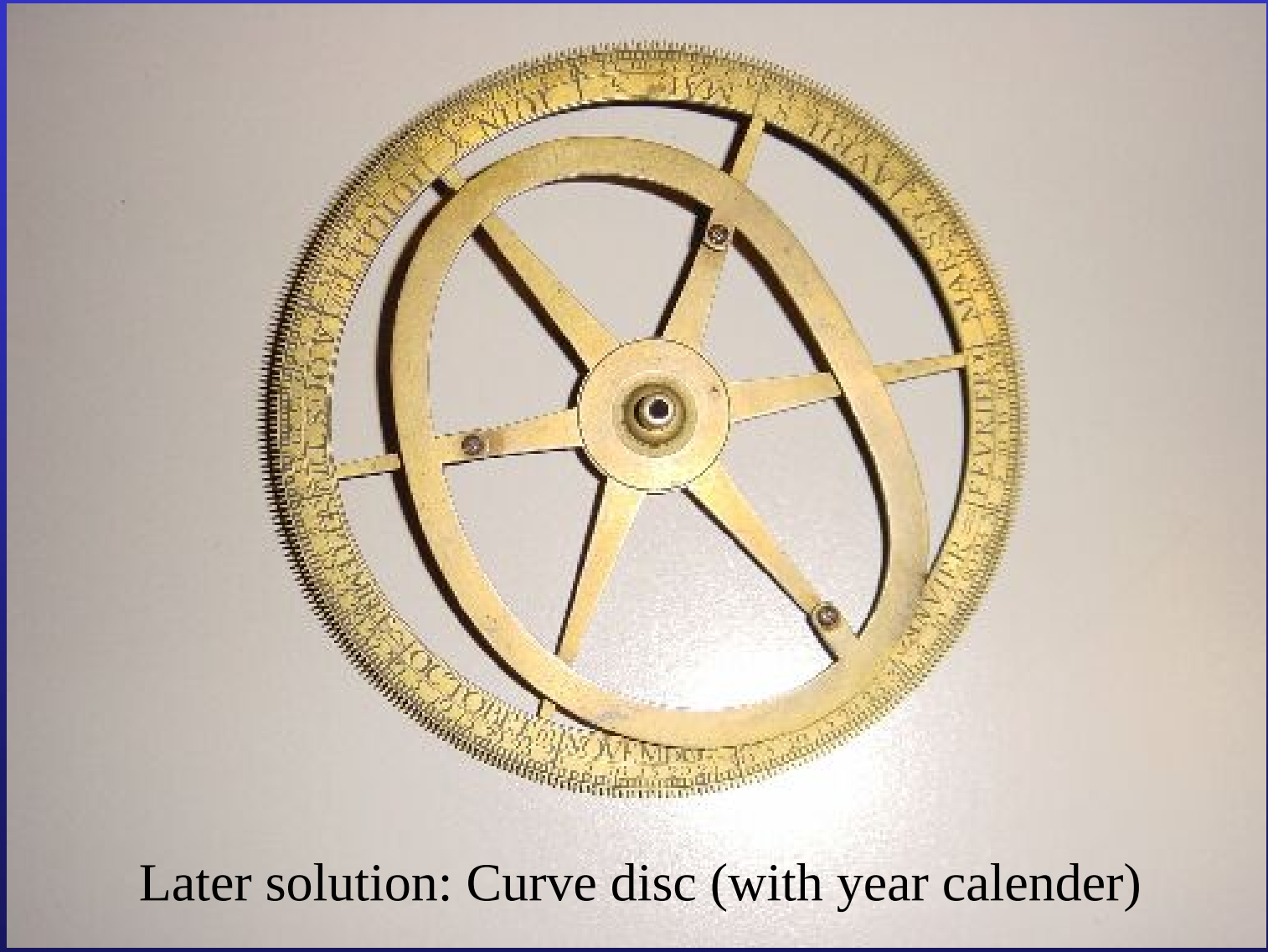
5 sun diameters ($2.5\ ^0$)

Equation of Time

Jobst Bürgi, Kassel 1594

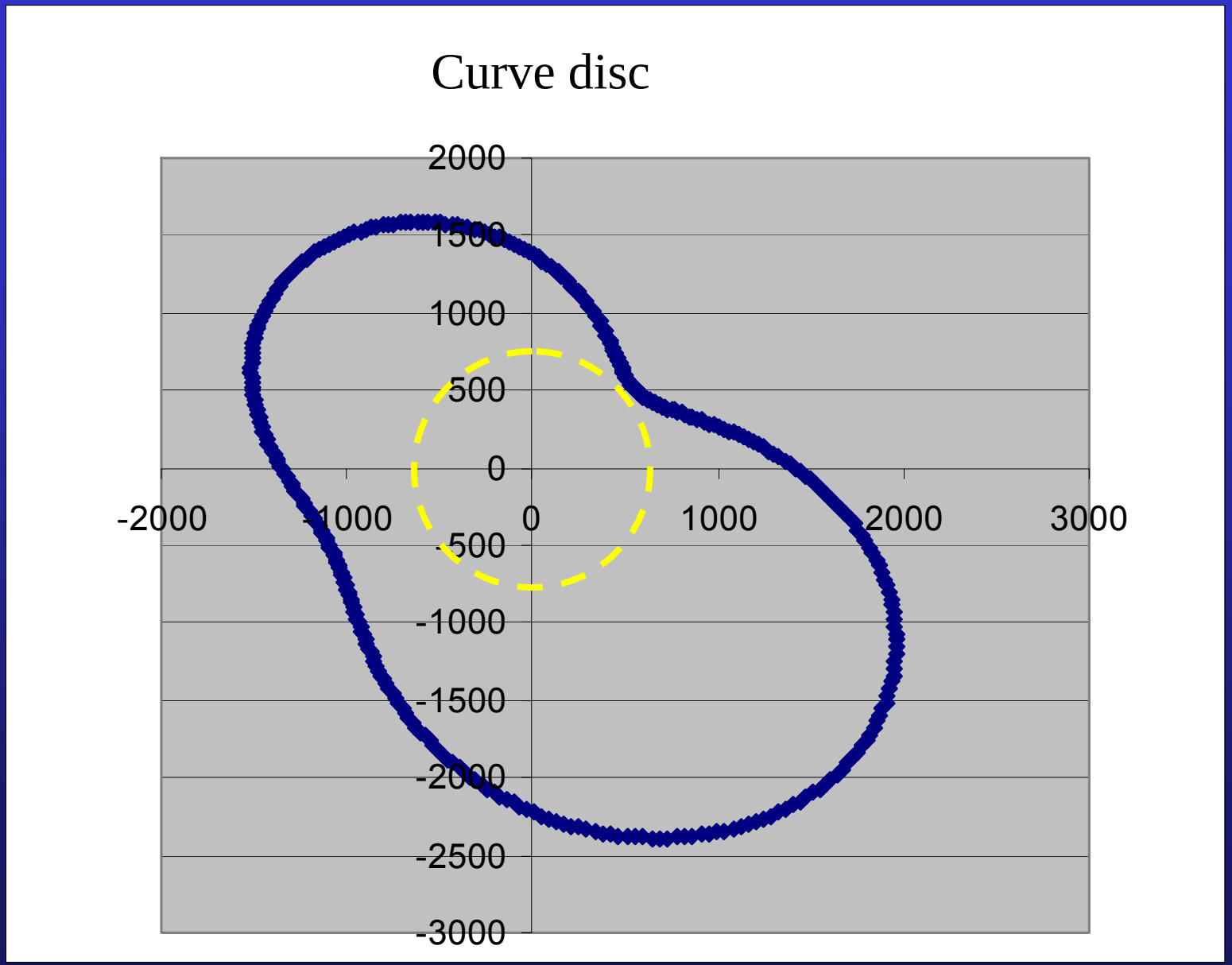


Equation table

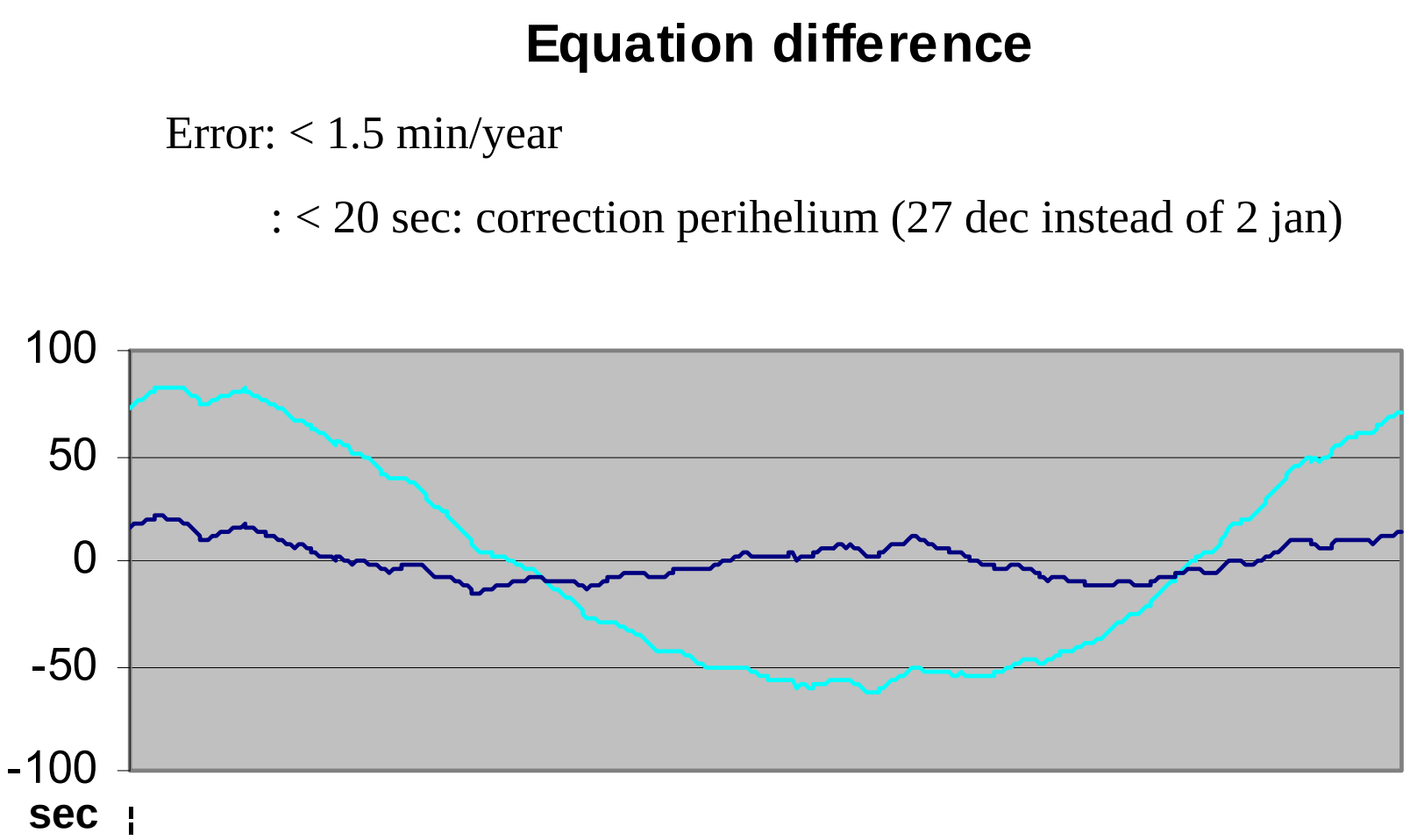


Later solution: Curve disc (with year calender)

Equation table; Computation Curve Disc Huygens



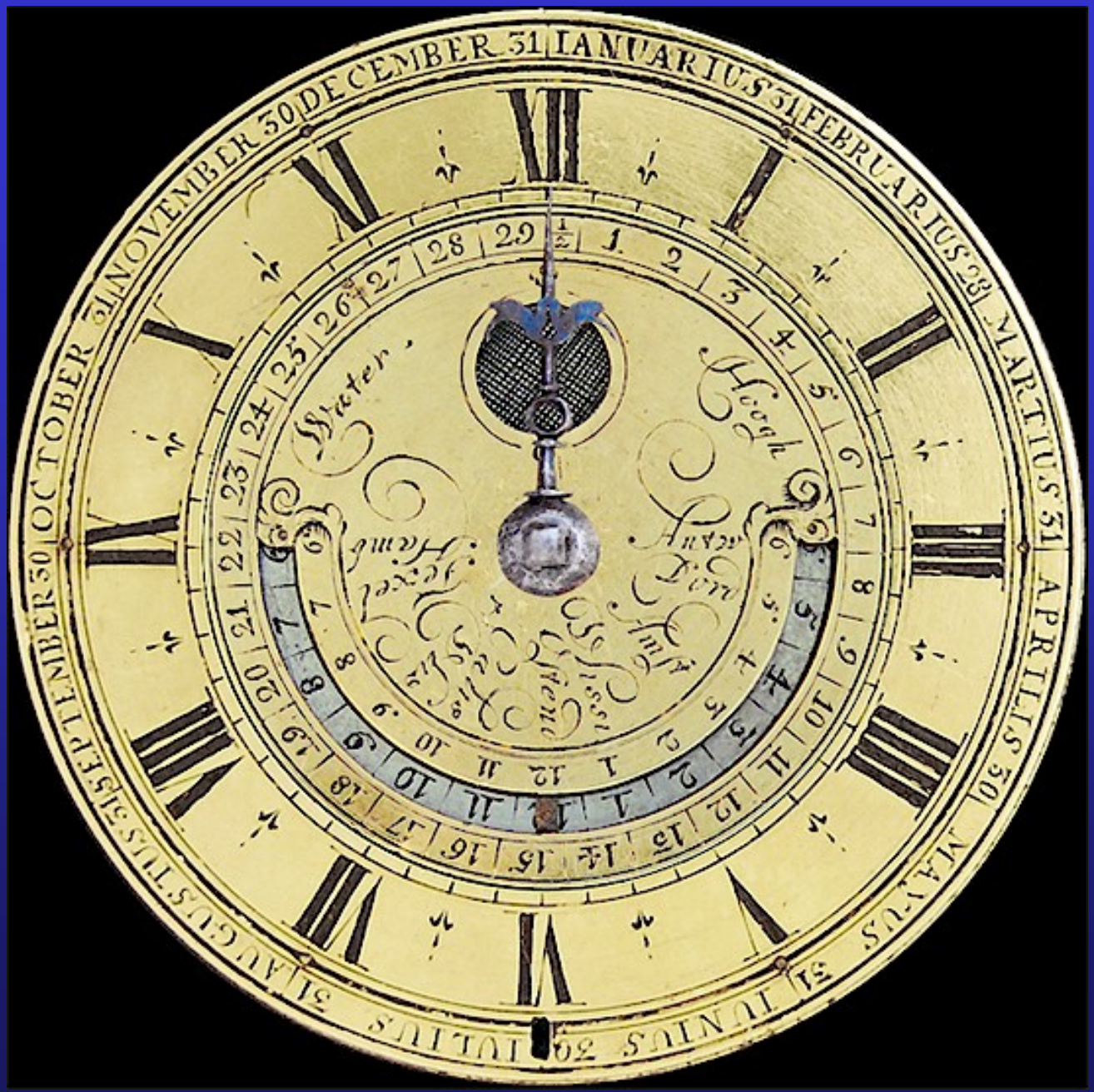
Equation Table; Huygens: Hor. Oscill. 1673



Tides

“Captain’s Watch”

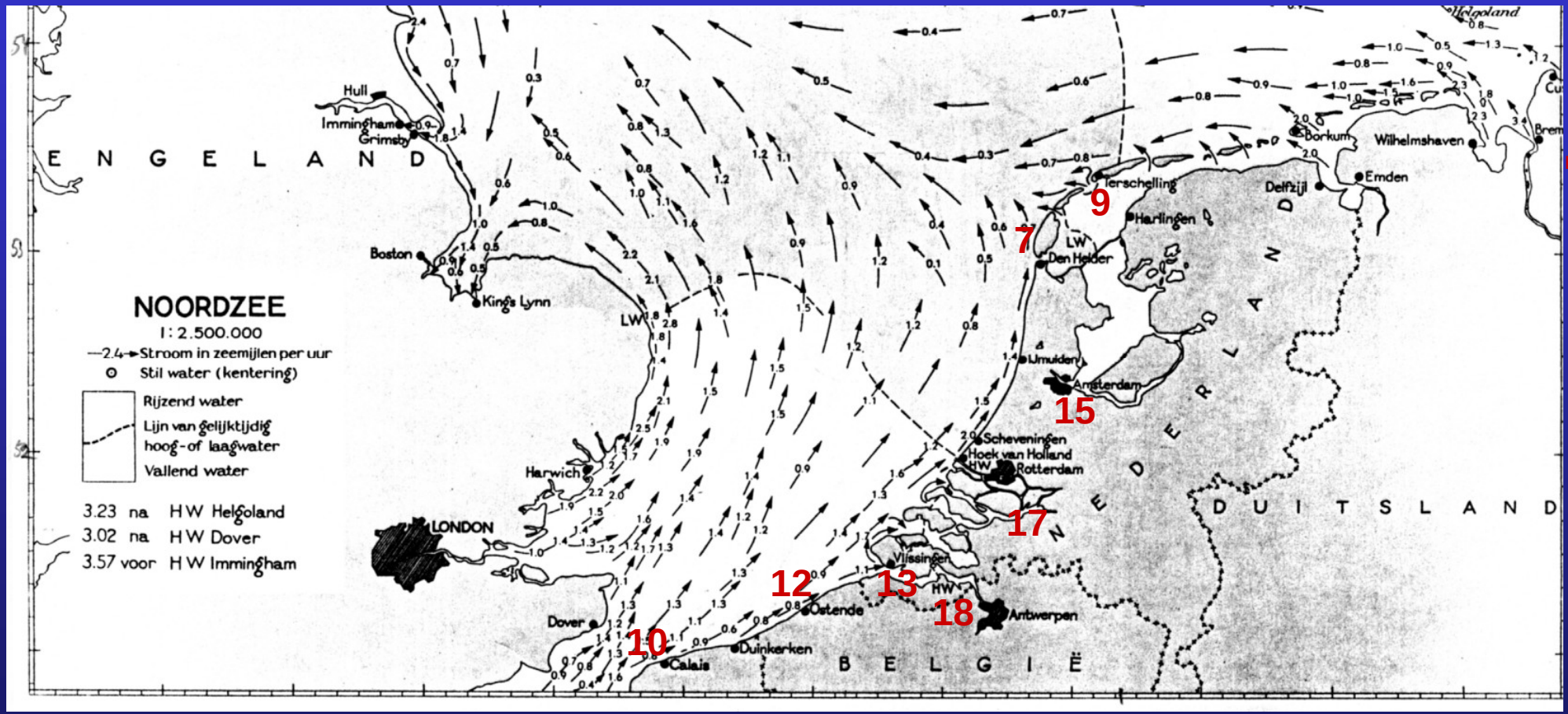
G.J. Nauta,
Leuwaerden“
c. 1720.




SEPTEMBER 30 OCTOBER 31 NOVEMBER 30 DECEMBER 31 IANUARIUS 31 FEBRUARIUS 28 MARTIUS 31 APRILIS 30 MAIUS 31 IUNIUS 30 IULIUS 31 AUGUSTUS 31
Water.


Gravenhage

Tides “Captain’s Watch”



Tides

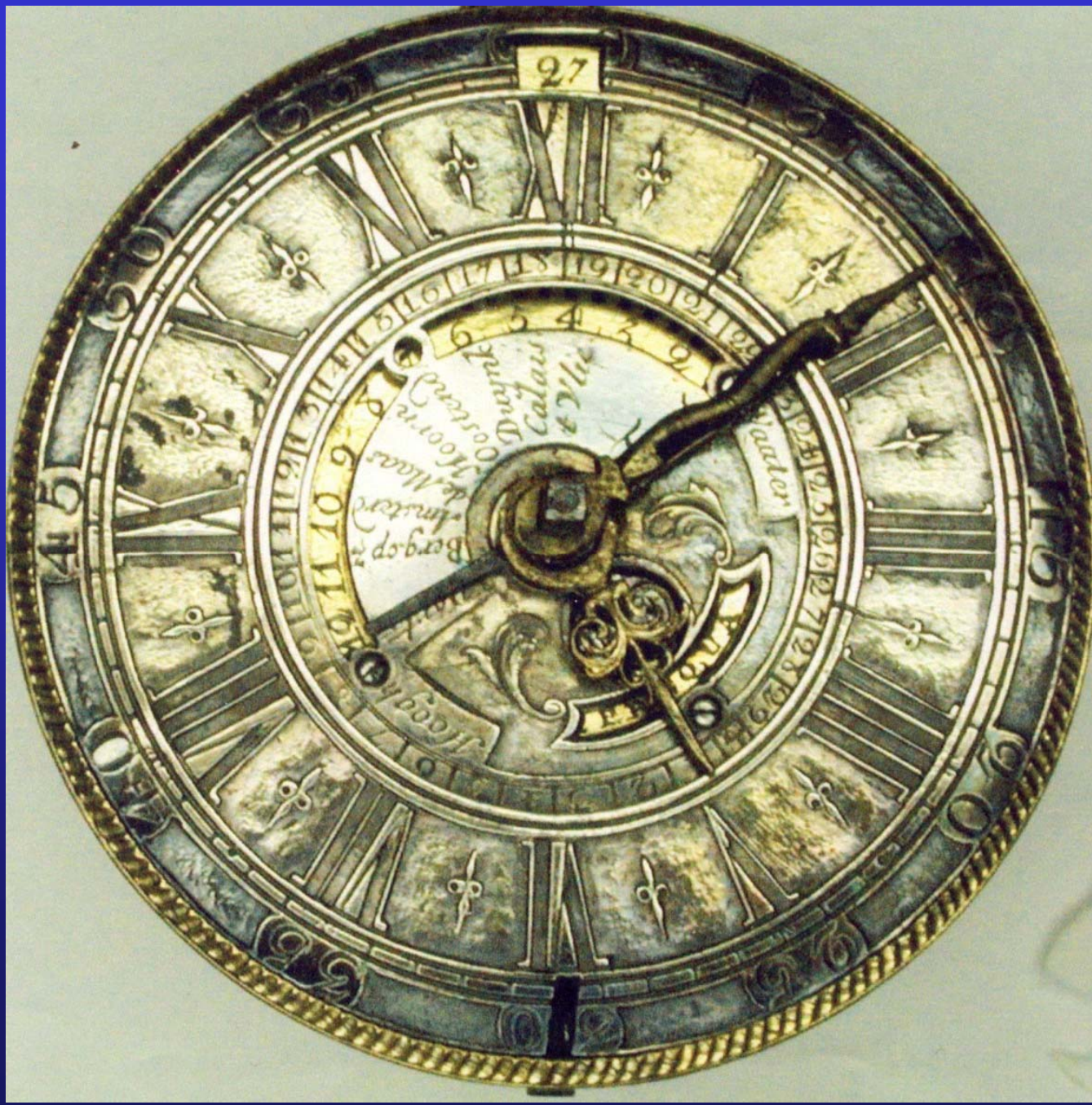
“Captain’s Watch”

Adrianus Haas

c. 1720

Ned. Goud-, Zilver-, en klokkenmuseum

Schoonhoven

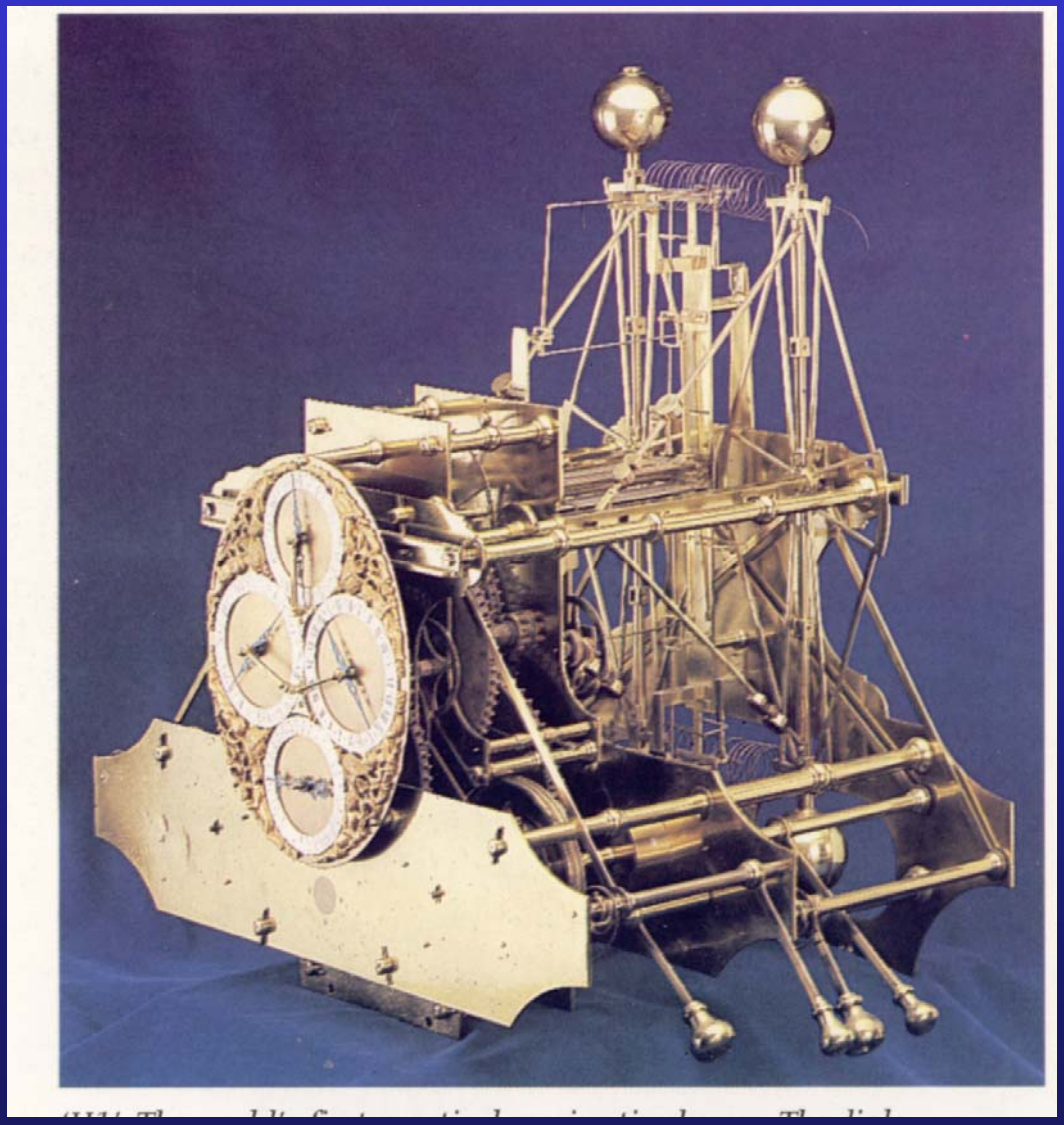


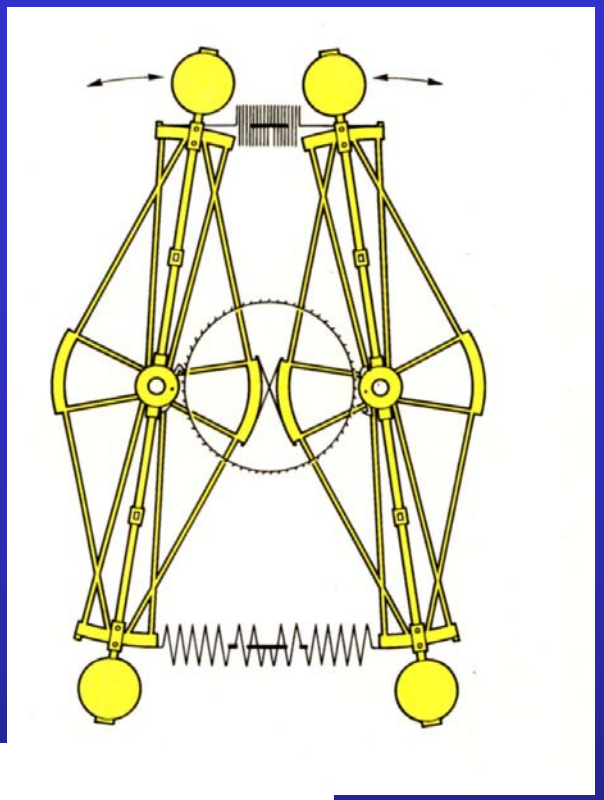
What Huygens missed

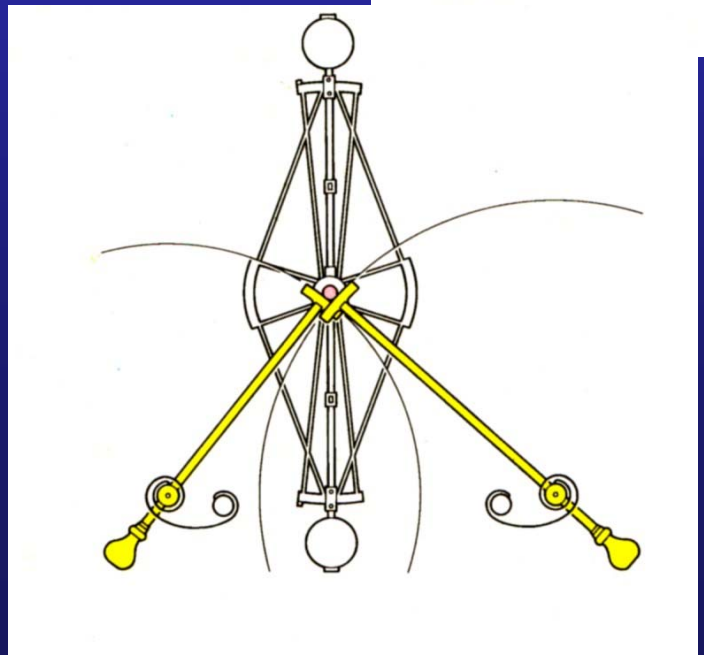
Determining longitude at sea

- Temperature dependence pendulum/Balance wheel
- Interaction with escapement (recoil vs resting escapements)
- Low frequency oscillation less accurate

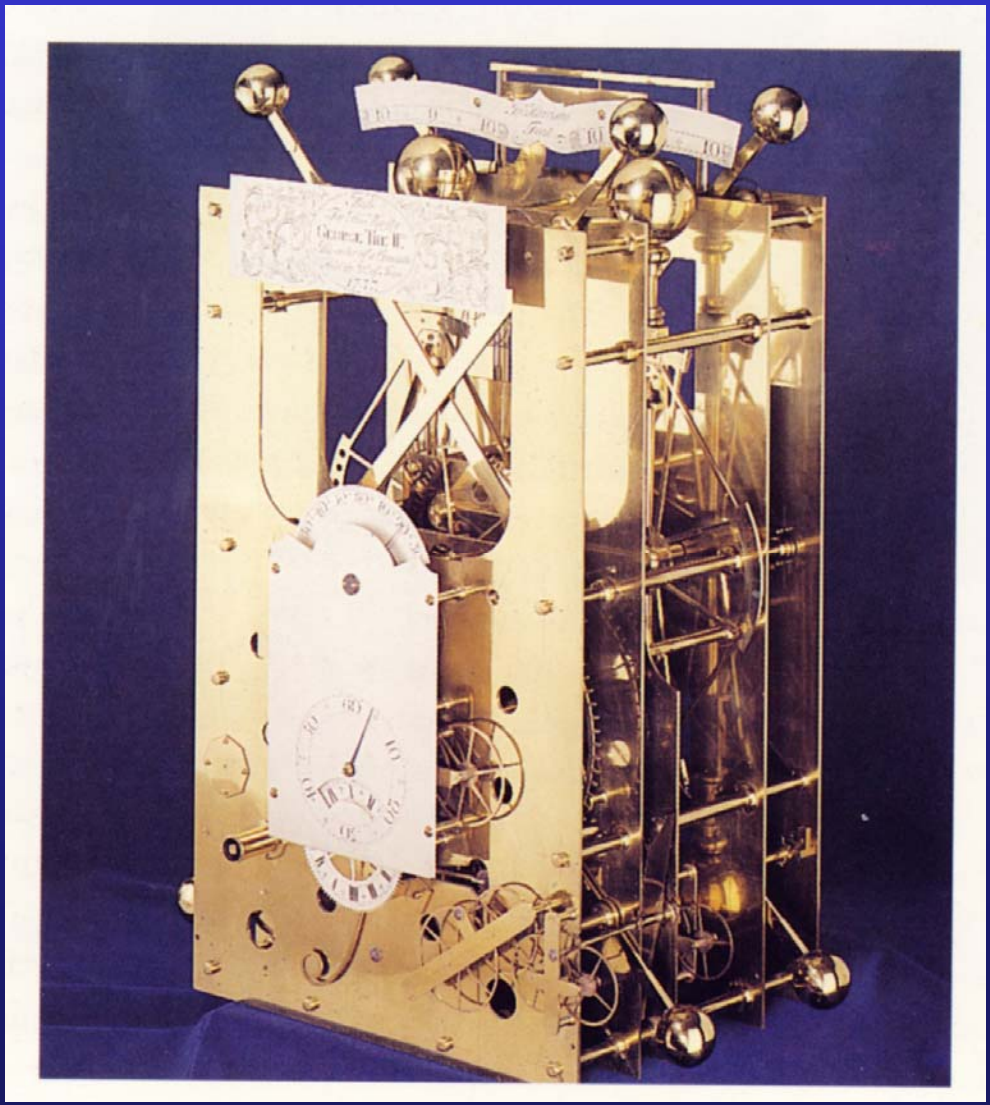
Determining longitude at sea H1

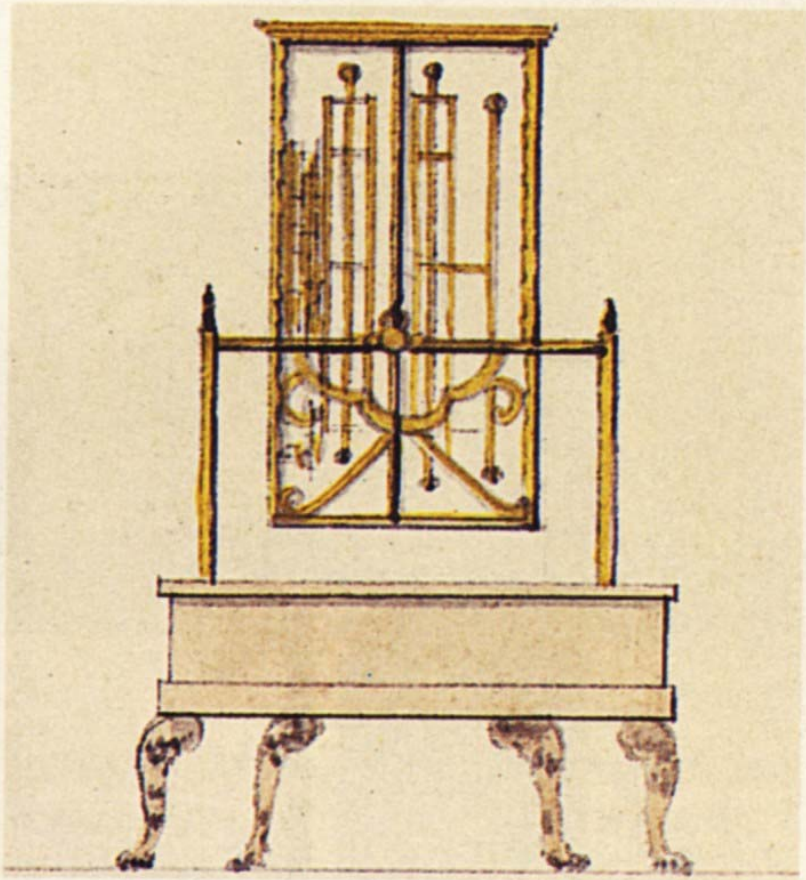






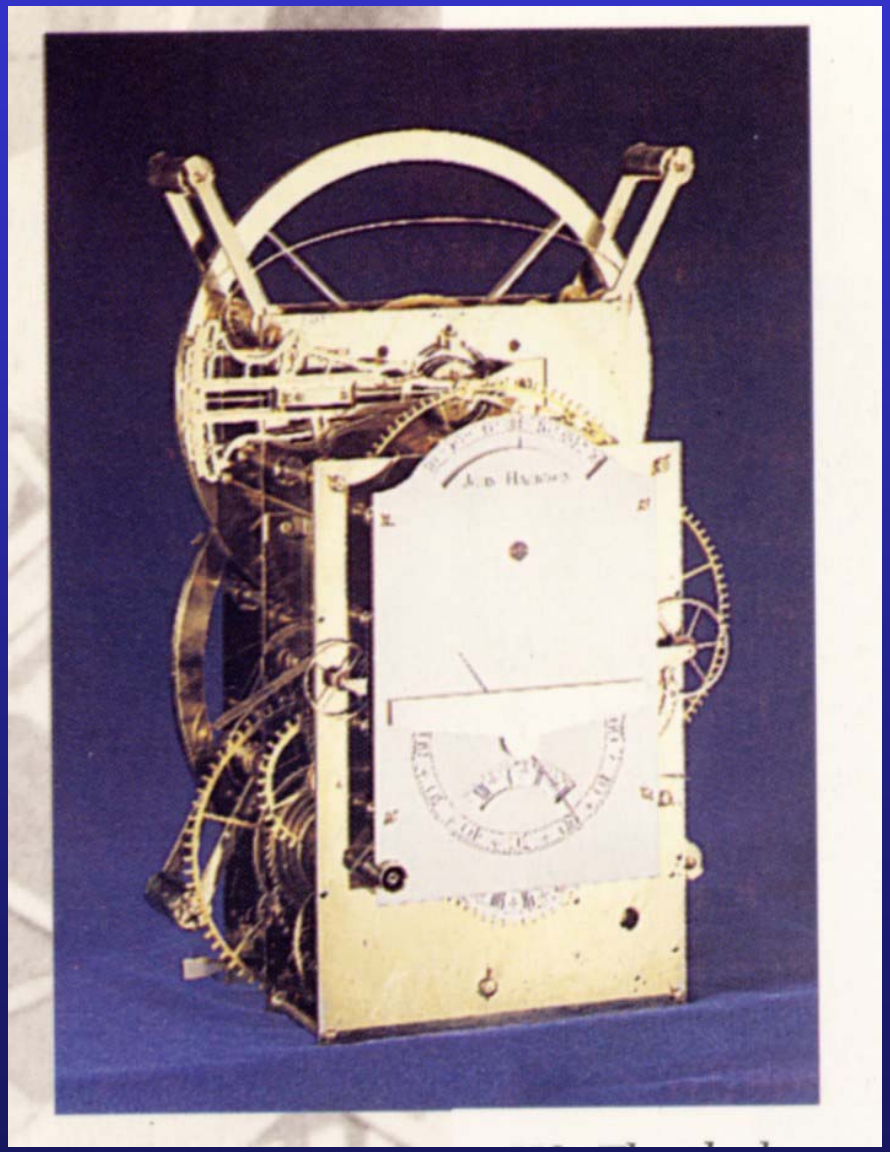
Determining longitude at sea H2

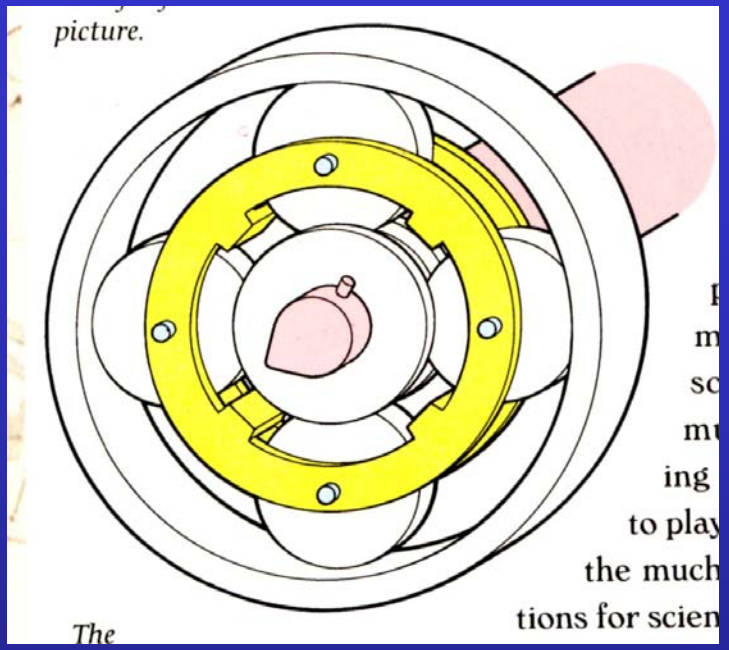


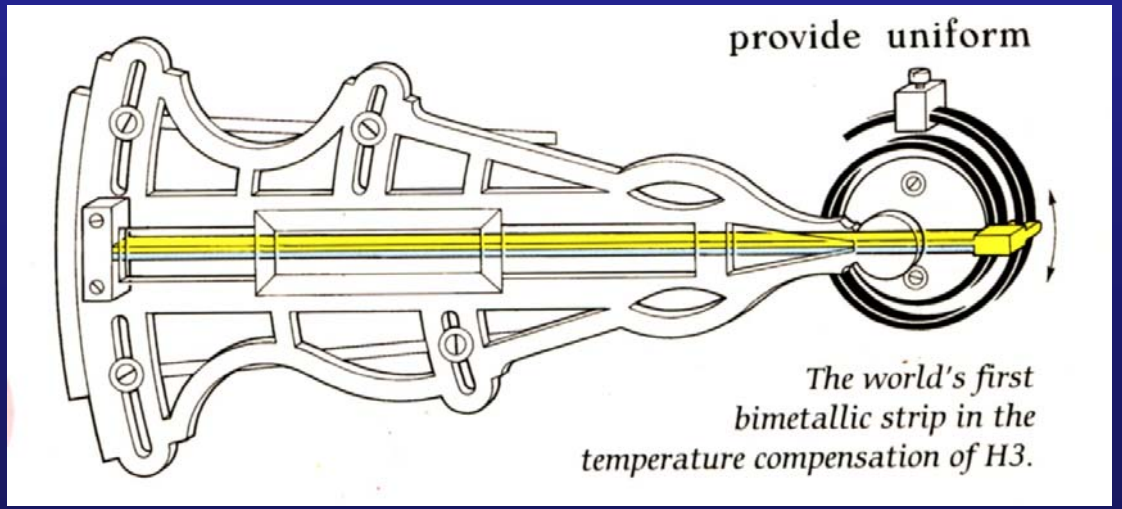


Small colour wash drawing of H2 by John Charnock, c 1770.

Determining longitude at sea H3

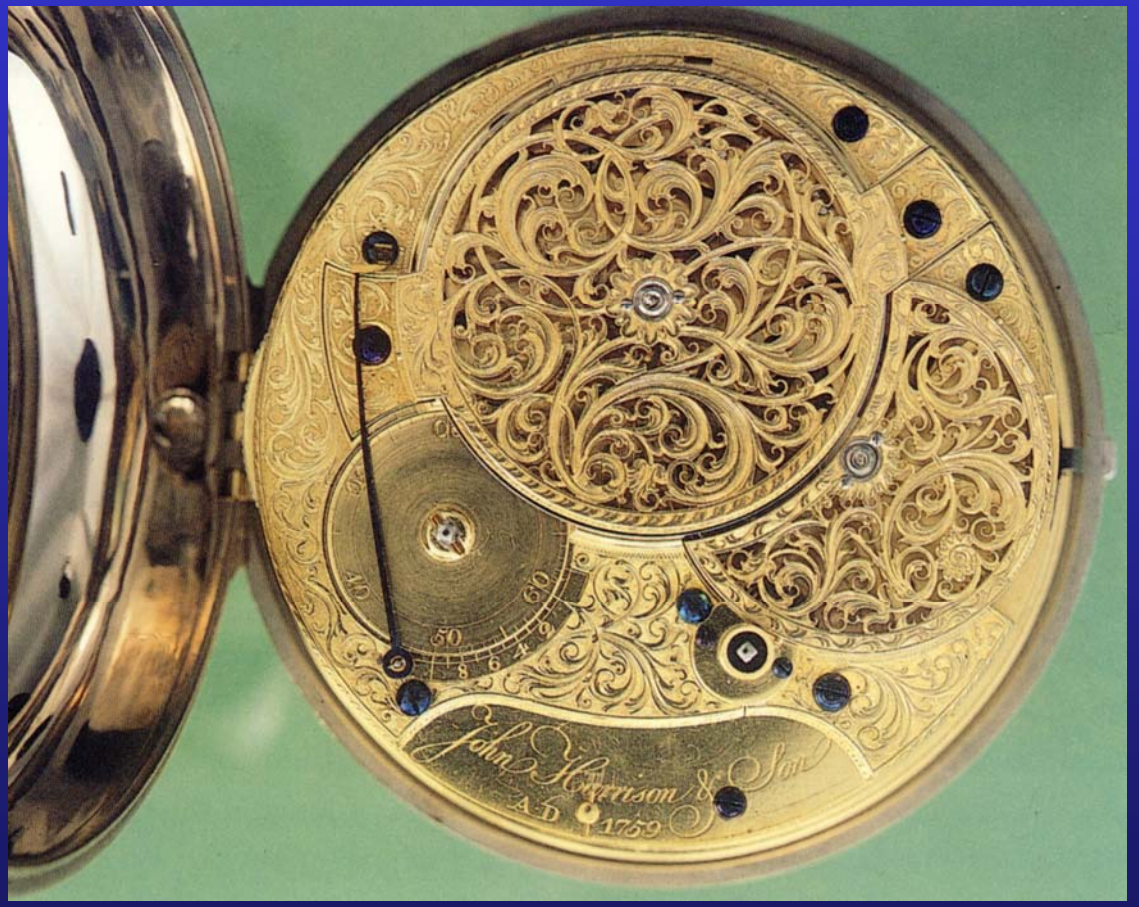




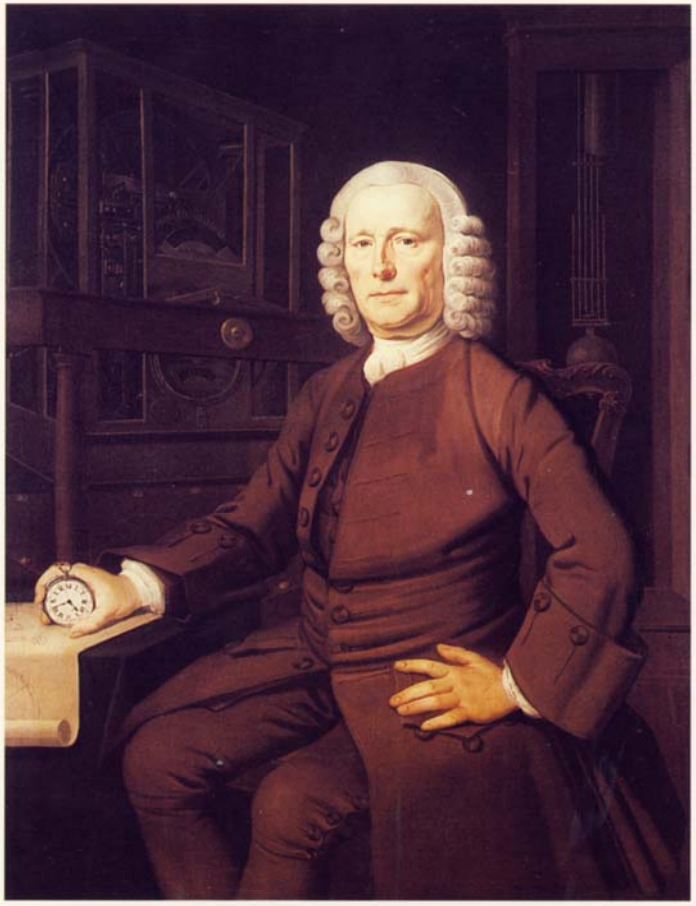


The world's first bimetallic strip in the temperature compensation of H3.

Determining longitude at sea H4: Longitude Prize 1773



Determining longitude at sea H4: Longitude Prize 1773



This book was published to accompany the special exhibition on John Harrison which opened to the public on 24th March 1993 – the 300th anniversary of his birth – at the Old Royal Observatory, Greenwich.

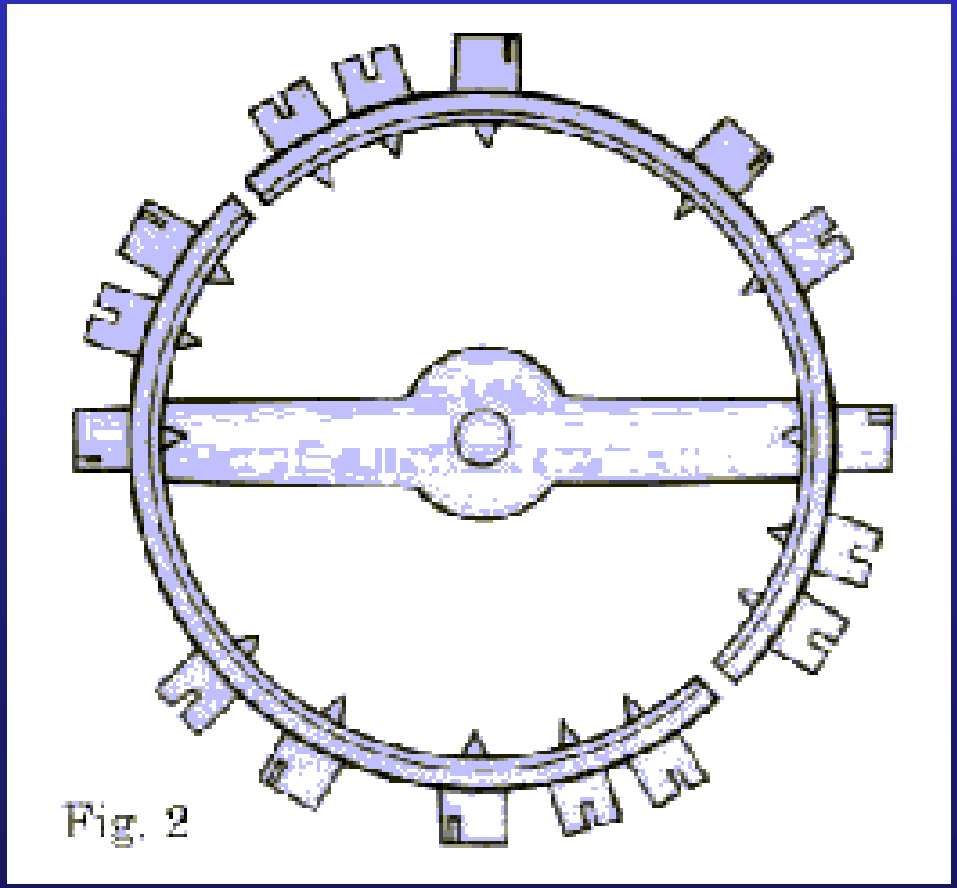
Temperature dependence

Iron: $11.10^{-6}/^{0}C$ → -.5 s/dag/^{0}C

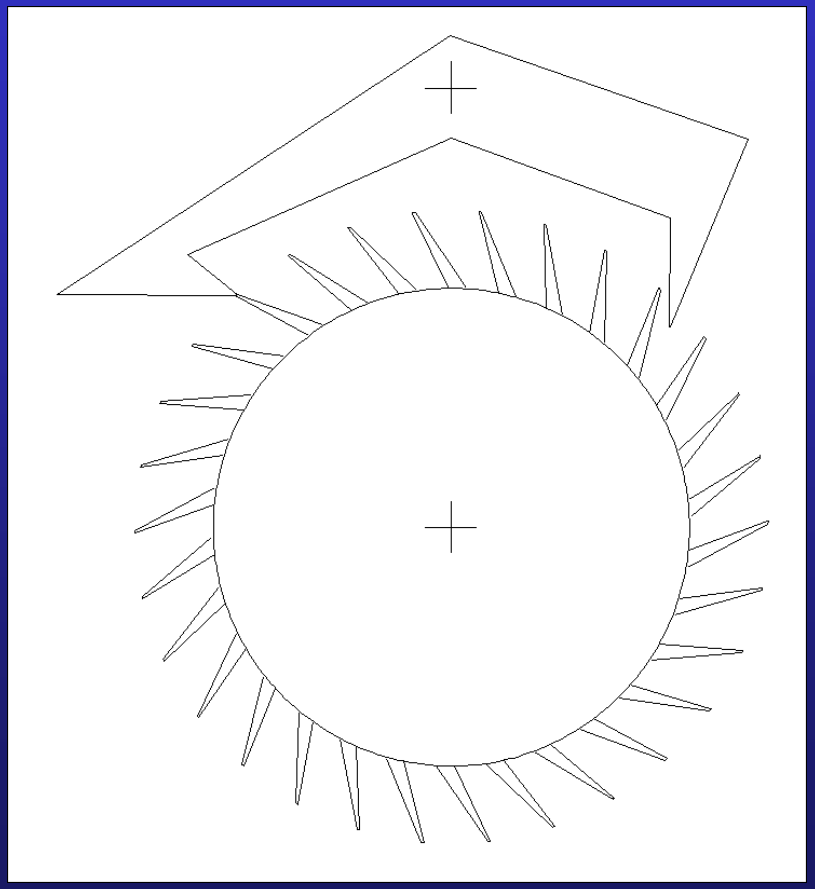
- Mercury pendulum (Graham 1726)
- Gridiron pendulum (Harrison 1729)
- Bimetal (Harrison 1730)
- Invar (C. E. Guillaume 1890, Noble Prize Physics 1920)
- Elinvar (C. E. Guillaume 1890, Noble Prize Physics 1920)

Temperature dependence

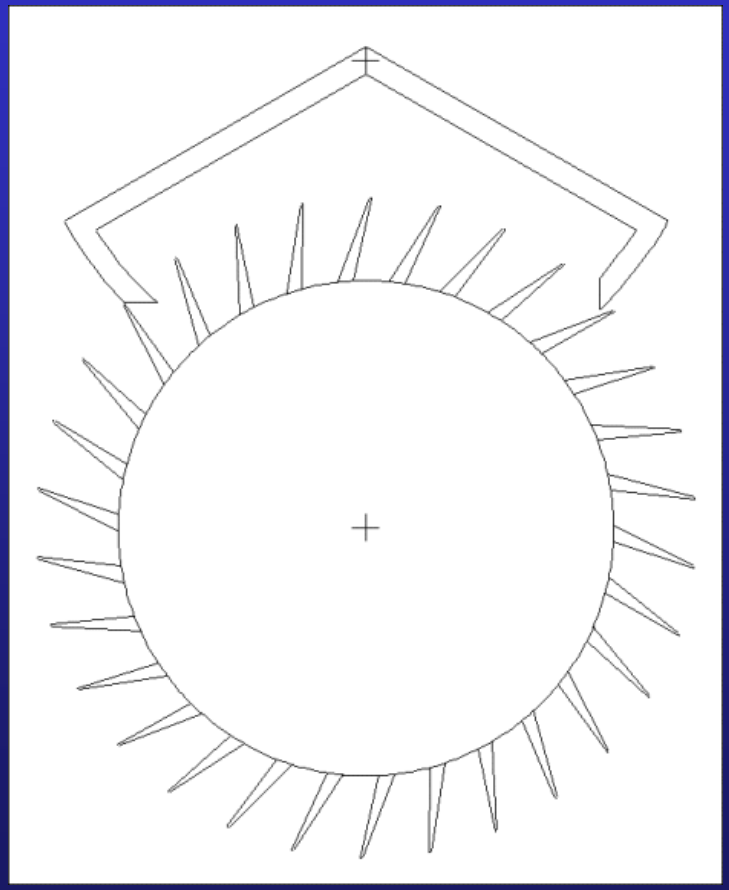
Bimetal: Balance Wheel, Compensation coil spring properties



Interaction with escapement (recoil vs resting escapements)

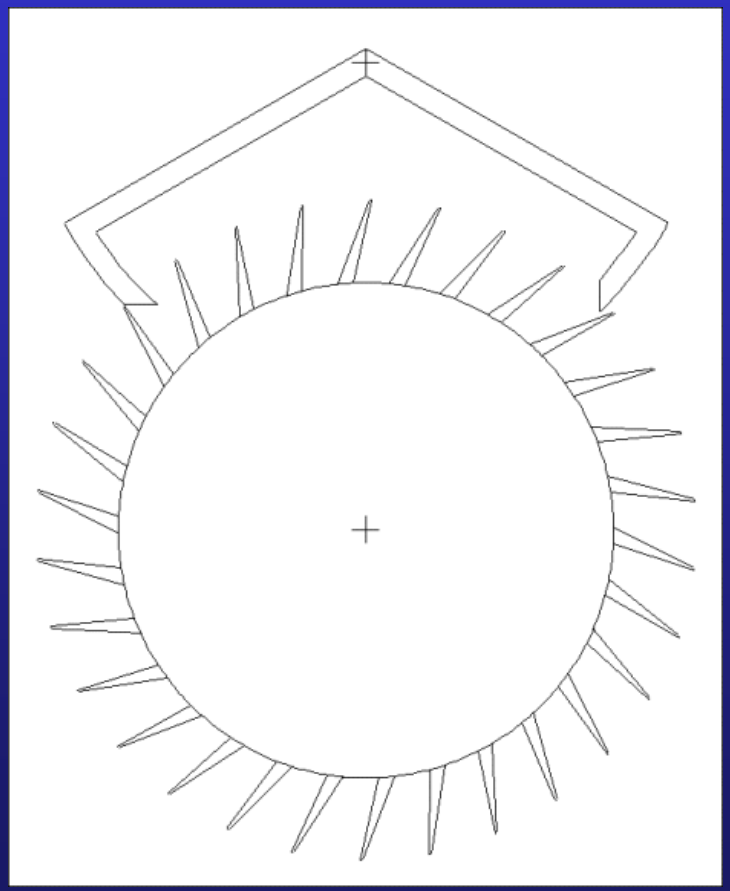


Anchor escapement (Clement)

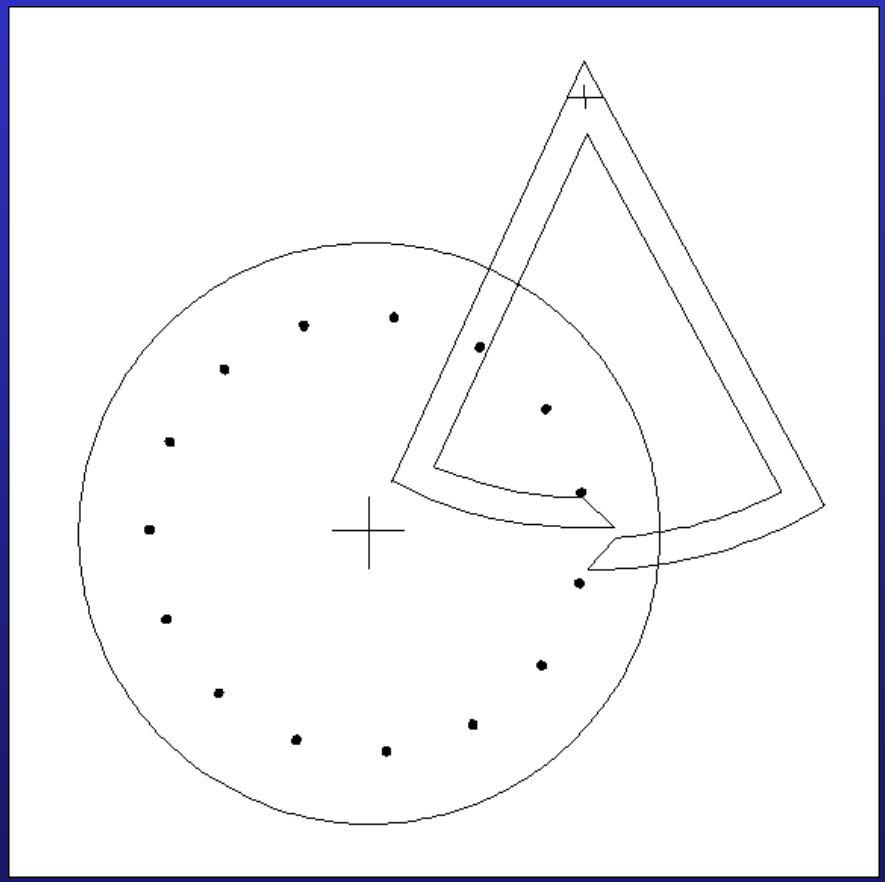


Dead beat escapement (Graham)

Interaction with escapement (resting escapements)



Dead beat escapement (Graham)



Pin wheel escapement (Lepaute)

Low frequency oscillation less accurate

- Balance wheel with coil spring (f_{osc} > 2 Hz)
- Quartz Oscillator (f_{osc} > 2 MHz): +/- 1 ppm
- Atomic clock (f_{osc} > 2 GHz): Cesium 9.2 GHz +/- 10^{-14}

Discussion

- Huygens man of science with insight in second order **(resonant) systems**
- Using this insight he produced two major break-throughs in timekeeping: The **pendulum** and the **balance spring**
- His science was stimulated by **important practical questions**
- Produced **solutions for most limitations** in time-keeping, except for temperature dependence

Christiaan Huygens: Appreciation

Eppo Doeve 1955

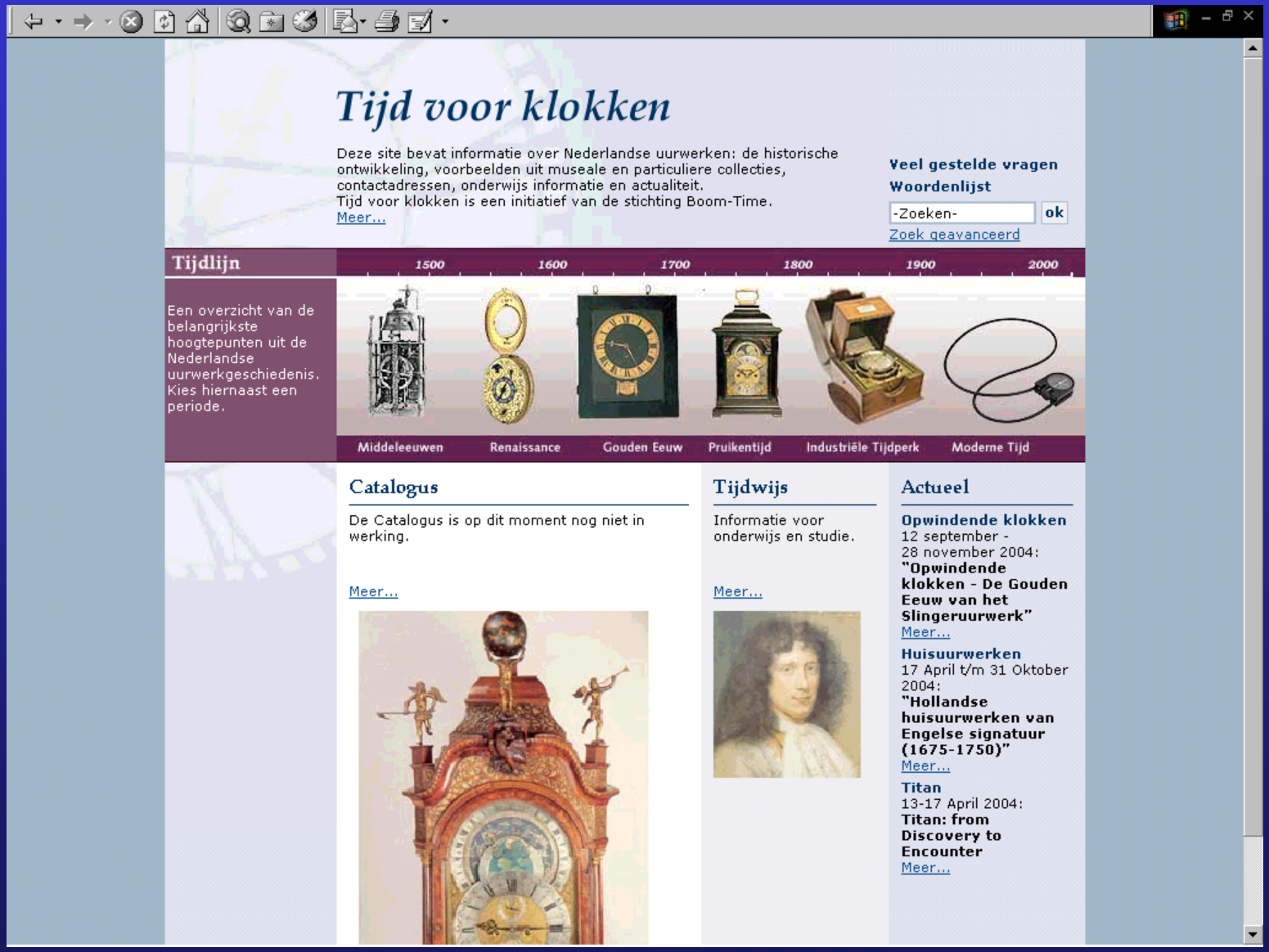
Exhibition Paleis, Het Loo Apeldoorn

September 12 - November 28, 2004

HUYGENS' LEGACY

The Golden Age of the pendulum clock

Horology Projects Boom-Time Foundation



Website:

www.tijdvoorklokken.nl/