

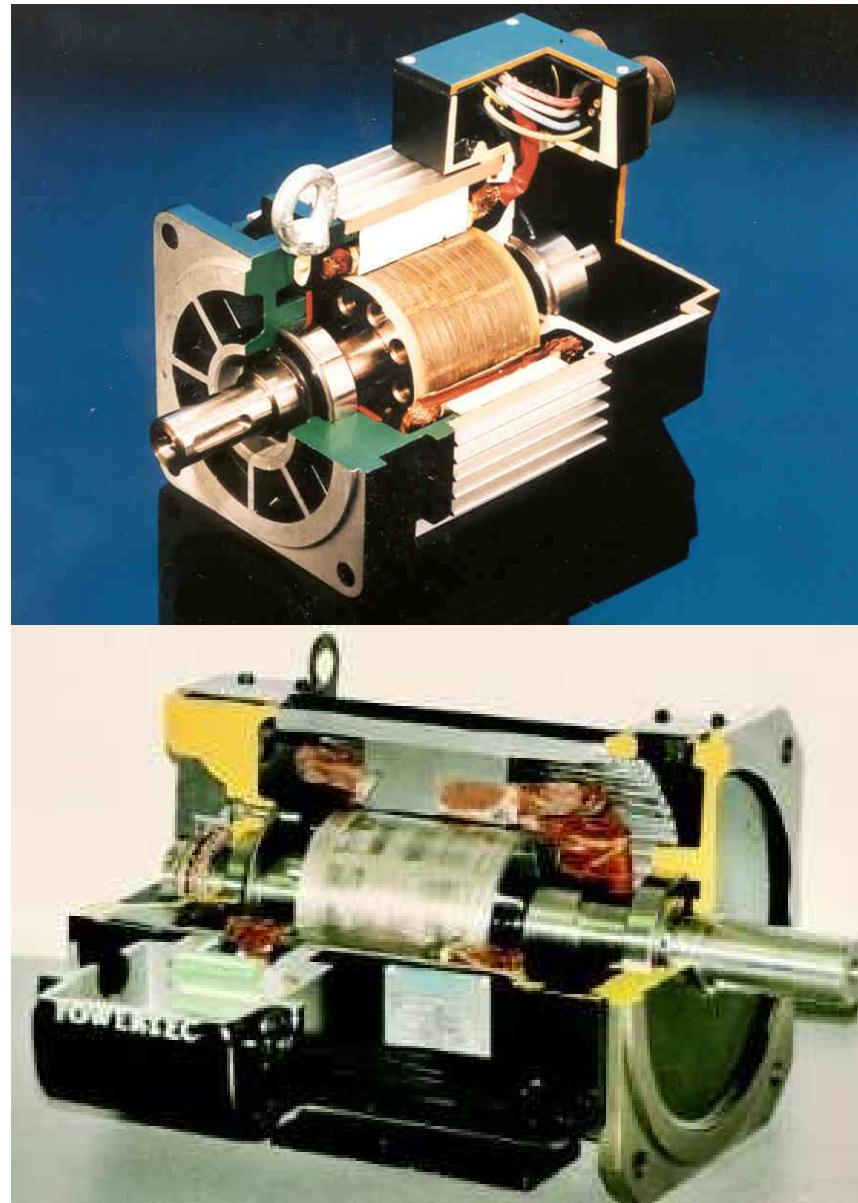
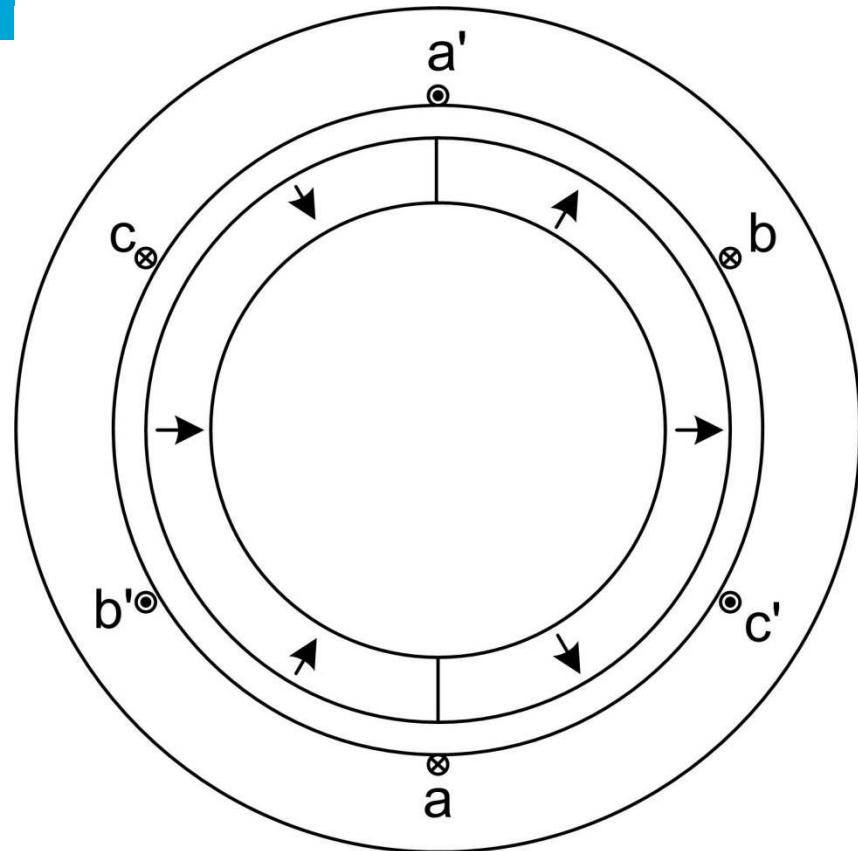
# Overview Electrical Machines and Drives

- 7-9 1: Introduction, Maxwell's equations, magnetic circuits
- 11-9 1.2-3: Magnetic circuits, Principles
- 14-9 3-4.2: Principles, DC machines
- 18-9 4.3-4.7: DC machines and drives
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- 12-10 6.14-8.3: PMACM, other machines
- 19-10: rest, questions
- 9-11: exam

# Permanent magnet AC machines (6.13)

- Introduction
- Calculation example
- Brushless DC motor (rectangular / trapezoidal waveforms)
- PMSM (sinusoidal waveforms)
- For these machine types
  - Construction
  - Electromotive force
  - Voltage equations and equivalent circuit
  - Power balance
  - Force or torque

# PM AC machine



# Rotor layouts

1 surface mounted magnets

- $L_d \approx L_q$
- $L$  small because of large air gap

2 inset magnets

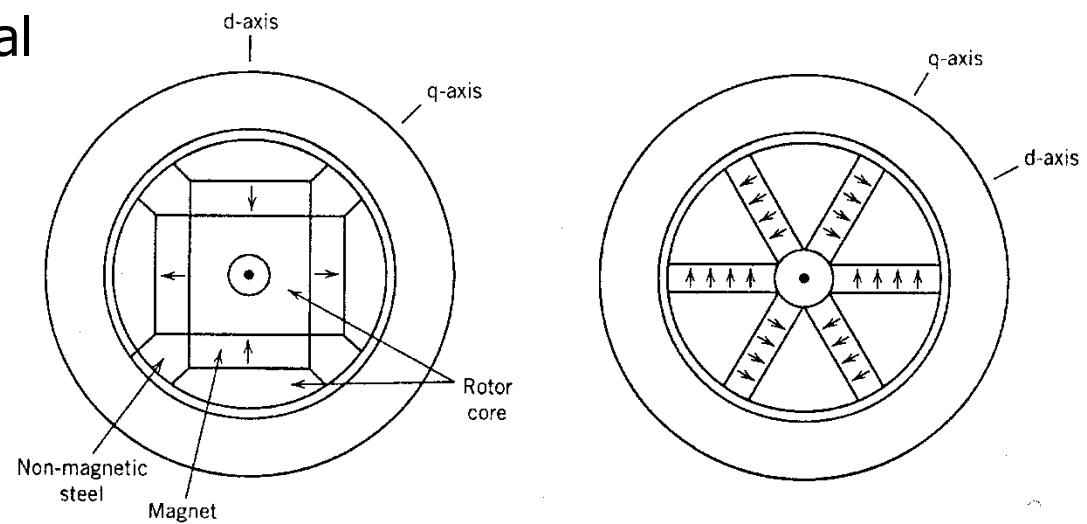
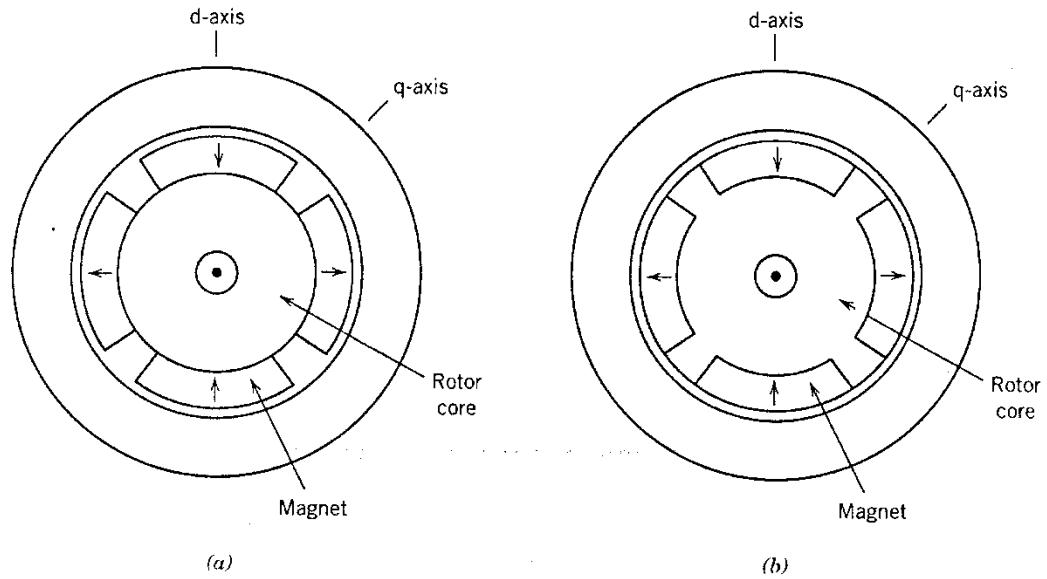
- $L_d < L_q$
- reluctance torque

3 embedded magnets, radial magnetization

- $L_d < L_q$
- reluctance torque

4 embedded magnets, circumferential magnetization

- flux concentration



# Permanent magnet AC machines (6.13)

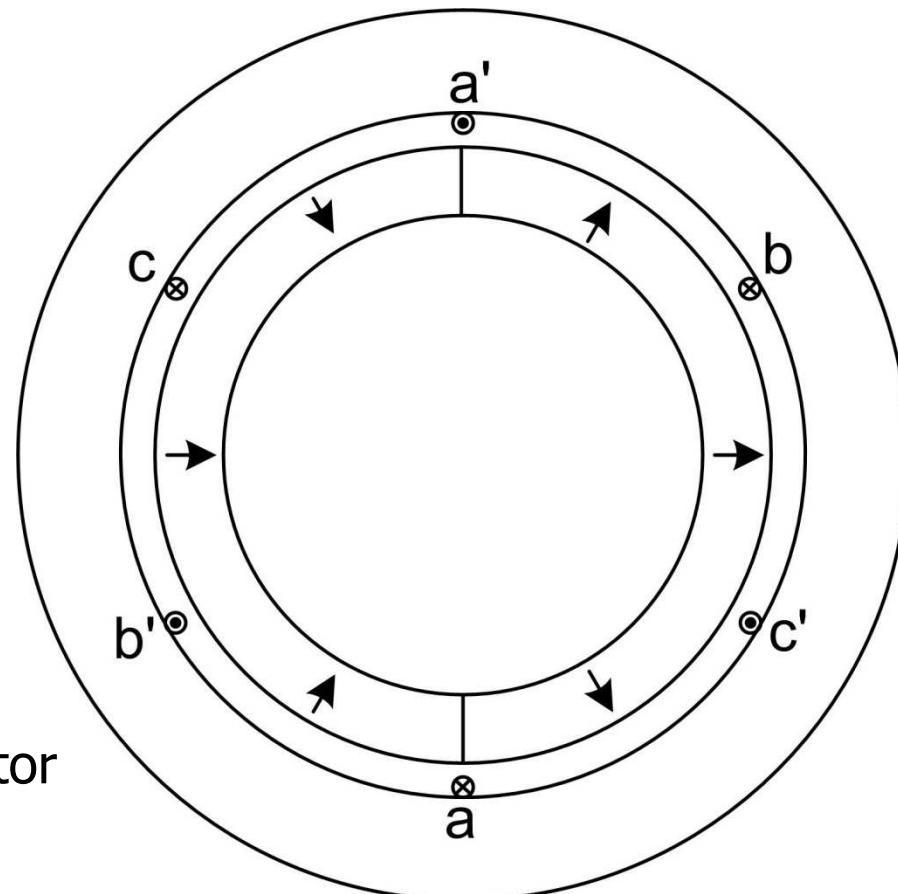
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# PM AC machine: calculation example

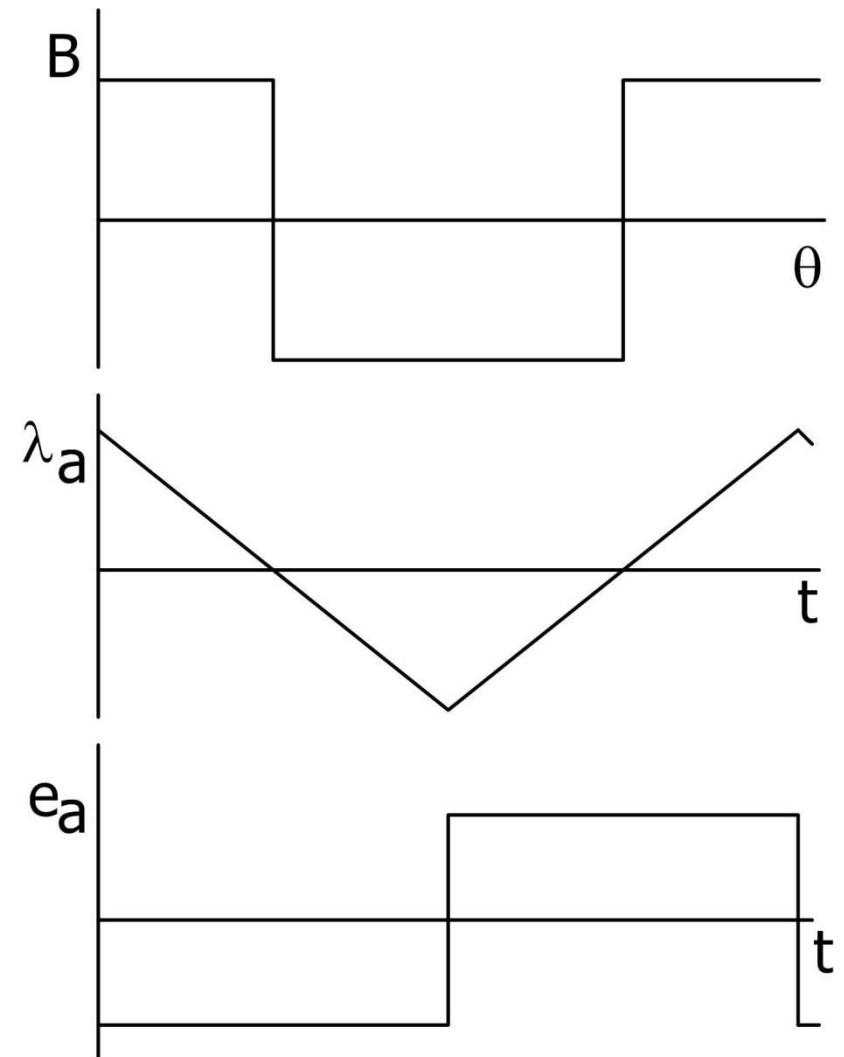
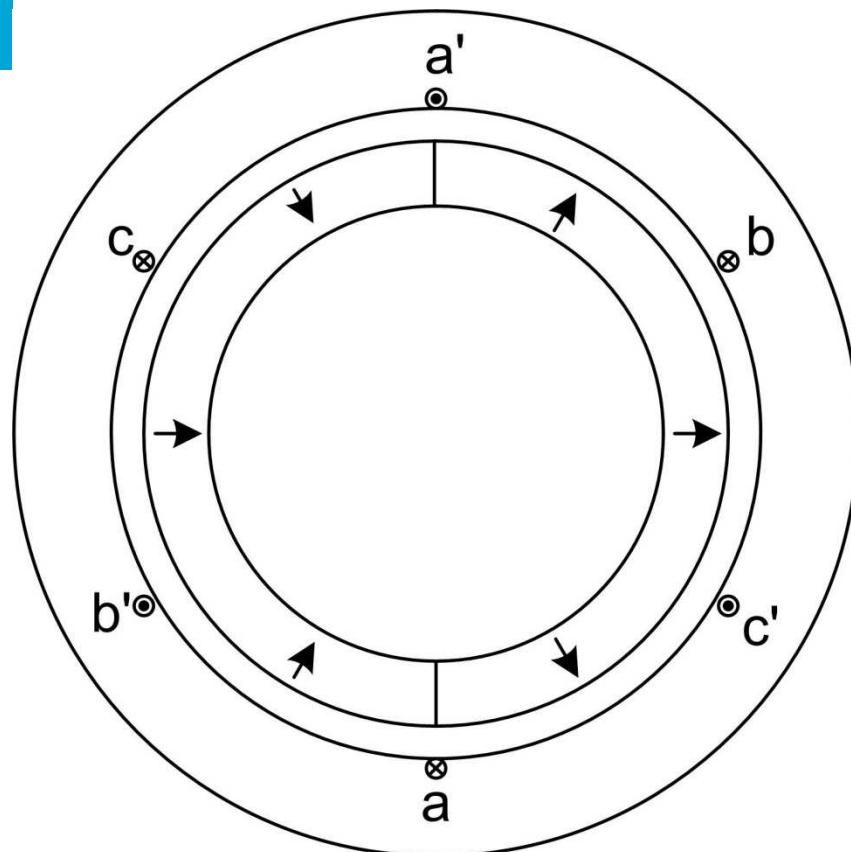
- air gap radius  $r_s$
- axial length  $l_s$
- magnet length  $l_m$
- air gap length  $l_g$
- number of turns  $N_s$
- remanent flux density  $B_{rm}$
- recoil permeability  $\mu_{rm}$
- rotor speed  $\omega_m$

Determine and sketch

- air gap flux density in gap
- flux linkage as a function of rotor position
- induced voltage



# PM AC machine



# PM AC machine

Using Ampère's law:

$$\oint_{C_m} \vec{H} \cdot \vec{\tau} \, d s = \iint_{S_m} \vec{J} \cdot \vec{n} \, d A \quad 2H_m l_m + 2H_g l_g = 0$$

Magnetic flux continuity

$$\iint_S \vec{B} \cdot \vec{n} \, d A = 0 \quad B_m = B_g$$

BH curve of magnet

$$B_m = \mu_0 \mu_{rm} H_m + B_r$$

BH curve of air

$$B_g = \mu_0 H_g$$

Result

$$B_g = \frac{l_m}{l_m + \mu_{rm} l_g} B_r$$

# PM AC machine

For one phase without load:

$$\lambda_{\max} = NBA = N_s B_g \pi r_s l_s$$

$$E_{pm\max} = \frac{d\lambda}{dt} = 4f\lambda_{pm\max} = 2N_s B_g r_s l_s \omega_m$$

$$E_{pm\max} = Blv$$

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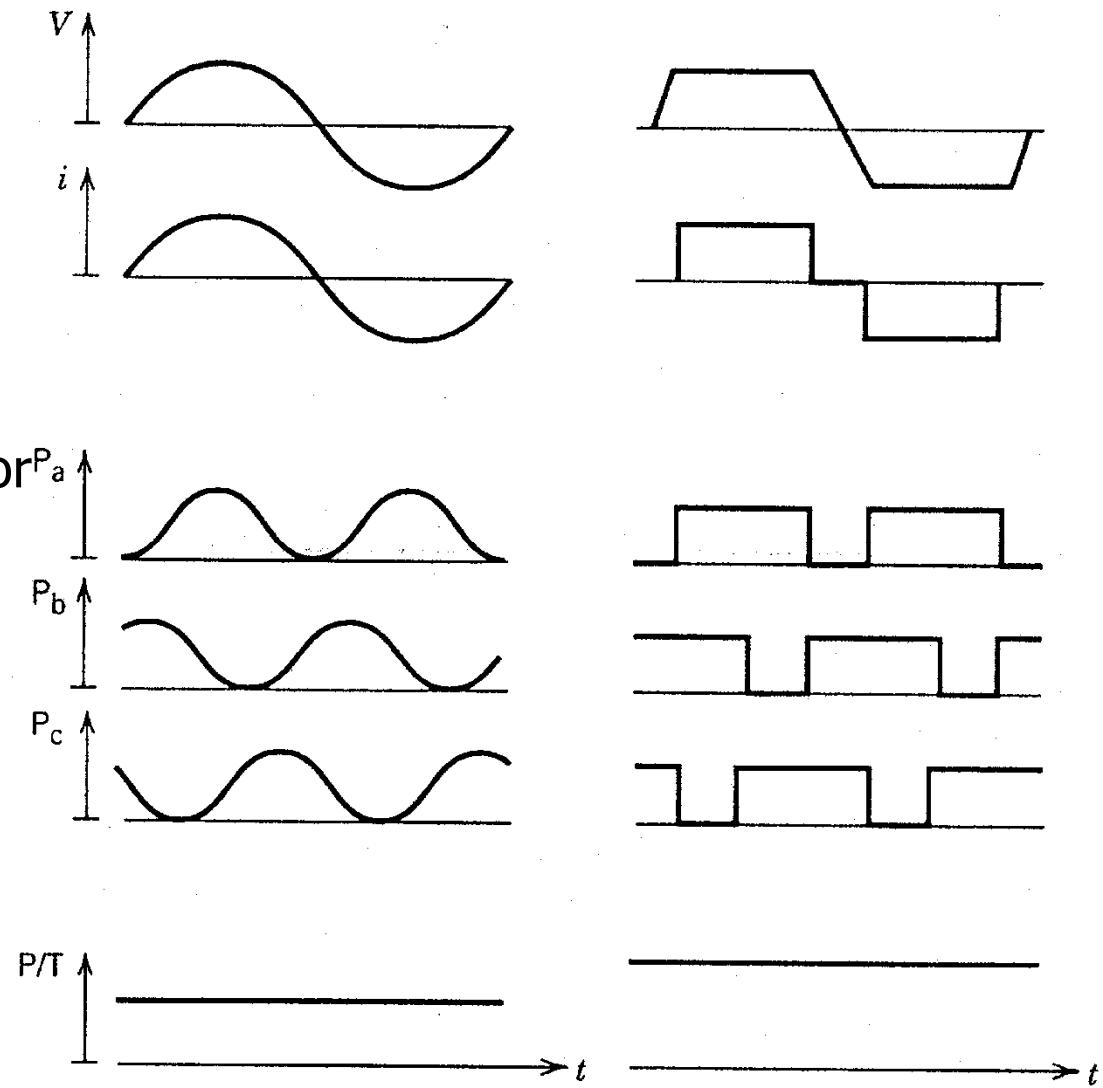
# Construction: PMAC or BDCM

PMSM:

- sinusoidal B
- distributed windings
- sinusoidal voltage
- sinusoidal currents
- continuous position sensor
- smooth force

BDCM:

- rectangular B
- concentrated windings
- trapezoidal voltage
- rectangular currents
- 6 step position sensor
- force ripple

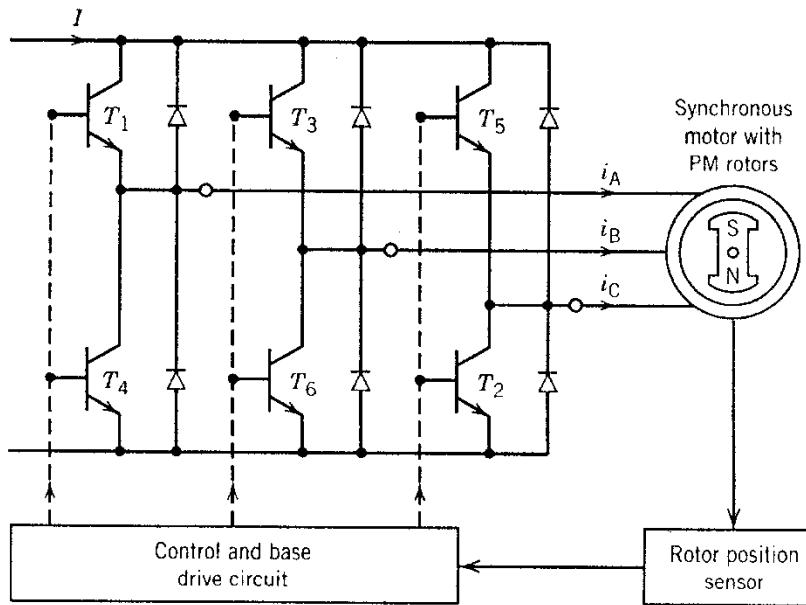


# Brushless DC machine

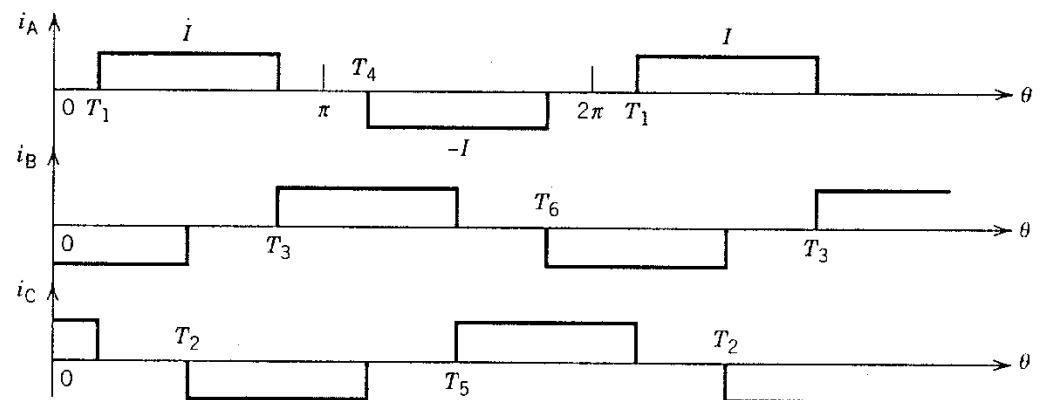
Name may be confusing

Idea:

- PM excitation on rotor:  
no brushes
- mechanic commutator  
replaced by converter

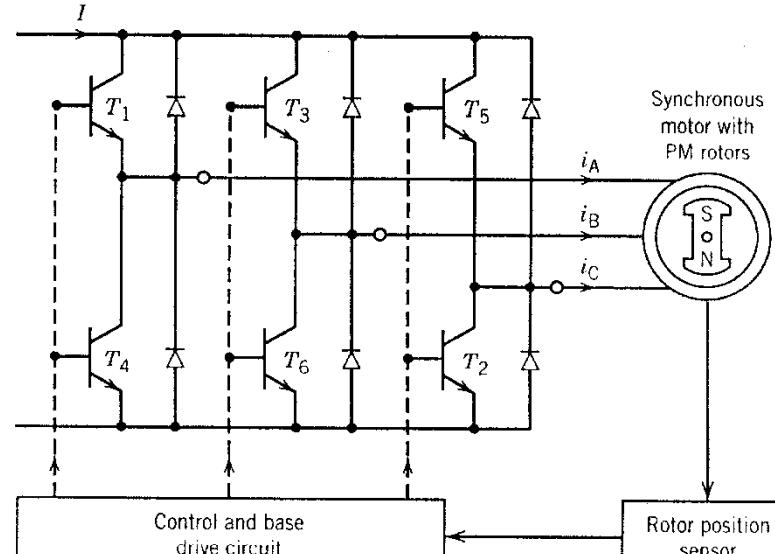
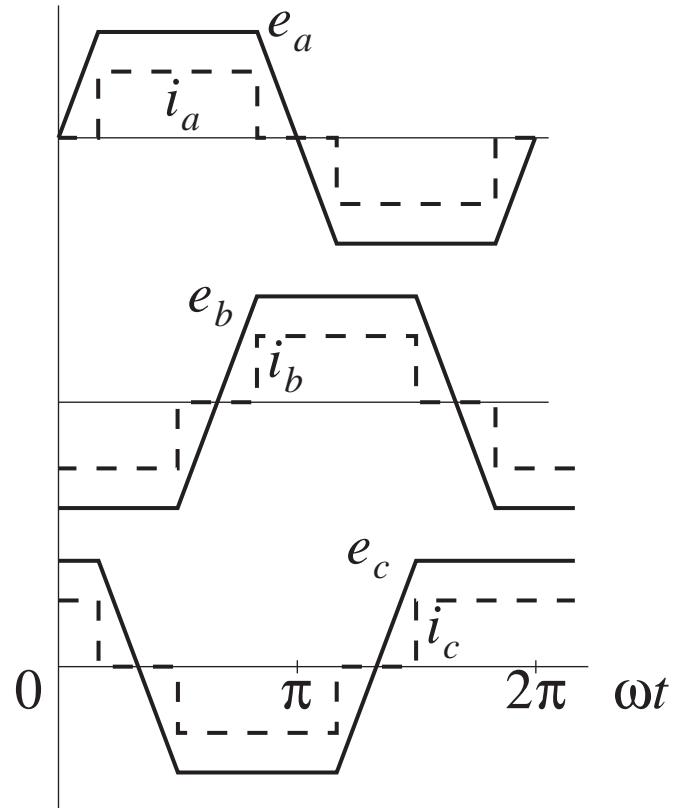


(a)

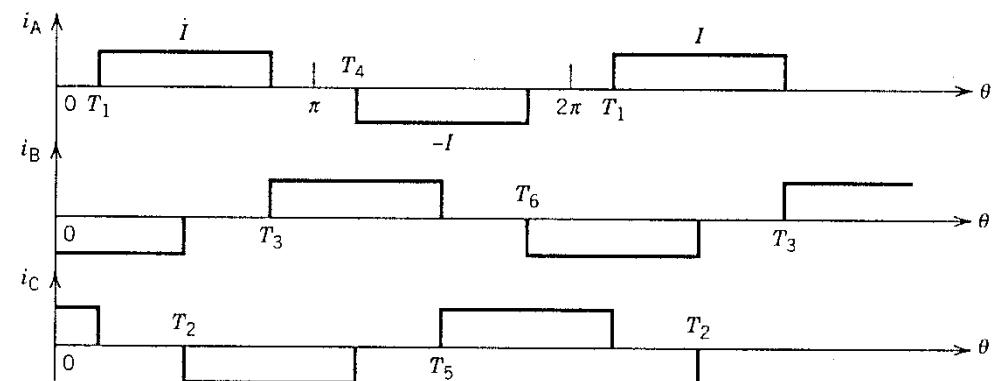


# Brushless DC machine

Why trapezoidal voltage?

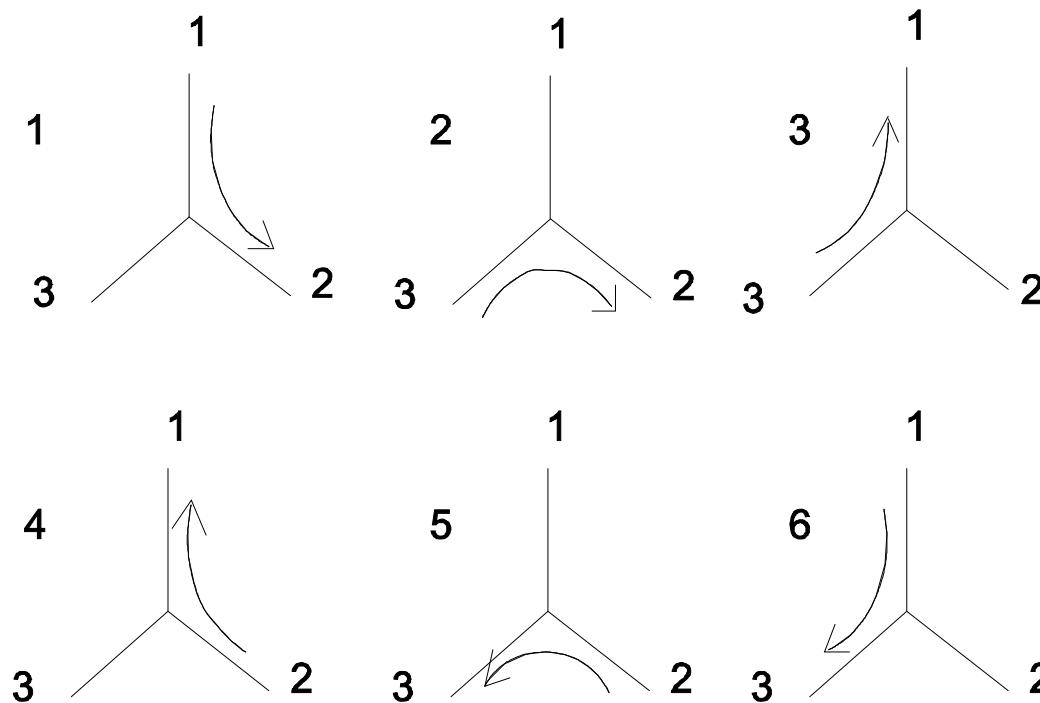


(a)



# Brushless DC machine operation

- Six step operation
- Six times per revolution position information necessary



# PM AC machine

For one phase without load:

$$\lambda_{\max} = NBA = N_s B_g \pi r_s l_s$$

$$E_{pm\max} = \frac{d\lambda}{dt} = 4f\lambda_{pm\max} = 2N_s B_g r_s l_s \omega_m$$

$$E_{pm\max} = Blv$$

# BDCM voltage equations

Maxwell, Faraday:  $u = Ri + \frac{d\lambda}{dt}$

$$\begin{cases} \lambda_{sa} = L_{sa}i_{sa} + M_{sab}i_{sb} + M_{sab}i_{sc} + \lambda_{pma}(\theta) \\ \lambda_{sb} = M_{sab}i_{sa} + L_{sa}i_{sb} + M_{sab}i_{sc} + \lambda_{pmb}(\theta) \\ \lambda_{sc} = M_{sab}i_{sa} + M_{sab}i_{sb} + L_{sa}i_{sc} + \lambda_{pmc}(\theta) \end{cases}$$

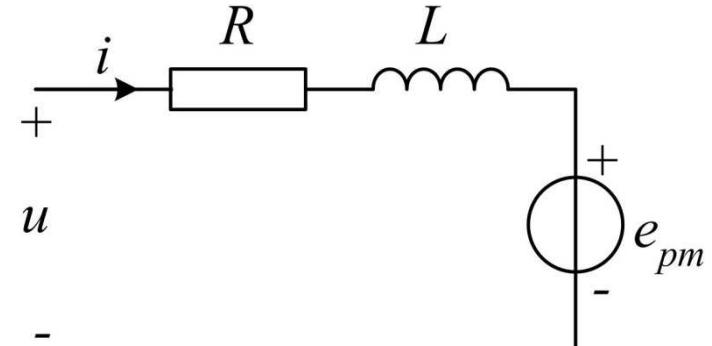
No star-point connection:  $i_{sa} + i_{sb} + i_{sc} = 0$

$$L_s = L_{sa} - M_{sab}$$

$$\begin{cases} \lambda_{sa} = (L_{sa} - M_{sab})i_{sa} + \lambda_{pma}(\theta) = L_s i_{sa} + \lambda_{pma}(\theta) \\ \lambda_{sb} = (L_{sa} - M_{sab})i_{sb} + \lambda_{pmb}(\theta) = L_s i_{sb} + \lambda_{pmb}(\theta) \\ \lambda_{sc} = (L_{sa} - M_{sab})i_{sc} + \lambda_{pmc}(\theta) = L_s i_{sc} + \lambda_{pmc}(\theta) \end{cases}$$

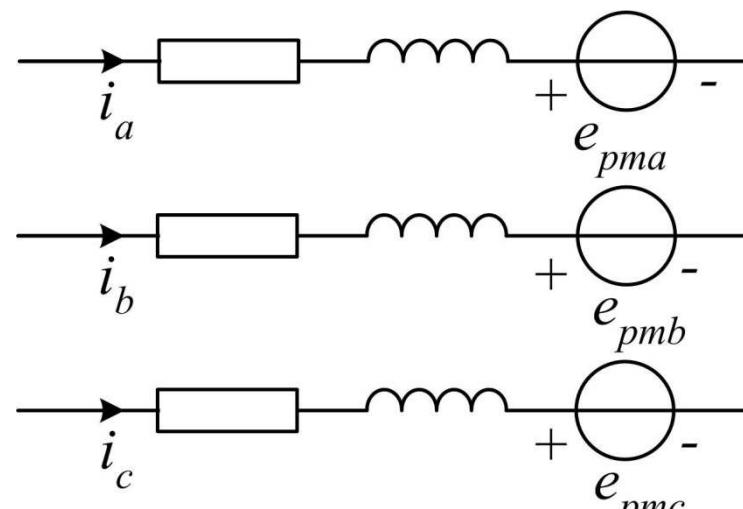
# BDCM voltage equations

$$u_a = R i_a + \frac{d \lambda_a}{dt}$$



Single-phase equivalent circuit

$$\begin{cases} u_a = R i_a + L_s \frac{d i_a}{dt} + e_{pma} \\ u_b = R i_b + L_s \frac{d i_b}{dt} + e_{pmb} \\ u_c = R i_c + L_s \frac{d i_c}{dt} + e_{pmc} \end{cases}$$



Three-phase equivalent circuit

# BDCM torque

Always two phases conducting:

$$P = u_x i_x + u_y i_y = 2RI^2 + 2 \frac{d^{\frac{1}{2}} L_s I^2}{dt} + 2E_{pm\max} I$$

Electrical input power = Losses + Increase stored energy + Mechanical output power

Torque:

$$T = \frac{P}{\omega_m} = \frac{2IE_{pm\max}}{\omega_m} = 4B_g N_s l_s r_s I \quad T = r_s F = r_s BlI$$

Power balance gives same result as Lorentz

# BDCM pros and cons

- Main problem:
  - Six times per period irregularities in the torque because
    - Hall sensors different and positioned with limited accuracy
    - Motor coils are slightly different
    - The EMF (voltage induced by magnets) are different
    - Controller branches are different
    - Current can not be a square wave. Why not?
  - Consequences
    - Not nice for position servo
    - Can be improved by hysteresis in Hall sensors
- Strengths BDCM
  - Simple controller with six step position sensor
  - Sensorless operation possible at higher speeds.

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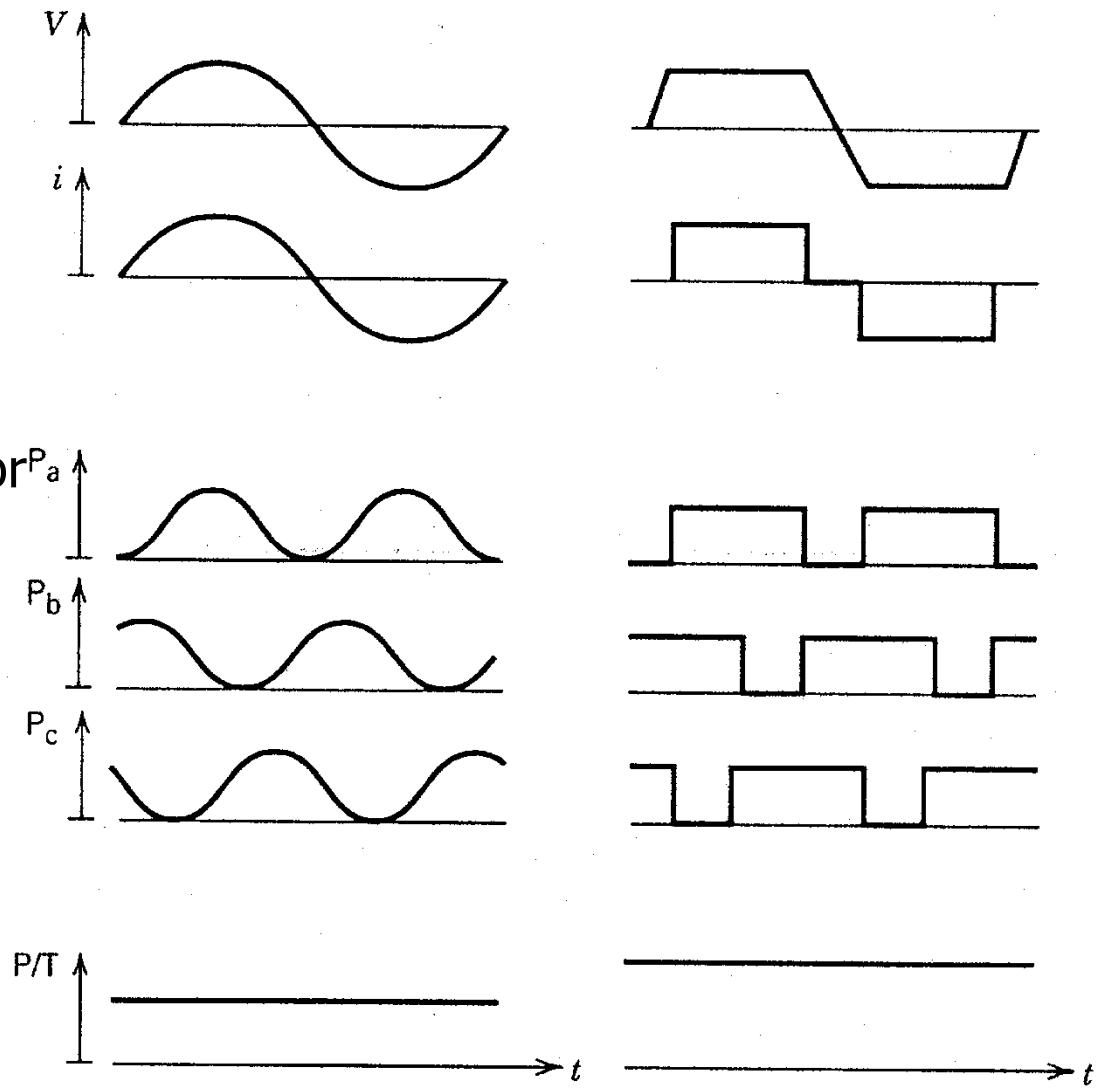
# Construction: PMAC or BDCM

PMSM:

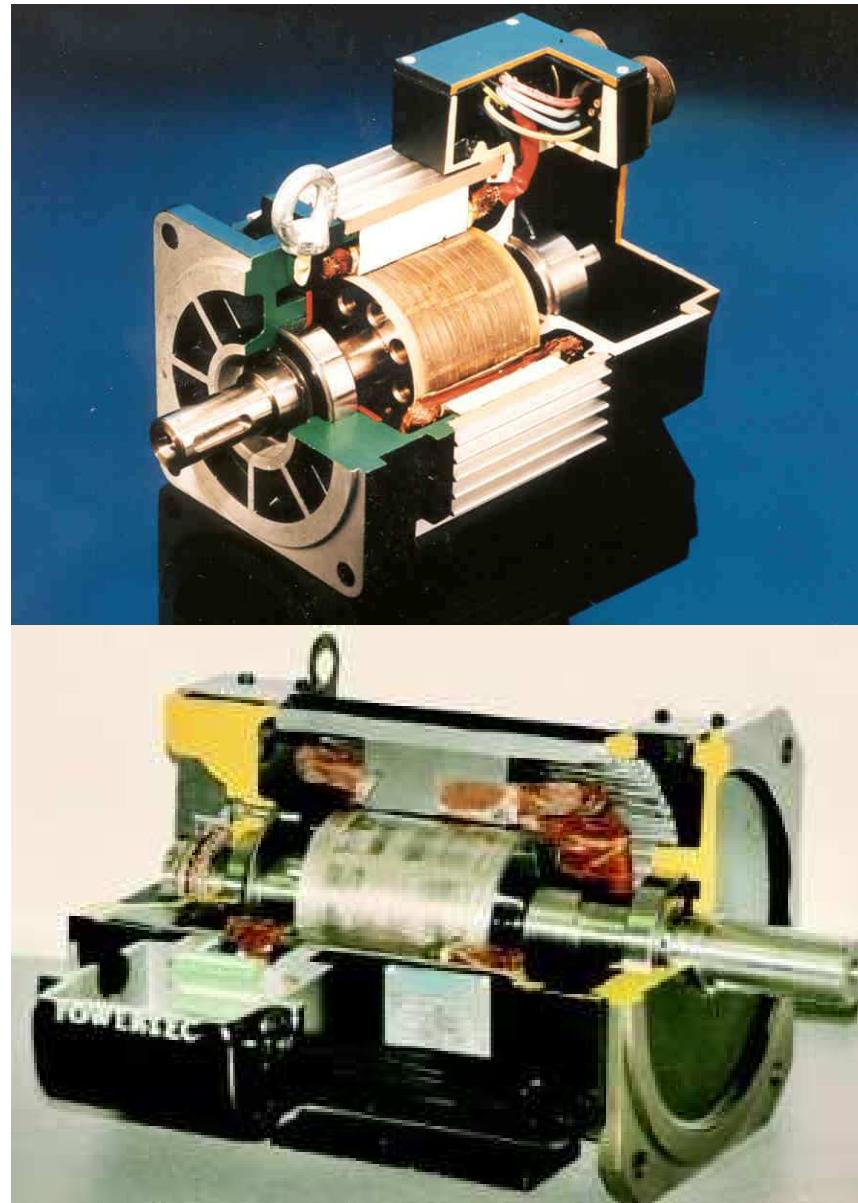
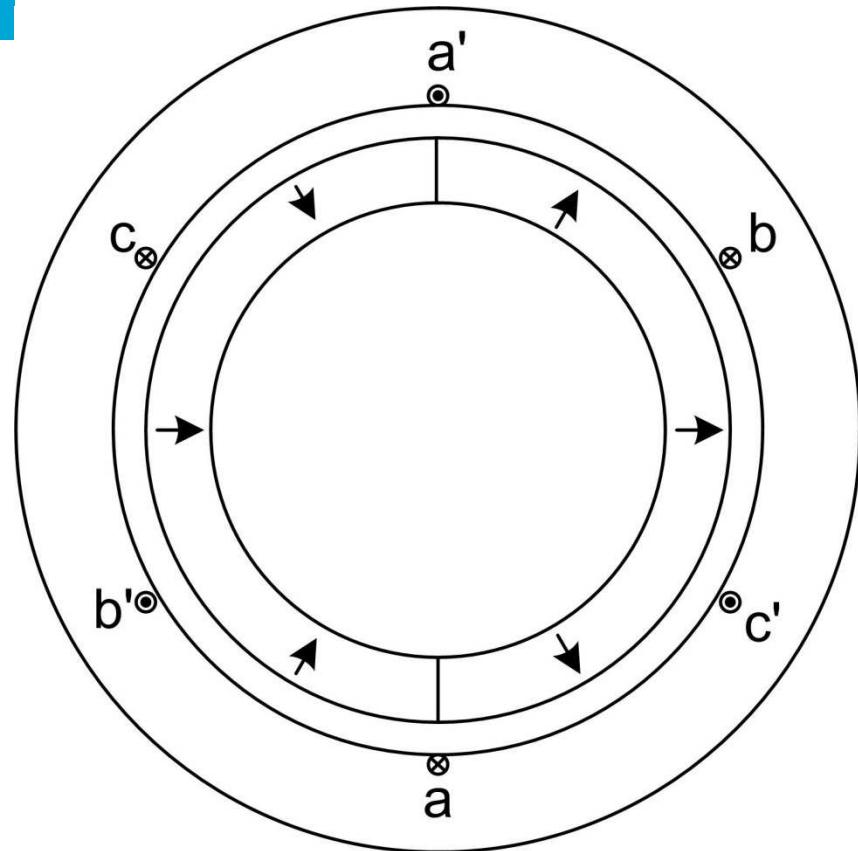
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- sinusoidal currents
- continuous position sensor
- smooth force

BDCM:

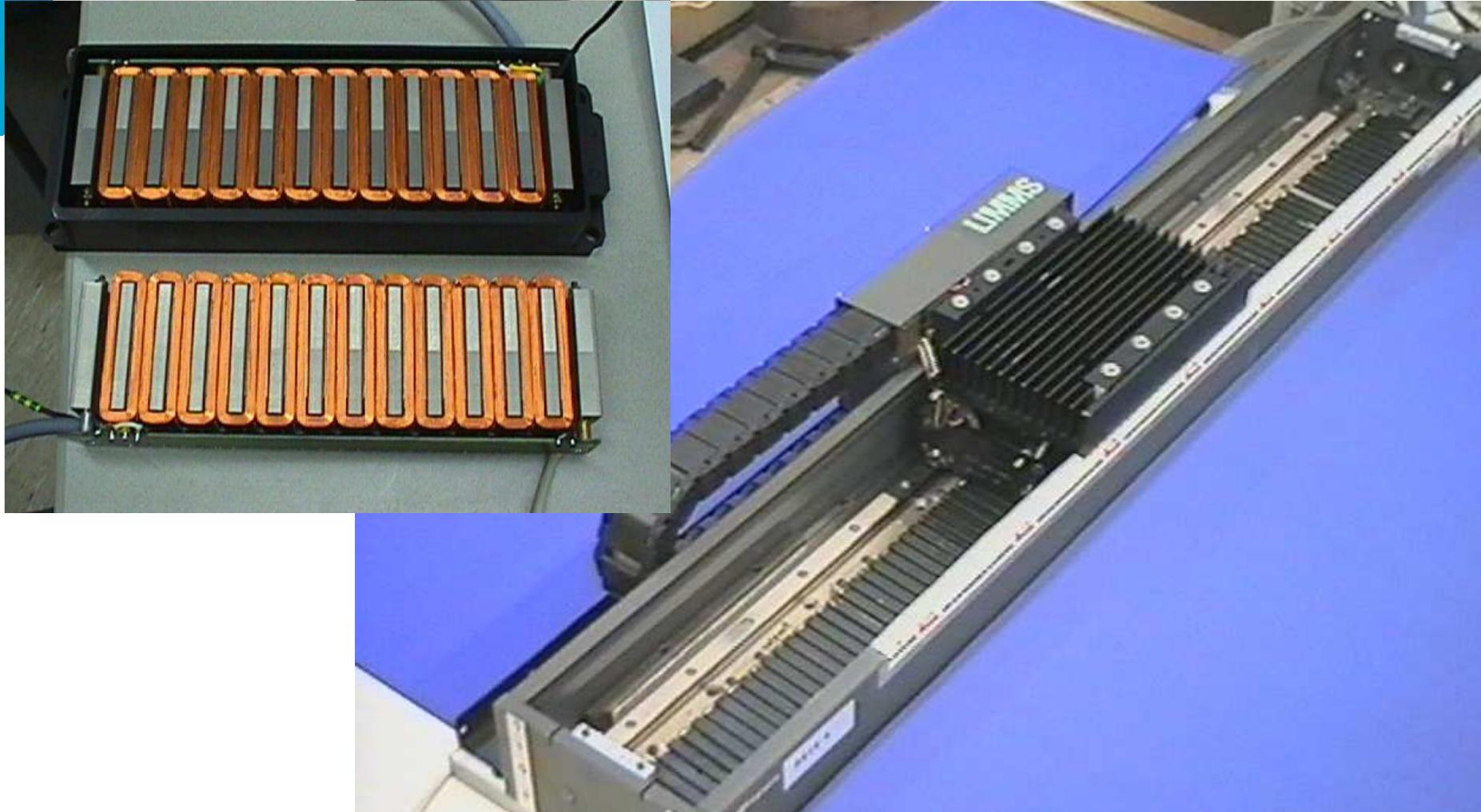
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# PM AC machine



# PM synchronous machine



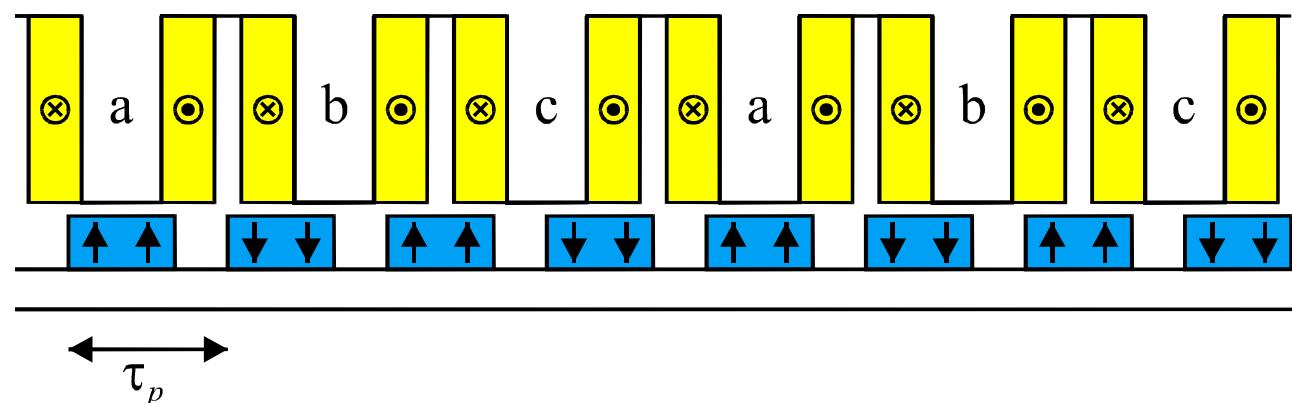
# PM synchronous machine

$$\begin{cases} \Phi_{pma} = -\hat{\Phi}_{pm} \cos(\frac{\pi}{\tau_p} x) \\ \Phi_{pmb} = -\hat{\Phi}_{pm} \cos(\frac{\pi}{\tau_p} x - \frac{2}{3}\pi) \\ \Phi_{pmc} = -\hat{\Phi}_{pm} \cos(\frac{\pi}{\tau_p} x - \frac{4}{3}\pi) \end{cases}$$

Cosinusoidal because of

- skewing
- end teeth

How can the voltage be calculated?



# PMSM voltage equations

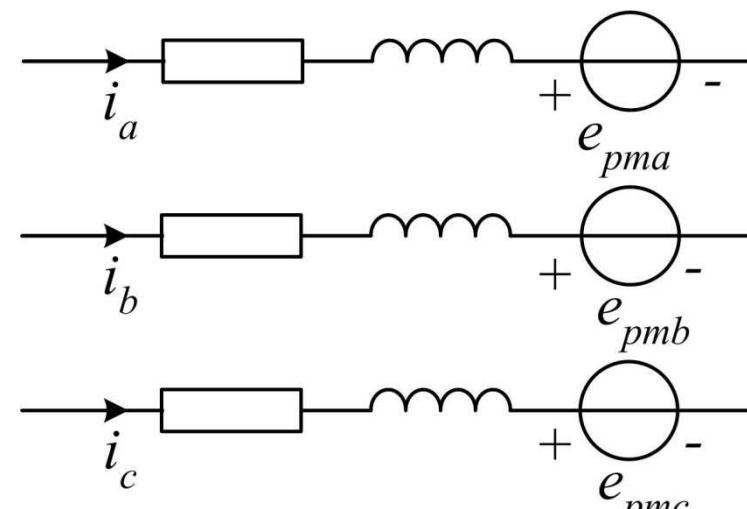
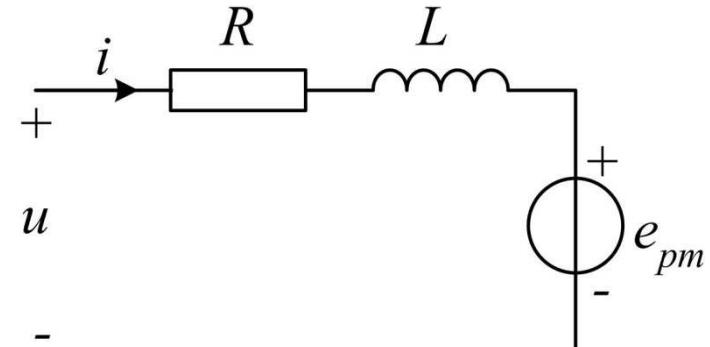
$$u_a = R i_a + \frac{d \lambda_a}{dt}$$

$$\lambda_a = L_{sa} i_a + M_{sab} i_b + M_{sab} i_c + N \Phi_{pma}$$

— Single-phase equivalent circuit

$$L_s = L_{sa} - M_{sab}$$

$$\begin{cases} u_a = R i_a + L_s \frac{d i_a}{dt} + e_{pma} \\ u_b = R i_b + L_s \frac{d i_b}{dt} + e_{pmb} \\ u_c = R i_c + L_s \frac{d i_c}{dt} + e_{pmc} \end{cases}$$



Three-phase equivalent circuit

# PMSM voltage equations

$$\lambda_a = L_{sa} \dot{i}_a + M_{sab} \dot{i}_b + M_{sac} \dot{i}_c + N\Phi_{pma}$$

$$= L_s \dot{i}_a + N\Phi_{pma}$$

$$\Phi_{pma} = -\hat{\Phi}_{pm} \cos\left(\frac{\pi}{\tau_p} x\right)$$

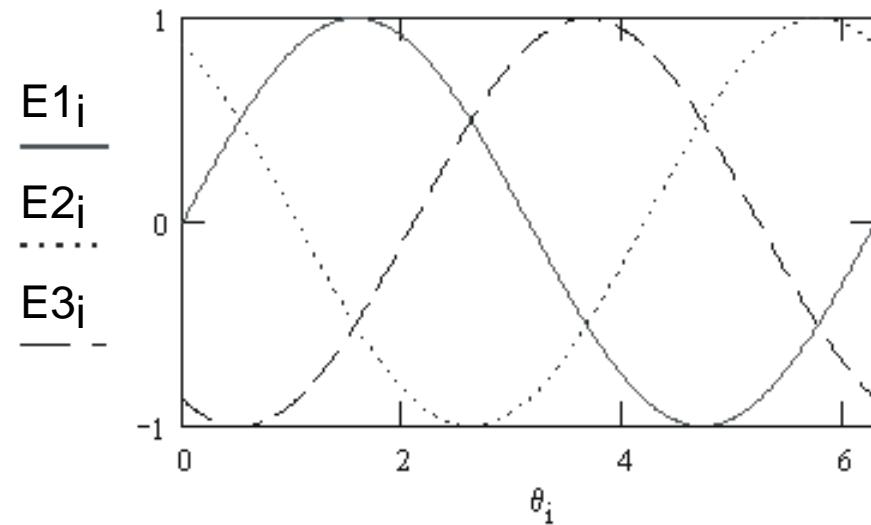
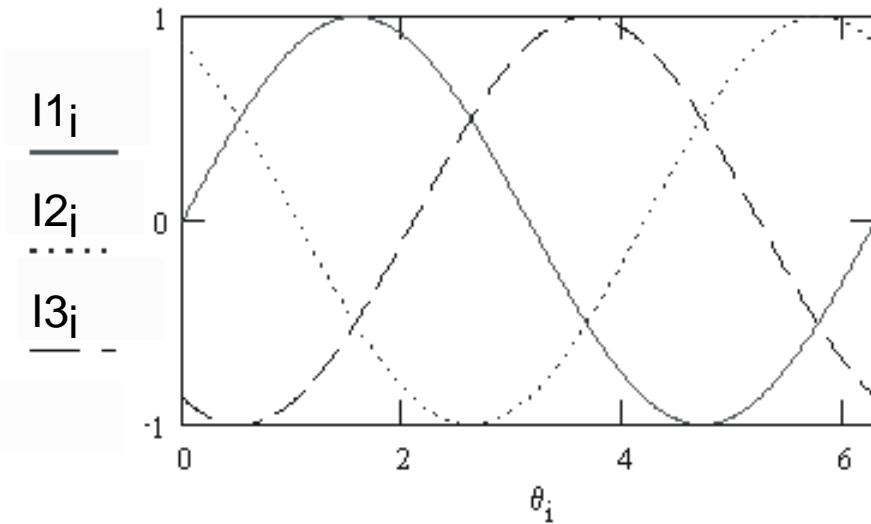
$$\begin{cases} u_a = R i_a + L_s \frac{d i_a}{d t} + e_{pma} \\ u_b = R i_b + L_s \frac{d i_b}{d t} + e_{pmb} \\ u_c = R i_c + L_s \frac{d i_c}{d t} + e_{pmc} \end{cases}$$

$$\begin{cases} e_{pma} = \hat{e}_{pm} \sin\left(\frac{\pi}{\tau_p} x\right) \\ e_{pmb} = \hat{e}_{pm} \sin\left(\frac{\pi}{\tau_p} x - \frac{2}{3}\pi\right) \\ e_{pmc} = \hat{e}_{pm} \sin\left(\frac{\pi}{\tau_p} x - \frac{4}{3}\pi\right) \end{cases}$$

What should be the current phase to make maximum force?

# PMSM currents

$$\begin{cases} i_a = \hat{i} \sin\left(\frac{\pi}{\tau_p} x\right) \\ i_b = \hat{i} \sin\left(\frac{\pi}{\tau_p} x - \frac{2}{3}\pi\right) \\ i_c = \hat{i} \sin\left(\frac{\pi}{\tau_p} x - \frac{4}{3}\pi\right) \end{cases}$$



# PMSM power and force

$$p = u_a i_a + u_b i_b + u_c i_c$$

$$p = R i_a^2 + R i_b^2 + R i_c^2 + \frac{d \frac{1}{2} L (i_a^2 + i_b^2 + i_c^2)}{dt} + e_{pma} i_a + e_{pmb} i_b + e_{pmc} i_c$$

$$p = \frac{3}{2} R \hat{i}^2 + \frac{d W_f}{dt} + \frac{3}{2} \hat{e}_{pm} \hat{i} = p_{Cu} + p_f + p_{mech}$$

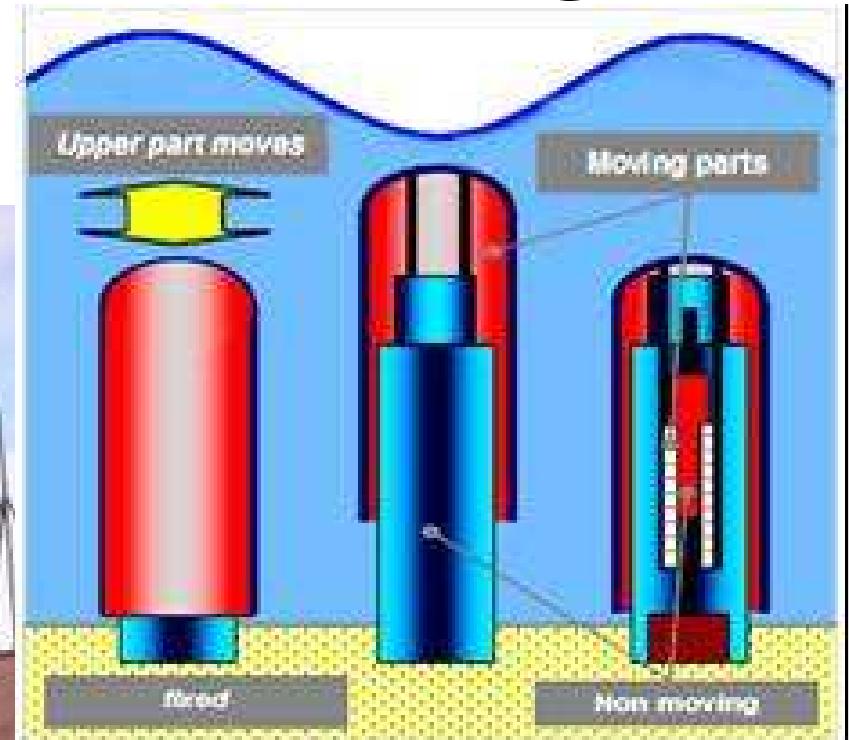
Maximum three-phase force:

$$F = \frac{p_{mech}}{\nu} = \frac{3 \hat{e}_{pm} \hat{i}}{2\nu} = \frac{3}{2} \frac{\pi}{\tau_p} N \hat{\Phi}_{pm} \hat{i}$$

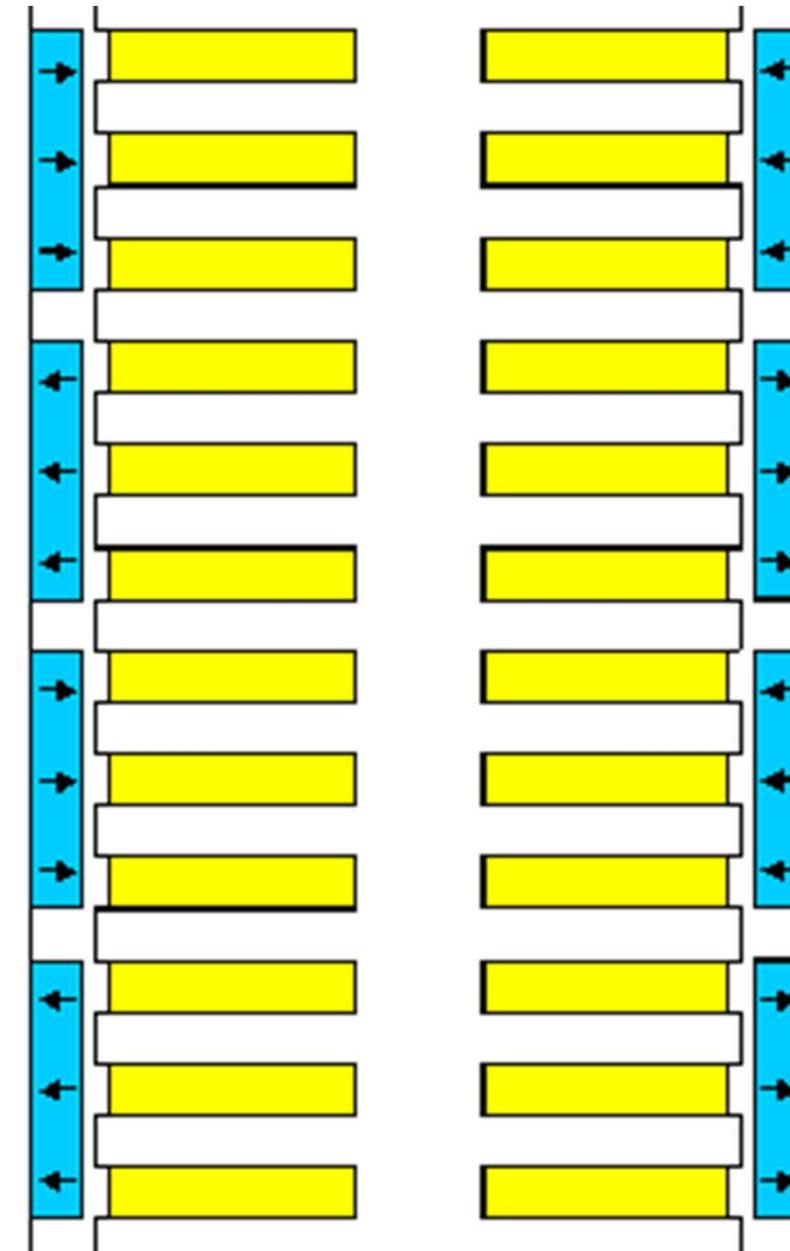
Voltage

$$\hat{u} = \sqrt{(R \hat{i} + \omega N \hat{\Phi}_{pm})^2 + (\omega L \hat{i})^2}$$
$$\omega = \frac{\pi}{\tau_p} \frac{dx}{dt}$$

# Example: Archimedes Wave Swing

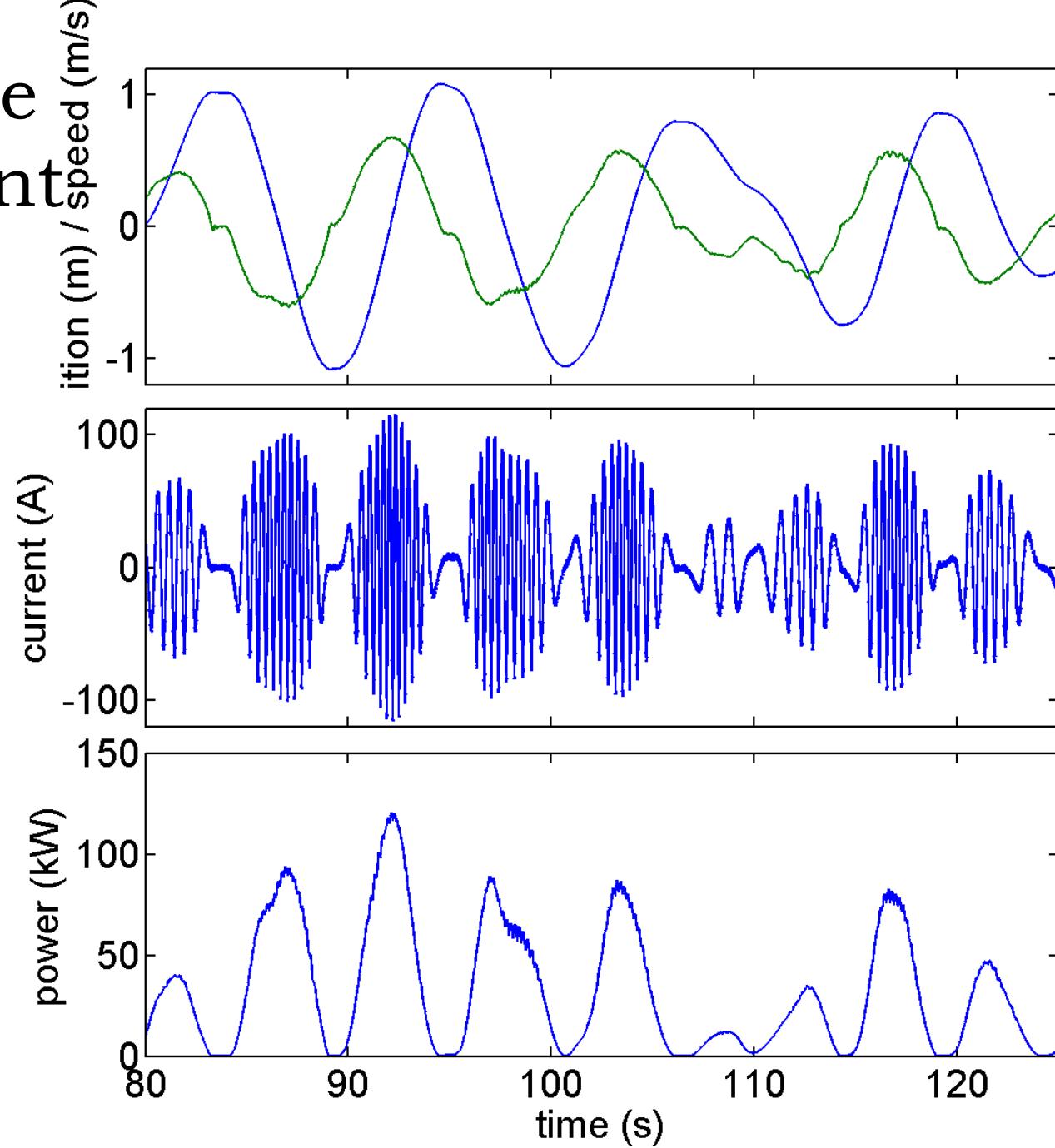


# Linear AWS generator





# Measure ment



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