Overview Electrical Machines and Drives

- 7-9 1: Introduction, Maxwell's equations, magnetic circuits
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Maxwell's equations / magnetic circuits

- Introduction of Maxwell's equations (for quasi-static fields)
- Ampere's law used to calculate flux densities (1.1)
  - Around a wire in air
  - In magnetic circuit
  - In a magnetic circuit with an gap
- Second of Maxwell's equations used to calculate voltages (1.1)
- Soft magnetic materials: hysteresis and eddy currents (1.2)
- Hard magnetic materials: permanent magnets (1.4)

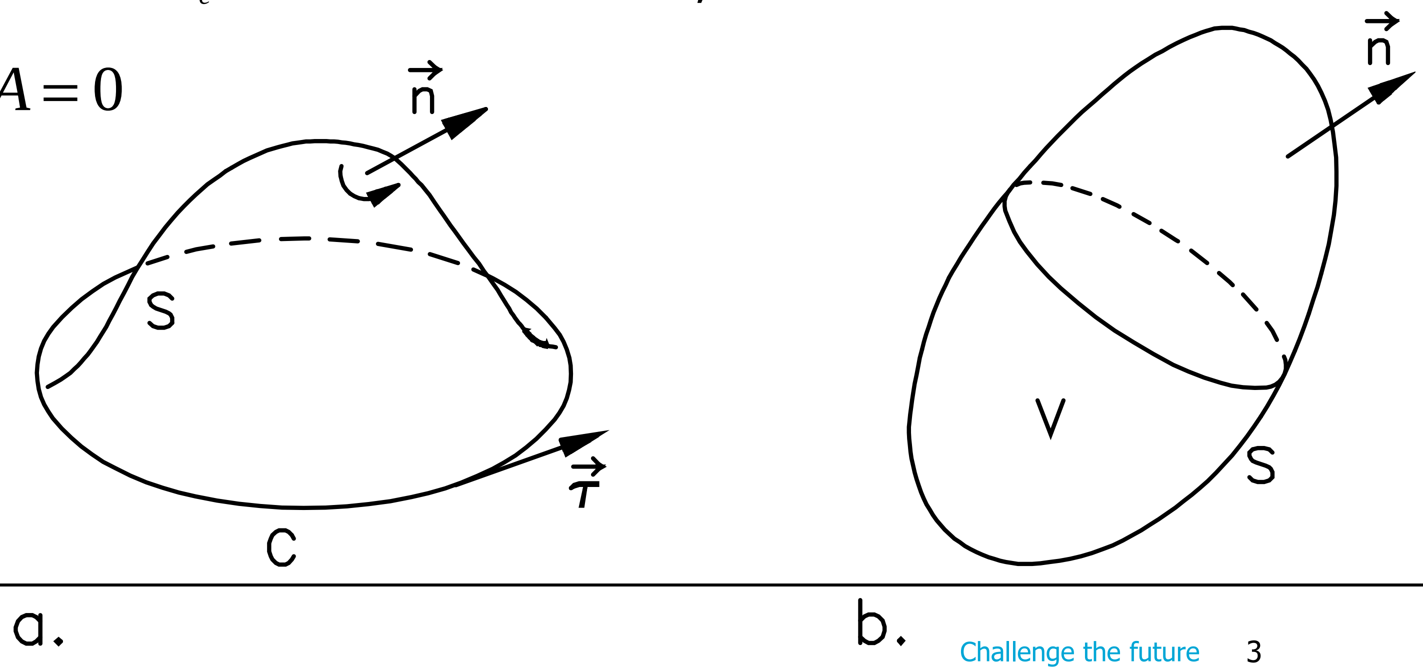
Maxwell's equations for quasi-static fields: Know by heart!

$$\oint_{C_m} \vec{H}\cdot\vec{\tau}\,\mathrm{d}s = \iint_{S_m} \vec{J}\cdot\vec{n}\,\mathrm{d}A$$

$$\oint_{C_e} \vec{E}\cdot\vec{\tau}\,\mathrm{d}s = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{S_e} \vec{B}\cdot\vec{n}\,\mathrm{d}A$$

$$\oiint_{S} \vec{B}\cdot\vec{n}\,\mathrm{d}A = 0$$

H : magnetic field intensity
J : current density
E : electric field intensity
B : magnetic flux density
n, τ: unit vectors



Magnetic field in a magnetic circuit

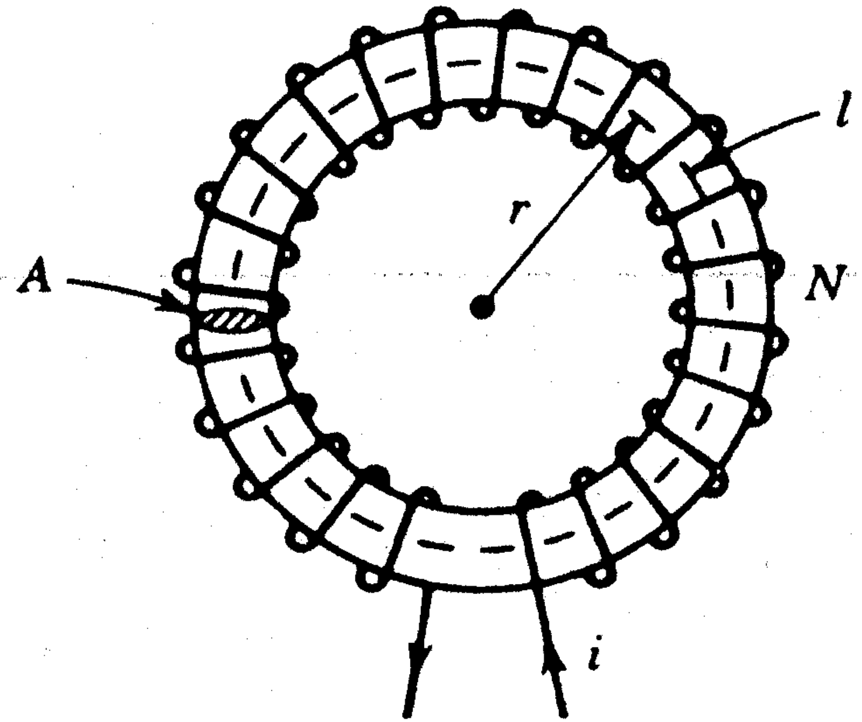
Contour follows magnetic path:

$$\oint_{C_m} \vec{H}\cdot\vec{\tau}\,\mathrm{d}\,s = \iint_{S_m} \vec{J}\cdot\vec{n}\,\mathrm{d}\,A$$

$$Hl = Ni \quad\Rightarrow\quad H = \frac{Ni}{l}$$

$$B = \mu_0\mu_r H = \mu_0\mu_r\frac{Ni}{l}$$

$$\Phi = BA_{core} = \frac{Ni}{R_m} = \frac{Ni}{\dfrac{l}{\mu_0\mu_r A_{core}}}$$



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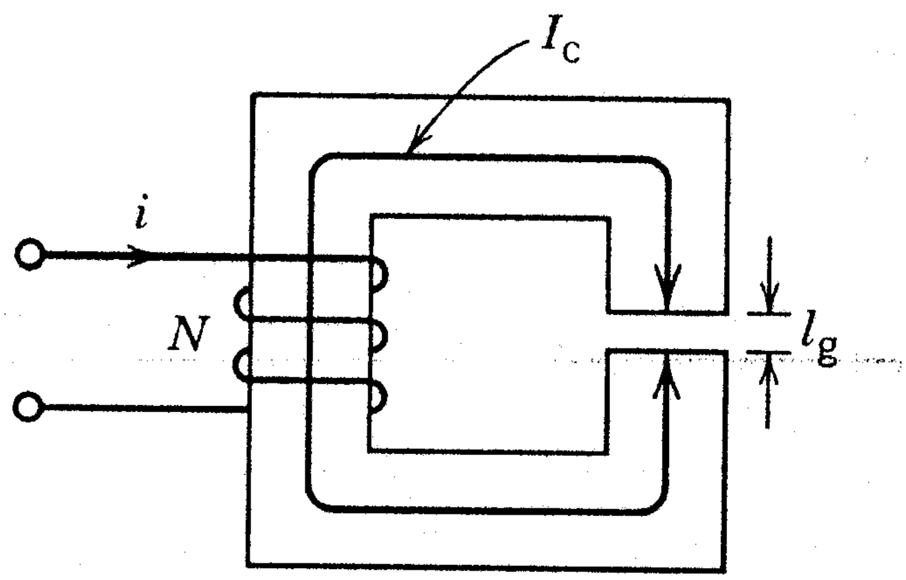
Magnetic circuit with air gap

$$\oint_{C_m} \vec{H} \cdot \vec{\tau} \, \mathrm{d}s = \iint_{S_m} \vec{J} \cdot \vec{n} \, \mathrm{d}A$$

$$H_c l_c + H_g l_g = Ni$$

$$\frac{B_c}{\mu_0 \mu_{rc}} l_c + \frac{B_g}{\mu_0} l_g = Ni$$



$$\oiint_{S} \vec{B} \cdot \vec{n} \, \mathrm{d}A = 0$$

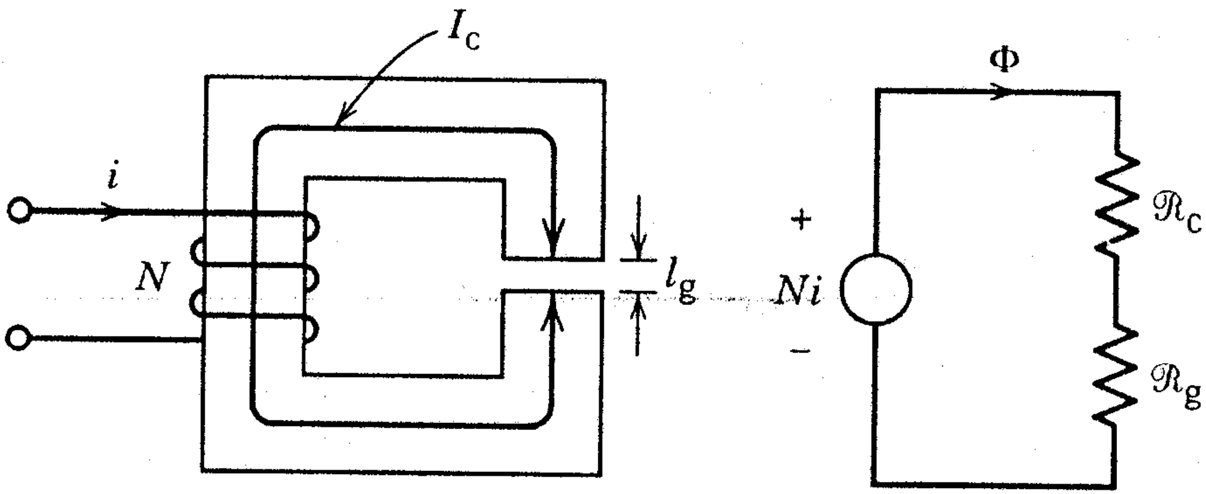
$$B_c A_c = B_g A_g$$

Two equations, two unknowns, can be solved

Magnetic circuit with air gap

$$R_{mc} = \frac{l_c}{\mu_0 \mu_r A_c}$$

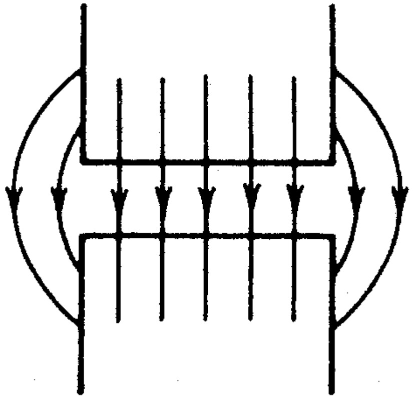
$$R_{mg} = \frac{l_g}{\mu_0 \sigma A_g}$$



$$\Phi = \frac{Ni}{R_{mg} + R_{mc}}$$

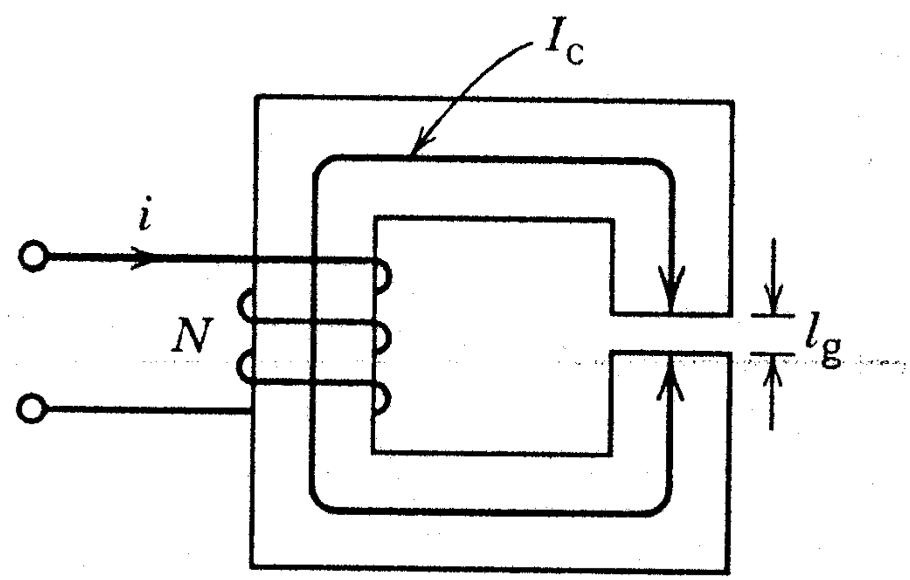
Which reluctance is dominating?

The factor σ



Example magnetic circuit with air gap

- Given
  - $N = 50$
  - $I = 1$ A
  - $\mu_{ry} = 5000$
  - $\mu_0 = 4\pi 10^{-7}$ H/m
  - $l_c = 0.2$ m
  - $l_g = 1$ mm
  - $A_c = 4$ cm^2
  - $\sigma = 1$
- Calculate
  - Reluctances
  - Flux
  - Flux density



Example magnetic circuit with air gap

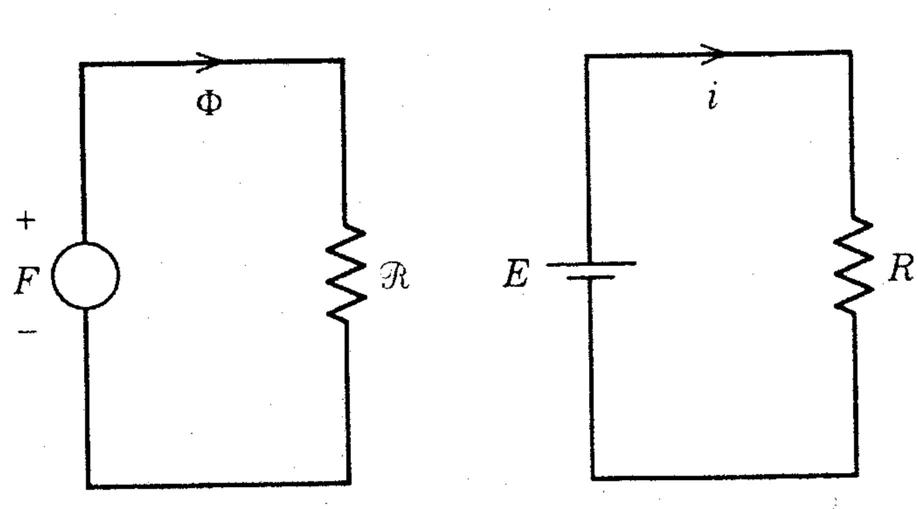
$$R_{mc} = \frac{l_c}{\mu_0 \mu_r A_c} = 79.6 \frac{\text{kA}}{\text{Wb}}$$

$$R_{mg} = \frac{l_g}{\mu_0 \sigma A_g} = 1.99 \frac{\text{MA}}{\text{Wb}}$$

$$\Phi = \frac{Ni}{R_{mg} + R_{mc}} = 24.2 \mu\text{Wb}$$

$$B = \frac{\Phi}{A_c} = 61\text{mT}$$

Comparison of circuits



Circuit	Water	Electric	Magnetic
Driving force	Force F Pressure	Voltage V Electric field E	Mmf Ni Magnetic field H
Produces	Flow density Water flow	Current density J Current I	Flux density B Flux Φ
Limited by	Resistance	Resistance R	Reluctance R_m
Isolators	Yes	Yes	No
Physical	Water	Electrons	?
Power	Fv	VI	0

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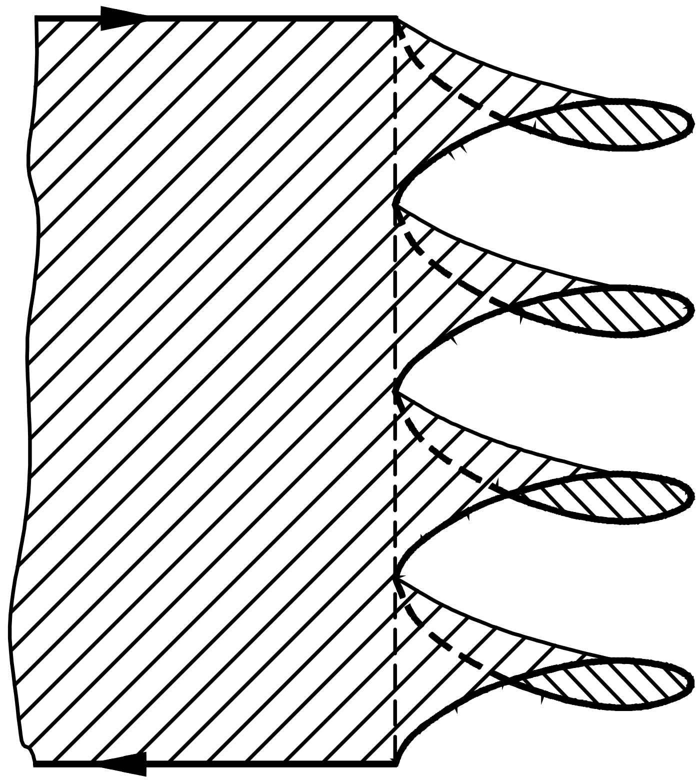
Second of Maxwell's equations

$$\oint_{C_e} \vec{E} \cdot \vec{\tau} \, \mathrm{d}s = -\frac{\mathrm{d}}{\mathrm{d}t} \iint_{S_e} \vec{B} \cdot \vec{n} \, \mathrm{d}A$$

Contour is chosen in the electric path, in the wire!



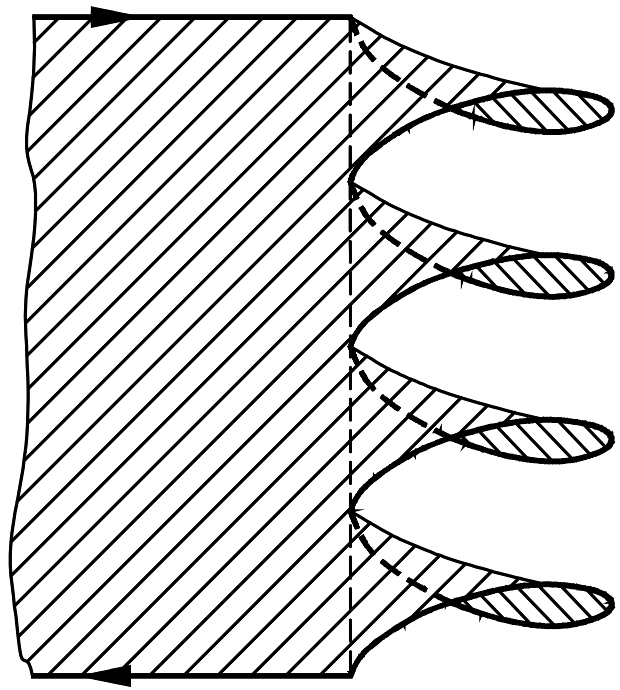
Calculation of voltage

$$\oint_{C_e} \vec{E}\cdot\vec{\tau}\,\mathrm{d}s = \int_{+term}^{-term} \vec{E}\cdot\vec{\tau}\,\mathrm{d}s + \int_{-term}^{+term} \vec{E}\cdot\vec{\tau}\,\mathrm{d}s$$

$$\oint_{C_e} \vec{E}\cdot\vec{\tau}\,\mathrm{d}s = \int_{+term}^{-term} \rho_{Cu}\vec{J}\cdot\vec{\tau}\,\mathrm{d}s - u = l_{Cu}\rho_{Cu}J - u$$

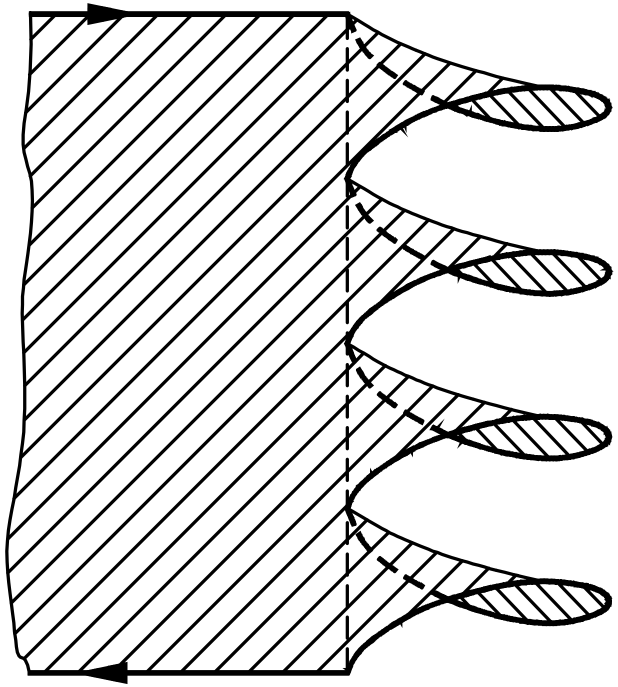
$$J = \frac{i}{A_{Cu}}$$

$$\oint_{C_e} \vec{E}\cdot\vec{\tau}\,\mathrm{d}s = \frac{\rho_{Cu}l_{Cu}}{A_{Cu}}i - u = Ri - u$$



Calculation of voltage

$$\oint_{C_e} \vec{E} \cdot \vec{\tau} \, \mathrm{d}s = -\frac{\mathrm{d}}{\mathrm{d}t} \iint_{S_e} \vec{B} \cdot \vec{n} \, \mathrm{d}A$$



With the flux linkage

$$\lambda = \iint_{S_e} \vec{B} \cdot \vec{n} \, \mathrm{d}A \approx N\Phi$$

This can be worked out to

$$u = Ri + \frac{\mathrm{d}\lambda}{\mathrm{d}t} \approx Ri + N\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

For coils

Induced voltage: $$e = \frac{\mathrm{d}\lambda}{\mathrm{d}t} \approx N \frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

For a coil: $$\lambda = N\Phi = \frac{N^2}{R_m} i = Li$$

$$L = \frac{N^2}{R_m}$$

$$e = \frac{\mathrm{d}\lambda}{\mathrm{d}t} = N \frac{\mathrm{d}\Phi}{\mathrm{d}t} = L \frac{\mathrm{d}i}{\mathrm{d}t}$$

Special cases

$$u = Ri + \frac{\mathrm{d}\lambda}{\mathrm{d}t}$$

- for DC:

  $$u = Ri$$

  - flux and flux linkage are determined by current

- for sinusoidal AC if resistance is negligible:

  $$u = \frac{\mathrm{d}\lambda}{\mathrm{d}t} \qquad U = \frac{1}{\sqrt{2}}\omega\hat{\lambda} = \frac{2\pi}{\sqrt{2}} f\hat{\lambda} = 4.44\, f\hat{\lambda}$$

  - flux and flux linkage are determined by voltage

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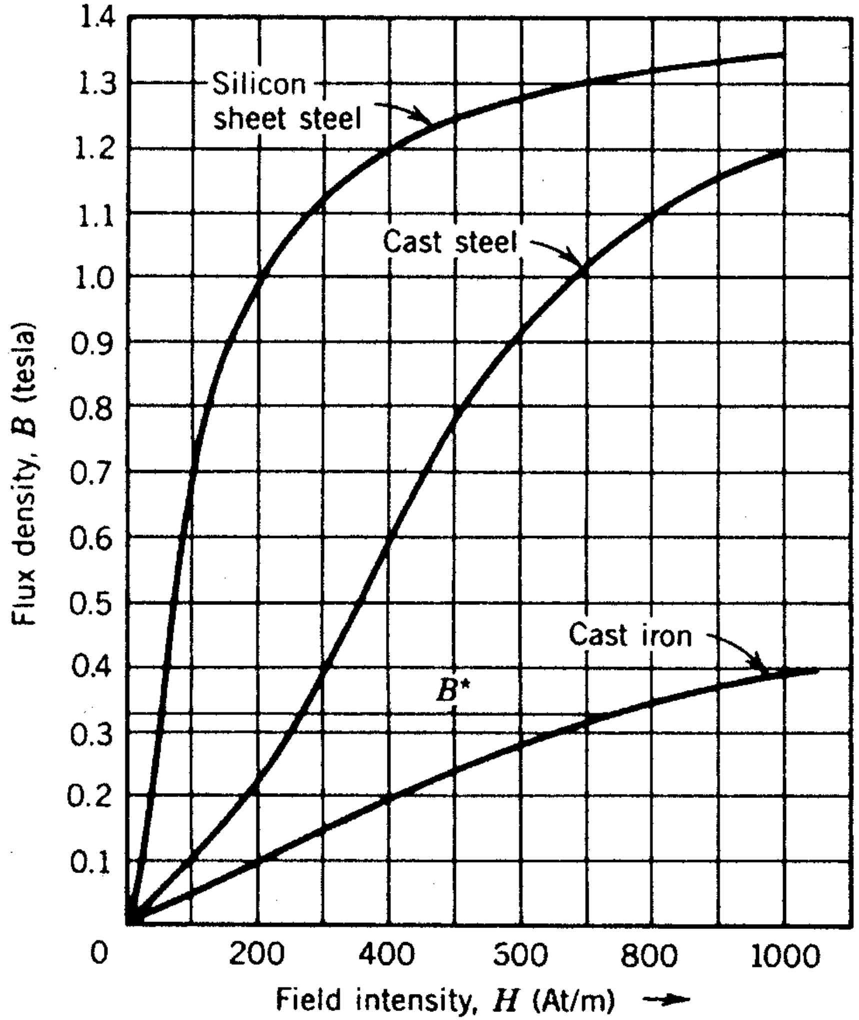
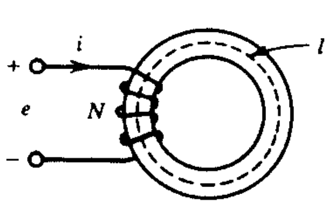
Magnetization curves

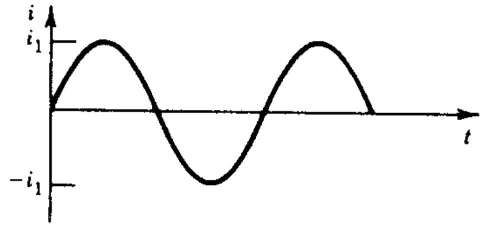


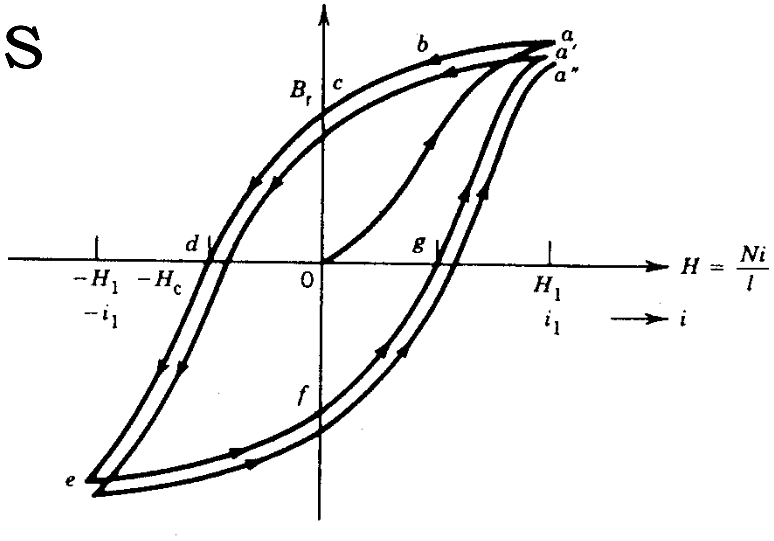
2T: heavily saturated silicon steel

limitation

Iron: Hysteresis losses







(b)

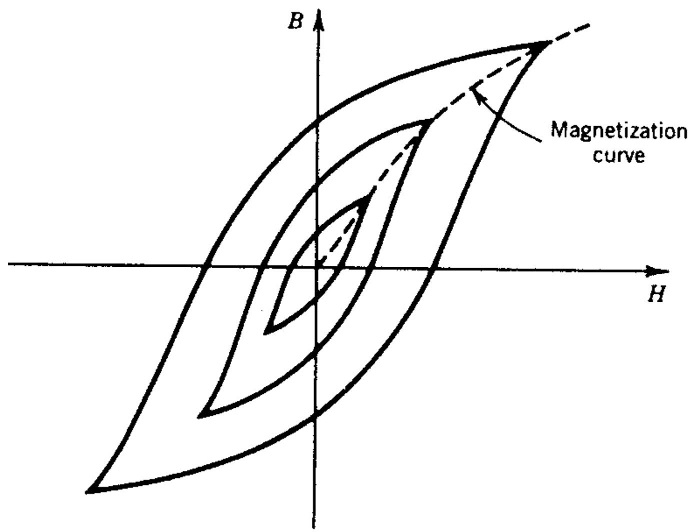
$$W = \int ui\,\mathrm{d}t = \int Ri^2 + i\frac{\mathrm{d}\lambda}{\mathrm{d}t}\mathrm{d}t$$

$$W_h = \int i\frac{\mathrm{d}\lambda}{\mathrm{d}t}\mathrm{d}t = \int i\,\mathrm{d}\lambda$$

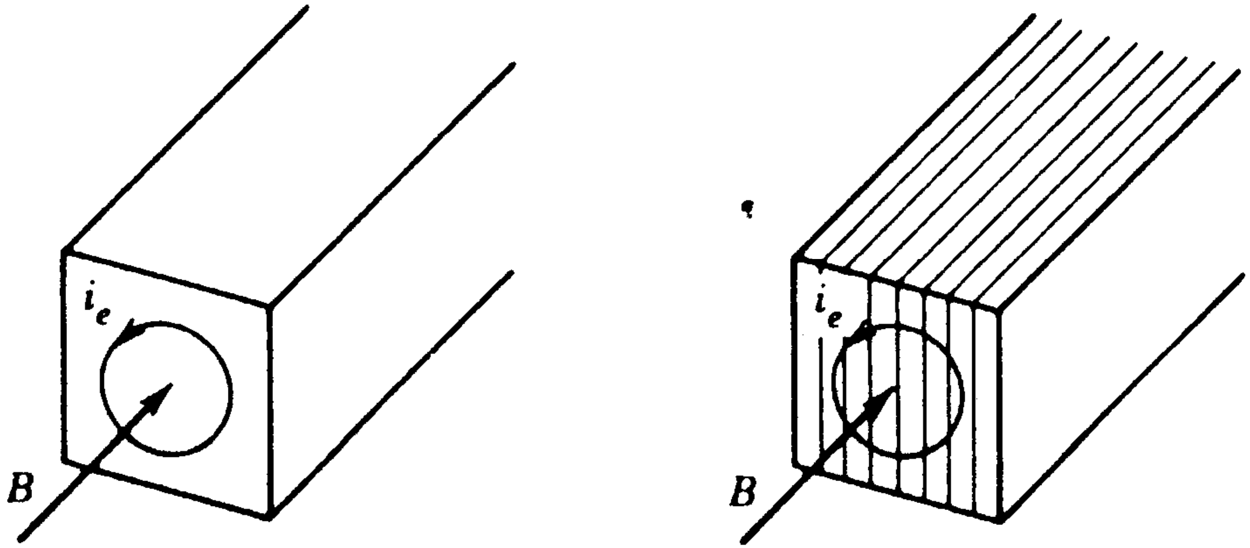
With $\lambda = NBA$ $\qquad i = \frac{Hl}{N}$

$$W_h = Al\int H\,\mathrm{d}B$$

$$P_h = \iiint k_h f\hat{B}^S\,\mathrm{d}V \qquad 1.5 < S < 2.3$$



Iron: eddy current losses



$$P_{Fe,e} \approx \iiint_V \frac{b_{Fe}^2 \omega^2 B^2}{12\rho_{Fe}} \mathrm{d}V \approx k_e \iiint_V \left(\frac{\omega}{\omega_0} \frac{B}{B_0} \right)^2 \mathrm{d}V$$

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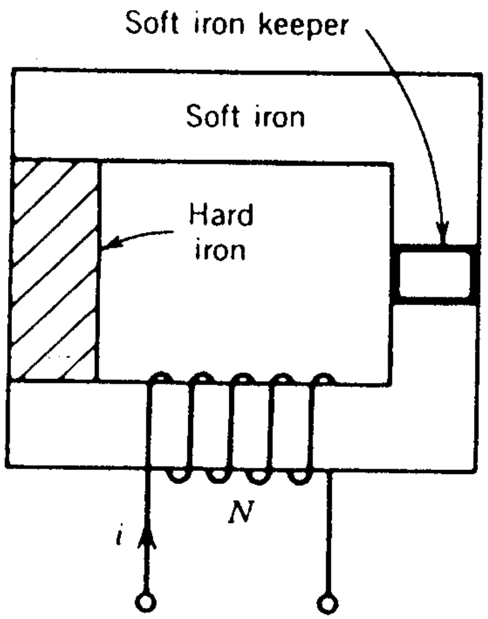
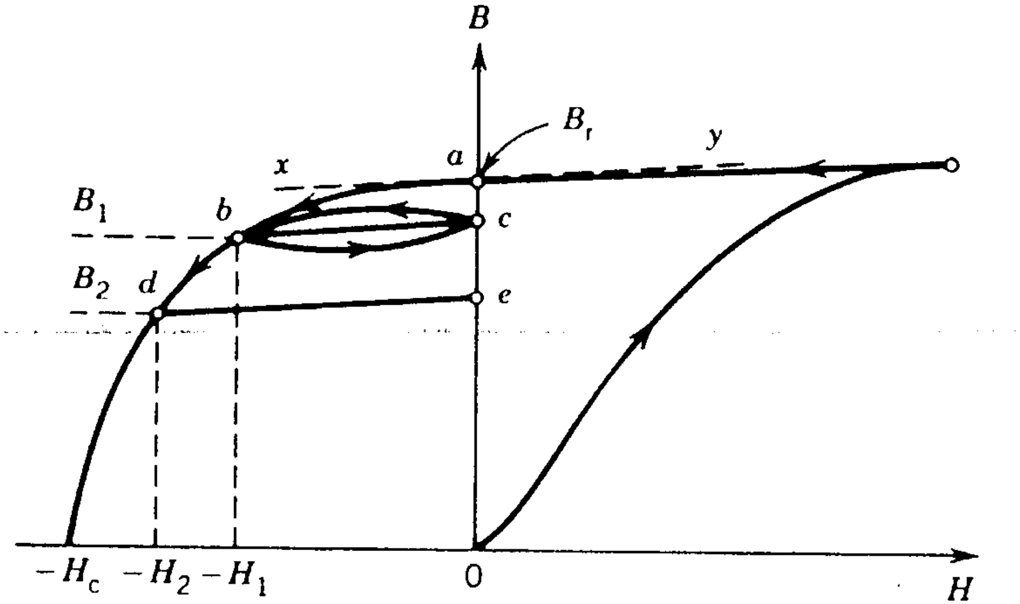
Magnetic circuits with permanent magnets

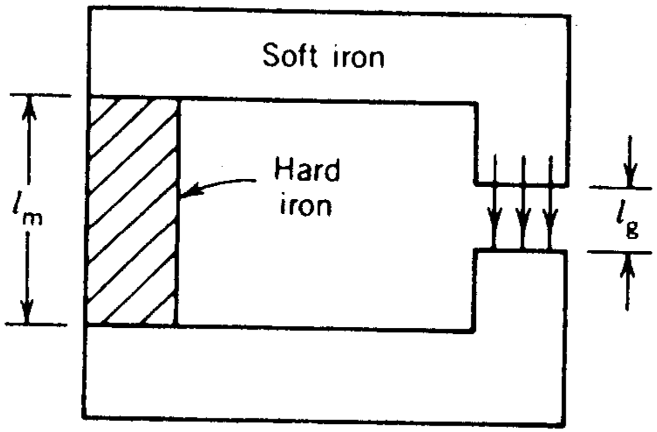
- Why increase of permanent magnet use?
  - field without current: no losses, no windings, small volume
  - new strong rare-earth magnets become cheaper

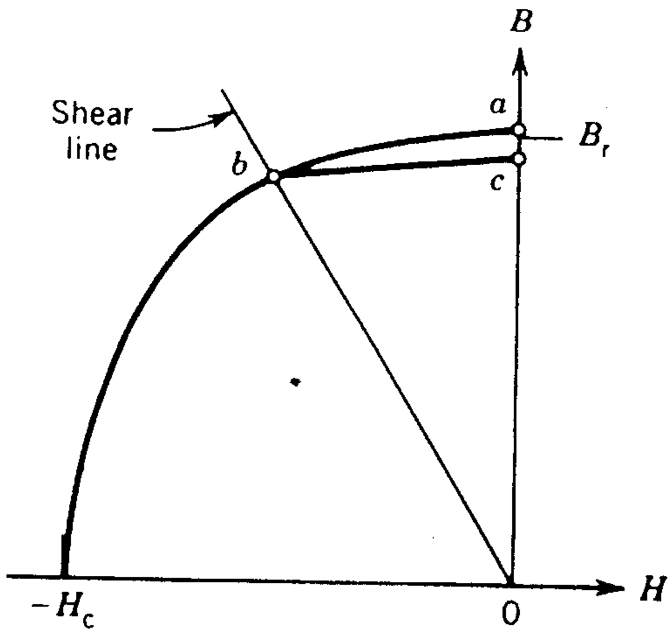
Magnetizing magnets





Calculating magnetic fields with magnets





$$\oint_{C_m} \vec{H} \cdot \vec{\tau}\, \mathrm{d}s = \iint_{S_m} \vec{J} \cdot \vec{n}\, \mathrm{d}A \qquad H_m l_m + H_g l_g = 0$$

permeability iron assumed infinite

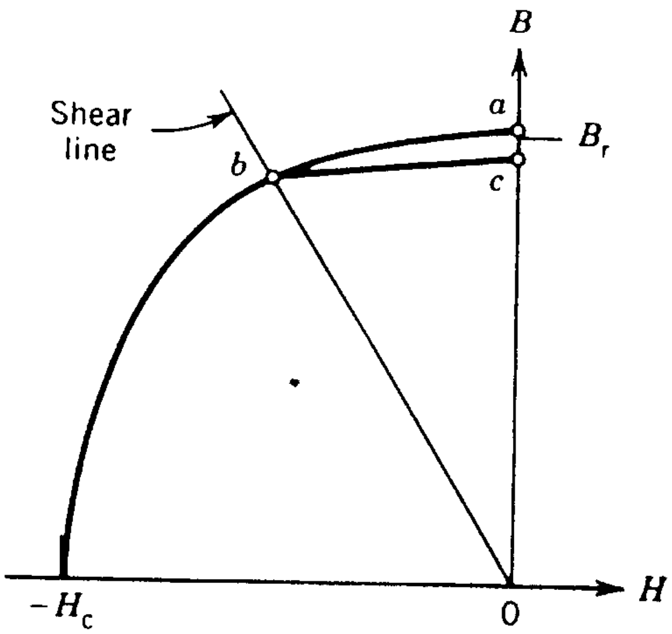
Calculating magnetic fields with magnets

$$H_m = -H_g \frac{l_g}{l_m}$$

$$B_g = \mu_0 H_g$$

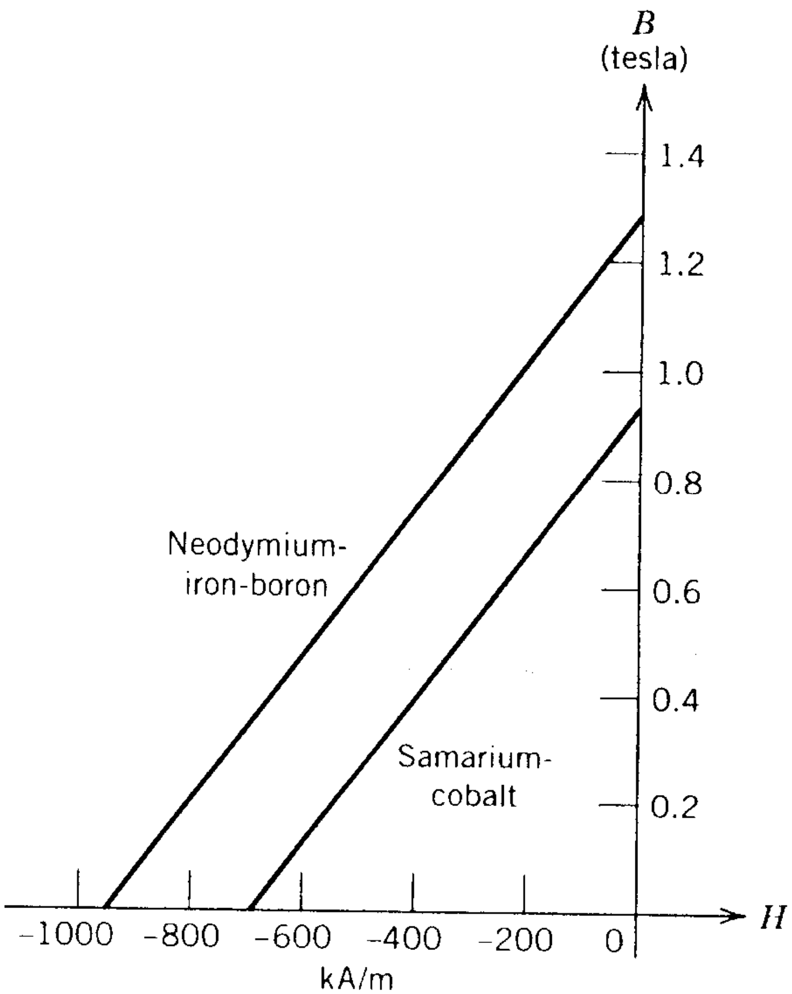
$$\oiint_S \vec{B} \cdot \vec{n}\, \mathrm{d}A = 0 \qquad B_g A_g = B_m A_m$$

$$B_m = -\mu_0 \frac{A_g l_m}{A_m l_g} H_m$$



Defines shear line (in Sen 1.47, a minus sign is missing)
Operating point: intersection of shear line and BH characteristic

Rare earth magnets



$$B_m = \mu_0 \mu_{rm} H_m + B_r$$

Calculation with magnets

Combine shear line with BH curve:

$$B_m = \mu_0 \mu_{rm} H_m + B_r$$

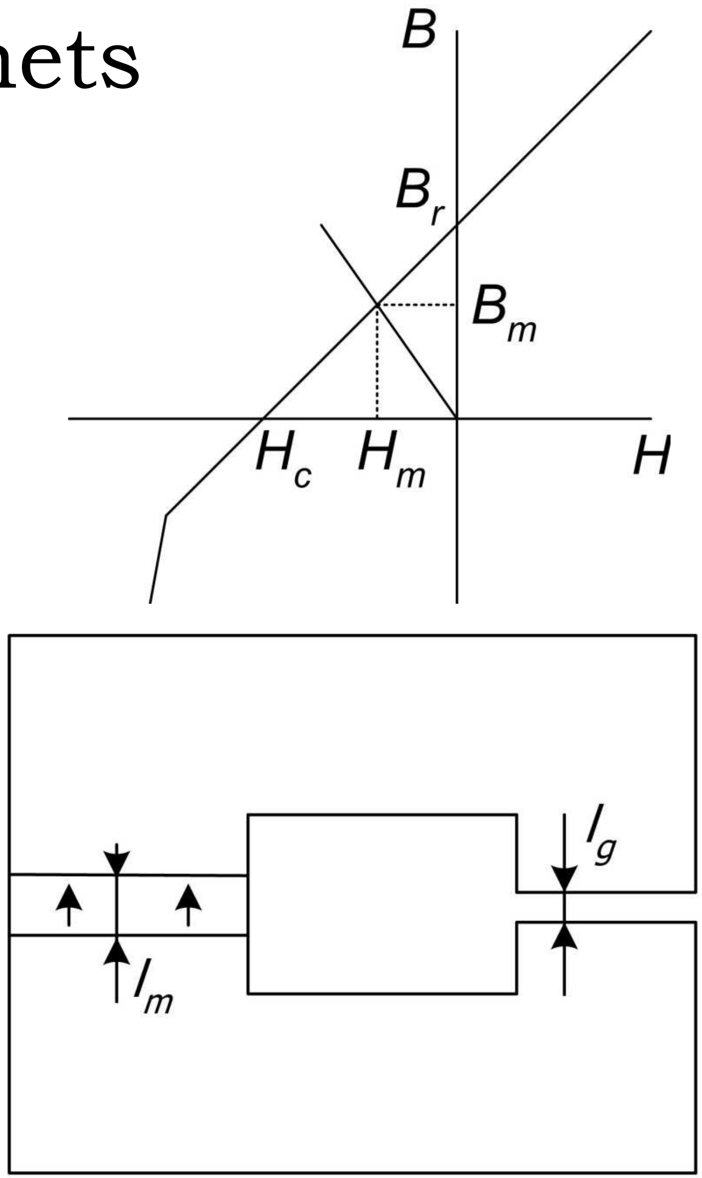
$$B_m = -\mu_0 \frac{A_g l_m}{A_m l_g} H_m$$

Result: $$B_g = \frac{l_m A_m}{l_m A_g + \mu_{rm} l_g A_m} B_r$$

$$A_m = A_g$$ If

$$B_g = \frac{l_m}{l_m + \mu_{rm} l_g} B_r$$

What is the effect of increasing the gap?



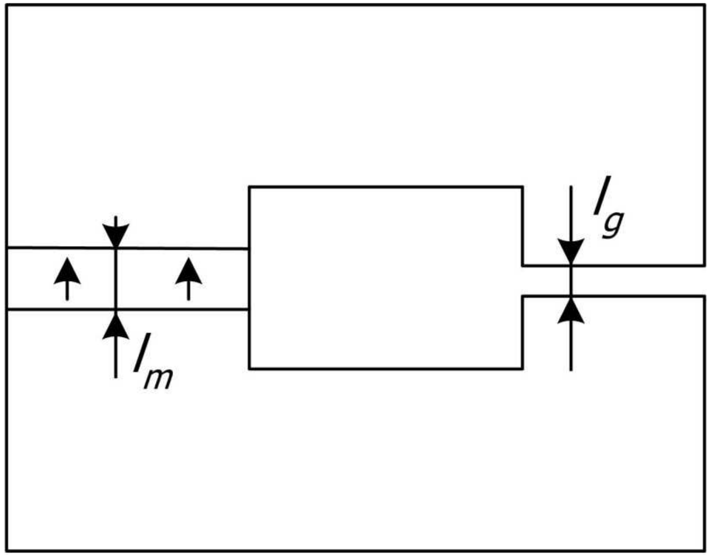
Alternative: using magnetic circuit theory

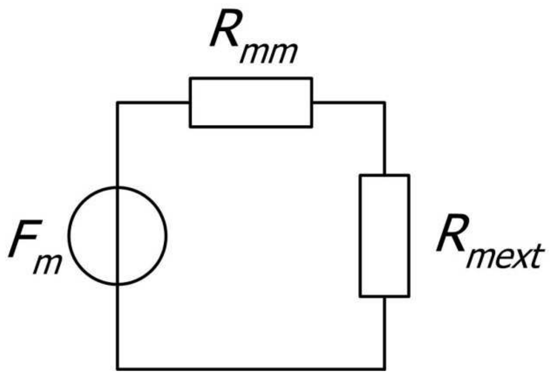
Magnetomotive force of a magnet:

$$F_m = -H_c l_m = \frac{B_r}{\mu_0 \mu_{rm}} l_m$$

$$R_{mm} = \frac{l_m}{\mu_0 \mu_r A_m} \qquad R_{mext} = \frac{l_g}{\mu_0 \mu_r A_g}$$

$$\Phi = \frac{-H_c l_m}{R_{mm} + R_{mg}}$$





A magnet can be replaced by an air coil with length l_m and cross section A_m and magnetomotive force $H_c l_m$

Permanent magnet characteristics

- Characteristics:
  - H_c coercive force (Coërcitief veldsterkte) [A/m],
  - B_r remanent flux density (remanente inductie) [T]
  - μ_{rm} relative permeability (1.05 ...1.15)

	B_r (T)	H_{cB} (kA/m)	dB_r/dT (%/K)	dH_{cB}/dT (%/K)	ρ (μΩm)	Cost (€/kg)
Ferrite	0,4	-250	-0,2	+0,34	10^{12}	2
Alnico	1,2	-130	-0,05	-0,25	0,5	20
SmCo	1,0	-750	-0,02	-0,03	0,5	100
NdFeB	1,4	-1000	-0,12	-0,55	1,4	75

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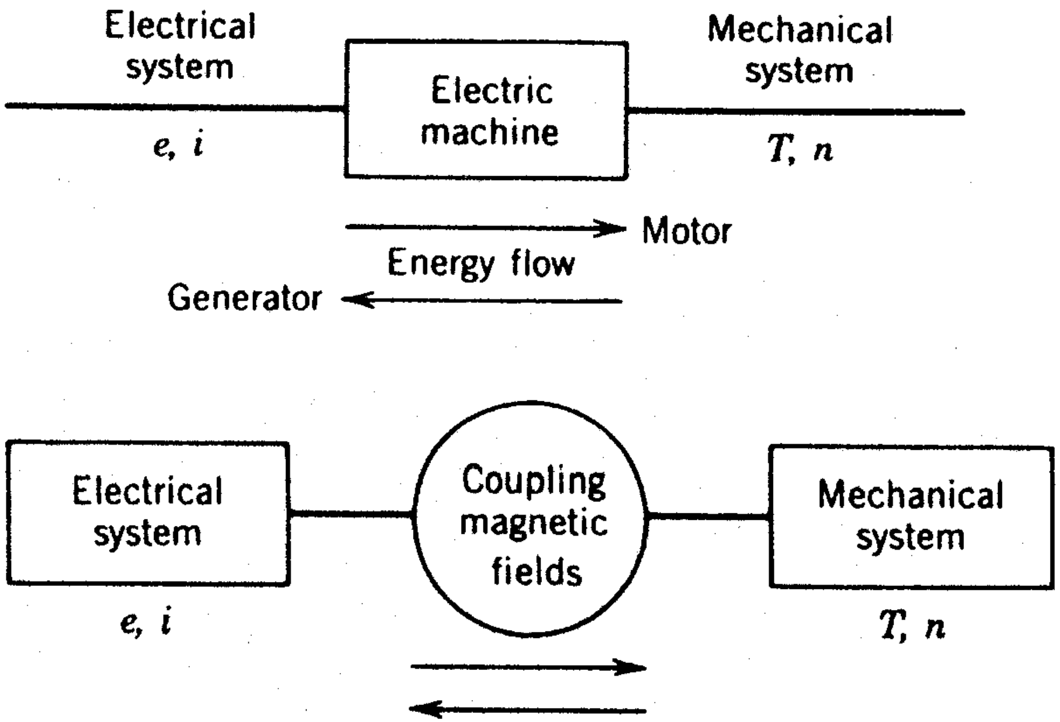
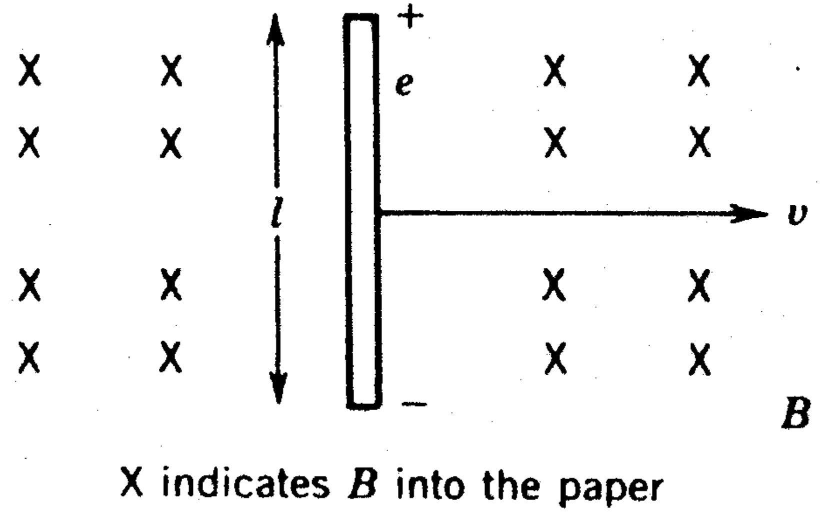
Principles of electromechanics (3)

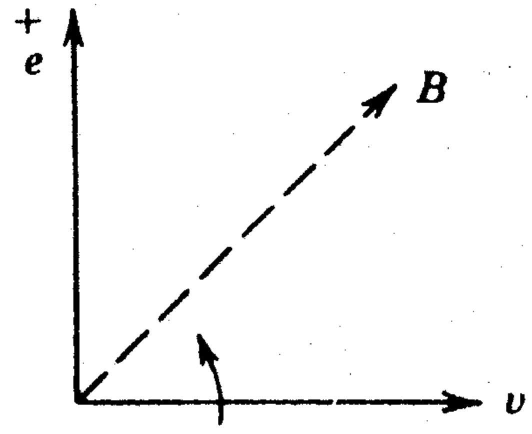
- Lorentz force, induced voltage (4.1)
- Energy or power balance (3.1)
- Energy and coenergy (3.2)
- Calculation of force from (co)energy (3.3)
- Application to actuators and rotating machines (3.4, 3.5)

Electromagnetic energy conversion (4.1)



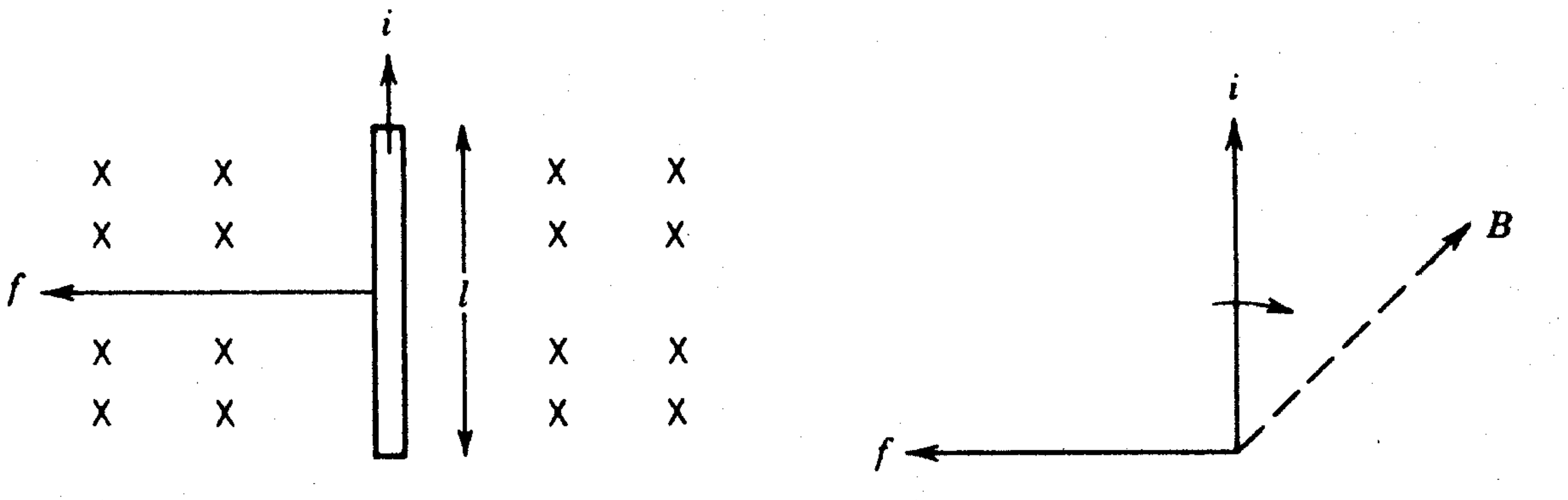
Induced voltage





$$E = Blv$$

Lorentz force

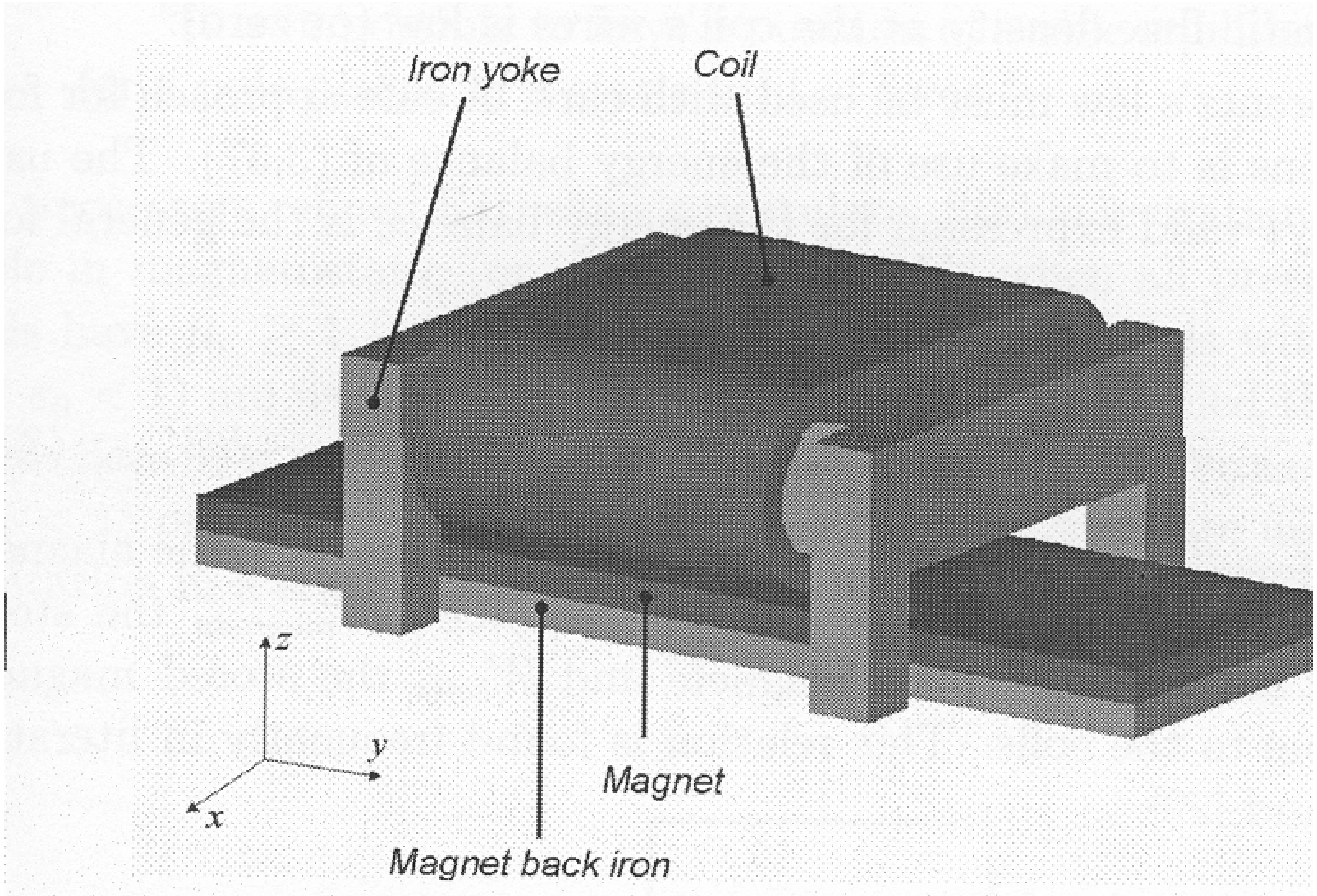


(a) (b)

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \qquad F = Bli$$

Power balance holds: $P = Ei = Blvi = Fv$

Lorentz force



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