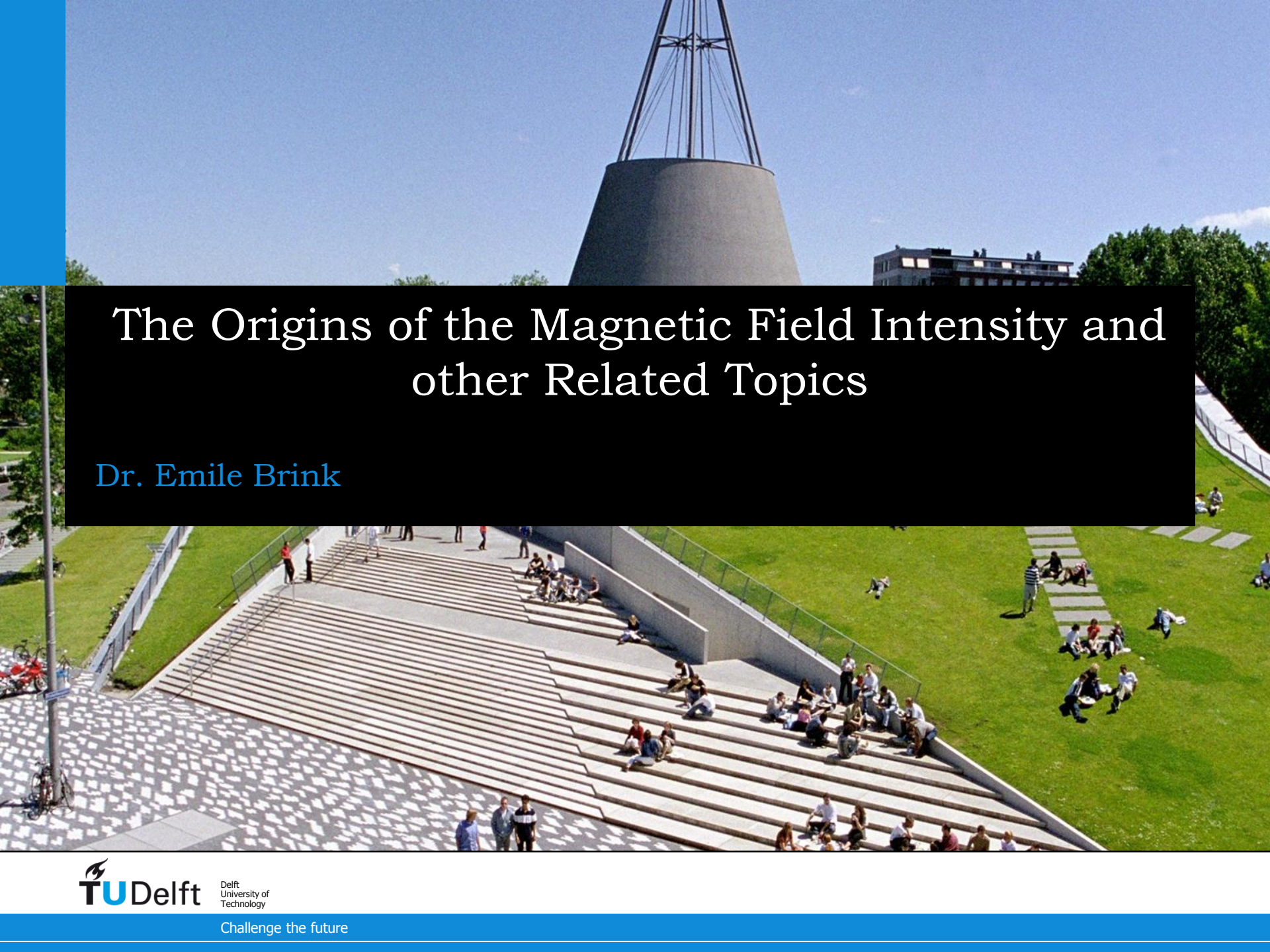




The Origins of the Magnetic Field Intensity and other Related Topics

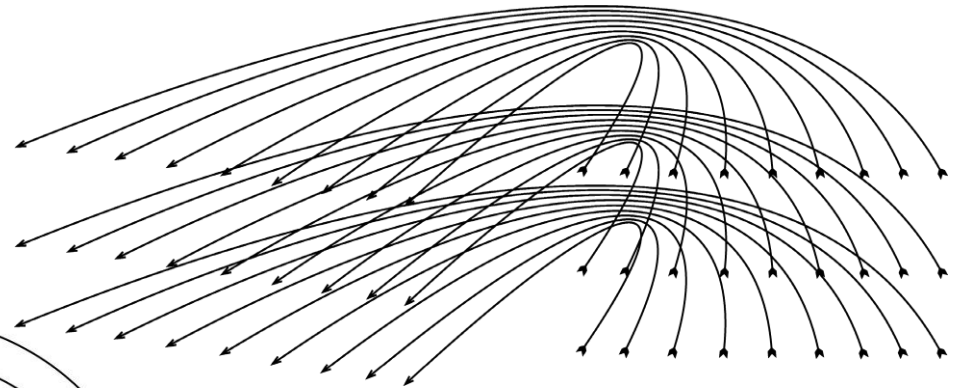
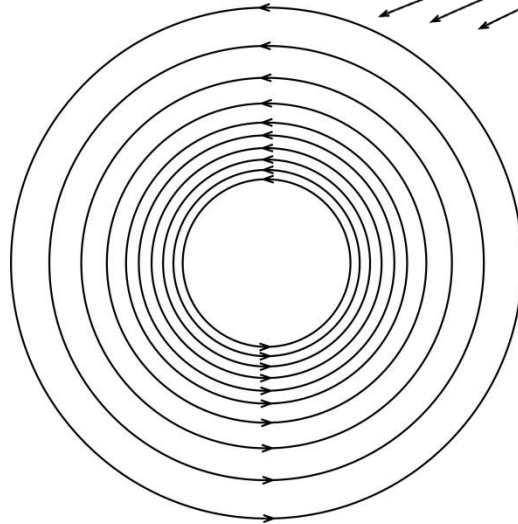
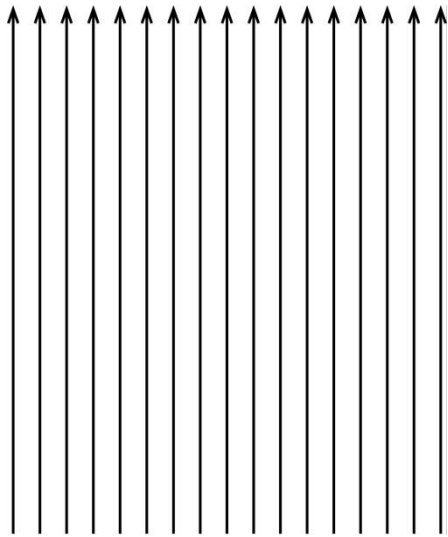
Dr. Emile Brink

Contents

- Origins of the Magnetic Field Intensity (H)
 - Vector field, curl and Stoke's Theorem
 - Point form of Ampere's law
 - Applied to non-magnetic conductor
 - Ferromagnetic materials
 - Magnetic field intensity (H)
 - Hysteresis curve and permanent magnets
- Inductance

Vector Field F

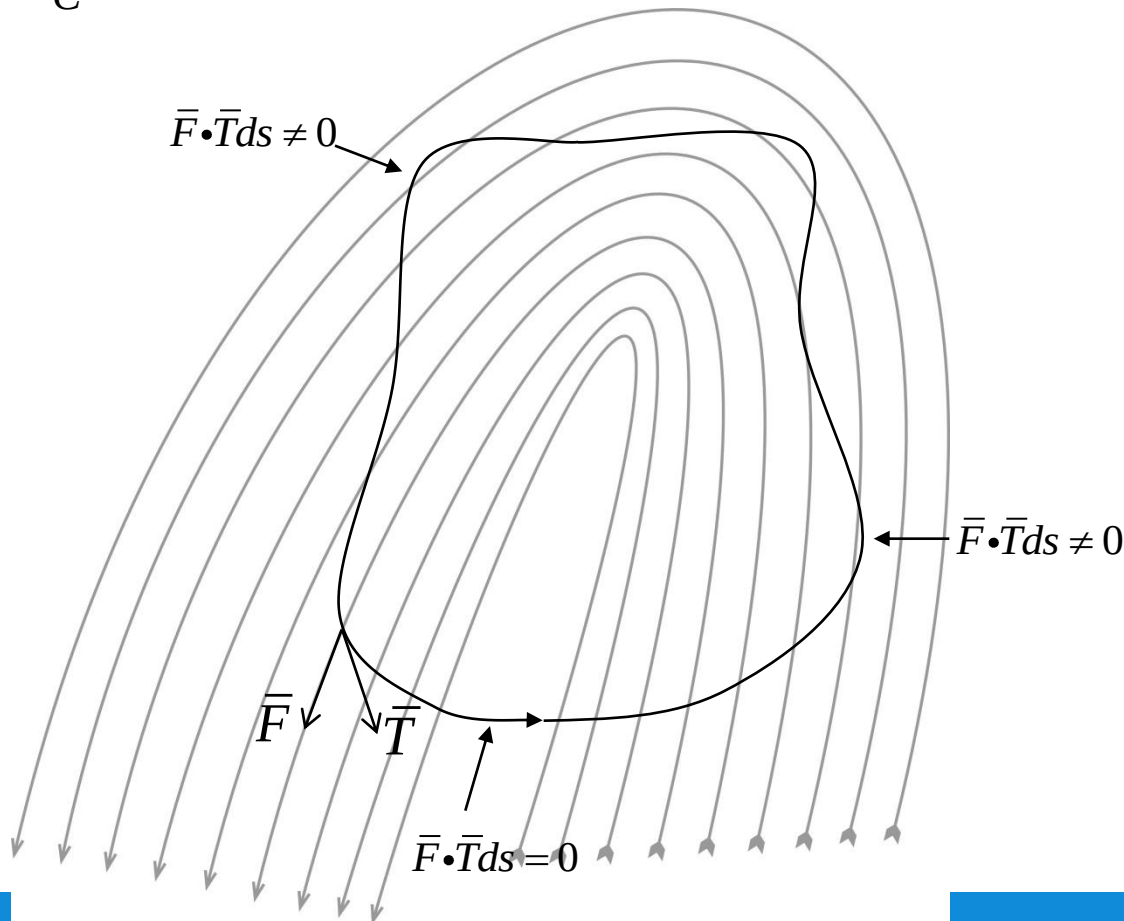
$$\bar{F} = M\hat{i} + N\hat{j} + P\hat{k}$$



Circulation of $\vec{F}$ around C

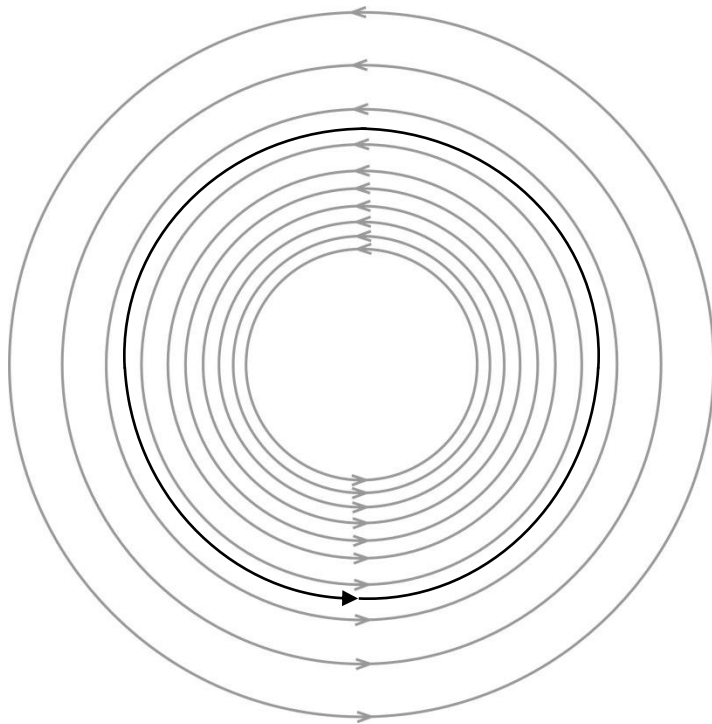
$\oint_C \vec{F} \cdot \vec{T} ds = \text{circulation of } \vec{F} \text{ around path } C$

$$\oint_C \vec{F} \cdot \vec{T} ds \neq 0$$



Circulation of $\vec{F}$ around C

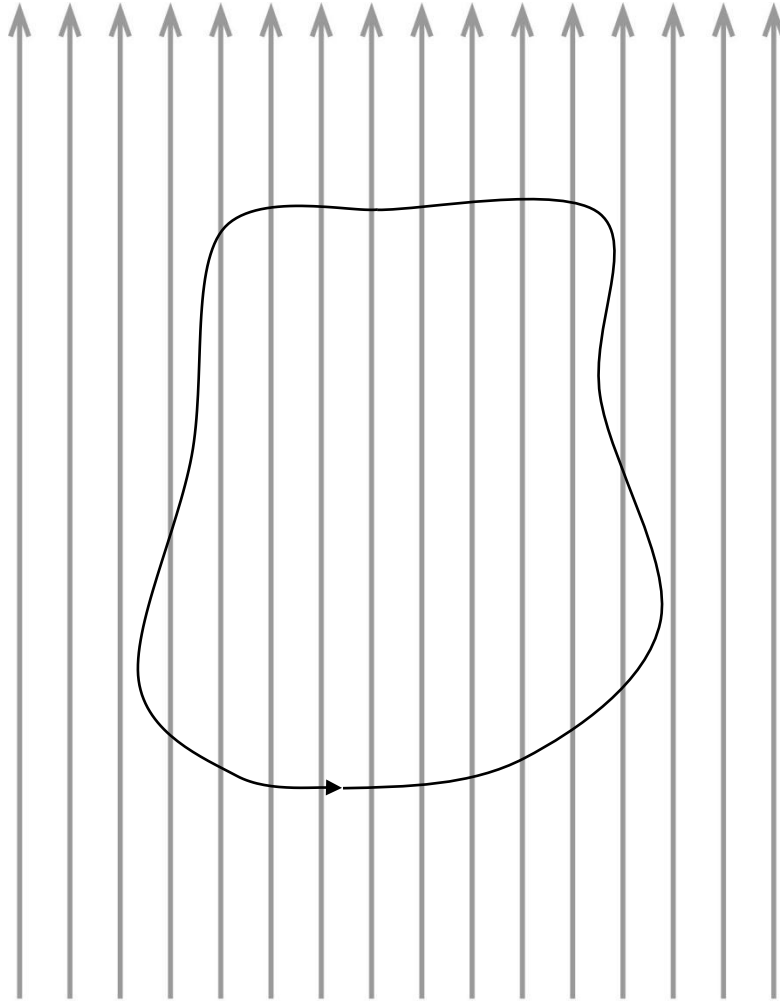
$$\oint_C \vec{F} \cdot \vec{T} ds = \text{circulation of } \vec{F} \text{ around path } C$$



$\vec{F} \parallel \vec{T}$ at all points along the path
 F is constant along the path

$$\oint_C \vec{F} \cdot \vec{T} ds = F \cdot 2\pi R$$

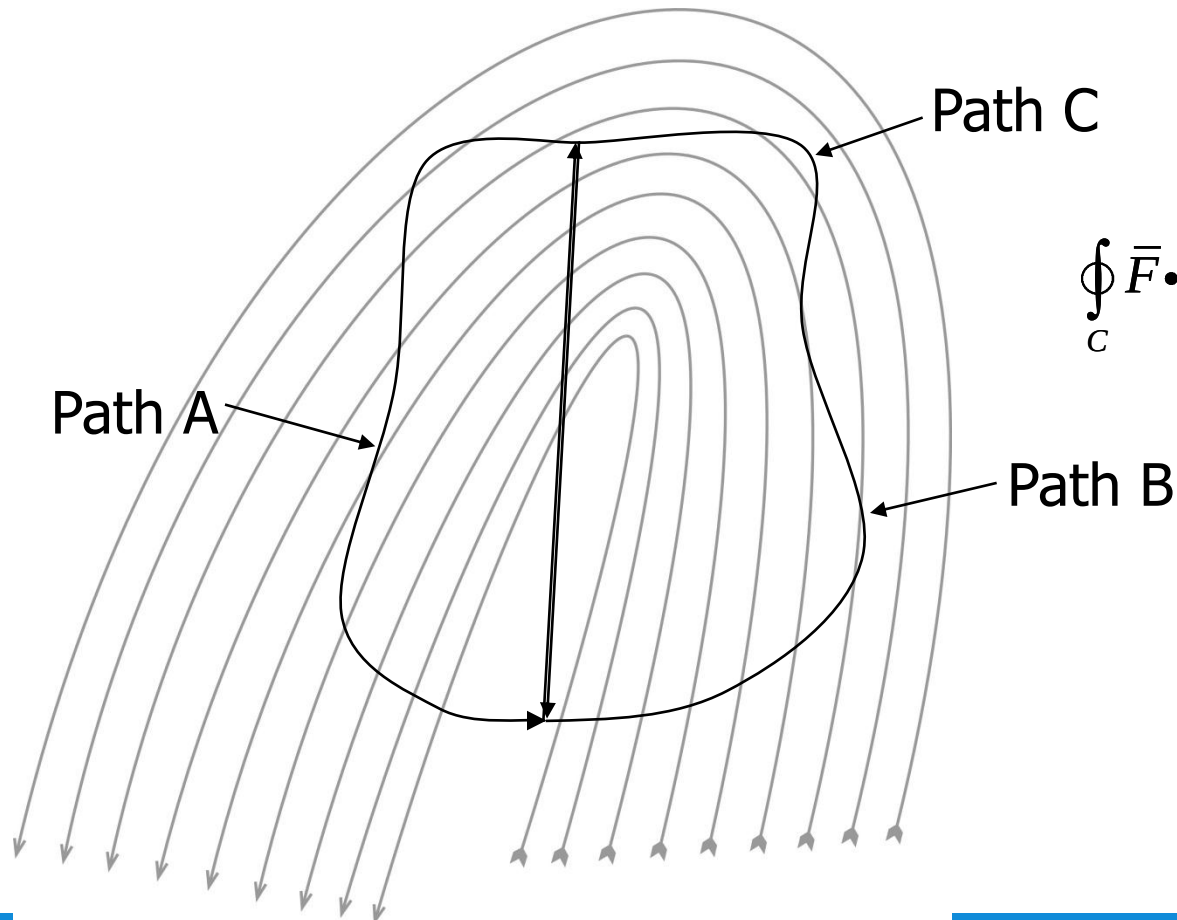
Circulation of $\vec{F}$ around C



$$\oint_C \vec{F} \cdot \vec{T} ds = 0$$

Stoke's Theorem

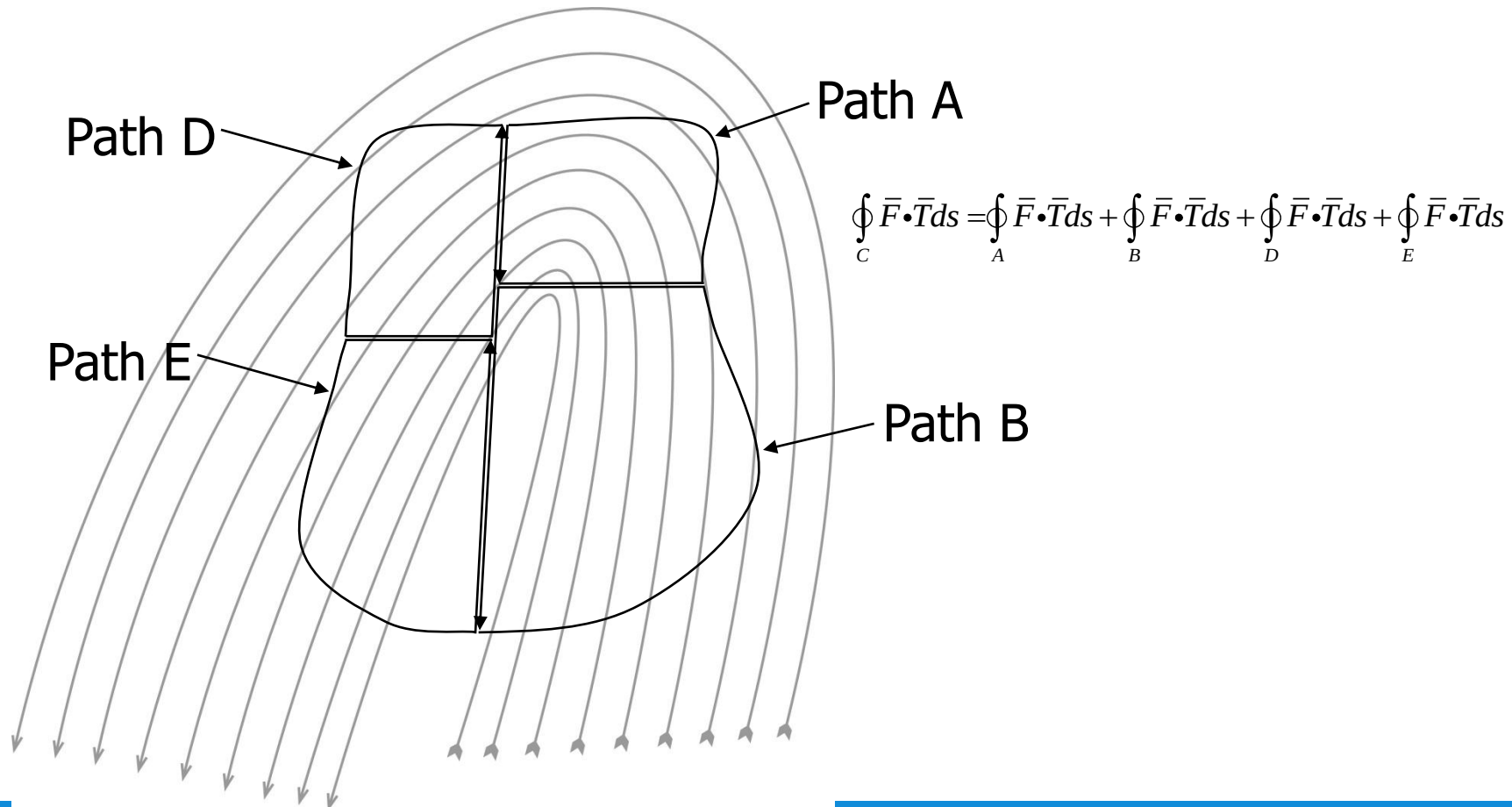
Stoke's Theorem $\rightarrow$ 2D $\rightarrow$ Green's Theorem (First alternative form)



$$\oint_C \vec{F} \cdot \vec{T} ds = \oint_A \vec{F} \cdot \vec{T} ds + \oint_B \vec{F} \cdot \vec{T} ds$$

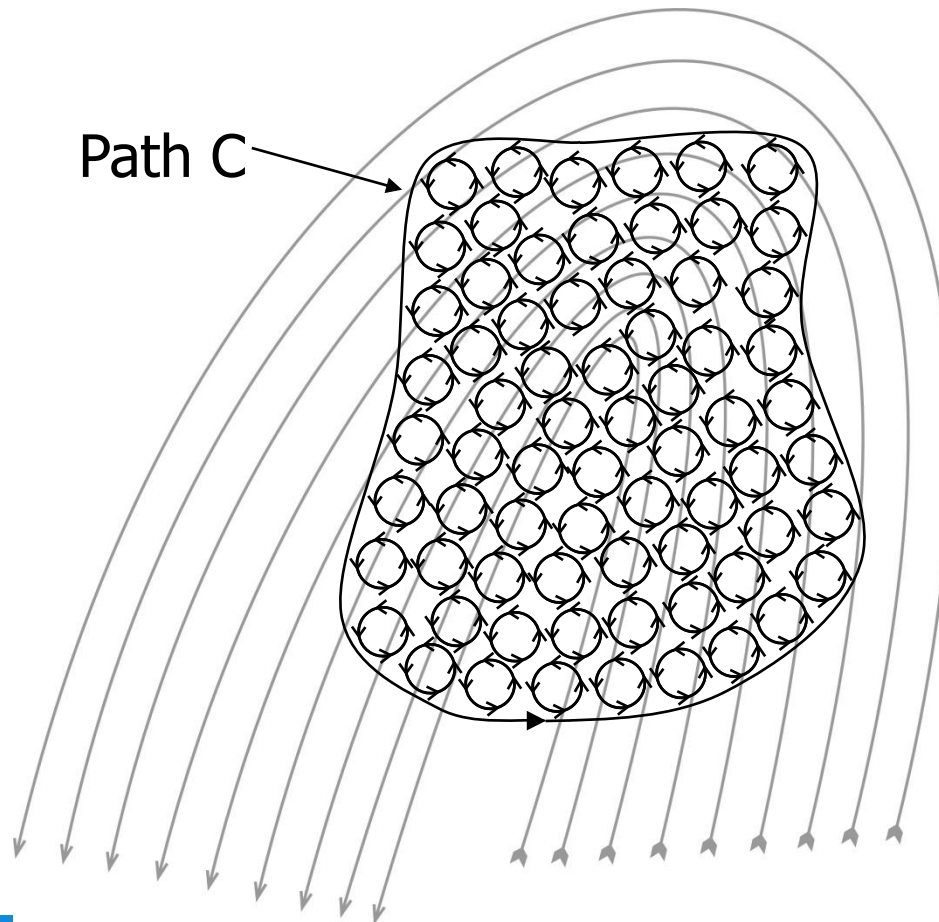
Stoke's Theorem

Stoke's Theorem $\rightarrow$ 2D $\rightarrow$ Green's Theorem (First alternative form)



Stoke's Theorem

Stoke's Theorem $\rightarrow$ 2D $\rightarrow$ Green's Theorem (First alternative form)



Taking the limit:

Amount of paths $\rightarrow \infty$

Path length $\rightarrow 0$

Results:

Addition of the circulation of $\mathbf{F}$ around an infinite number of points bound by the path C

$$\oint_C \bar{\mathbf{F}} \cdot \bar{\mathbf{T}} ds = \sum_{1}^{\infty} \oint_N \bar{\mathbf{F}} \cdot \bar{\mathbf{T}} ds$$

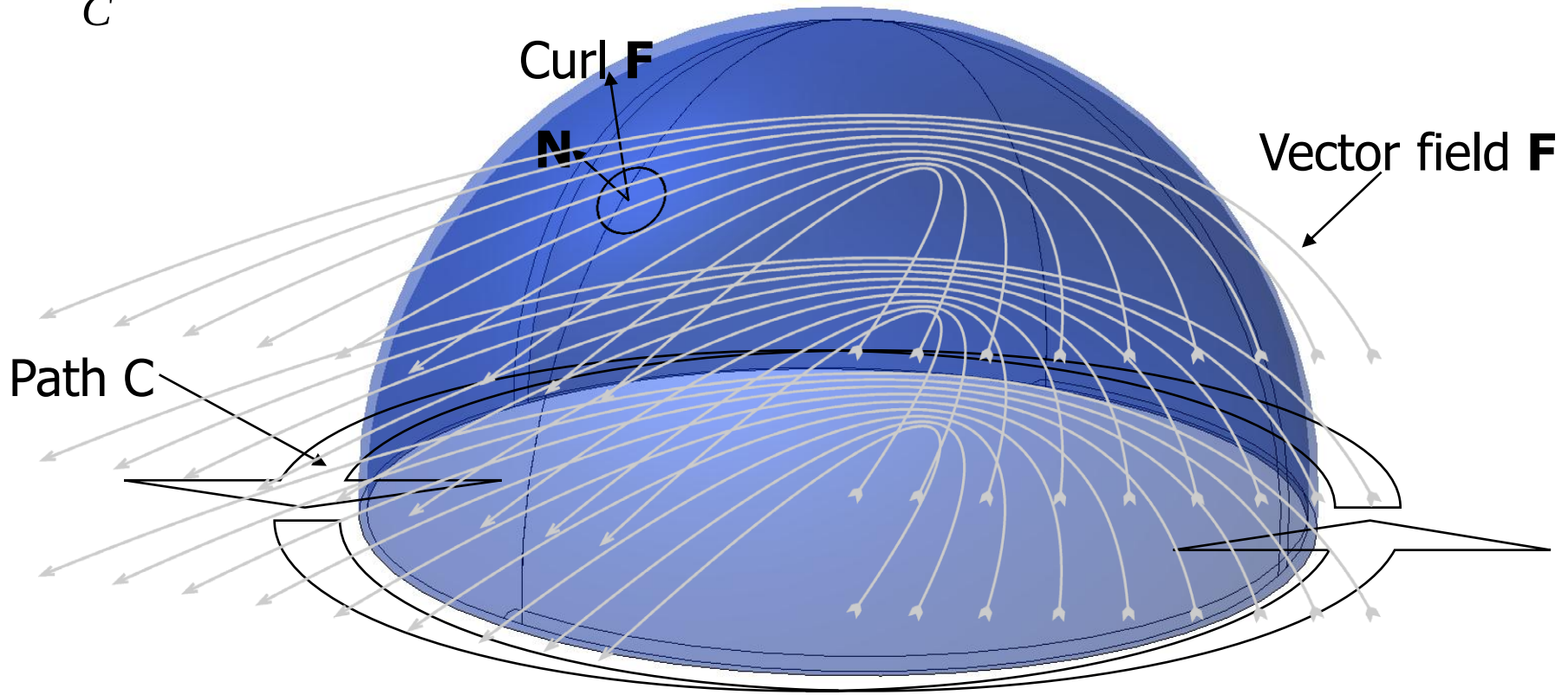
Rotation of $\mathbf{F}$ about a point (x,y,z) :

$$(\nabla \times \bar{\mathbf{F}}(x, y, z)) \cdot \bar{\mathbf{N}}$$

$$\oint_C \bar{\mathbf{F}} \cdot \bar{\mathbf{T}} ds = \iint_A (\nabla \times \bar{\mathbf{F}}) \cdot \bar{\mathbf{N}} da$$

Stoke's Theorem

$$\oint_C \vec{F} \cdot \vec{T} ds = \iint (\nabla \times \vec{F}) \cdot \vec{N} da$$



Curl of F

$$\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$$

$$\nabla \times \bar{F} = \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \times \left(M\hat{i} + N\hat{j} + P\hat{k} \right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \hat{i} \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z} \right) + \hat{j} \left(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x} \right) + \hat{k} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right)$$

Maxwell's Equations

In the absence of magnetic and dielectric mediums

$$\nabla \cdot \bar{E} = \frac{\rho}{\epsilon_0}$$

Divergence Theorem →

$$\iint_A \bar{E} \cdot \bar{N} da = \frac{Q}{\epsilon_0}$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

Stoke's Theorem →

$$\oint_C \bar{E} \cdot \bar{T} ds = -\frac{\partial \phi}{\partial t}$$

$$\nabla \cdot \bar{B} = 0$$

Divergence Theorem →

$$\iint_A \bar{B} \cdot \bar{N} da = 0$$

$$c^2 \nabla \times \bar{B} = \frac{\bar{J}}{\epsilon_0} + \frac{\partial \bar{E}}{\partial t}$$

Stoke's Theorem →

$$\oint_C \bar{B} \cdot \bar{T} ds = \frac{1}{c^2} \left(\frac{I}{\epsilon_0} + \frac{\partial \psi}{\partial t} \right)$$

$$\bar{E} = -\nabla V - \frac{\partial \bar{A}}{\partial t}$$

$$\bar{B} = \nabla \times \bar{A}$$

Point form of Ampere's law

$$c^2 \nabla \times \bar{B} = \frac{\bar{J}}{\epsilon_0} + \frac{\partial \bar{E}}{\partial t} \longrightarrow c^2 \epsilon_0 \nabla \times \bar{B} = \bar{J}$$

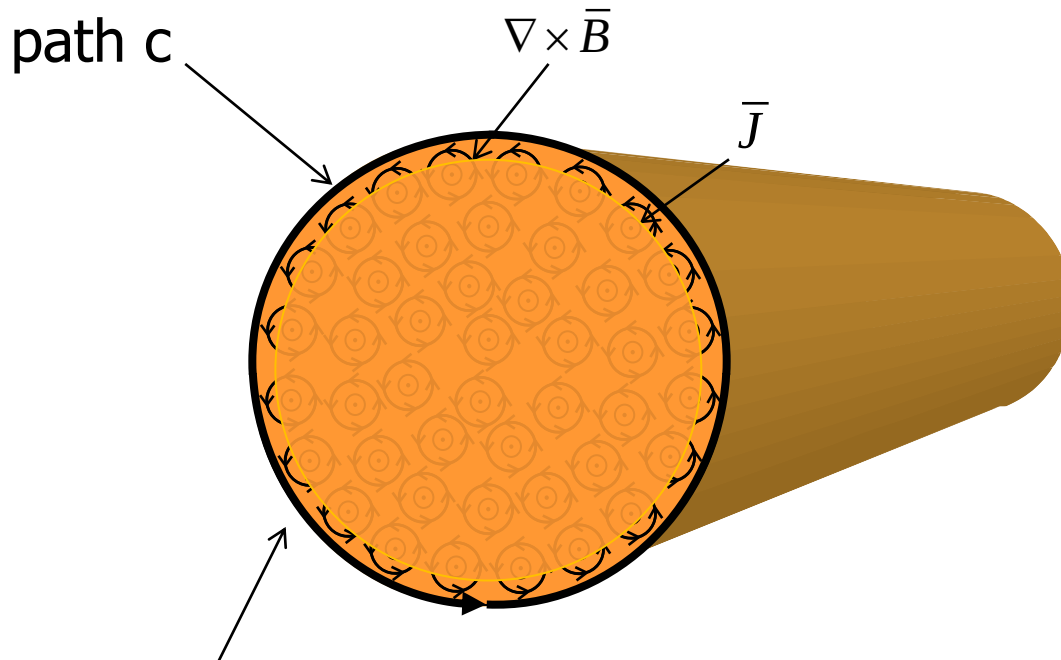
Stoke's Theorem
around path c

$$\oint_c \bar{B} \cdot d\bar{s} = \iint_s (\nabla \times \bar{B}) \cdot \bar{N} da$$

$$\oint_c \bar{B} \cdot d\bar{s} = \frac{1}{c^2 \epsilon_0} \iint_s \bar{J} \cdot \bar{N} da$$

$\bar{B} \parallel \bar{T}$ at all points along the path
B is constant along the path

$$2\pi r B = \frac{I}{c^2 \epsilon_0}$$



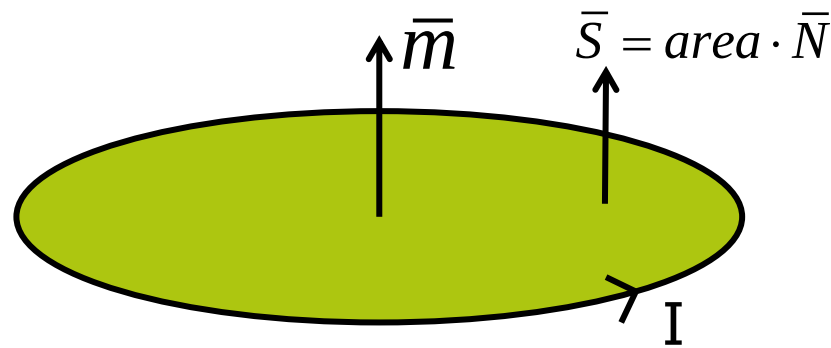
Non-magnetic conductor carrying constant current I

Magnetism

- Can only be completely explained through Quantum Mechanics
- Classical model gives, however, adequate explanation
 - Therefore used
- Diamagnetic materials
 - Weakly repelled from a magnetic field
 - E.g. Bismuth
- Paramagnetic materials
 - Weakly attracted to a magnetic field
 - E.g. Aluminium
- Antiferromagnetic materials
 - No net magnetic moment within the material
- Ferrimagnetic materials
 - E.g. Ferrites used for high-frequency inductor and transformer cores
- Ferromagnetic materials
 - Strongly attracted to magnetic fields

Ferromagnetic materials

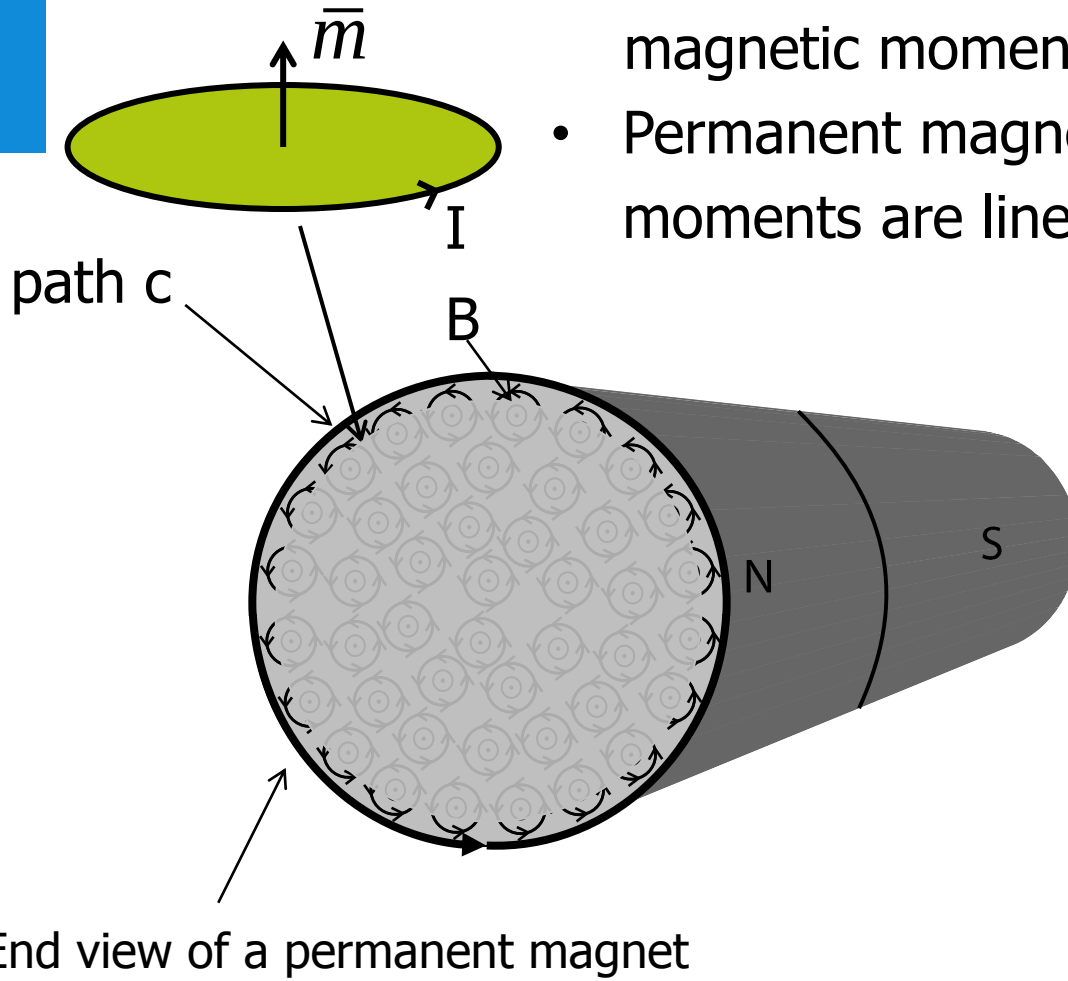
- Within an atom magnetic field is caused by:
 - Orbiting electrons around nucleus
 - Spin of electrons
 - Orbiting protons within the nucleus
 - Spin of the protons
 - Spin of the neutrons
- Lead to magnetic moments within the material



$$\vec{m} = I\vec{S} \quad (A \cdot m^2)$$

Ferromagnetic materials

- Each atom can be represented by a magnetic moment, $\bar{m}$
- Permanent magnet - all the magnetic moments are lined up



Ferromagnetic materials

- Define magnetization vector, $\bar{M}$
 - Average magnetic moment per unit volume

$$\bar{M} = \frac{1}{vol} \sum_{vol} \bar{m}_i$$

Diagram illustrating the units of magnetization $\bar{M}$:

- The denominator vol has units m^3 .
- The numerator $\sum_{vol} \bar{m}_i$ has units $A \cdot m^2$.
- The resulting magnetization $\bar{M}$ has units A/m .

Ferromagnetic materials

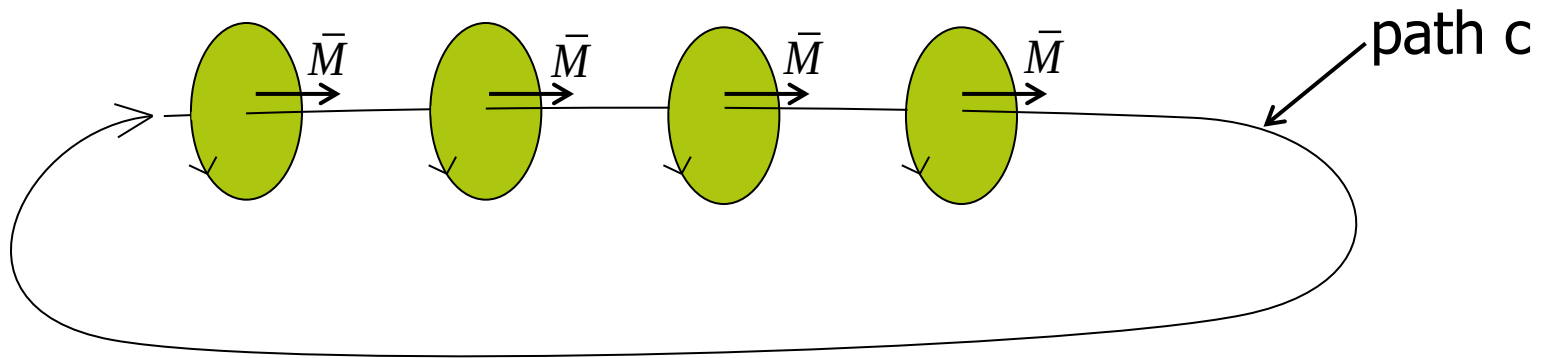
$$\bar{\mathbf{M}} = \frac{1}{\text{vol}} \sum_{\text{vol}} \bar{\mathbf{m}}_i \left(\frac{\text{A}}{\text{m}} \right)$$

$$\oint_c \bar{\mathbf{M}} \cdot d\bar{\mathbf{l}} = \sum I_M = I_{M_T}$$

$$\oint_c \bar{\mathbf{M}} \cdot d\bar{\mathbf{l}} = \iint_S (\nabla \times \bar{\mathbf{M}}) \cdot \bar{\mathbf{N}} da = I_{M_T}$$

Differentiating w.r.t area

$$\nabla \times \bar{\mathbf{M}} = \bar{\mathbf{J}}_M$$



Maxwell's equations in the presence of Ferromagnetic materials

The magnetic fields due to the winding current, I_{cond} , lines up the magnetic moments in the core

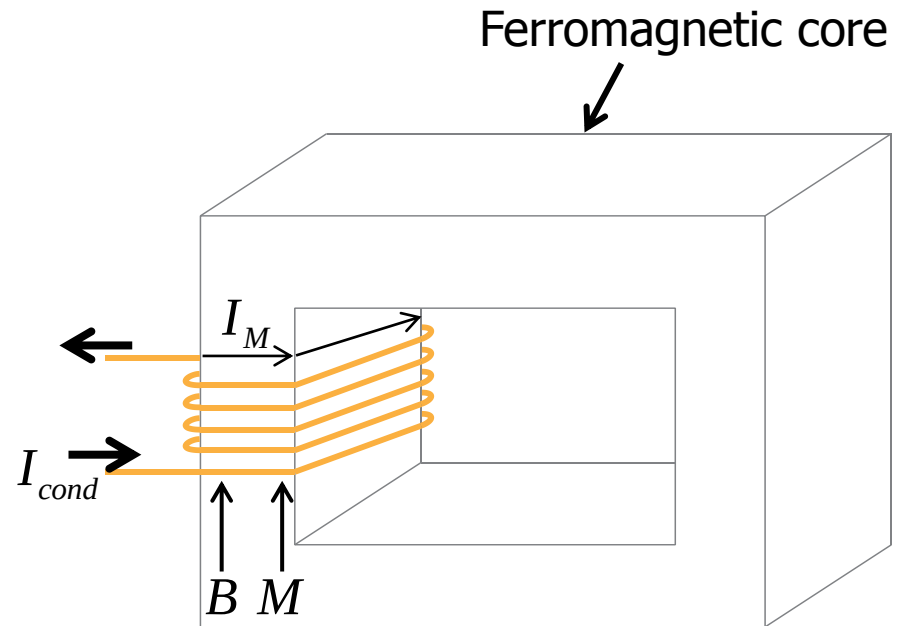
Result in an additional current, I_M , circulating on the core surface

Point form of Ampere's law

$$c^2 \nabla \times \bar{B} = \frac{\bar{J}}{\epsilon_0} + \frac{\partial \bar{E}}{\partial t}$$

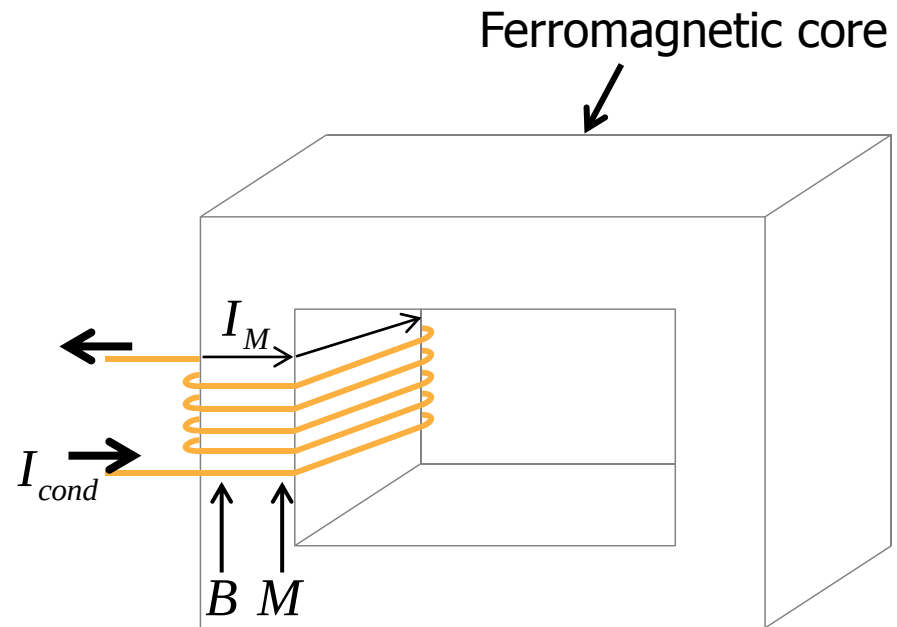


$$c^2 \nabla \times \bar{B} = \frac{\bar{J}}{\epsilon_0}$$



Maxwell's equations in the presence of Ferromagnetic materials

$$\begin{aligned}
 c^2 \nabla \times \bar{B} &= \frac{\bar{J}}{\epsilon_0} \longrightarrow \bar{J} = \bar{J}_{cond} + \bar{J}_M \longrightarrow \bar{J} = \bar{J}_{cond} + \nabla \times \bar{M} \\
 &\downarrow \\
 c^2 \nabla \times \bar{B} &= \frac{1}{\epsilon_0} (\bar{J}_{cond} + \bar{J}_M) \\
 &\downarrow \\
 c^2 \nabla \times \bar{B} &= \frac{1}{\epsilon_0} (\bar{J}_{cond} + \nabla \times \bar{M}) \\
 &\downarrow \\
 \epsilon_0 c^2 \nabla \times \bar{B} - \nabla \times \bar{M} &= \bar{J}_{cond} \\
 \nabla \times (\epsilon_0 c^2 \bar{B} - \bar{M}) &= \bar{J}_{cond} \\
 &\downarrow \\
 \bar{H} = \epsilon_0 c^2 \bar{B} - \bar{M} &\longrightarrow \nabla \times \bar{H} = \bar{J}_{cond}
 \end{aligned}$$



Maxwell's equations in the presence of Ferromagnetic materials

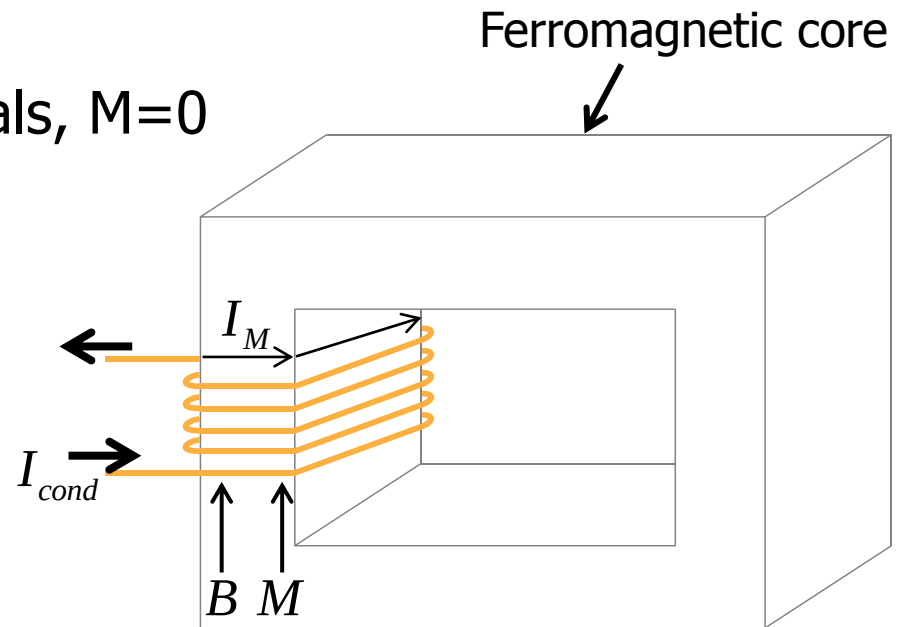
$$\nabla \times (\epsilon_0 c^2 \bar{B} - \bar{M}) = \bar{J}_{cond} \quad \nabla \times \bar{H} = \bar{J}_{cond}$$

$$\bar{H} = \epsilon_0 c^2 \bar{B} - \bar{M}$$

In the absence of magnetic materials, $M=0$

$$\downarrow$$
$$\bar{H} = \epsilon_0 c^2 \bar{B}$$

$$\downarrow$$
$$\mu_0 = \frac{1}{\epsilon_0 c^2}$$



Maxwell's equations in the presence of Ferromagnetic materials

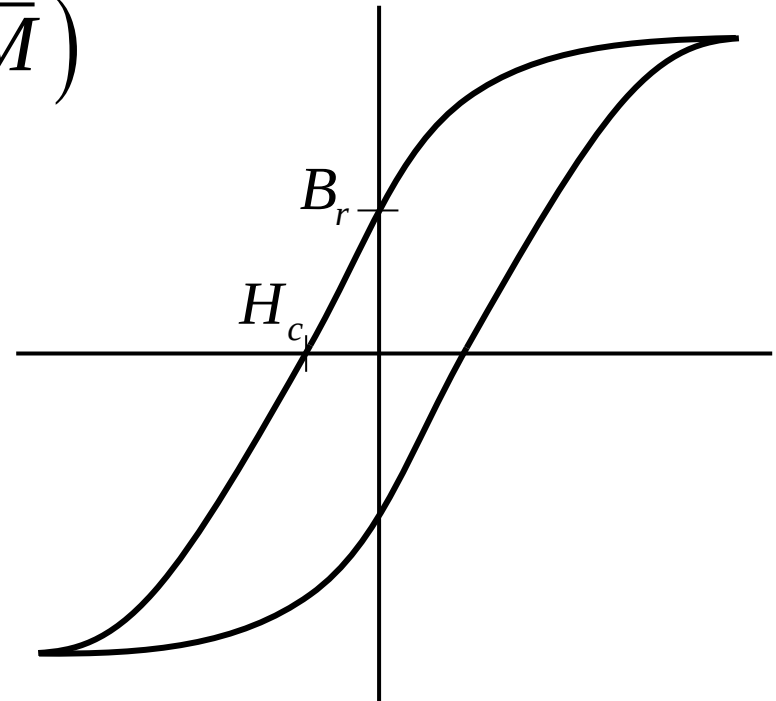
$$\nabla \times (\epsilon_0 c^2 \bar{B} - \bar{M}) = \bar{J}_{cond} \quad \nabla \times \bar{H} = \bar{J}_{cond}$$

$$\bar{H} = \epsilon_0 c^2 \bar{B} - \bar{M} \quad \bar{B} = \frac{1}{\epsilon_0 c^2} (\bar{H} + \bar{M})$$

BH curve $\rightarrow$

No direct relation between B and M

- Depends on past history



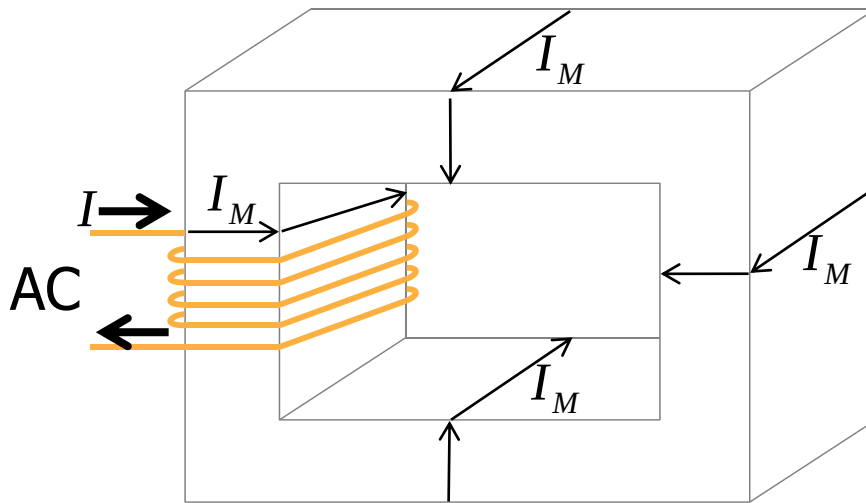
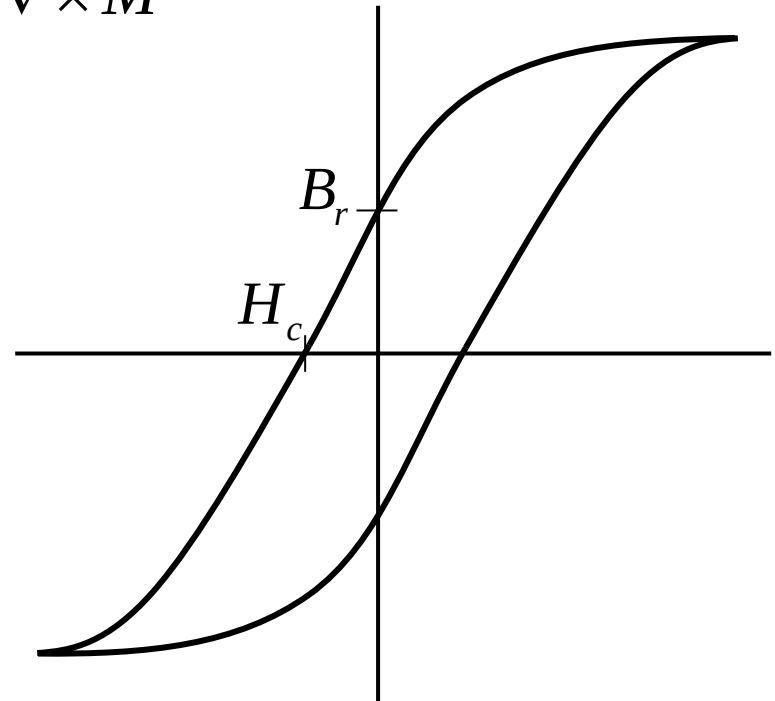
Maxwell's equations in the presence of Ferromagnetic materials

$$\nabla \times (\varepsilon_0 c^2 \bar{B} - \bar{M}) = \bar{J}_{cond} \quad \text{No external excitation} \quad I=0$$

$$\nabla \times (\varepsilon_0 c^2 \bar{B} - \bar{M}) = 0 \quad \nabla \times \varepsilon_0 c^2 \bar{B} = \nabla \times \bar{M}$$

$$\nabla \times \varepsilon_0 c^2 \bar{B} = \bar{J}_M$$

Thought of:
MMF equal to $\sum I_M$
producing B_r



Maxwell's equations in the presence of Ferromagnetic materials

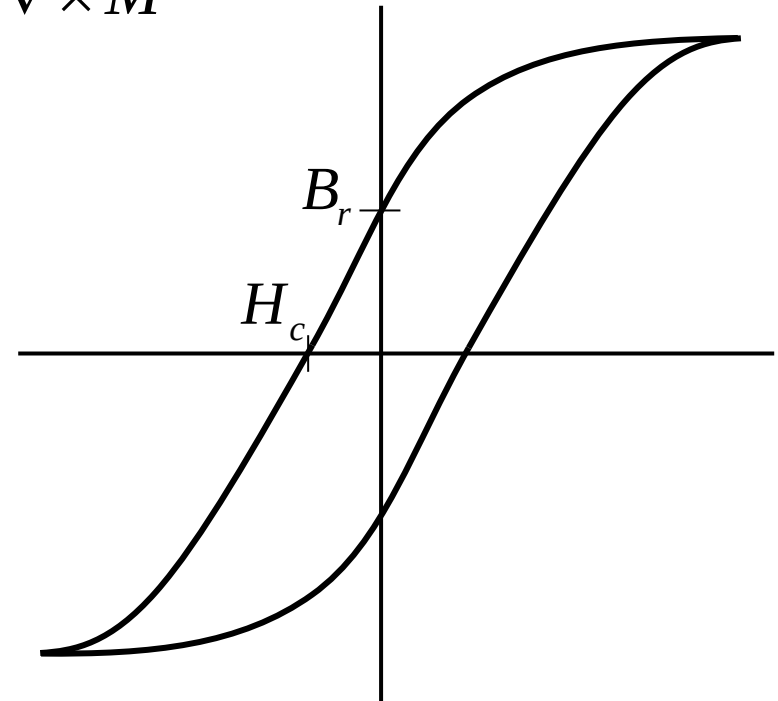
$$\nabla \times (\varepsilon_0 c^2 \bar{B} - \bar{M}) = \bar{J}_{cond} \quad \text{No external excitation} \\ I=0$$

$$\nabla \times (\varepsilon_0 c^2 \bar{B} - \bar{M}) = 0 \quad \nabla \times \varepsilon_0 c^2 \bar{B} = \nabla \times \bar{M}$$

$$\nabla \times \varepsilon_0 c^2 \bar{B} = \bar{J}_M$$

H_c can be thought of as the material's ability to maintain I_M under loaded conditions

Thought of:
MMF equal to $\sum I_M$
producing B_r



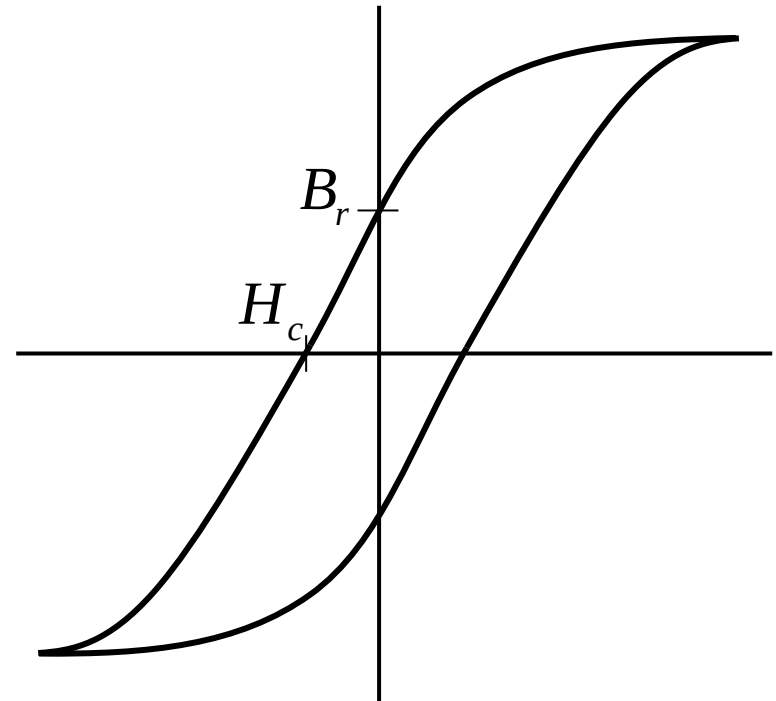
Maxwell's equations in the presence of Ferromagnetic materials

Soft magnetic materials:

- High remanent magnetization, B_r
- Small coercivity, H_c

Hard magnetic materials:

- High remanent magnetization, B_r
- Large coercivity, H_c

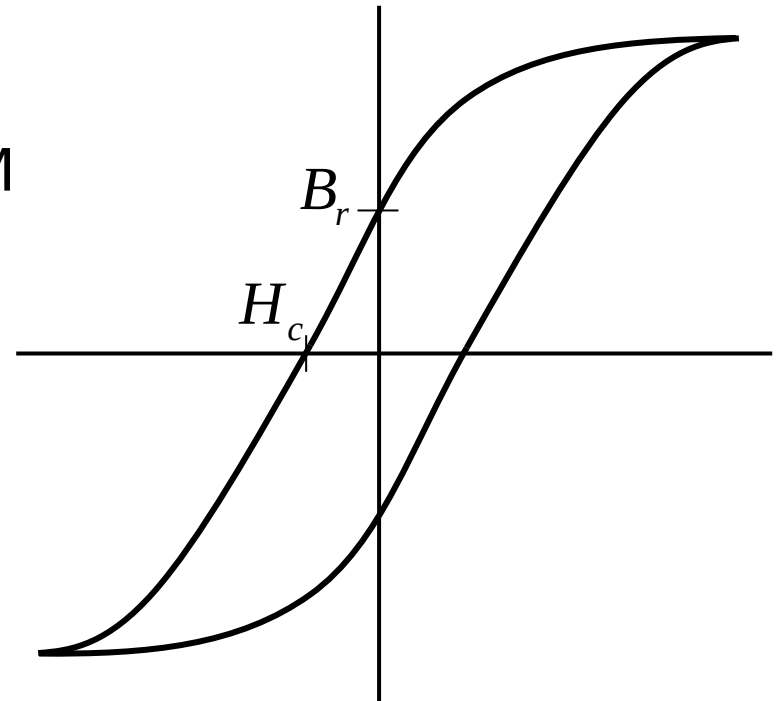


Maxwell's equations in the presence of Ferromagnetic materials

Engineering applications

↳ Linear relation between B and M

↳ $\bar{B} = \mu_0 \mu_r \bar{H}$



Maxwell's equations in the presence of Ferromagnetic materials

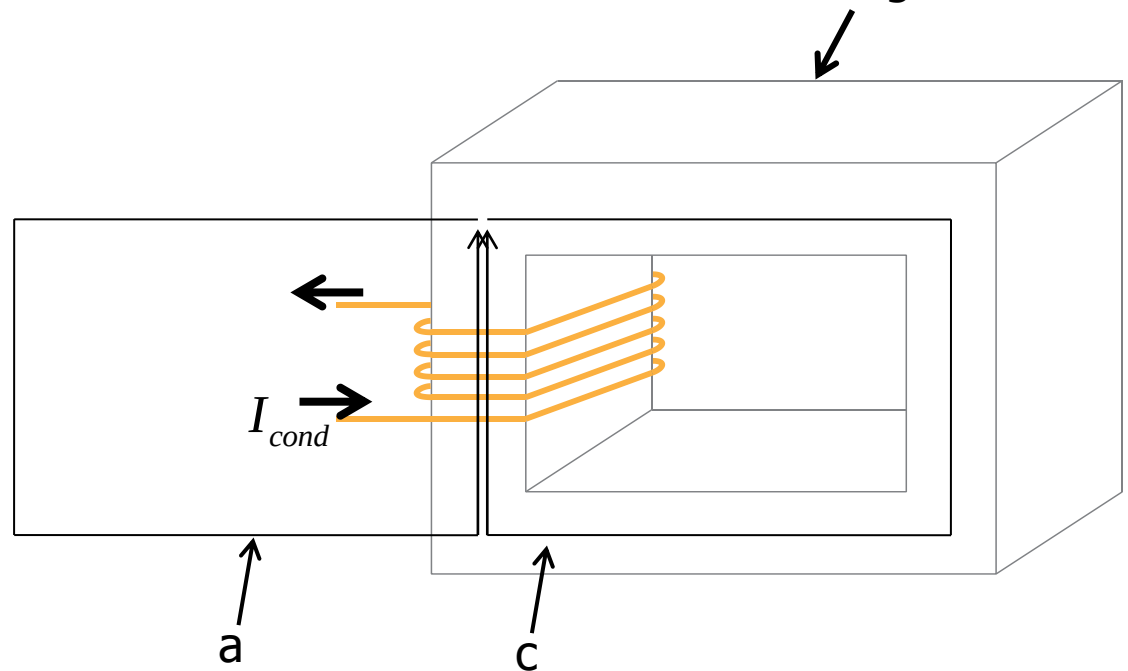
Determine inductance

$$\nabla \times \bar{H} = \bar{J}_{cond}$$

$$\bar{H} = \varepsilon_0 c^2 \bar{B} - \bar{M}$$

$$\bar{B} = \mu_0 \mu_r \bar{H}$$

Ferromagnetic core



Maxwell's equations in the presence of Ferromagnetic materials

Determine inductance

$$\nabla \times \bar{H} = \bar{J}_{cond}$$

$$\bar{H} = \varepsilon_0 c^2 \bar{B} - \bar{M}$$

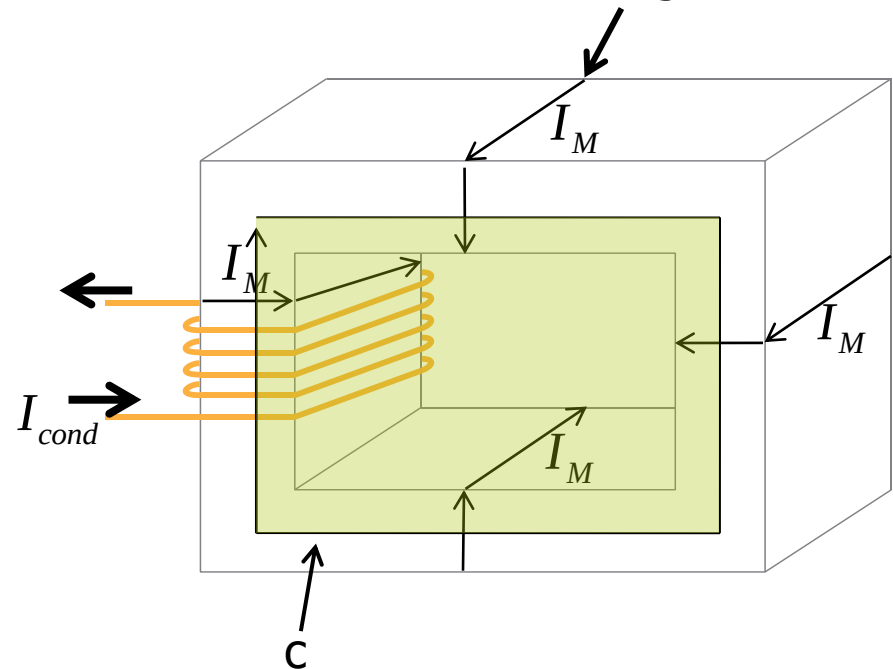
$$\bar{B} = \mu_0 \mu_r \bar{H}$$

Ferromagnetic core

$$\oint_c \bar{H} \cdot d\bar{l} = \iint_{S_c} (\nabla \times \bar{H}) \cdot \bar{N} da = \iint_{S_c} \bar{J} \cdot \bar{N} da$$

$$H l_c = \text{turns} \cdot I \quad B = \mu_0 \mu_r H$$

$$B = \frac{\mu_0 \mu_r \cdot \text{turns} \cdot I_{cond}}{l_c}$$



Maxwell's equations in the presence of Ferromagnetic materials

Determine inductance

$$\nabla \times \bar{H} = \bar{J}_{cond}$$

$$\bar{H} = \varepsilon_0 c^2 \bar{B} - \bar{M}$$

$$\bar{B} = \mu_0 \mu_r \bar{H}$$

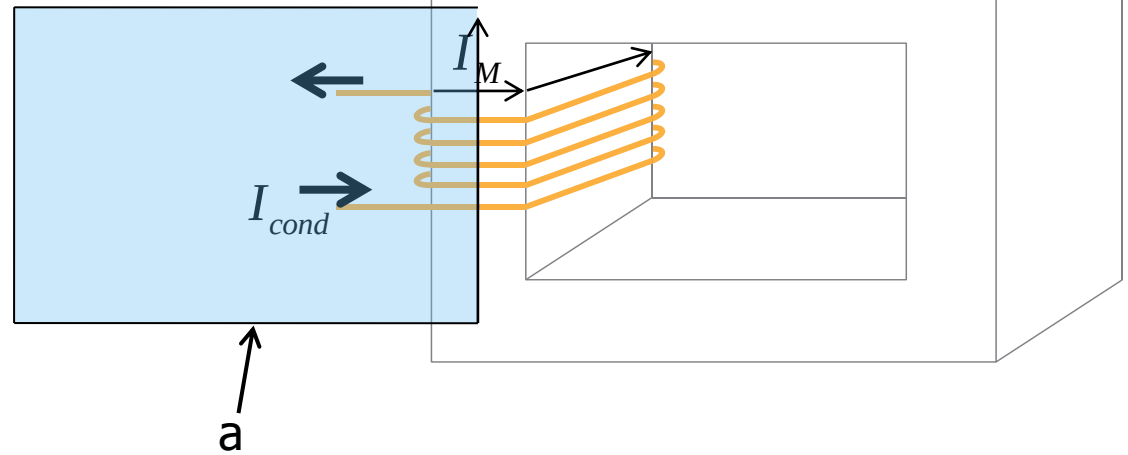
Ferromagnetic core

$$\oint_a \bar{H} \cdot d\bar{l} = \iint_{S_a} (\nabla \times \bar{H}) \cdot \bar{N} da = \iint_{S_a} \bar{J} \cdot \bar{N} da$$

$$H_{air} l_{air} + H_{core} l_{a_{core}} = turns \cdot I$$

$$\nabla \cdot \bar{B} = 0 \rightarrow B = const$$

$$B_c = \frac{turns \cdot I}{\frac{l_{air}}{\mu_0} + \frac{l_{a_{core}}}{\mu_0 \mu_r}} \approx \frac{\mu_0 turns \cdot I}{l_{air}}$$



Maxwell's equations in the presence of Ferromagnetic materials

Determine inductance

$$\nabla \times \bar{\bar{H}} = \bar{\bar{J}}_{cond}$$

$$\bar{H} = \varepsilon_0 c^2 \bar{B} - \bar{M}$$

$$\bar{B} = \mu_0 \mu_r \bar{H}$$

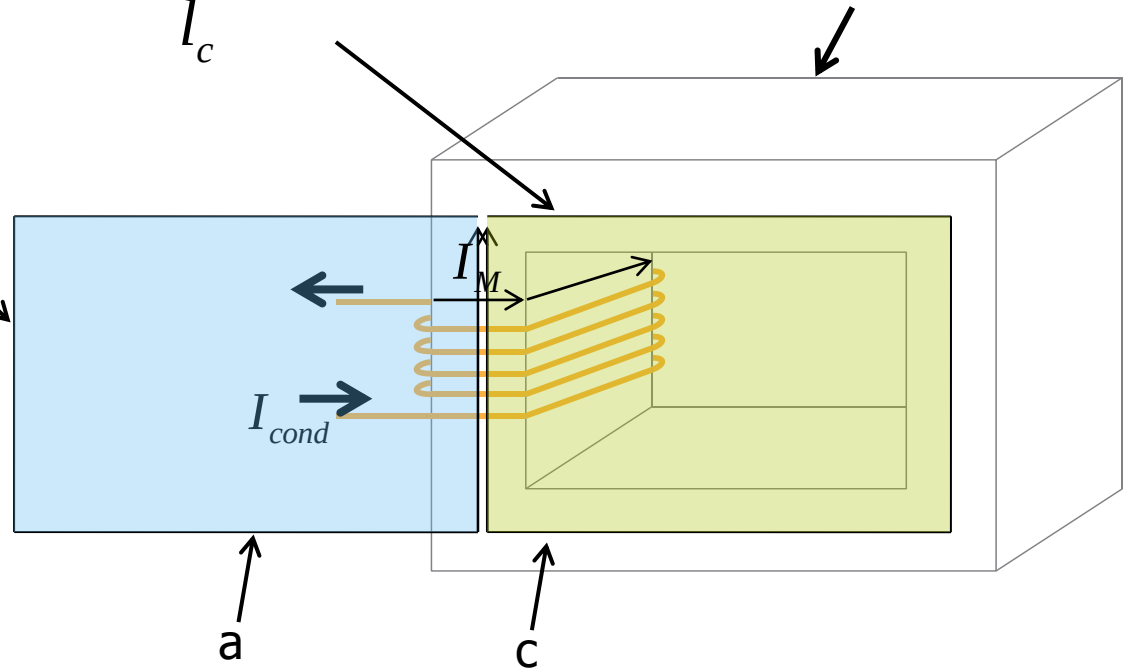
Ferromagnetic core

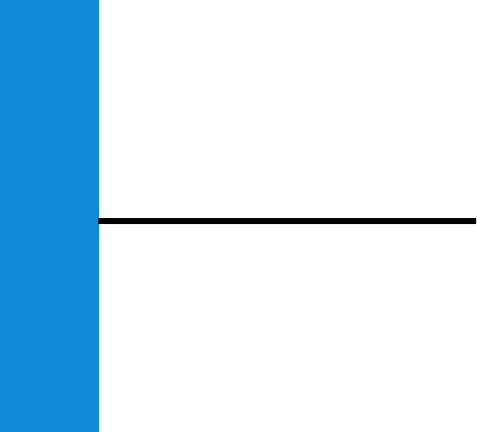
$$B_a \approx \frac{\mu_0 \text{turns} \cdot I}{l_{a_{air}}}$$

$$B_c = \frac{\mu_0 \mu_r \cdot \text{turns} \cdot I_{cond}}{l_c}$$

$$B_c \gg B_a \rightarrow B_{total} \approx B_c$$

$$L = \frac{\lambda}{I}$$





Questions