Examination Electrical Machines and Drives Et4-117
Thursday, October 30, 2003 from 9.00 to 12.00

This examination consists of 6 problems.
The number before a problem indicates how many points can be earned with this problem.

15 Problem 1

a Sketch a cross-section of a dc motor with interpoles. Indicate the current directions in the excitation winding, the winding around the interpoles and the armature with dots (for currents coming out of the plane of the drawing) and + (for currents going into the plane of the drawing).

A variable-speed drive system uses a dc motor which is supplied from a variable-voltage source. The drive speed is varied from 0 to 1500 rpm (base speed) by varying the terminal voltage from 0 to 500 V (rated value) with the field current maintained constant. The speed beyond (above) the base speed is obtained by field weakening while the armature voltage is held constant at 500 V (rated value). The rated value of the torque below 1500 rpm (base speed) is 300 Nm.
Armature reaction, iron losses and the voltage drop over the armature resistance may be neglected.

b Determine the motor armature current if the torque is held constant at the rated value of 300 Nm below 1500 rpm (base speed).

c Determine the torque available at a speed of 3000 rpm if the armature current is held constant at the value obtained in question b.

d Sketch the maximum torque and the maximum power as a function of the speed for a speed range of 0 to 3000 rpm.

11 Problem 2

A three-phase, 500 MVA 20.8 kV four-pole star-connected synchronous machine has negligible stator resistance and a synchronous reactance of 0.8 Ohm per phase at rated terminal voltage. The machine is operated as a generator connected to a three-phase 20.8 kV infinite bus.
Note: 20.8 kV is the line voltage.

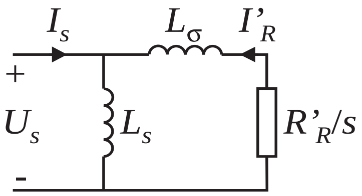
a Give the per-phase equivalent circuit of the synchronous machine.

b Calculate the phase voltage.

c Sketch the phasor diagram when the machine is delivering rated MVA at a power factor of 0.8 lagging.

d Determine the excitation voltage and the power angle when the machine is delivering rated MVA at a power factor of 0.8 lagging.

24 Problem 3



This problem deals with a 2-pole three-phase induction machine connected to a 50 Hz supply ($\omega_s = 2\pi 50$ rad/s). The figure gives an equivalent circuit where the stator resistance is neglected. This equivalent circuit can be mathematically derived from the T-scheme (the IEEE recommended equivalent circuit).

a Use this equivalent circuit to derive an expression for the torque-speed characteristic as a function of the parameters of the equivalent circuit, the voltage, the angular frequency and the slip (L_σ, $R_R^{/}$, U_s, ω_s, s).

b Mention at least 6 important assumptions used in the derivation other than that Maxwell's equations and the power balance hold.

c Give an expression for the slip where the torque is maximum (the derivation is not required).

The parameters of the equivalent circuit can be determined from the no-load test and the short circuit test.
In the no-load test, the rotor rotates synchronously $(\omega_m = \omega_s)$ at the rated voltage. The voltage and the current are measured as $U_s = 230$ V and $I_s = 50$ A.

d Determine the value of L_s.

In the short-circuit test, the rotor is blocked ($\omega_m = 0$) and voltage, current and power are measured at a reduced voltage. The results are $U_s = 40$ V, $I_s = 200$ A en $P = 1500$ W. (N.B. the power is measured three-phase).

e Determine L_σ and $R_R^{/}$. It may be assumed that the current through L_s is negligible.

f The machine works as a motor at $\omega_m = \omega_s/2$. Determine the efficiency according to the used equivalent circuit.

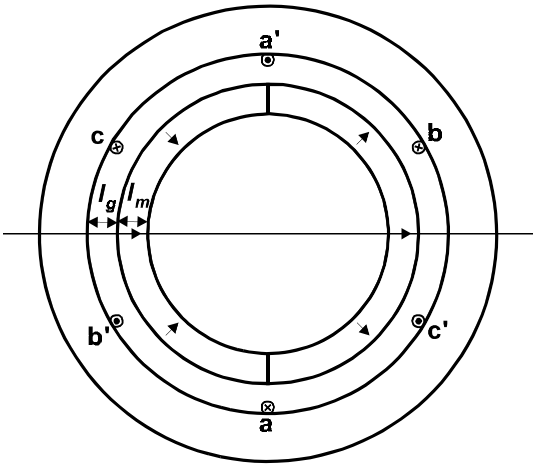
21 Problem 4

The Dutch railway system has a DC supply. This problem deals with the comparison of three types of electrical motors for railway traction applications, namely DC motors, three-phase squirrel-cage induction motors and three-phase permanent-magnet synchronous motors.
We assume the voltage of this DC supply to be constant.

a For the three motor types, sketch the power electronic converter (switches and diodes) between the DC supply and the motor. If you think the same converter can drive more machine types, one sketch of that converter is enough.

b Compare the three motor drives (motor and converter) for this application by giving at least 6 advantages or disadvantages per motor type by paying attention to e.g. cost, maintenance, risk of failures, size and weight, efficiency, complexity of the power electronic converter and the control, field weakening

22 Problem 5



The depicted brushless DC motor has the following characteristics.
The magnetic permeability of iron is infinite.
The axial stack length (in the direction perpendicular to the plane of the drawing) is l_s.
The air gap radius is r_s.
The air gap length is l_g.
The magnet length in the direction of magnetization is l_m.
The magnets have remanent flux density B_r and relative recoil permeability μ_{rm} and the BH-characteristic is a straight line in the second quadrant of the BH-plane.
The stator windings are concentrated windings with N_s turns.
The air gap length and the magnet length may be assumed so small compared to the air gap radius ($l_g + l_m << r_s$) that the flux density in the air gap and the magnets does not change in radial direction and that the flux may be assumed to cross the air gap perpendicular.
In questions a to f, the stator currents are zero.

a Give Ampère’s law and sketch a contour and a surface where Ampere’s law holds.
b Give an equation describing the BH-characteristic of the magnet.
c Using the given characteristics, give an expression for the maximum value of the magnetic flux density in the air gap B_g using Ampère’s law.

Use further that the magnetic flux density in the gap is B_g.
d Using the given characteristics, give an expression for the maximum flux of phase a (conductors aa’).
e Sketch the flux linkage of coil aa’ as a function of the rotor position θ.

The machine rotates with an angular speed ω_m.
f Calculate the maximum excitation voltage of phase a (the voltage induced by the magnetic field of the magnets).

In practice, the no-load (excitation) voltage waveforms of a brushless DC machine are trapezoidal due to skewing. The maximum phase current is *I*.
g Sketch the three-phase trapezoidal no-load (excitation) voltage waveforms and the idealized current wave forms as a function of time for one cycle.
h Calculate the torque.

7 Problem 6

a Sketch a cross-section of a shaded-pole single-phase induction machine
b Explain why it starts rotating when it is connected to a single-phase supply.

Answers to the examination of October 30, 2003

15 Problem 1

4 a see figure 4.10. The flux due to the currents around the interpoles has to oppose the flux due to the armature current.

4 b The armature resistance is neglected, therefore, $V_t = E_a = 500\ \text{V}$.

$$E_a = K\Phi\omega_m \text{ , therefore } K\Phi = \frac{E_a}{\omega_m} = \frac{60\,E_a}{2\pi N} = 3.1831\ \text{Vs/rad}$$

$$T = K\Phi I_a \text{ , therefore } I_a = \frac{T}{K\Phi} = 94.25\ \text{A}$$

3 c $E_a = K\Phi\omega_m$, therefore $K\Phi = \frac{E_a}{\omega_m} = \frac{60\,E_a}{2\pi N} = 1.5916\ \text{Vs/rad}$

$$T = K\Phi I_a = 150\ \text{Nm}$$

4 d See figure 4.53 d

11 Problem 2

2 a See figure E6.3, generator convention is used, but motor convention is also allowed.

2 b $V_t = \frac{20.80}{\sqrt{3}}\ \text{kV} = 12.01\ \text{kV}$

2 c See figure E6.3

5 d $I = \frac{S}{3V_t} = 13.88\ \text{kA}$

The power factor is 0.8 lagging. Therefore, if the terminal voltage phasor is chosen in the real axis ($\underline{V}_t = 12.01\ \text{kV}$), the phasor for the current can be written as $\underline{I} = 11.10 - \text{j}\,8.33\ \text{kA}$

Therefore, the phasor for the excitation voltage can be written as

$$\underline{E}_f = \underline{V}_t + \text{j}X_s\underline{I} = 12.01 + 6.66 + \text{j}\,8.88\ \text{kV} = 18.67 + \text{j}\,8.88\ \text{kV}$$

$$E_f = \sqrt{18.67^2 + 8.88^2}\ \text{kV} = 20.68\ \text{kV}$$

The power angle can be calculated as

$$\delta = \arctan\frac{8.88}{18.67} = 0.444\ \text{rad} = 25.43\ ^\circ$$

24 Problem 3

7 a The expression is derived from the power balance and the Thevenin equivalent circuit. According to this equivalent circuit, the rotor current is given by

$$I_R^{/} = \frac{U_s}{\sqrt{\left(\omega_s L_\sigma\right)^2 + \left(\frac{R_R^{/}}{s}\right)^2}}$$

A steady-state condition is assumed, which means that
- the rotor speed is constant
- the stored magnetic energy does not change

Therefore, all power is dissipated in the rotor resistance or delivered as mechanical output power. According to the per-phase equivalent circuit, the three-phase stator input power is given by

$$P_s = 3{I'_R}^2 \frac{R'_R}{s} = 3{I'_R}^2 R'_R + 3{I'_R}^2 R'_R \frac{1-s}{s}$$

The power dissipated in the rotor resistance is

$$P_{diss} = 3{I'_R}^2 R'_R.$$

The rest is the electromechanic output power P_{em}. Therefore, the torque can be written as

$$T_e = \frac{P_{em}}{\omega_m} = \frac{3{I'_R}^2 R'_R \frac{1-s}{s}}{(1-s)\,\omega_s} = 3{I'_R}^2 \frac{R'_R}{s\,\omega_s}.$$

Substitution of the expression for the rotor current gives:

$$T_e = 3\,\frac{R'_R}{s\,\omega_s}\,\frac{U_s^2}{(\omega_s L_\sigma)^2 + \left(\frac{R'_R}{s}\right)^2}$$

6 b Used assumptions:

- stator windings are sinusoidally distributed
- stator windings have axis which are 120° shifted with respect to each other
- the stator voltages (and the resulting currents) are sinusoidal (with constant amplitude and frequency, steady state)
- the stator voltages (and the resulting currents) have a phase shift of 120° with respect to each other
- the mechanical speed is constant (steady state)
- the stored magnetic energy is constant (steady state)
- stator copper losses are neglected,
- iron losses are neglected
- mechanical (friction and windage losses) are neglected.

2 c $s_{T_{\max}} = \dfrac{R'_R}{\omega_s L_\sigma}$

2 d $L_s = \dfrac{U_s}{\omega_s I_s} = 14.64\ \text{mH}$

4 e $R'_R = \dfrac{P}{3I_s^2} = 12.50\ \text{m}\Omega$

The voltage can be written as $U_s = \sqrt{(\omega_s L_\sigma I_s)^2 + (R'_R I_s)^2}$

Therefore $L_\sigma = \dfrac{\sqrt{U_s^2 - (R'_R I_s)^2}}{\omega_s I_s} = 0.6354\ \text{mH}$

3 f In the used equivalent circuit, only the resistance takes active power P. sP Is dissipated in the rotor winding, while $(1-s)P$ is converted into electromechanical power.

Therefore, $\eta = \dfrac{1}{2}$

21 Problem 4

6 a For DC motor: see fig. 10.34 of Sen

For induction motor and permanent-magnet synchronous motor: see fig. 10.43 of Sen

15 b

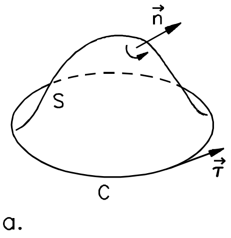
	DC motor	IM	PMSM
cost	- expensive due to commutator	+ cheap	- expensive due to magnets
maintenance	- commutator and brushes need regular maintenance	+ low maintenance no mechanical contact with rotor	+ low maintenance no mechanical contact with rotor
risk of failures	- commutator failures are a risk	+ extremely robust	- demagnetization of the magnets due to high temperatures or high currents is a risk
size and weight	- large and heavy	+ more compact	++ smallest
efficiency	- not very high due to losses in rotor, stator and commutator	- not very high because of losses in stator and rotor -not very high due to low power factor	+ best: no losses in rotor
complexity control	+ chopper simpler than inverter, less switches	- complex inverter, more switches	- complex inverter, more switches
field weakening	+ possible	+ possible	- more difficult

22 Problem 5

3 a $\oint_{C_m} \vec{H}\cdot d\vec{l} = \iint_{S_m} \vec{J}\cdot d\vec{A}$ or

$\oint_{C_m} \vec{H}\cdot\vec{\tau}\,ds = \iint_{S_m} \vec{J}\cdot\vec{n}\,dA$

where $\vec{\tau}$ is the unit vector parallel to the path C_m
and $\vec{n}$ is a unit vector perpendicular to the surface S_m



2 b $B_m = \mu_0\mu_{rm}H_m + B_r$

5 c Applying Ampère’s law results in $2(l_mH_m + l_gH_g) = 0$

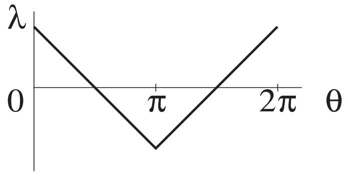
In the air gap: $B_g = \mu_0 H_g$

$\oiint_S \vec{B}\cdot d\vec{A} = 0$ therefore $B_g = B_m$

Substitution gives $B_g = \dfrac{l_m}{l_m + \mu_{rm}l_g}B_r$

2 d $\lambda_{\max} = N_s\Phi = N_sAB_g = N_sl_s\pi r_sB_g$

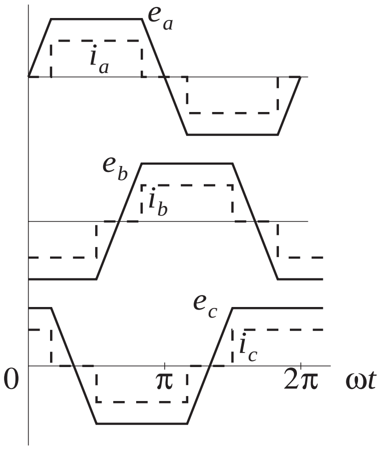
2 e triangular



2 f $E_{\max} = \dfrac{d\lambda}{dt} = 4f\lambda_{\max} = 2N_sB_gl_sr_s\omega_m$

May also be derived from $E = Blv$.

4 g see figure 6.39 and 6.40b



2 h $T = \dfrac{2E_{\max}I}{\omega_m} = 4N_sB_gl_sr_sI$

May also be derived from $T = rF = rBlI$.

7 Problem 6

3 a See figure 7.13 of Sen

4 b The winding around the poles results in a pulsating flux in the air gap. There is a short-circuited ring around a part of the pole. When the flux through this ring is changing, a current is induced in the ring. This current opposes the change of the flux. Therefore, the flux through this part of the pole is delayed with respect to the flux through the rest of the pole. This gives a rotating component in the pulsating flux. This rotating component of the flux induces voltages in the rotor bars. The rotor bars are short-circuited so that currents flow. The combination of these currents with the rotating component of the flux result in a torque which is enough to start the machine.