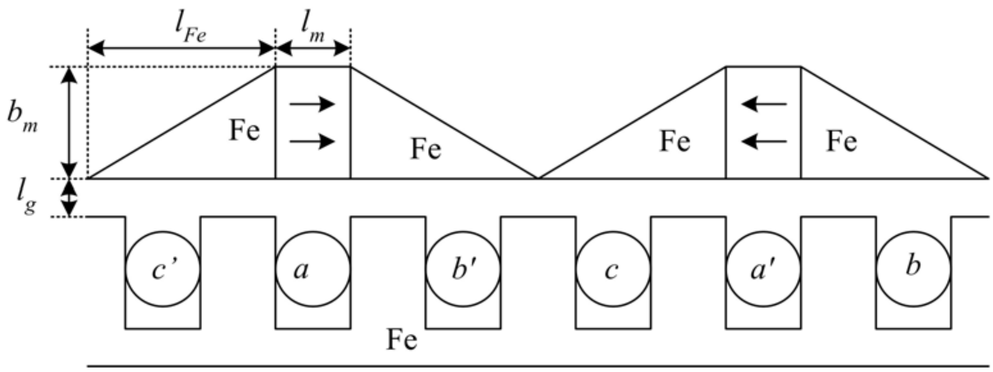
Examination Electrical Machines and Drives (ET4117)
4 November 2004 from 9.00 to 12.00.
This examination consists of 3 problems on 4 pages.
The number before a question indicates how many credits you can earn with the question.
This examination has to be made without using a book, old examinations or notes.

26 Problem 1

The figure below depicts a linearized cross-section of two pole-pitches of the magnetic circuit of a permanent magnet motor for a solar car. The upper part is the rotor with permanent magnets, the lower part is the stator with coils. The parts with Fe in it, are iron. The parts with arrows in it are magnets. The copper conductors are round with an indication for the phase (a, b or c).



The machine has the following characteristics (most of them are in the figure).
- The magnet length in the direction of magnetization is l_m.
- The air-gap length is l_g.
- The width of the magnet is b_m.
- The width of a flux concentrator is l_{Fe}.
- The axial stack length (in the direction perpendicular to the plane of the drawing) is l_s.
- The magnets have a remanent flux density B_{rm}, a relative recoil permeability μ_{rm} and the BH-characteristic in the second quadrant of the BH-plane is a straight line.

In the calculations, it may be assumed that
- The magnetic permeability of iron is infinite.
- The flux density crosses the magnets and the air gap perpendicularly.
- For the calculation of the flux density, the stator slots may be neglected, which means that you can calculate as if the air gap were constant (l_g) and the conductors are in the air gap.
- The wires of the turns are very thin.

2 a Write down Ampere's law (a simplified form of the first of Maxwell's equations).
3 b Sketch a contour and a surface in the cross section to which you can apply Ampere's law to calculate the magnetic flux density in the air gap.

In questions c to e, the stator currents are zero.

2 c Give an equation for the BH characteristic of the magnet in the second quadrant.
5 d Derive an expression for the air-gap flux density B_g using Ampere's law.

In question e, the air-gap flux density is B_g.

2 e Give an expression for the flux per pole, the pole flux.

The objective of the rest of this problem is to derive an equation for the voltage between the two terminals of the winding of phase *a* of this machine. This voltage equation can be derived from the second of Maxwell's equations or Faradays law.

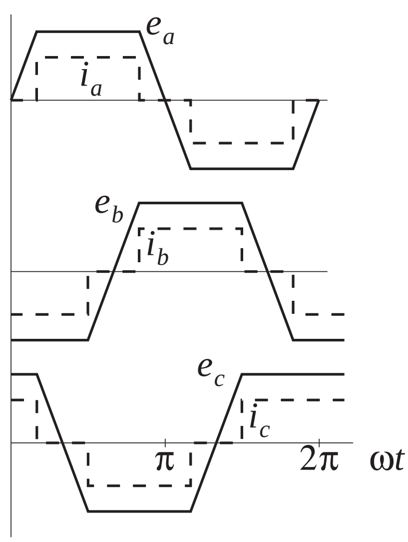
2 f Write down the second of Maxwell's equations or Faraday's law.
3 g Describe the contour and the surface to which you can apply the second of Maxwell's equations to derive the voltage equation of this winding.
2 h Give the general form of the equation for the voltage between the two machine terminals that can be derived from the second of Maxwell's equations (it is not necessary to give the derivation).

We assume that the self inductance of a phase winding and the mutual inductance between two phase windings are not a function of the rotor position.
It is given that

- the self-inductance of a stator phase winding is L
- the mutual inductance between two different stator phase windings is M
- the no-load flux linkage of a stator phase (due to the magnets) varies cosinusoidally with maximum $\lambda_{pm\max}$: $\lambda_{pma} = \lambda_{pm\max}\cos\theta_e$
- the sum of the three stator phase currents is zero.

5 i Work the general voltage equation out to the voltage equation of stator phase a.

33 Problem 2



In this problem, we again consider a solar car. If the solar car drives horizontally at a constant speed, the load for the motor is formed by the friction forces. In this problem it is assumed that the only friction force is air friction, and the air friction force is given by $F = K_f v^2$ where $K_f = 0.07\,\text{Ns}^2/\text{m}^2$.
The diameter of the driving wheel is 0.50 m.
The motor is a brushless DC motor. The no-load (excitation) phase voltage waveforms of the brushless DC motor for a solar car are trapezoidal as indicated in the figure. The maximum value of this phase voltage is given by $E_s = K_s\omega_m$ where $K_s = 0.6\,\text{Vs/rad}$.

The idealized currents are 120° degrees current blocks as indicated in the figure. The maximum value is I_s.
The stator phase resistance is $R_s = 0.05\,\Omega$.
In this problem, it is assumed that the iron losses are given by $P_{Fe} = d\omega_m^2$ where $d = 2.5\,\text{mNms/rad} = 2.5 \cdot 10^{-3}\,\text{mNms/rad}$.
Other losses in the motor are neglected.
The number of poles is 14.

3 a Give and expression for the electromagnetically generated torque using the power balance (it is not necessary to work out the power balance in detail, the result is enough).

The speed is 100 km/h (27.78 m/s).

3 b Calculate the mechanical shaft power necessary to drive at this speed.
3 c Calculate the mechanical angular frequency of the rotor ω_m and electrical angular frequency of the fundamental wave in the stator ω_e.
2 d Calculate the shaft torque at this speed.

It is assumed that the torque that has to be generated electromagnetically is the sum of the shaft torque and the torque due to the iron losses.

2 e Calculate the additional torque due to iron losses.
2 f Calculate the current necessary to generate the total electromagnetic torque.
4 g Calculate the efficiency.

The solar cells deliver a DC voltage. A battery is used as energy buffer. The voltage is 170 V.

4 h Sketch a suitable power electronic converter that converts the DC of the battery to a suitable voltage for the motor (on a level of active switches and diodes).
3 i Calculate the maximum speed the motor can reach on the DC battery voltage.
2 j The current waveforms are idealized. How do they change in practice when they are connected to a suitable power electronic converter?

3 k The expression for the iron losses used in this problem is not complete and not completely correct. Give more complete expressions for the iron losses (indicating what they depend on) and indicate what is missing in the expression for the iron losses in this problem.
2 l The model takes into account iron losses and copper losses. Are there other losses in the motor that are neglected? If yes, which?

31 Problem 3

This problem deals with induction machines. First a few general questions.

3 a Sketch a cross-section of a three-phase induction machine.
3 b Sketch the IEEE recommended equivalent circuit of the induction machine.
3 c Sketch the torque-speed characteristic of an induction machine and indicate the three modes of operation.

Questions d to f are about an induction machine with a name plate telling it is a 50 Hz induction machine with a rated speed of 960 rpm.

2 d What is the number of poles?
2 e What is the rated slip?
2 f What is the synchronous speed if the supply frequency is reduced to 25 Hz?

The stator winding of phase *a* has a winding distribution given by

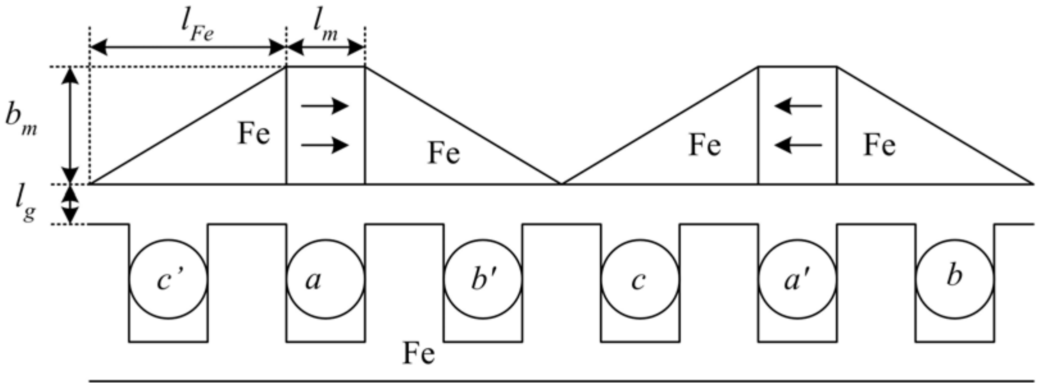
$$n_{sa}(\theta) = \frac{N_s}{2}\sin(\theta)$$

where n_{sa} is the number of conductors per radian, N_s is the number of turns of the phase winding and θ is the angular position.

4 g Derive and expression for the flux density in the air gap as a function of the angular position θ for the case that the current in this winding is i_a.
5 h Explain how the stator of a three-phase induction machine creates a rotating magnetic field in the air gap of the machine. In this explanation, give a mathematical derivation of such a rotating field, starting from expressions for the winding distributions and the currents. The structure of the derivation is more important than the correctness of all the mathematical steps.
Hint: $\cos(x)\cos(y) = \frac{1}{2}\cos(x+y) + \frac{1}{2}\cos(x-y)$

5 g Give at least 5 advantages or disadvantages of using an induction machine instead of a brushless DC machine in a solar car.
2 h Make a choice between the induction machine and the brushless DC machine if the main objective is winning the world solar challenge, a race over 3000 km purely on solar energy.

Page available answering problem 1b.

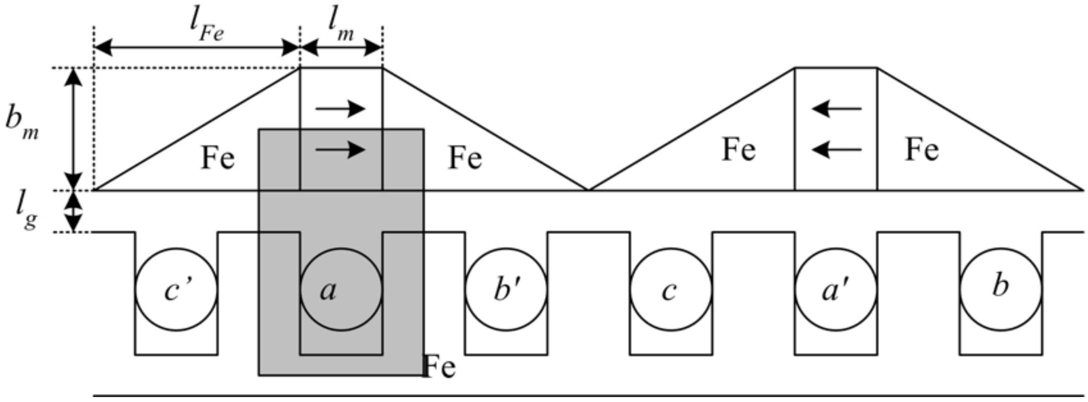


Answers to the examination of 4 November 2004

26 Problem 1

2 a $\oint_{C_m} \vec{H} \cdot d\vec{l} = \iint_{S_m} \vec{J} \cdot d\vec{A}$

3 b The surface area is made grey. The boundary of the surface is the contour.



2 c $B_m = \mu_0 \mu_{rm} H_m + B_{rm}$

5 d $2H_g l_g + H_m l_m = 0$

Using the BH relations for magnets and air:

$$\frac{2B_g l_g}{\mu_0} + \frac{(B_m - B_{rm})l_m}{\mu_0 \mu_{rm}} = 0$$

Flux continuity: $\oiint_A \vec{B} \cdot d\vec{A} = 0$

$$B_g A_g = B_m A_m \Rightarrow B_g l_{Fe} l_s = B_m b_m l_s \Rightarrow B_g l_{Fe} = B_m b_m$$

Substitution gives

$$\frac{2B_g l_g}{\mu_0} + \frac{(B_g \frac{l_{Fe}}{b_m} - B_{rm})l_m}{\mu_0 \mu_{rm}} = 0$$

$$B_g = \frac{l_m b_m}{2l_g b_m \mu_{rm} + l_m l_{Fe}} B_{rm}$$

2 e $\Phi_p = 2B_g l_s l_{Fe}$

2 f $\oint_{C_e} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_{S_e} \vec{B} \cdot d\vec{A}$

3 g The contour is the boundary of the surface. The contour is chosen in the electrical circuit, in the wire.

2 h $u_a = R_s i_a + \frac{d\lambda_a}{dt}$

5 i The flux linkage is a linear superposition of the different contributions:

$$\lambda_a = \lambda_{pm\max} \cos\theta_e + Li_a + Mi_b + Mi_c$$

The sum of the currents is zero, therefore

$$\lambda_a = \lambda_{pm\max} \cos\theta_e + Li_a + Mi_b + Mi_c - Mi_a - Mi_b - Mi_c = \lambda_{pm\max} \cos\theta_e + (L - M)i_a$$

Substitution in the general voltage equation gives:

$$u_a = Ri_a + \frac{d}{dt}\lambda_{pm\max}\cos p\theta + (L-M)\frac{di_a}{dt}$$

$$u_a = e_a + Ri_a + (L-M)\frac{di_a}{dt}$$

where

$$e_a = \frac{d}{dt}\lambda_{pm\max}\cos\theta_e = -\lambda_{pm\max}\sin\theta_e\frac{d\theta_e}{dt}$$

33 Problem 2

3 a $T_{em} = \frac{P_{em}}{\omega_m} = \frac{2E_s I_s}{\omega_m} = 2K_s I_s$

3 b $F = K_f v^2 = 54.01\,\text{N}$

$P_m = Fv = K_f v^3 = 1500\,\text{W}$

3 c $\omega_m = \frac{v}{r} = 111.1$ rad/s, $\omega_e = \frac{p}{2}\omega_m = 777.7\,\text{rad/s}$

2 d $T_{shaft} = \frac{P_m}{\omega_m} = 13.50\,\text{Nm}$

2 e $T_{Fe} = d\omega_m = 0.2778\,\text{Nm}$

2 f $I_s = \frac{T_{em}}{2K_s} = \frac{T_{shaft} + T_{Fe}}{2K_s} = 11.48\,\text{A}$

4 g $P_{Fe} = d\omega_m^2 = 30.86\,\text{W}$

There are always two phases conducting: $P_{Cu} = 2R_s I_s^2 = 13.19\,\text{W}$

$$\eta = \frac{P_m}{P_m + P_{Fe} + P_{Cu}} = 97.15\%$$

4 h See fig. 10.43.

3 i The inverter is not able to control the current and to make motor torque when the line voltage of the machine is larger than the battery voltage: $\omega_{m\max} = \frac{V_{battery}}{2K_s} = 141.7\,\text{rad/s}$.

The speed is then $v_{\max} = r\omega_{m\max} = 35.42\,\text{m/s} = 127.5\,\text{km/h}$.

2 j - There is a ripple on the current due to the pulse width modulation (PWM) of the converter

- Because of the inductance of the windings, the currents can not be switched on instantaneously.

3 k The iron losses consist of hysteresis losses and eddy-current losses:

$P_{hyst} = K_h B^S f$ where S is between 1.5 and 2.5

$P_{eddy} = K_e B^2 f^2$

In this problem, hysteresis losses and the effect of the stator currents on the magnetic field have been neglected

2 l Mechanical losses in the motor have also been neglected: friction losses in bearings, air friction in the motor, windage losses.

31	**Problem 3**
3 a	See fig. 5.3: cylindrical stator bore, three phases, 120° displacement between axes of phase windings, cylindrical rotor
3 b	See fig. 5.15
3 c	see fig. 5.17
2 d	6 poles (synchronous speed is 1000 rpm)
2 e	rated slip: 4%
2 f	synchronous speed at 25 Hz: 500 rpm
4 g	Using Amperes law, the flux density in the air gap due to a current in phase a can be calculated:

$$\oint_{C_m} \vec{H} \cdot d\vec{l} = \iint_{S_m} \vec{J} \cdot d\vec{A}$$

$$2l_g H_g(\theta) = \int_{\theta}^{\theta+\pi} n_{sa}(\theta) i_{sa} d\theta = N_s i_{sa} \cos(\theta)$$

$$B_{ga}(\theta) = \mu_0 H_{ga}(\theta) = \mu_0 \frac{N_s}{2l_g} i_{sa} \cos(\theta)$$

5 h The phase windings are sinusoidally distributed with 120 degrees displacement between the axes of the windings:

$$n_{sa}(\theta) = \frac{N_s}{2} \sin(\theta)$$

$$n_{sb}(\theta) = \frac{N_s}{2} \sin(\theta - \frac{2}{3}\pi)$$

$$n_{sb}(\theta) = \frac{N_s}{2} \sin(\theta - \frac{4}{3}\pi)$$

By using Ampere's law, the flux densities due to the currents in the different phases are calculated as

$$B_{ga}(\theta) = \mu_0 \frac{N_s}{2l_g} i_{sa} \cos(\theta)$$

$$B_{gb}(\theta) = \mu_0 \frac{N_s}{2l_g} i_{sb} \cos(\theta - \frac{2}{3}\pi)$$

$$B_{gc}(\theta) = \mu_0 \frac{N_s}{2l_g} i_{sc} \cos(\theta - \frac{4}{3}\pi)$$

The phase currents are sinusoidal currents with 120 degrees phase shift between the three phases:

$$i_{sa} = \hat{i} \cos(\omega t)$$

$$i_{sb} = \hat{i} \cos(\omega t - \frac{2}{3}\pi)$$

$$i_{sb} = \hat{i} \cos(\omega t - \frac{4}{3}\pi)$$

Substitution of the currents in the expression for the flux density gives

$$B_{ga}(\theta) = \mu_0 \frac{N_s \hat{i}}{2l_g} \cos(\theta) \cos(\omega t)$$

$$B_{gb}(\theta) = \mu_0 \frac{N_s \hat{i}}{2l_g} \cos(\theta - \frac{2}{3}\pi)\cos(\omega t - \frac{2}{3}\pi)$$

$$B_{gc}(\theta) = \mu_0 \frac{N_s \hat{i}}{2l_g} \cos(\theta - \frac{4}{3}\pi)\cos(\omega t - \frac{4}{3}\pi)$$

These three waves are standing waves, that can be described as the sum of two traveling waves:

$$B_{ga}(\theta) = \mu_0 \frac{N_s \hat{i}}{4l_g}(\cos(\omega t - \theta) + \cos(\omega t + \theta))$$

$$B_{gb}(\theta) = \mu_0 \frac{N_s \hat{i}}{4l_g}(\cos(\omega t - \theta) + \cos(\omega t + \theta - \frac{4}{3}\pi))$$

$$B_{gc}(\theta) = \mu_0 \frac{N_s \hat{i}}{4l_g}(\cos(\omega t - \theta) + \cos(\omega t + \theta - \frac{8}{3}\pi))$$

If we add these waves, we get

$$B_g(\theta) = \mu_0 \frac{3N_s \hat{i}}{4l_g} \cos(\omega t - \theta)$$

The waves traveling in negative direction cancel each other.

5 i advantages of the induction machine:

- field weakening is possible so that
  - higher speeds are possible (e.g. downhill)
  - reduction of iron losses is possible (e.g. downhill)
- cheaper
- no magnets that might be demagnetized
- no rotor position sensor necessary

disadvantages of the induction machine (more important)

- on average less efficient (also currents in rotor)
- heavier
- larger

2 j For winning the world solar challenge, the most important issues are a good efficiency and a low weight. Therefore, a PM machine is chosen.