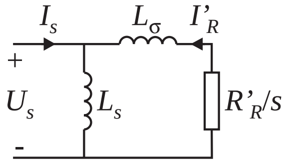
Examination Electrical Machines and Drives (ET4117)

3 November 2006 from 9.00 to 12.00.
This examination consists of 4 problems on 3 pages. Page 4 can be used to answer question 2b.
The number before a question indicates how many credits you can earn with the question.
This examination has to be made without using a book, old examinations, notes, dictionaries or programmable calculators.

25 Problem 1



This problem deals with a 2-pole three-phase induction machine connected to a 50 Hz supply (ω_s=2π50 rad/s). The figure gives an equivalent circuit where the stator resistance is neglected. This equivalent circuit can be mathematically derived from the T-scheme (the IEEE recommended equivalent circuit). The Thevenin equivalent circuit of this equivalent circuit is this equivalent circuit where L_s has been omitted.

8 a Use this equivalent circuit to derive an expression for the torque-speed characteristic as a function of the parameters of the equivalent circuit, the voltage, the angular frequency and the slip (L_σ, R_R', U_s, ω_s, s).

5 b Mention at least 6 important assumptions used in the derivation other than that Maxwell's equations and the power balance hold.

3 c Sketch the torque-speed characteristic and indicate the three modes of operation.

The parameters of the equivalent circuit can be determined from the no-load test and the short circuit test. In the no-load test, the rotor rotates synchronously ($\omega_m=\omega_s$) at the rated voltage. The voltage and the current are measured as
U_s =230 V and
I_s=20 A.

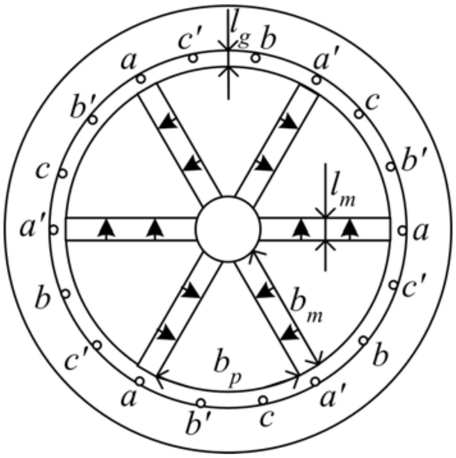
2 d Determine the value of L_s.

In the short-circuit test, the rotor is blocked (ω_m=0) and voltage, current and power are measured at a reduced voltage. The results are
U_s =40 V
I_s=80 A
P =600 W (N.B. the power is measured three-phase).

4 e Determine L_σ and R_R'. It may be assumed that the magnetizing current through L_s during the blocked rotor test is negligible.

3 f The machine works as a motor at $\omega_m=\omega_s/2$ while still connected to the 50 Hz grid. Determine the efficiency according to the used equivalent circuit.

32 Problem 2



The figure depicts a cross-section of the magnetic circuit of a permanent magnet motor. The parts with arrows in it are magnets. The conductors are drawn in the air gap with an indication for the phase (a, b or c).
The machine has the following characteristics (most of them are in the figure).

- The magnet length in the direction of magnetization is l_m=10 mm.
- The air-gap length is l_g=1 mm.
- The width of the magnet is b_m=60 mm.
- The width of a rotor pole is b_p=60 mm.
- The axial stack length (in the direction perpendicular to the plane of the drawing) is l_s=150 mm.
- The total number of turns of each of the stator windings is 18.
- The remanent flux density of the magnets is B_{rm}=1.2 T.
- The relative recoil permeability of the magnets μ_{rm}=1.
- The BH-characteristic in the second quadrant of the BH-plane is a straight line.

In the calculations, it may be assumed that

- The magnetic permeability of iron is infinite.
- The flux density crosses the magnets and the air gap perpendicularly.
- The wires of the turns are very thin.
- The stator currents are zero.

2 a Write down Ampere's law (a simplified form of the first of Maxwell's equations).
3 b Sketch a contour and a surface in the cross section to which you can apply Ampere's law to calculate the magnetic flux density in the air gap.
2 c Give an equation for the BH characteristic of the magnet in the second quadrant.
5 d Derive an expression for the air-gap flux density B_g using Ampere's law.
2 e Calculate the air-gap flux density B_g.
2 f Write down the second of Maxwell's equations or Faraday's law.
3 g Describe the contour and the surface to which you can apply the second of Maxwell's equations to derive the voltage equation of this winding.

In the next questions, the air-gap flux density is B_g=1.4 T.
The machine rotates at 6000 rpm.

2 h Calculate the maximum flux per pole, the pole flux.
2 i Calculate the frequency of the voltages induced in the stator windings.
2 j Calculate the maximum no-load voltage induced in the stator windings.
2 k Is an air gap flux density larger than the remanent flux density realistic? Why or why not?
5 l Sketch a power electronic converter that can be used to drive this machine (at least the active switches, the diodes and the important connections) from a DC source.

20 Problem 3

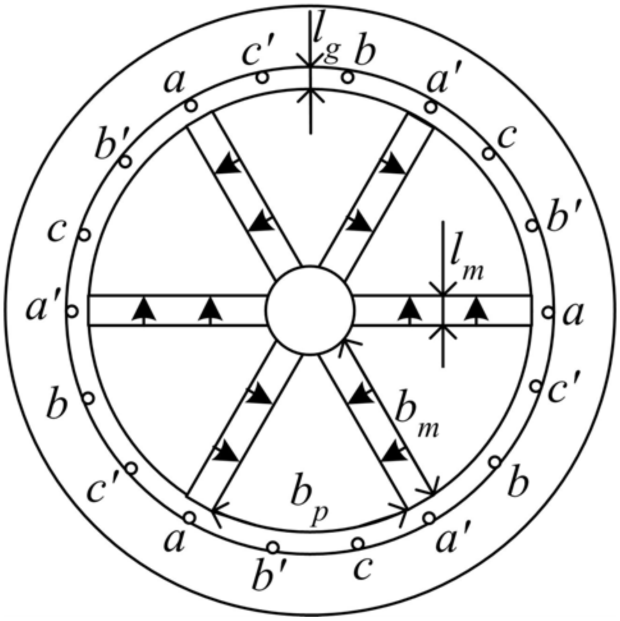
A three-phase synchronous generator is driven by a gas turbine.
The synchronous reactance is X=2 Ω.
The losses in the generator may be neglected.
It has been just been connected to an infinite bus with a phase voltage of 20 kV and a frequency of 50 Hz in a proper way.

2 a Give the equivalent circuit of the synchronous machine.
4 b Which 4 conditions that have to be satisfied before a synchronous machine can be connected to the grid?
2 c Calculate the stator current just after the proper grid connection.
5 d The excitation current is doubled while the power remains the same. Calculate the stator current phasor (value and angle or real and imaginary part) and sketch the phasor diagram.
7 e The shaft power is increased to 600 MW while the excitation current remains the same at double the value during grid connection. Sketch the phasor diagram and calculate the stator current phasor (value and angle or real and imaginary part) and the load angle.

13 Problem 4

6 a Sketch a cross section of a switched reluctance motor and explain the principle of operation.
7 d Explain the principle of operation of a single-phase hysteresis motor and explain how it can be started.

Page available answering problem 2b.



25 **Problem 1**

8 a The expression is derived from the power balance and the Thevenin equivalent circuit. According to this equivalent circuit, the rotor current is given by

$$I_s = \frac{U_s}{\sqrt{(\omega_s L_\sigma)^2 + \left(\frac{R'_R}{s}\right)^2}}$$

Steady-state is assumed, which means that the rotor speed is constant and the stored magnetic energy does not change.

Therefore, all power is dissipated in the rotor resistance or delivered as mechanical output power. According to the per-phase equivalent circuit, the three-phase stator input power is given by

$$P_s = 3I_R^2 \frac{R'_R}{s} = 3I_R^2 R'_R + 3I_R^2 R'_R \frac{1-s}{s}$$

The power dissipated in the rotor resistance is

$$P_{diss} = 3I_R^2 R'_R$$

The rest is the electromechanic power P_{em} from which the torque can be calculated:

$$T_{em} = \frac{P_{em}}{\omega_m} = \frac{3I_R^2 R'_R \frac{1-s}{s}}{(1-s)\omega_s} = \frac{3I_R^2 R'_R}{s\omega_s}$$

Substitution of the earlier calculated current gives

$$T_{em} = \frac{P_{em}}{\omega_m} = \frac{3I_R^2 R'_R \frac{1-s}{s}}{(1-s)\omega_s} = \frac{3R'_R}{s\omega_s} \frac{U_s^2}{(\omega_s L_\sigma)^2 + \left(\frac{R'_R}{s}\right)^2}$$

5 b Used assumptions:

- stator windings are sinusoidally distributed
- stator windings have axis which are 120° shifted with respect to each other
- the stator voltages (and the resulting currents) are sinusoidal (with constant amplitude and frequency, steady state)
- the stator voltages (and the resulting currents) have a phase shift of 120° with respect to each other
- the mechanical speed is constant (steady state)
- the stored magnetic energy is constant (steady state)
- stator copper losses are neglected,
- iron losses are neglected
- mechanical (friction and windage losses) are neglected.

3 c See figure 5.17.

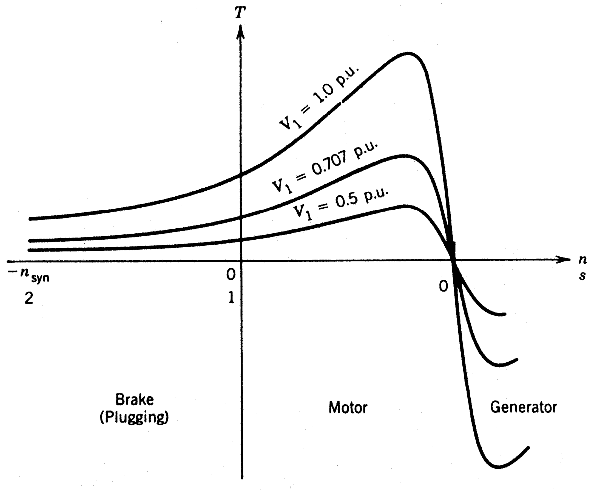


FIGURE 5.17 Torque–speed profile at different voltages.

2 d $$L_s = \frac{U_s}{\omega_s I_s} = 36.6mH$$

4 e $$R'_R = \frac{P}{3I_R^2} = 31.25m\Omega$$

During the blocked-rotor test:

$$U_s = I_s \sqrt{R'^2_R + (\omega_s L_\sigma)^2}$$

Therefore

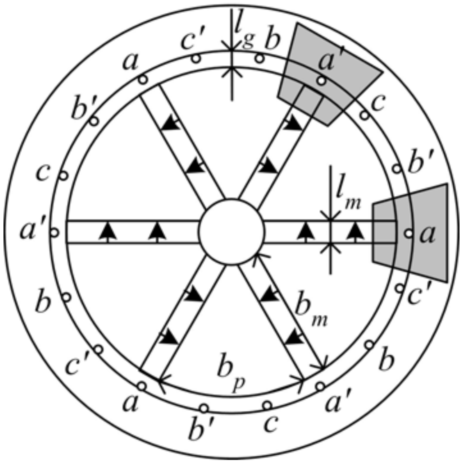
$$L_\sigma = \frac{\sqrt{U_s^2 - (I_s R_R')^2}}{\omega_s I_s} = 1.588mH$$

3 f In the used equivalent circuit, only the rotor resistance takes active power P. sP is dissipated in the rotor winding, while (1-s)P is converted into electromechanical power. Therefore,

$$\eta = \frac{1}{2}$$

32 Problem 2

2 a $\oint_{C_m} \vec{H} \cdot d\vec{l} = \iint_{S_m} \vec{J} \cdot d\vec{A}$



3 b Two possible contours and two possible surfaces are indicated. The boundary of the surface is the contour.

2 c $B_m = \mu_0 \mu_{rm} H_m + B_{rm}$

5 d $2H_g l_g + H_m l_m = 0$

Using the BH relations for magnets and air:

$$\frac{2B_g l_g}{\mu_0} + \frac{(B_m - B_{rm}) l_m}{\mu_0 \mu_{rm}} = 0$$

Flux continuity: $\oiint_A \vec{B} \cdot d\vec{A} = 0$

$$B_g A_g = B_m A_m \Rightarrow B_g l_s \frac{1}{2} b_p = B_m b_m l_s \Rightarrow B_g l_{Fe} = 2B_m b_m$$

Substitution gives

$$\frac{2B_g l_g}{\mu_0} + \frac{(B_g \frac{b_p}{2b_m} - B_{rm}) l_m}{\mu_0 \mu_{rm}} = 0$$

$$B_g = \frac{2 l_m b_m}{4\mu_{rm} l_g b_m + b_p l_m} B_{rm}$$

2 e $B_g = 1.5\,\mathrm{T}$

2 f $\oint_{C_e} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_{S_e} \vec{B} \cdot d\vec{A}$

3 g The contour is chosen in the electrical circuit, in the wire. The contour is the boundary of the surface.

2 h $\Phi_p = B_g l_s b_p = 12.6\,\mathrm{mWb}$

2 i $f = \frac{n}{60} \frac{p}{2} = 300\,\mathrm{Hz}$

2 j The stator bore radius can be calculated as

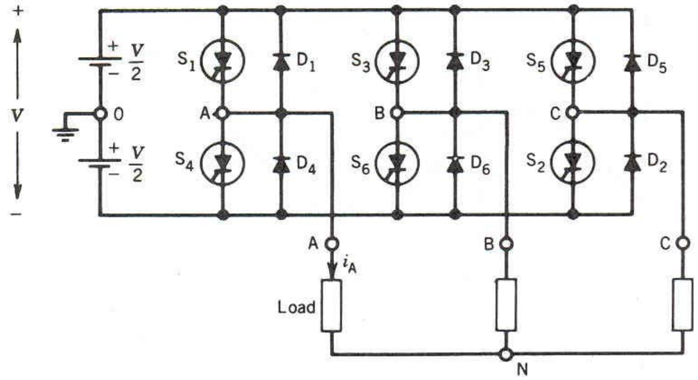
$$r_s = \frac{p(b_p + l_m)}{2\pi} + l_g = 67.85\,\mathrm{mm}$$

Using this, the induced voltage can be calculated as

$$e_{\max} = 2N_s B_g l_s \omega_m r_s = 322.3\,\mathrm{V}$$

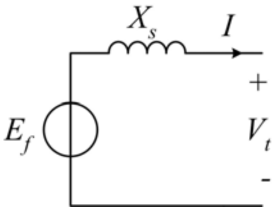
2 k Yes, an air gap flux density larger than the remanent flux density is realistic because of flux concentration: the flux going through the magnet is concentrated in a part of the rotor pole that is smaller than the magnet.

5 l Figure 10.43 depicts an inverter.



20 Problem 3

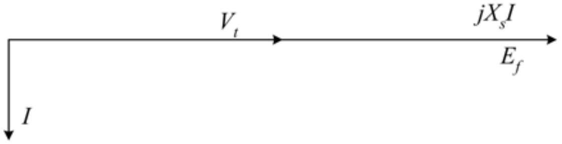
2 a equivalent circuit



4 a Before connecting a synchronous machine to the infinite bus, the following 4 conditions have to be met:
- voltage of machine and bus are equal
- frequency of machine and bus are equal
- phase order of machine and bus are equal
- phase of machine and bus are equal

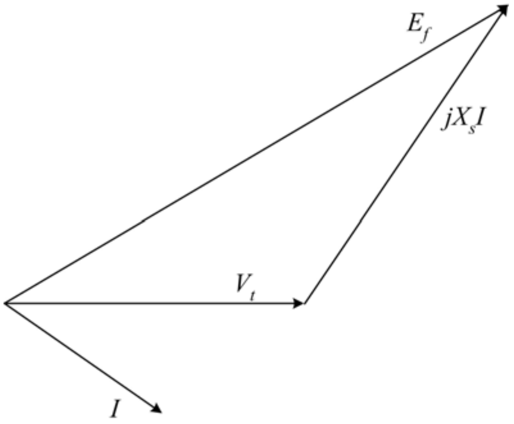
2 c Before grid connection, the rotor runs synchronously with the grid (frequency of machine and bus are equal). The power level is just high enough to overcome friction friction and windage losses and iron losses, there is no additional torque. Voltage of machine and bus are equal: $E_f = V_t$.

Therefore $I = \frac{E_f - V_t}{jX_s} = 0$



5 d $E_f = 2V_t$, therefore

$$I = \frac{E_f - V_t}{jX_s} = \frac{V_t}{jX_s} = -j10\,\text{kA}$$



7 e The phasor diagram will have a form like sketched.

$$P = \frac{3E_fV_t}{X}\sin(\delta)$$

Therefore, the load angle can be calculated as

$$\delta = \arcsin\left(\frac{XP}{3E_fV_t}\right) = \frac{\pi}{6} = 30°$$

Using this:

$$I = \frac{E_f - V_t}{jX} = \frac{34.64 + j20 - 20}{2j} = 10 - j7.32\,\text{kA}$$

13 Problem 4

6 a See figure 6.42. When two stator poles that are 180 degrees displaced (e.g. phase A in the figure) are excited, the rotor poles that are most close, are attracted and move to the aligned position. As soon as the aligned position is reached, the excitation is removed from this phase and the next phase is excited, e.g. phase B in the figure. This will attract poles b of the rotor. So if the stator poles are excited counter-clockwise, the rotor will move clockwise and at a much lower speed.

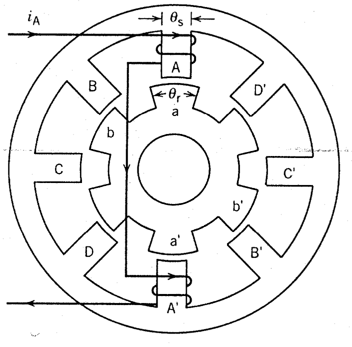


FIGURE 6.42 Cross section of a switched reluctance motor (SRM).

7 b The principle of operation of a hysteresis motor is that there is an angle between the field of the semi-hard magnetic material in the rotor and the field of the stator when there is a torque. When the motor runs asynchronous, the magnetization of the semi-hard magnetic material in the rotor rotates with the magnetic field of the stator, but before the magnetic field of the rotor changes, there must be a difference with the magnetic field of the stator, which creates a torque. When the rotor runs synchronous, it runs like a permanent-magnet synchronous machine.

A single-phase hysteresis motor is started in the same way as a single-phase induction motor: there has to be a winding that gives the magnetic field a rotating component besides the pulsating component of the main winding. This could be done with shaded poles in the stator, or with an auxiliary winding in the stator that has a different impedance from the main stator winding and is also displaced in position. The difference in impedance can be increased by using a capacitor in series with the auxiliary winding.