

Examination Electrical Machines and Drives (ET4117)

2 November 2007 from 9.00 to 12.00.

This examination consists of 5 problems on 4 pages. Page 5 can be used to answer problem 4 question b.

The number before a question indicates how many credits you can earn with the question.

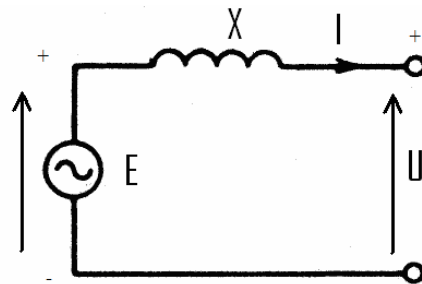
This examination has to be made without using a book, old examinations, notes, dictionaries or programmable calculators; a pocket calculator may be used.

5 Problem 1

- 3 a Give the expressions for the no-load armature voltage E_a and the torque T as a function of the pole flux Φ , the motor constant K , the speed ω_m and the current I .
- 2 b Why is it dangerous to disconnect of the field winding of a separately excited DC machine that is running in no-load?

12 Problem 2

In the AC transmission network, most of the energy is generated by synchronous generators. This question is about the synchronous generator of a power plant, which is connected to the grid, which is assumed infinitely strong. The machine is in generator operation and in steady state. A simple single phase-equivalent circuit is given below. E is the generator excitation phase voltage and U is the phase grid voltage (the terminal voltage).



The rotor current is so that the RMS value of the excitation voltage E is twice the RMS value of the terminal voltage: $E = 2U$.

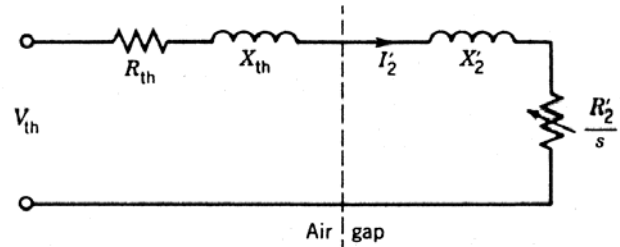
The generator delivers maximal power P . (Hint: this has implications for the load angle.)

- 3 a Sketch the components E , U and I in a phasor diagram for the case the voltages and the current are as given in the figure.
- 3 b Calculate the angle φ between the terminal voltage and the phase current and calculate the power factor.
- 2 c Do you think an operation situation like this is realistic in practice? Explain.
- 2 d A capacitor bank delivers reactive power, a reactor consumes reactive power. In the situation defined in this question, does the generator deliver or consume reactive power?
- 2 e Describe what action is needed in general in a power plant to change the amount of (i) active power P and (ii) reactive power Q .

28 Problem 3

- 4 a Describe why an asynchronous machine (or induction machine) starts to rotate when the 3 phase voltage is applied to the stator. Explain also why the torque is zero if the rotor runs synchronously.

The Thevenin equivalent circuit of an induction machine is shown in the figure.



- 4 b Derive expressions for the copper losses in the rotor P_{Cu} and for the electrical power that is converted into mechanical power P_{mech} as a function of the air gap power P_{ag} and the slip s .
- 4 c Give an expression for the torque-speed characteristic of an induction machine that can be derived from the Thevenin equivalent circuit. (It is not necessary to give the derivation).
- 4 d Neglect the Thevenin resistance ($R_{th} = 0$) and sketch the torque speed characteristic of the induction machine in one figure for four different cases
- Original (rated frequency, rated voltage, normal rotor resistance)
 - Halve the rated stator frequency (and also halve the rated stator voltage)
 - Halve the stator voltage (stator frequency remains the rated frequency)
 - Double the rotor resistance (via slip rings)

In most modern wind turbines, a doubly-fed induction machine is used as generator. We are now going to consider the basic operation principle of such a machine, without going into very detailed calculations. The doubly-fed induction machine is a wound-rotor induction machine. There is a power electronic converter between the grid and the rotor slip rings. By controlling the power electronics, any electrical stator frequency can be obtained. In a wind turbine, the converter is controlled in such a way that the frequency induced in the stator windings is 50 Hz. Therefore, the stator windings can be connected to the 50 Hz grid.

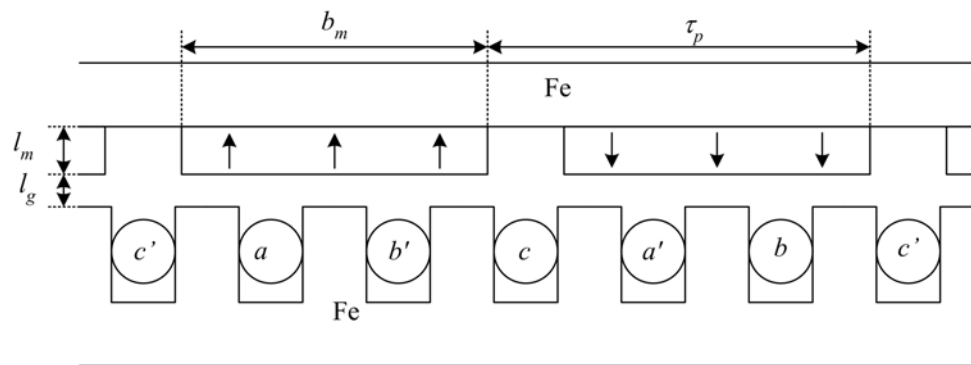
This feature enables the wind turbine to operate at low rotor speed (below synchronous speed) when the wind speed is low and at high operating speed (above synchronous speed) when the wind speed is high.

The doubly-fed induction generator of a wind turbine has 2 pole pairs (4 poles) and is connected to the 50 Hz AC grid. It operates as a generator at a speed of $n = 1200$ rpm.

- 3 e Calculate the slip.
- 2 f Determine the frequency that has to be produced by the power electronic converter connected to the rotor slip rings.
- 2 g What is the sign of the air-gap power P_{ag} and of the expression for the copper losses in the rotor P_{Cu} (positive or negative)? N.B.1: compare question c. N.B.2: motor convention is used in the figure with the equivalent circuit.
- 2 h What is the direction of the power flow between the power electronic converter and the rotor?
- 3 i What kind of power electronic converter do we need between the rotor slip rings and the grid? Pay attention both at the grid side and the machine side.

37 Problem 4

Direct-drive generators in wind turbines often are of a permanent-magnet type. The figure below depicts a linearized cross-section of two pole-pitches of the magnetic circuit of such a permanent-magnet generator. The upper part is a part of the rotor with the permanent magnets, the lower part is the stator with coils. The parts with Fe in it, are iron. The parts with arrows in it are magnets. The copper conductors are round with an indication for the phase (a, b or c).



The machine has the following characteristics (most of them are in the figure).

- The magnet length in the direction of magnetization is $l_m = 15$ mm
- The air-gap length is $l_g = 5$ mm.
- The width of the magnet is $b_m = 80$ mm
- The pole pitch is $\tau_p = 100$ mm
- The width of a slot is equal to the width of a tooth: $b_s = bt = 16.67$ mm.
- The axial stack length (in the direction perpendicular to the plane of the drawing) is $l_s = 1.2$ m.
- The magnets have a remanent flux density of $B_{rm} = 1.2$ T a relative recoil permeability $\mu_{rm} = 1$ and the BH-characteristic in the second quadrant of the BH-plane is a straight line.
- The number of poles is $p = 120$.
- The total number of turns of a phase winding is $N_s = 60$. The number of conductors per slot is $2N_s/p = 1$.
- The nominal speed of the wind turbine is 20 rpm.
- The nominal power of the wind turbine is 2 MW.

In the calculations, it may be assumed that

- The magnetic permeability of iron is infinite.
- The flux density crosses the magnets and the air gap perpendicularly.
- For the calculation of the flux density, the stator slots may be neglected, which means that you can calculate as if the air gap were constant (l_g) and the conductors are in the air gap.
- The wires of the turns are very thin and the windings are concentrated (which means that the winding factor is 1).

- 2 a Write down Ampere's law (a simplified form of the first of Maxwell's equations).
- 3 b Sketch a contour and a surface in the cross section of the machine to which you can apply Ampere's law to calculate the magnetic flux density in the air gap due to the

magnets. You can use the cross section of the machine in the figure on the last page of the exam.

In questions c to m, the stator currents are zero.

- 2 c Give an equation for the BH-characteristic of the magnet in the second quadrant.
- 4 d Derive an expression for the air-gap flux density B_g using Ampere's law and calculate the value of the air-gap flux density B_g .

In questions e to m, the air-gap flux density is $B_g = 0.85$ T.

- 2 e Calculate the flux density in a tooth (you can assume that the flux density in the slot is zero because of the high permeability of the iron of the slot).
- 2 f Is this a realistic value? Why or why not? What is the main limitation?
- 2 g Write down the second of Maxwell's equations or Faraday's law.
- 2 h Describe the contour and the surface to which you can apply the second of Maxwell's equations to derive the voltage induced in phase c.
- 2 i Calculate the electrical frequency.
- 2 j Sketch the flux linkage of phase c as a function of time if the rotor rotates with a constant speed.
- 2 k Sketch the no-load phase voltage (electromotive force, EMF) of phase c as a function of time if the translator moves with a constant speed.
- 2 l Calculate the maximum value of the flux linkage.
- 2 m Calculate the maximum value of the no-load phase voltage (electromotive force, EMF).

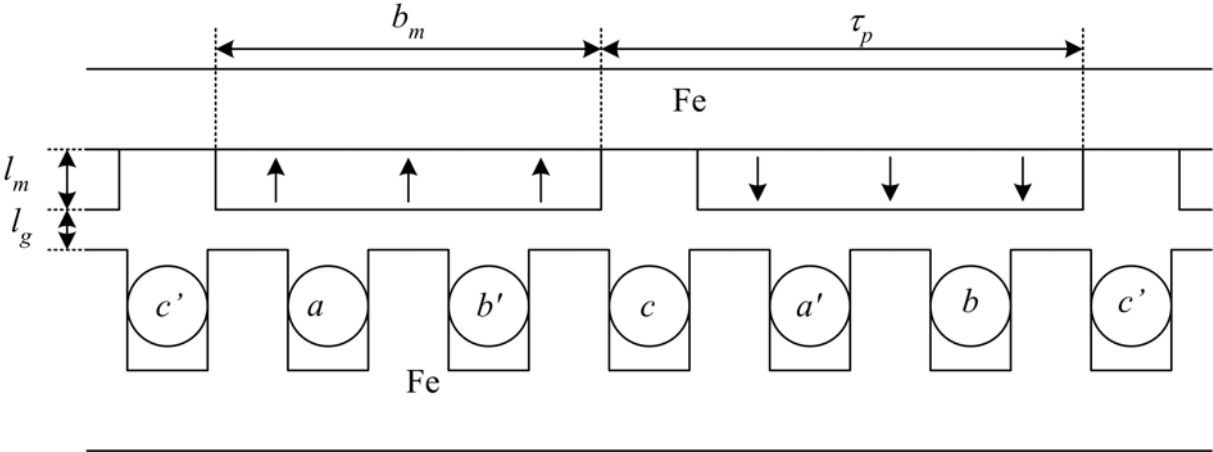
From now, it is assumed that the no-load phase voltage is sinusoidal (because of skewing and flux fringing) with an RMS value of the phase voltage of 350 V. The machine is used as a PM synchronous machine with sinusoidal currents.

- 2 n Calculate the minimum current necessary to make a power of 2 MW.
- 2 o What type of converter can be used to connect this generator to the three-phase grid?
- 3 p Sketch such a converter or a phase leg of such a converter.

8 Problem 5

- 2 a A single phase induction machine is normally connected between a phase line (L) and the neutral (N). We are now looking at the split-phase motor machine where an auxiliary winding is provided to start the machine. What happens to the rotation direction of this machine if one exchanges the L and N?
- 2 b For some purposes, the starting torque of the split-phase motor is too low. What can be done to give the machine a higher starting torque?
- 4 c A universal motor is a motor that can be fed from both AC and DC sources, and is used in, for example, drilling machines and vacuum cleaners. Sketch a simple equivalent circuit of the universal machine and tell what is needed here to change the rotation direction.

Page available answering problem 4 b.



Answers to the examination of 2 November 2007

5 Problem 1

3 a $E_a = K_a \Phi \omega_m$

$$T = K_a \Phi I_a$$

- 2 b If the field is disconnected, Φ will become really low (there is still some Φ present). Because E_a becomes due to this also lower, the current I_a becomes higher and ω_m increases to an extremely high value.

12 Problem 2

- 3 a The maximum power P is generated when the load angle δ (the angle between excitation voltage and terminal voltage) is $\pi/2$. This appears for example from the equation for the three-phase power

$$P = \frac{3V_t E}{X} \sin \delta$$

- 3 b From the phasor diagram follows that:

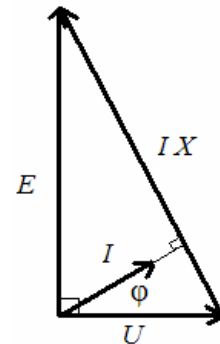
$$jXI = (-1 + 2j)U$$

$$I = (2 + j) \frac{U}{X}$$

$$\varphi = \arctan(1/2) = 0.464 = 26.6^\circ$$

power factor: $\cos \varphi = 0.894$

- 2 c This is not realistic in practice, because the generator is operating on the boundary between stable operation and instable operation. The protections of the generator do not allow a critical operation point like this.
- 2 d When generator convention is used, the current in a reactor leads the voltage by 90 degrees. In this phasor diagram, the current also has a leading component, which means that the generator consumes reactive power.
- 2 e (i) vary applied rotor torque (by changing the amount of fuel or steam) to vary P
(ii) vary applied rotor current to vary Q



28 Problem 3

- 4 a The axes of the stator windings have a spatial displacement of 120 degrees. The stator currents have a phase shift of 120 degrees. A three-phase current in a three-phase stator creates a rotating magnetic field in the air gap. This rotating magnetic field induces voltages in the rotor bars. Because these bars are short-circuited, currents are flowing in these bars. These rotor currents produce an electric field that wants to align with the rotating field of the stator, resulting in a torque. In other words, the combination of the rotating magnetic field and the rotor currents results in a torque. Therefore the machine starts rotating. If the rotor rotates synchronously, the no voltages and currents are induced in the rotor, so there is no torque.

- 4 b From the power balance and the equivalent circuit

$$P_{ag} = P_{mech} + P_{Cu}$$

$$P_{Cu} = 3I_2'^2 R'_R$$

$$P_{ag} = 3I_2'^2 \frac{R'_R}{s}$$

$$P_{mech} = P_{ag} - P_{Cu} = 3I_2'^2 R'_R \frac{(1-s)}{s}$$

Therefore,

$$P_{Cu} = sP_{ag}$$

$$P_{mech} = (1-s)P_{ag}$$

- 4 c The expression is derived from the power balance and the Thevenin equivalent circuit. According to this equivalent circuit, the rotor current is given by

$$I'_2 = \frac{V_{th}}{\sqrt{(X_{th} + X'_2)^2 + \left(R_{th} + \frac{R'_R}{s}\right)^2}}$$

Steady-state is assumed, which means that the rotor speed is constant and the stored magnetic energy does not change.

Therefore, all power is dissipated in the rotor resistance or delivered as mechanical output power. According to the per-phase equivalent circuit, the three-phase stator input power is given by

$$P_s = 3I_2'^2 \frac{R'_R}{s} = 3I_2'^2 R'_R + 3I_2'^2 R'_R \frac{1-s}{s}$$

The power dissipated in the rotor resistance is

$$P_{diss} = 3I_2'^2 R'_R$$

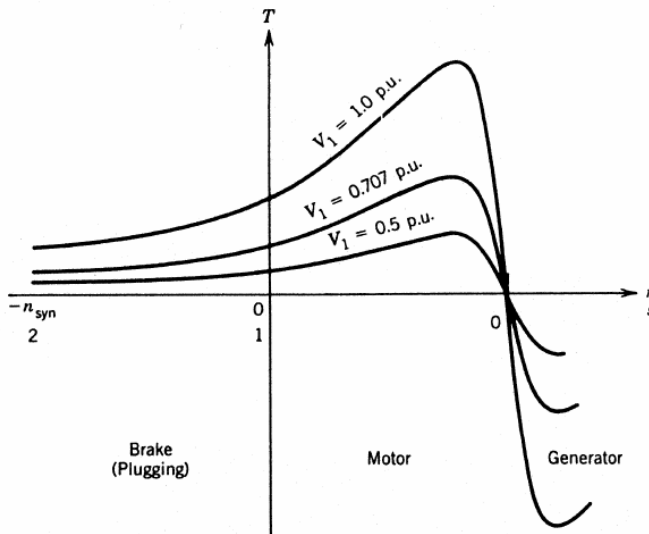
The rest is the electromechanic power P_{em} from which the torque can be calculated:

$$T_{em} = \frac{P_{em}}{\omega_m} = \frac{3I_2'^2 R'_R \frac{1-s}{s}}{(1-s)\omega_s} = \frac{3I_2'^2 R'_R}{s\omega_s}$$

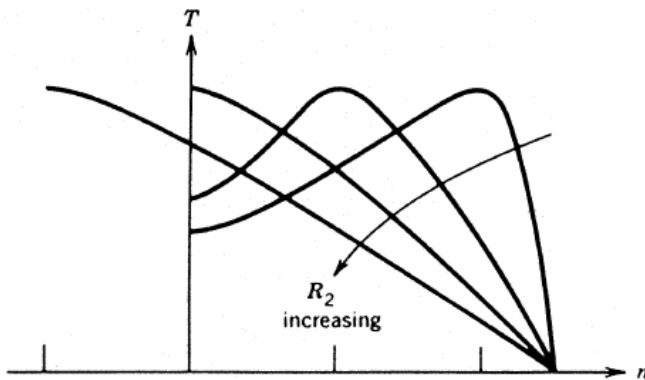
Substitution of the earlier calculated current gives

$$T_{em} = \frac{P_{em}}{\omega_m} = \frac{3I_2'^2 R'_R \frac{1-s}{s}}{(1-s)\omega_s} = \frac{3R'_R}{s\omega_s} \frac{V_{th}^2}{(X_{th} + X'_2)^2 + \left(R_{th} + \frac{R'_R}{s}\right)^2}$$

- 4 d In case (i) $V_1 = 1$ pu and the synchronous speed is 1 pu.
 In case (ii) $V_1 = 0.5$ pu, and the synchronous speed is 0.5 pu. The shape of the torque and the maximum value remain the same as for case (i), namely 1 pu
 In case (iii) $V_1 = 0.5$ pu and the synchronous speed is 1 pu. See below, the maximum value of the torque is 0.25 pu.



Case (iv) is shown below.



3 e If p is the number of poles,

$$n_s = \frac{120f}{p} = 1500 \text{ rpm}$$

$$s = \frac{n_s - n}{n_s} = 0.2$$

2 f $f_{rotor} = sf_{stator} = 10 \text{ Hz}$

2 g The machine is generating power, while motor convention is used. Therefore, the air gap power and the mechanical power must be negative.

We have seen that

$$P_{Cu} = sP_{ag} \text{ and}$$

$$P_{mech} = (1-s)P_{ag}$$

For a slip of 0.2, this means that also the power dissipated in the rotor (and the power electronics) is negative, which means that power is supplied by the power electronic converter to the rotor.

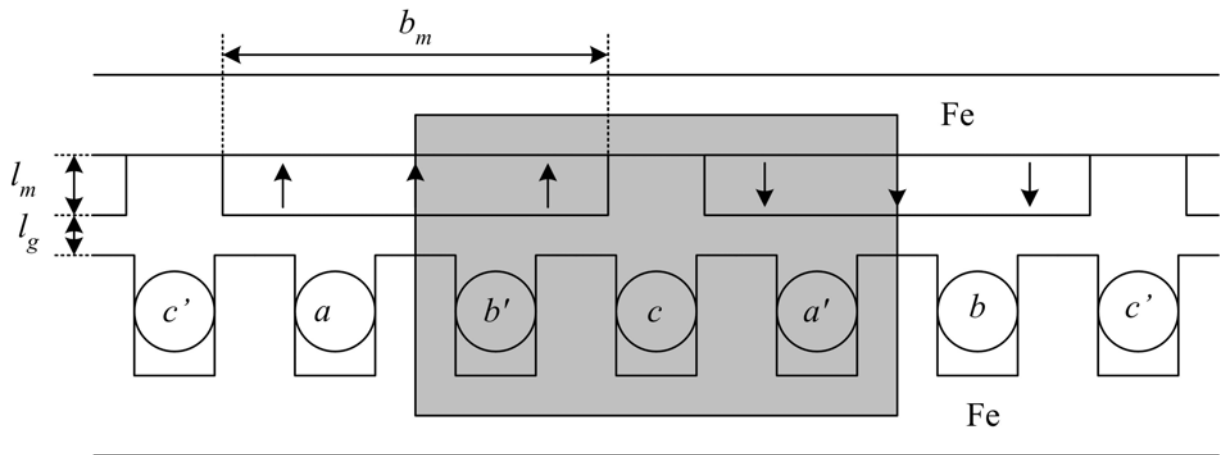
2 h From converter to rotor.

3 i A back-to-back voltage source inverter, which means that the grid side of the converter and the rotor side of the converter have a full inverter with active switching elements. This is necessary because the direction of the power flow is from grid to rotor at speeds below synchronous speed, and from rotor to grid at speeds above synchronous.

38 Problem 4

2 a
$$\oint_{C_m} \vec{H} \cdot d\vec{l} = \iint_{S_m} \vec{J} \cdot d\vec{A}$$

3 b The surface area is made grey. The boundary of the surface is the contour.



2 c $B_m = \mu_0 \mu_{rm} H_m + B_{rm}$

4 d $2H_g l_g + 2H_m l_m = 0$

Using the BH relations for magnets and air:

$$\frac{B_g l_g}{\mu_0} + \frac{(B_m - B_{rm}) l_m}{\mu_0} = 0$$

Flux continuity: $\oint \vec{B} \cdot d\vec{A} = 0$

$$B_g A_g = B_m A_m \Rightarrow B_g = B_m$$

Substitution gives

$$\frac{B_g l_g}{\mu_0} + \frac{(B_g - B_{rm}) l_m}{\mu_0} = 0$$

$$B_g = \frac{l_m}{l_g + l_m} B_{rm} = 0.9 \text{ T}$$

2 e $B_t = \frac{b_s + b_t}{b_t} B_g = 1.7 \text{ T.}$

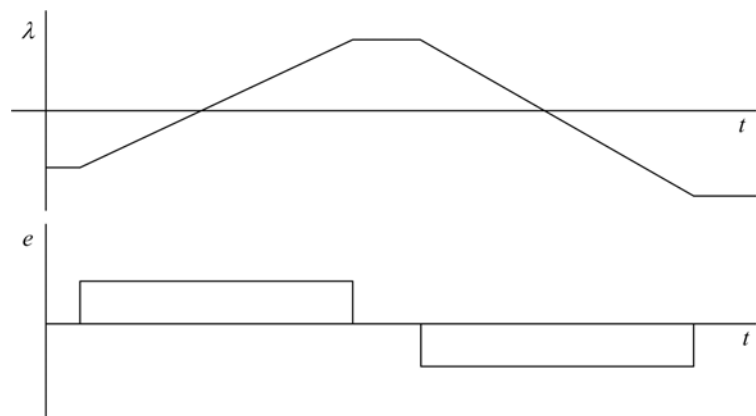
2 f This is a realistic value, because it is just below saturation, which is the main limitation.

2 g $\oint_{C_c} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_{S_c} \vec{B} \cdot d\vec{A}$

2 h The contour is the boundary of the surface. The contour is chosen in the electrical circuit, in the wire.

2 i $f = \frac{n p}{60 \cdot 2} = 20 \text{ Hz}$

2 j
2 k



2 l $\lambda_{\max} = N_s B_g l_s b_m = 4.896 \text{ Wb}$

2 m The flux linkage changes from λ_{\max} to $-\lambda_{\max}$ minimum in 40% of the electrical period.

Therefore: $e_{\max} = \frac{d\lambda}{dt} = 2.5 f 2\lambda_{\max} = 489.6 \text{ V}$

This could also be calculated as $e_{\max} = \frac{d\lambda}{dt} = 2.5 f 2\lambda_{\max} = 489.6,$

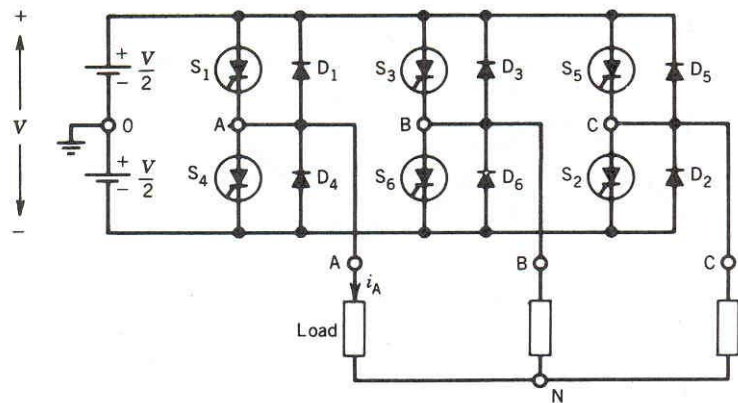
where $r_s = \frac{p\tau_p}{2\pi} = 1.910 \text{ m}$

2 n $I = \frac{P}{3E} = 1905 \text{ A.}$

2 o voltage source inverter

3 p Figure 10.27 depicts a rectifier.

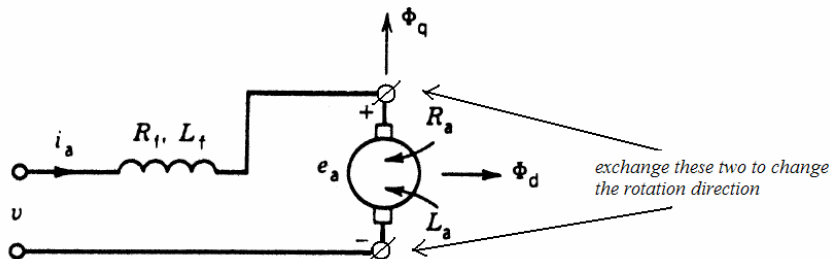
Figure 10.43 depicts an inverter.



8 Problem 5

2 a Nothing, the rotation direction remains the same.

2 b Create a magnetic field with a larger rotating component, which means that the current in the auxiliary winding needs to be larger in amplitude, and that the phase difference between the main winding and the auxiliary winding needs to be larger. Therefore, adding a capacitor in series with the auxiliary winding is effective.



4 c