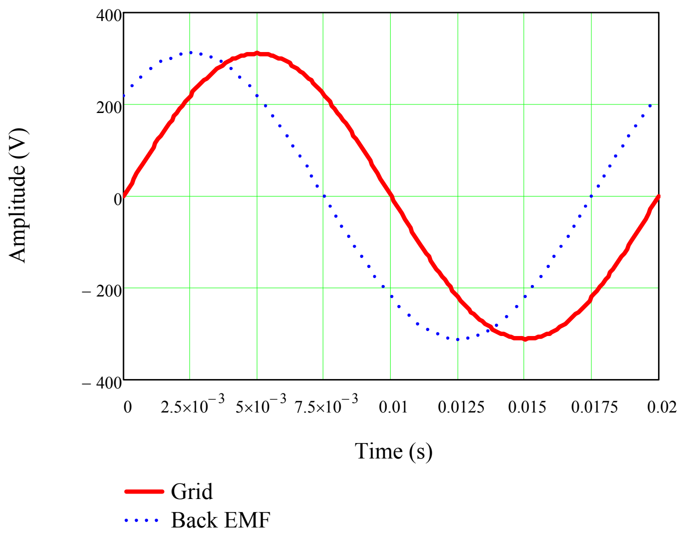
Exam Electrical Machines and Drives (ET4117)
6 November 2009 from 9.00 to 12.00.
This exam consists of 4 problems on 4 pages.
Page 5 can be used to answer problem 2 question b.
The number before a question indicates how many credits you can earn by answering that question. A partly correct answer may give a part of the credits.
This examination has to be made without using a book, old examinations, notes, dictionaries or programmable calculators; a pocket calculator may be used.

15 Problem 1

A synchronous machine has 3 electrical phases and 4 magnetic poles. It is suitable to be connected to the 50Hz grid with a phase voltage of 230 V.
The stator resistance can be neglected.

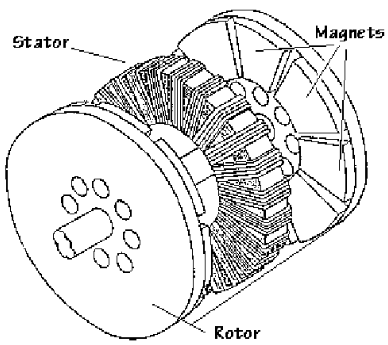
3 a Draw the equivalent circuit diagram of the synchronous machine.
2 b What is the expected synchronous speed of the machine?

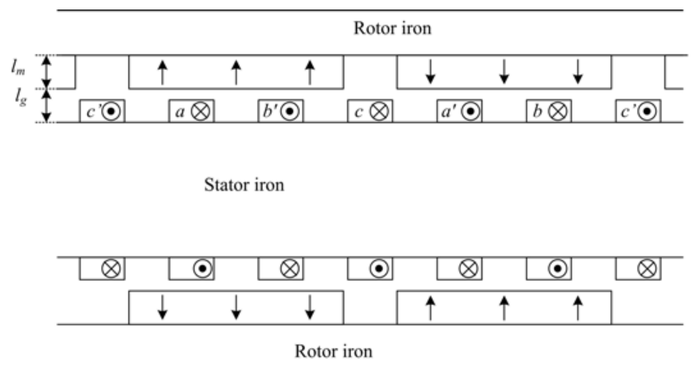
The synchronous machine is connected to the supply grid and rotates at the synchronous speed. The figure shows the phase terminal voltage (or grid voltage) and the phase excitation voltage (or back emf). Both voltages have an RMS value of 230V.
4 c Sketch the phasor diagram (with the phasors for the terminal voltage, the back emf and the current) for this situation.
4 d Calculate how much power is flowing between the grid and the machine if the synchronous reactance is 1Ω.
2 e Does the machine operate as a generator or as a motor?



30 Problem 2

The figure below on the left depicts an axial flux permanent magnet machine. The rotor consists of two discs with permanent magnets. The picture on the right depicts a linearized cross section of two pole pitches. The arrows depict the direction of the magnetisation of the magnets. The stator consists of stator iron with a three-phase winding wound around it. The machine has an air gap winding; the stator has no slots.





The machine has the following characteristics (most are in the figure):
- The magnet length in the direction of magnetization is l_m = 5 mm.
- The air-gap length (including the windings) is l_g = 5 mm.
- The machine has 8 poles.
- The width of a pole is 80% of the width of a pole pitch ($b_m = 0.8\tau_p$).
- The outer radius of the magnets and the stator iron is r_{so} = 50 mm.
- The inner radius of the magnets and the stator iron is r_{si} = 30 mm.
- Each phase winding consists of 8 coils; each coil has 50 turns, so the number of turns of a phase winding is N_s = 400.
- The remanent flux density of the magnets is B_{rm} = 1.2T
- The relative recoil permeability of the magnets μ_{rm} = 1
- The BH characteristic in the second quadrant of the BH plane is a straight line

In the calculations, it may be assumed that:
- The magnetic permeability of iron is infinite
- The flux density crosses the magnets and the air gap perpendicularly
- The wires of the turns are very thin

In the questions a through i, the stator currents are zero.

2 a Write down Amperes law (a simplified form of the first of Maxwell's equations).

2 b Sketch a contour and a surface in the cross section to which you can apply Amperes law to calculate the magnet flux density in the air gap.

2 c Give an equation for the BH characteristic of the magnet in the second quadrant.

4 d Derive an expression for the air gap flux density B_g using Ampere's law.

2 e Write down the second of Maxwell's equations or Faradays law.

2 f Describe the contour and the surface to which you can apply the second of Maxwell's equations to derive the voltage equation of this winding.

In the rest of this problem, use that the air gap flux density is B_g = 0.6 T.

3 g Calculate the flux of one pole.
In the rest of the problem, use that the flux of one pole is 0.3 mWb.
2 h Calculate the maximum flux linkage of a turn (of phase c in the left figure).
2 i Calculate the maximum flux linkage of a phase winding.

This machine is operated as a brushless DC machine. The current is 2 A.
4 j Calculate the torque. You are allowed to use the power balance. You are also allowed to use the Lorentz force.

3 k How must the stator be laminated in order to limit the eddy current losses?

One of the rotor discs is rotated over one pole pitch (45 degrees), while the other stays in position.
2 l What is the effect of this change on the motor performance (e.g. the induced voltage)?

20 Problem 3

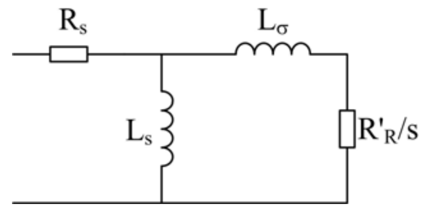
A DC machine with independent electrical excitation is connected to a DC voltage source. Armature reaction is neglected. The voltage is V_t, the motor constant is K, the armature resistance is R, the pole flux is Φ_0.

3 a Draw the equivalent circuit and give the (steady state) voltage equation.
2 b Give an expression for the no-load speed.
2 c Give an expression for the stall torque (torque at zero speed with locked rotor).
2 d Sketch the torque-speed characteristic including the no-load speed and the stall torque.
2 e Now sketch the torque-speed characteristic including the no-load speed and stall torque if the pole flux is reduced to $\Phi_0/2$.

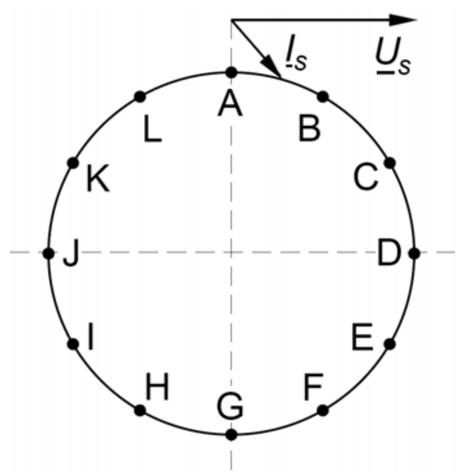
When DC machines are loaded, there is an armature reaction. Problems due to armature reaction can be reduced by adding interpoles and sometimes compensation windings. Including the armature reaction and:

4 f Sketch a DC machine cross section. Include the interpoles and the compensating windings. Indicate current direction with dots and crosses.
3 g Explain which problems may arise due to armature reaction.
2 h Why is it dangerous to disconnect the field winding of a separately excited DC machine running under no-load conditions?

35 Problem 4



This problem deals with a 4-pole three-phase induction machine connected to a symmetrical three-phase supply with a frequency of 50 Hz and a phase voltage of 230 V.
The parameters of this equivalent circuit are given by
L_s = 81 mH;
L_σ = 9 mH;
R_R' = 0.3 Ω;
R_s = 0.
Iron losses are neglected.
The depicted circle diagram gives the stator current phasor trajectory as a function of the slip.



2 a In which of the operating points A to L is the machine in generator operation?

2 b Calculate the no-load current phasor (the rotor rotates synchronously).

16 c For point D, calculate
- the slip,
- the angular frequency of the electrical quantities in the rotor,
- the mechanical speed (in rpm),
- the current through the rotor resistance,
- the stator power,
- the copper losses in the rotor,
- the efficiency, and
- the electromagnetic torque.

Note: in point D, the resistance and the reactance of the rotor branch of the equivalent circuit are equal.

3 d Sketch the torque-speed characteristic of a three-phase induction machine.

2 e Indicate the three modes of operation in the torque-speed characteristics.

5 f Draw the electrical diagram (diodes and active switches) of the speed controlling mechanism that can be used to control the speed of three-phase AC motors.

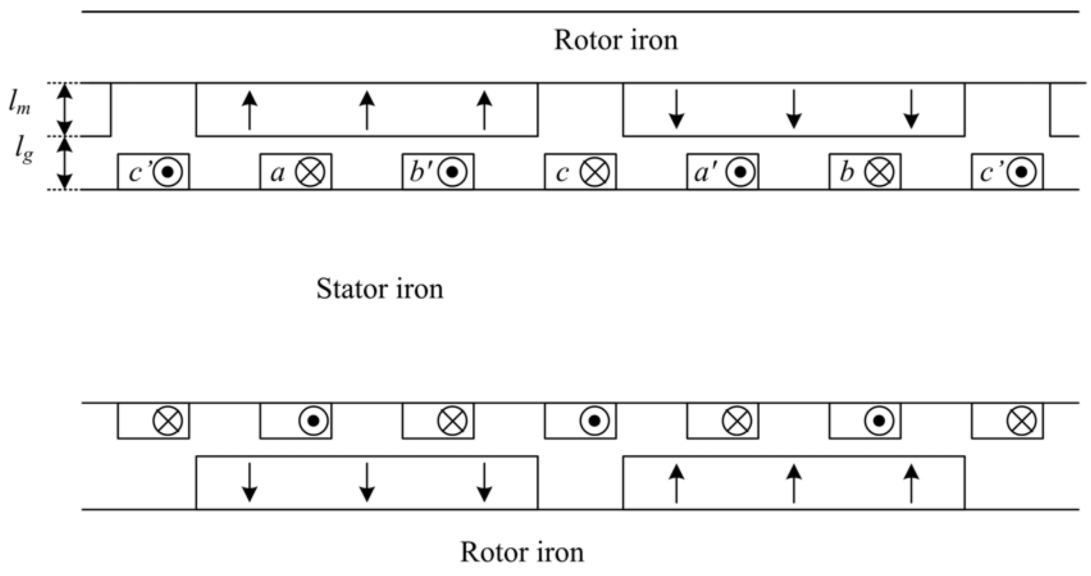
5 g Describe how a single phase induction machine can be made to rotate.

Examination Electrical Machines and Drives ET4117
Friday, November 6, 2009 from 9.00 to 12.00

Name:

Student number:

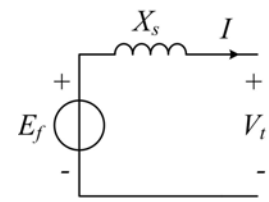
Answer to question 2b



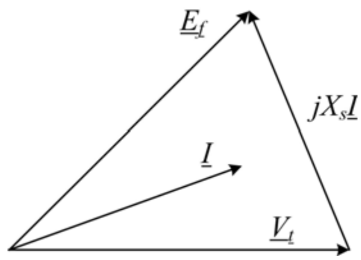
Answers to the exam Electrical Machines and Drives ET4117

15 Problem 1

3 a



2 b $n = \dfrac{2 \cdot 60 \cdot f}{p} = \dfrac{2 \cdot 60 \cdot 50}{4} \text{rpm} = 1500 \text{rpm}$



4 c

4 d If the terminal voltage is chosen in the real axis, the excitation voltage can be written as

$$\underline{E}_f = E_f(\cos\delta + \mathrm{j}\sin\delta)$$

$$S = P + \mathrm{j}Q = 3\underline{V}_t\underline{I}^* = 3V_t\left(\frac{E_f(\cos\delta + \mathrm{j}\sin\delta) - V_t}{\mathrm{j}X_s}\right)^* = \frac{3V_tE_f\sin\delta}{X_s} + \frac{3\,\mathrm{j}(V_t\,E_f\cos\delta - V_t^2)}{X_s}$$

The angle between excitation voltage (or back emf) and terminal voltage is $45°$. Therefore,

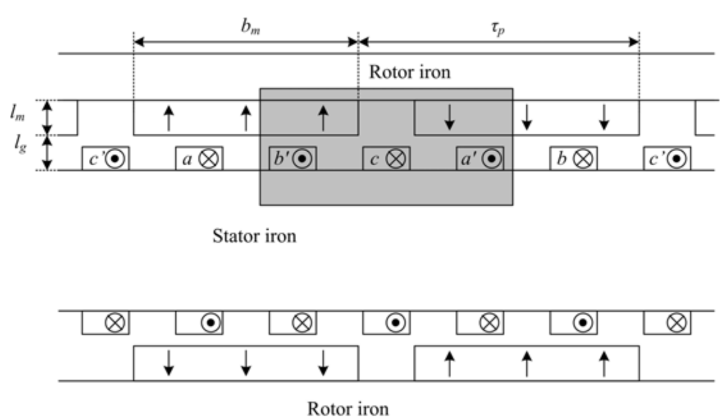
$$P = 3\frac{V_tE_f}{X_s}\sin\delta = 3\frac{230^2}{1}\sin 45° \text{W} = 112.2\,\text{kW}$$

2 e The grid voltage is lagging the machine back emf, therefore the machine is operating as a generator and the machine is delivering power to the grid.

30 Problem 2

2 a $\oint_{C_m} \vec{H} \cdot \vec{\tau}\,\mathrm{d}s = \iint_{S_m} \vec{J} \cdot \vec{n}\,\mathrm{d}A$

The magnetic field intensity *H*, the unit vector in the direction of the contour τ, the current density *J*, and the normal vector *n* are vectors. The dots mean that the inproducts of these vectors are taken.



2 b A possible contour and a possible surface are indicated. The boundary of the surface is the contour. The contour should approximately follow the field lines.

2 c $B_m = \mu_0\mu_{rm}H_m + B_{rm}$

4 d $2H_gl_g + 2H_ml_m = 0$

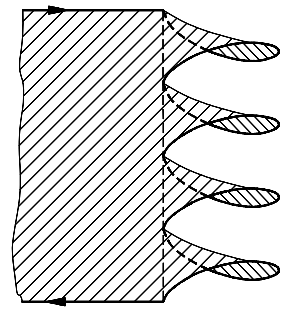
Using the constitutive relations for magnets and air: $\dfrac{B_gl_g}{\mu_0} + \dfrac{(B_m - B_{rm})l_m}{\mu_0\mu_{rm}} = 0$

Flux continuity: $\oiint_A \vec{B} \cdot \vec{n}\,\mathrm{d}A = 0$. Therfore, $B_gA_g = B_mA_m \Rightarrow B_g = B_m$

Substitution gives $\frac{B_g l_g}{\mu_0} + \frac{(B_g - B_{rm}) l_m}{\mu_0 \mu_{rm}} = 0$

$$B_g = \frac{l_m}{l_g + l_m} B_{rm}$$

2 e $\oint_{C_e} \vec{E} \cdot \vec{\tau}\, \mathrm{d}s = -\frac{\mathrm{d}}{\mathrm{d}t} \iint_{S_e} \vec{B} \cdot \vec{n}\, \mathrm{d}A$



2 f The contour is chosen in the electrical circuit, in the wire. The contour is the boundary of the surface.

3 g $A_m = \frac{0.8\pi(r_{so}^2 - r_{si}^2)}{p} = 502.6\,\mathrm{mm}^2$

$$\Phi_p = A_m B_g = 0.3016\,\mathrm{mWb}$$

2 h The flux linkage of a stator turn is equal to the pole flux. Half of the flux of a magnet goes to the left and the other half to the right, so half of the pole flux of one pole links with a turn on the stator. However, the machine is double sided, so the magnets on the other side also give a contribution of half the pole flux.

$$\lambda_{turn} = \Phi_p = A_m B_g = 0.300\,\mathrm{mWb}$$

2 i $\lambda_{\max} = N_s \Phi_p = 0.120\,\mathrm{Wb}$

4 j The force on a conductor is constant, independent from the radius. The torque depends on the radius. We can calculate the torque using the average radius:

$$T = Fr = F\frac{(r_{so} + r_{si})}{2} = 4 N_s B_g I (r_{so} - r_{si}) \frac{(r_{so} + r_{si})}{2} = 2 N_s B_g I (r_{so}^2 - r_{si}^2) = 1.536\,\mathrm{Nm}$$

The factor 4 comes from the fact that every turn has two turn sides that make a torque and from the fact that in a brushless DC machine, always 2 phases are conducting.
The other way of calculating the torque is via the power balance:

$$T = \frac{P_{em}}{\omega_m} = \frac{2 E_{\max} I}{\omega_m}$$

In 80% of half an electrical period, the flux linkage changes from the maximum flux linkage to the minimum flux linkage. Therefore:

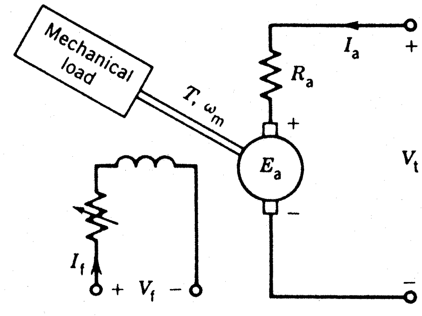
$$E_{\max} = \frac{\mathrm{d}\lambda}{\mathrm{d}t} = \frac{4 f \lambda_{\max}}{0.8} = 5 f \lambda_{\max}$$

$$T = \frac{P_{em}}{\omega_m} = \frac{10 \omega_e \lambda_{\max} I}{2\pi\omega_m} = \frac{5 p \lambda_{\max} I}{2\pi} = 1.528\,\mathrm{Nm}$$

The results are slightly different, because the first is based on an air gap flux density of 0.6 T, while the second is based on a maximum flux linkage of 0.12 Wb.

3 k The laminations should be wound like tape on a roll.

2 l The machine will not work anymore, the induced voltage becomes zero because the flux linkage becomes zero



20 Problem 3

3 a $V_t = R_a I + E_a$

2 b In no-load, the current is zero, therefore

$$V_t = E_a = K\Phi\omega$$

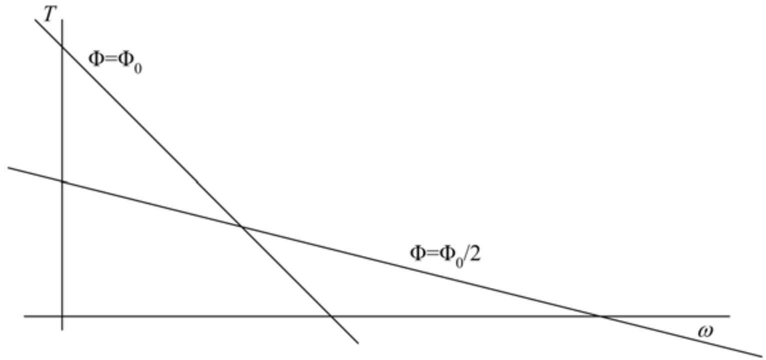
Therefore, $\omega = \frac{V_t}{K\Phi}$

2 c The stall torque is the torque when the speed is zero, therefore, $V_t = R_a I$.

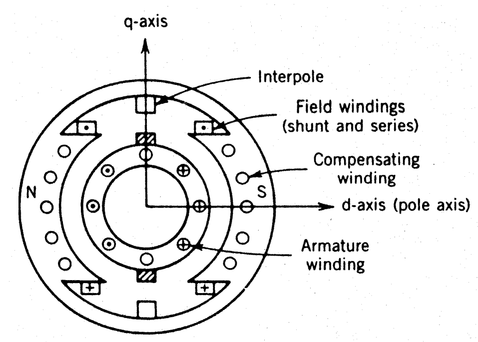
The torque is then $T = K\Phi I = K\Phi \dfrac{V_t}{R_a}$

2 d

2 e



4 f The direction of the currents in the compensating windings and the currents around the interpoles is opposite to the direction of the current in the armature. Therefore, in the right half of the machine, there should be dots in the compensating windings and around the interpoles, and in the left half of the machine there should be stars.



3 g 1. Saturation, which leads to a decrease of the pole flux.

2. Commutation problems due to the fact that the commutating coil is not in the field-free zone while commutating, which leads to sparks and additional wear.

2 h When the field winding is disconnected, the machine speed will increase, theoretically until infinity. In practice, mostly something else happens before infinity is reached. Mostly that is damaging and dangerous.

35 Problem 4

2 a Motor convention is used in the equivalent circuit, so in the points H, I, J, K, and L, the power is negative, so the machine is in generator operation.

2 b $\underline{I} = \dfrac{\underline{U}}{\mathrm{j}\omega L_s} = -\,\mathrm{j}9.038\,\mathrm{A}$

16 c In operating point D, $R_R' = s\omega_s L_\sigma$

Therefore,

$$s = \frac{R_R'}{\omega_s L_\sigma} = 0.1061$$

$$\omega_r = s\omega_s = \frac{R_R'}{L_\sigma} = 33.33\,\mathrm{rad/s}$$

$$n = (1-s)n_s = 1341\,\mathrm{rpm}$$

$$I_R' = \frac{U}{\sqrt{\left(\dfrac{R_R'}{s}\right)^2 + (\omega_s L_\sigma)^2}} = \frac{U}{\sqrt{2}\omega_s L_\sigma} = 57.52\,\mathrm{A}$$

$P_{Cur} = 3R_R' I_R'^2 = 2.978\,\text{kW}$

$P_s = \dfrac{3R_R' I_R'^2}{s} = 28.06\,\text{kW}$

If there are no other losses, the efficiency can be calculated as $\eta = 1 - s = 0.8939$

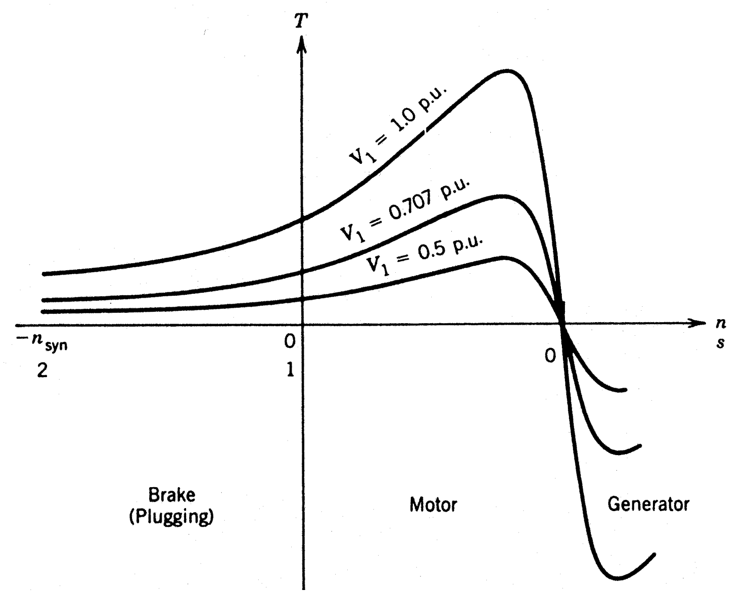
The electromagnetic torque can be calculated as

$$T_{mech} = \frac{P_m}{\omega_m} = 3\frac{p}{2}\frac{(1-s)P_s}{(1-s)\omega_s} = 3\frac{p}{2}\frac{R_R' I_R'^2}{s\omega_s} = 3\frac{p}{2\omega_s}\frac{U^2}{\left(\frac{R_R'}{s}\right)^2 + (\omega_s L_\sigma)^2}\frac{R_R'}{s}$$

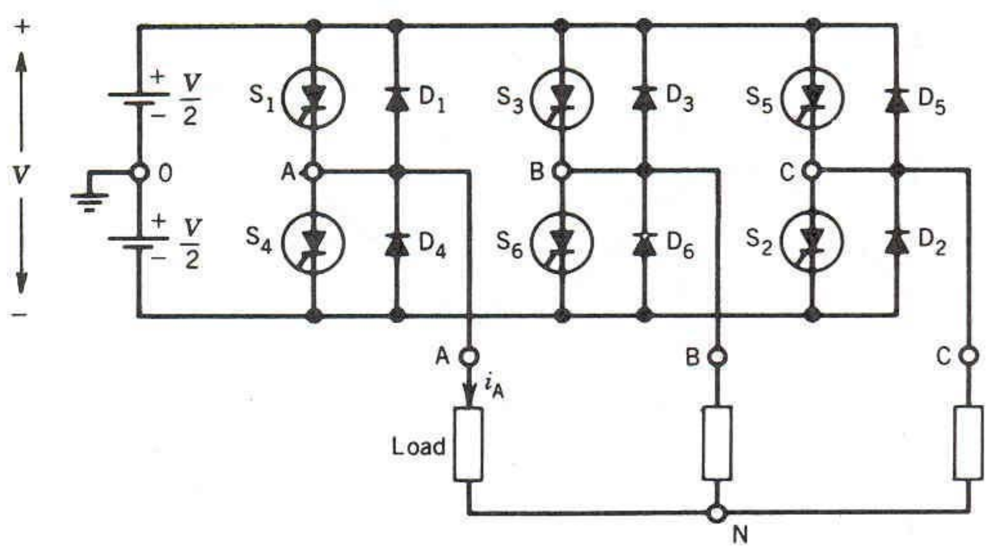
$$= 3\frac{p}{2\omega_s}\frac{U^2}{2(\omega_s L_\sigma)^2}\frac{R_R'}{s} = 178.7\,\text{Nm}$$

3 d

2 e



5 f Figure 10.43 depicts an inverter.



5 g When connected to an AC source, the stator of a single-phase induction machine creates a pulsating field. The pulsating magnetic field of a single phase must get a

rotating component. This can be done by adding an auxiliary winding. The axis of this winding is shifted 90 degrees with respect to the axis of the main winding. The current in this winding has a phase shift with respect to the current in the main winding because it has a different impedance or because a capacitor is added. Another way of creating a rotating component in a pulsating field is by making a split phase induction motor. Here the stator has an even number of poles with the single phase AC winding around them. Around a part of the pole, there is a short-circuited copper ring, which delays the change of flux compared to the rest of the pole, thus giving a rotating component.