Exam Electrical Machines and Drives (ET4117)
5 November 2010 from 9.00 to 12.00.
This exam consists of 5 problems on 4 pages.
Page 5 can be used to answer problem 3 question b.
The number before a question indicates how many credits you can earn by answering that question. A partly correct answer may give a part of the credits.
This examination has to be made without using a book, old examinations, notes, dictionaries or programmable calculators; a pocket calculator may be used.

16 Problem 1

A train is driven by a DC machine with independent electrical excitation. The machine runs at 1500 rpm. The armature voltage is 750 V. The current is 100 A. The armature resistance is 0.5Ω. The armature inductance is 50 mH.

a 3 Sketch the equivalent circuit of the DC machine.
b 2 Calculate the motor constant $K\Phi$.
c 2 Calculate the torque.

The DC source voltage is instantaneously reduced to 700 V.

d 2 Give an approximation of the armature current immediately (1 microsecond) after the change of voltage.

After some time, a new steady-state situation is reached. It can be assumed that the load torque is constant.

e 3 Calculate the speed of the machine.
f 4 Sketch a chopper (switches and diodes) that can be used to control the voltage on the machine terminals that enables regenerative braking.

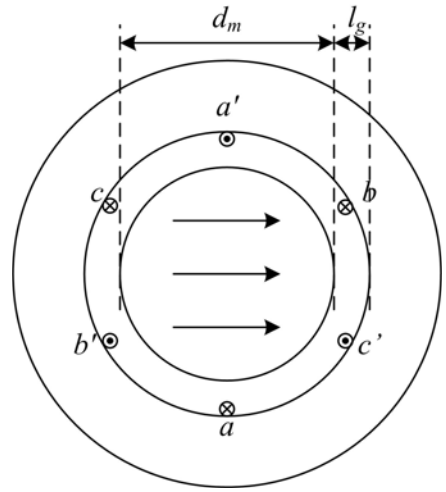
12 Problem 2

A three-phase synchronous generator is connected to a grid with a line voltage of 20 kV and a frequency of 50 Hz.
It delivers a power of 500 MW.
The power factor is $\cos\varphi = 0.9$.
The generator delivers a lagging current to the grid.
The synchronous reactance is X=1 Ω.
The stator resistance is negligible.

2 a Draw the equivalent circuit.
10 b Sketch the phasor diagram and calculate the phase current, the phase electromotive force (or the excitation voltage) and the load angle.

34 **Problem 3**

This problem deals with a high speed permanent magnet machine that drives a compressor. A cross section of this machine is given in the picture. The rotor consists of a cylindrical magnet, magnetized as indicated in the figure. To keep the rotor together at high speeds, a carbon fibre sleeve is used, which has not been sketched. This sleeve has the same electric and magnetic properties as air, so in this problem it can be considered as part of the air gap.
The stator has a three-phase distributed winding. In this figure, it is sketched as an air gap winding without slots.



The machine has the following characteristics (most of them are in the figure).

- The diameter of the magnet is d_m =30 mm
- The air-gap length is l_g = 5 mm.
- The inner stator radius is r_s = 20 mm.
- The stack length is l_s= 50 mm
- The magnets have a remanent flux density of B_{rm} = 1.2 T a relative recoil permeability $\mu_{rm} = 1$ and the BH-characteristic in the second quadrant of the BH-plane is a straight line.
- The number of turns of a phase winding is N_s= 20.
- The nominal speed of the machine is 120000 rpm.
- The amplitude of the rated current of this machine is $\hat{i} = 40\,\text{A}$.

In the calculations, it may be assumed that

- The magnetic permeability of iron is infinite.
- The magnetic flux density due to the magnet in the air gap is a sinusoidal function of the rotor circumference: $B_g = \hat{B}_g \cos(\theta_r)$
- The wires of the turns are very thin and the windings are concentrated (which means that the winding factor is 1).

2 a Write down Ampere's law (a simplified form of the first of Maxwell's equations).

2 b Sketch a contour and a surface in the cross section of the machine to which you can apply Ampere's law to calculate the magnetic flux density in the air gap due to the magnets. You can use the cross section of the machine in the figure on the last page of the exam.
In questions c to l, the stator currents are zero.

2 c Give an equation for the BH-characteristic of the magnet in the second quadrant.

5 d Derive an expression to estimate the air-gap flux density $\hat{B}_g$ using Ampere's law, explain the assumptions in the derivation and use it to calculate the air-gap flux density $\hat{B}_g$.

In questions e to m, the amplitude of the air-gap flux density is $\hat{B}_g = 0.6\text{T}$.

2 e Write down the second of Maxwell's equations or Faraday's law.

2 f Describe the contour and the surface to which you can apply the second of Maxwell's equations to derive the voltage induced in phase *a*.

2 g Calculate the electrical angular frequency ω in rad/s.

4 h Calculate the flux linkage of stator phase *a* as a function of time if the rotor rotates at the rated speed. At t=0, the rotor is in the position of the sketch.

3 i Calculate the no-load phase voltage (electromotive force, EMF) of phase *a* as a function of time if the rotor rotates with a the rated speed.

In a real machine, the conductors are mostly not placed in the air gap, but in slots.

2 j If the slot width is equal to the tooth width, calculate the flux density in a tooth (you can assume that the flux density in the slot is zero because of the high permeability of the iron of the slot).

At a frequency of 50 Hz and a flux density amplitude of 1.5 T, normal laminations have typical eddy current losses in the order of 1 W/kg and hysteresis losses in the order of 3 W/kg.

4 k Estimate the iron losses in this machine if the amplitude of the flux density in the laminations is 1.2 T and the weight of the laminations is 2 kg.

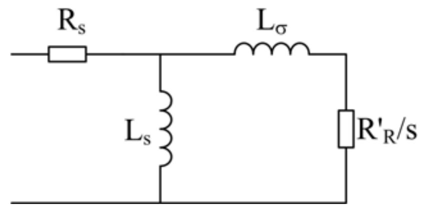
2 l What must be changed to reduce these losses?

3 m Calculate the maximum power the machine can deliver to the load at rated current.

6 Problem 4

Describe the machine on the picture as accurately as you can (machine type, number of poles, principle of operation).

32 Problem 5



This question deals with star-connected induction machines. The equivalent circuit is given.

The parameters of this equivalent circuit can be determined from a combination of a resistance measurement, the no-load test and the short-circuit test.
The stator phase resistance is measured as 30 mΩ.
During the no-load test, the rotor rotates at synchronous speed. The test results are the following: U=400 V, f=50 Hz, I=45 A, P=2000W.
The blocked-rotor test results are: U=60 V, f=50 Hz, I=150A, P=4000W.
NB1: the measured power is the three-phase power.
NB2: the measured voltage is the line voltage.
NB3: the current through the synchronous inductance L_s during the short-circuit test can be neglected.

a 9 Calculate the parameters of the equivalent circuit.
b 2 Calculate the copper losses during the no-load test.
c 2 Explain the difference between the calculated copper losses and the measured losses in the no-load test.

Another four-pole three-phase star-connected squirrel cage induction machine has the following parameters:
R'_R = 120 mΩ, L_σ = 2.5 mH, L_s = 50 mH.
The stator resistance is neglected: R_s = 0.
The rated line voltage is 400 V.
The rated current is 50 A.
The rated frequency is 50 Hz.
The machine is driven by a voltage source inverter.
The machine is at standstill.
The converter supplies the stator of the machine with a sinusoidal voltage with a frequency of 2.5 Hz and a line voltage of 20 V.

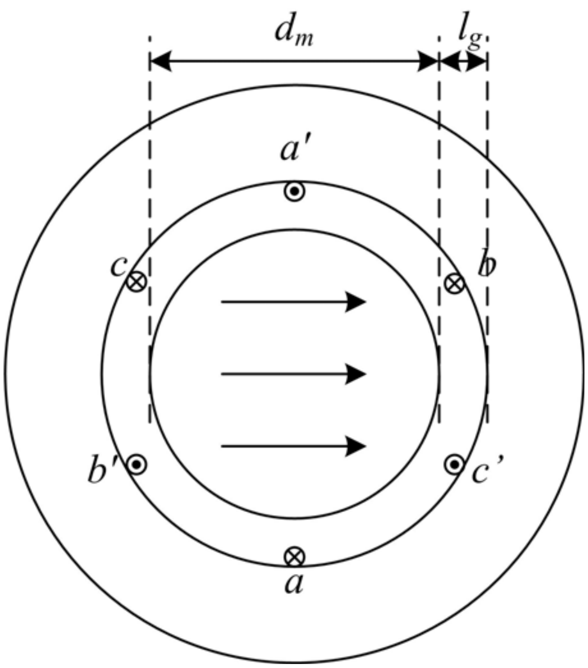
d 3 Calculate the current through the resistance R'_R.
e 9 Calculate the torque.
f 2 Why is the voltage level at this frequency reduced to 20 V?
g 2 What do you conclude from the ratio of the current in the machine to the rated current?
h 3 Calculate the angular frequency at which the torque is maximum (while the rotor is still at standstill).

Examination Electrical Machines and Drives ET4117
Friday, November 6, 2009 from 9.00 to 12.00

Name:

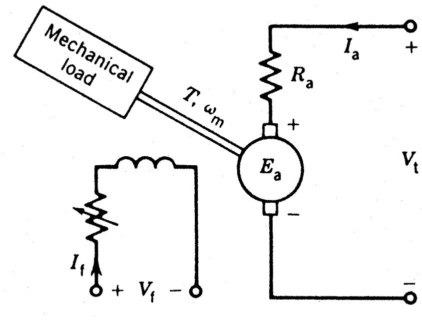
Student number:

Answer to question 3b



Answers to the exam Electrical Machines and Drives ET4117

16 Problem 1



a 3

b 2 $E_a = K\Phi\omega_m$

$$K\Phi = \frac{E_a}{\omega_m} = \frac{V_t - R_a I_a}{\omega_m} = 4.456\,\text{Vs/rad}$$

c 2 $T = K\Phi I_a = 445.6\,\text{Nm}$

d 2 The rate of change of the current is determined by the voltage equation of the machine

$$V_t = RI_a + L\frac{\mathrm{d}I_a}{\mathrm{d}t} + E_a$$

$$L\frac{\mathrm{d}I_a}{\mathrm{d}t} = RI_a + E_a - V_t$$

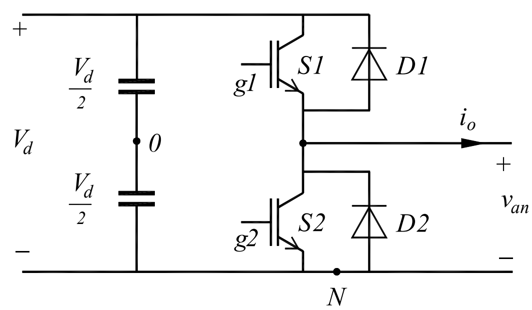
$$I_a = I_a(0) + \frac{1}{L}\int_0^t RI_a + E_a - V_t\,\mathrm{d}t \approx I_a(0) + \frac{1}{L}(RI_a + E_a - V)\Delta t = 100\,\text{A} - 0.001\,\text{A} \approx 100\,\text{A}$$

e 3 The load torque remains constant, so in a new steady state situation, the electromagnetic torque should again be equal to the load torque, and the current is again 100 A.

$$\omega_m = \frac{E_a}{K\Phi} = \frac{V_t - R_a I_a}{K\Phi}$$

$$= 145.9\,\text{rad/s} = 1393\,\text{rpm}$$

f 4



12 Problem 2

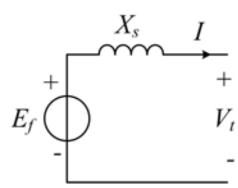
2 a

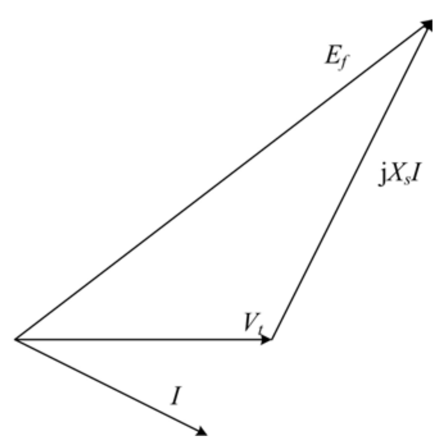
10 b The phase voltage is

$$V_t = \frac{20000}{\sqrt{3}}\,\text{V} = 11547\,\text{V}$$

The phase current is

$$I = \frac{P}{3V_t\cos\varphi} = 16038\,\text{A}$$





The terminal voltage phasor is chosen in the real axis: V_t=11547V .

The power factor is 0.9 with lagging current.

Therefore, the phasor for the current can be written as

$I = 14434 - \mathrm{j}6991\,\text{A}$

The phasor for the excitation voltage can be calculated as

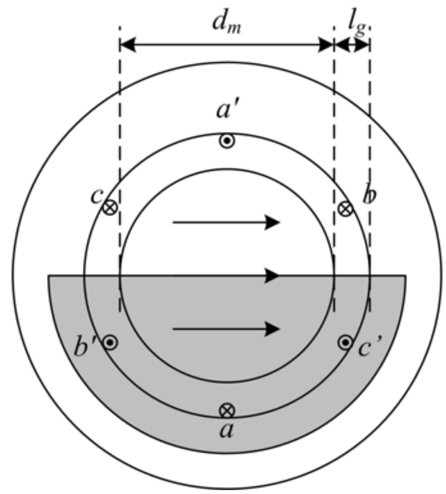
$$E_f = V_t + \mathrm{j}XI = 11547 + 6991 + \mathrm{j}14434\,\text{V} = 18538 + \mathrm{j}14434\,\text{V}$$

The RMS value of the excitation voltage is

$$E_f = \sqrt{18538^2 + 14434^2}\,\text{V} = 23494\,\text{V}$$

The load angle is

$$\delta = \arctan(\frac{14434}{18538}) = 0.6616\,\text{rad} = 37.91°$$



34 Problem 3

2 a $\oint_{C_m} \vec{H} \cdot \vec{\tau}\, \mathrm{d}s = \iint_{S_m} \vec{J} \cdot \vec{n}\, \mathrm{d}A$

2 b The surface area is made grey. The boundary of the surface is the contour.

2 c $B_m = \mu_0 \mu_{rm} H_m + B_{rm}$

4 d If we apply Ampere's law to the sketched contour and we assume H_g to be constant in the air gap, and we assume H_m to be constant in the magnet, we obtain

$$2H_g l_g + H_m d_m = 0$$

Using the BH relations for magnets and air:

$$2\frac{B_g l_g}{\mu_0} + \frac{(B_m - B_{rm})d_m}{\mu_0} = 0$$

Flux continuity: $\oiint_A \vec{B} \cdot \vec{n}\, \mathrm{d}A = 0$

$$B_g A_g = B_m A_m \Rightarrow B_g = B_m$$

Substitution gives

$$\frac{2B_g l_g}{\mu_0} + \frac{(B_g - B_{rm})d_m}{\mu_0} = 0$$

$$B_g = \frac{d_m}{2l_g + d_m} B_{rm} = 0.9\,\text{T}$$

This estimate is overestimating the flux density at the inside stator radius. The flux leaving the magnet surface at a smaller radius will enter the stator at a larger radius. Because of flux continuity, the flux density at the inner stator radius must be lower.

2 e $\oint_{C_e} \vec{E} \cdot \vec{\tau}\, \mathrm{d}s = -\frac{\mathrm{d}}{\mathrm{d}t} \iint_{S_e} \vec{B} \cdot \vec{n}\, \mathrm{d}A$

2 f The contour is the boundary of the surface. The contour is chosen in the electrical circuit, in the wire.

2 g $\omega = \frac{2\pi n}{60} = 12566\,\text{rad/s}$

4 h If the rotor is in the position of the sketch, the flux linkage of phase a is maximum, and can be calculated as:

$$\hat{\lambda}_{pm} = \iint_{S_e} \vec{B} \cdot \vec{n}\, \mathrm{d}A = N_s l_s \int_{-\pi/2}^{\pi/2} \hat{B}_{pm} r_s \cos(\theta)\, \mathrm{d}\theta = 2N_s l_s r_s \hat{B}_{pm} = 24.00\,\text{mWb}$$

The flux linkage as a function of time can be written as

$$\lambda_{pma} = \hat{\lambda}_{pm} \cos(\omega t)$$

3 i $e_{pma} = \frac{\mathrm{d}\lambda_{pma}}{\mathrm{d}t} = -\omega \hat{\lambda}_{pm} \sin(\omega t) = -\hat{e}_{pm} \sin(\omega t)$

where $\hat{e}_{pm} = 301.6\,\text{V}$

2 j All flux that enters the stator on a tooth pitch (tooth width b_t plus slot width b_s) has to go through the tooth. Therefore, $\hat{B}_t = \frac{b_s + b_t}{b_t}\hat{B}_{pm} = 1.2\,\mathrm{T}$.

4 k
$$P_{Feh} = M_{Fe}\left(\frac{\hat{B}}{\hat{B}_0}\right)^2\left(\frac{f}{f_0}\right)P_h = 153.6\,\mathrm{W}$$
$$P_{Fee} = M_{Fe}\left(\frac{\hat{B}}{\hat{B}_0}\right)^2\left(\frac{f}{f_0}\right)^2 P_e = 2048\,\mathrm{W}$$

2 l Better laminations have to be used which means that they have to be thinner and that the electrical and magnetic properties have to be chosen optimal for high frequencies.

3 m The electromechanically converted power can be calculated as
$$P_{em} = e_{pma}i_a + e_{pmb}i_b + e_{pmc}i_c$$
This power is maximum if the electromotive forces and the currents are in phase. In that case, the power is
$$P_{em} = \frac{3}{2}\hat{e}_{pm}\hat{i} = 18.1\,\mathrm{kW}$$

6 Problem 4

This machine is a 4-pole shaded-pole single phase induction motor.

32 Problem 5

a 9
$$U_{phase} = U_s/\sqrt{3} = 230.9\,\mathrm{V}$$
During the no-load test, $s = 0$, and therefore,
$$U_{phase} = I_s\sqrt{\omega_s^2 L_s^2 + R_s^2}$$
$$L_s = \frac{1}{\omega_s}\sqrt{\frac{U_{phase}^2}{I_s^2} - R_s^2} = 16.34\,\mathrm{mH}$$
During the short-circuit test, the current through the synchronous inductance can be neglected. Therefore,
$$R'_R = \frac{P}{3I_s^2} - R_s = 29.3\,\mathrm{m}\Omega$$
$$U_{phase} = U_s/\sqrt{3} = 34.64\,\mathrm{V}$$
$$U_{phase} = I_s\sqrt{\omega_s^2 L_\sigma^2 + (R_s + R'_R)^2}$$
$$L_\sigma = \frac{1}{\omega_s}\sqrt{\frac{U_{phase}^2}{I_s^2} - (R_s + R'_R)^2} = 0.7105\,\mathrm{mH}$$

b 2 $P = 3I_s^2 R_s = 182.5\,\mathrm{W}$.

c 2 The other losses are iron losses and mechanical losses

d 2
$$I_R = \frac{U_s}{\sqrt{3}\sqrt{\omega_s^2 L_\sigma^2 + R_R'^2}} = 91.45\,\mathrm{A}$$

e 9 Generally, the air gap power per phase can be written as
$$P_\delta = \frac{I_R^2 R'_R}{s} = I_R^2 R'_R + I_R^2 R'_R\frac{1-s}{s}$$
We also know that

$$P_\delta = P_{Cu} + P_{mech} = P_{Cu} + \omega_m T$$

Therefore,

$$\omega_m T = I_R^2 R_R' \frac{1-s}{s}$$

The mechanical speed can be written as

$$\omega_m = \frac{2}{p}(1-s)\omega_s$$

Therefore, the torque of a three-phase machine can be written as

$$T = 3 I_R^2 R_R' \frac{p}{2s\omega_s} = 383.6\,\text{Nm}$$

f 2 The voltage level is reduced to 20 V to avoid saturation; in order to keep the flux level at the rated flux level, the voltage should be proportional to the frequency.

g 2 The current is about twice the rated current. For a short time, that is allowed, but if the machine would continuously be used at this current level, it would become too hot and be destroyed.

h 3 $\omega_s = \frac{R_R'}{L_\sigma} = 48.00\,\text{rad/s} \Rightarrow f_s = 7.639\,\text{Hz}$.