Exam Electrical Machines and Drives (ET4117)
9 November 2012 from 14.00 to 17.00.
This exam consists of 3 problems on 3 pages.
Page 5 can be used to answer problem 4 question a.
The number before a question indicates how many credits you can earn by answering that question. A partly correct answer may give a part of the credits.
This examination has to be made without using a book, old examinations, notes, dictionaries or programmable calculators; a pocket calculator may be used.

18 Problem 1

A DC machine with independent electrical excitation is connected to a DC voltage source.

5 a Sketch a DC machine cross section. Include the interpoles and the compensating windings. Indicate current direction with dots and crosses.

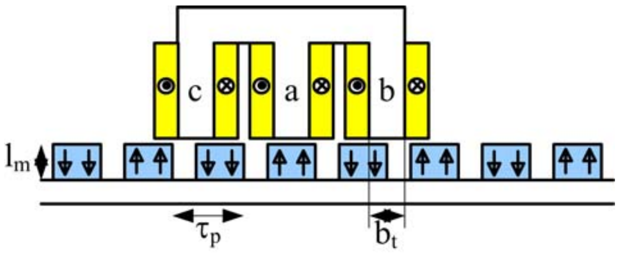
3 b Draw the equivalent circuit.

3 c Give at least 1 important advantage and 2 important disadvantages of using DC machine when compared to induction machines.

7 d Sketch a cross section of a switched reluctance machine and explain the principle of operation.

38 **Problem 2**

The figure depicts a linear permanent-magnet machine. The stator consists of back-iron and permanent magnets. The moving part (the translator) has three teeth and three coils wound around the teeth.



The width of a tooth is b_t = 6 mm.
The number of turns of one coil is N_s = 600.
The pole pitch (the distance between two magnets) is τ_p = 12 mm.
The stack length of the motor (the length of the magnets and the teeth in the plane perpendicular to the plane of the drawing) is l_s = 50 mm.
The flux linkage of the translator coils is a linear superposition of the flux linkage due to the magnets and the flux linkage due to the translator currents (i_a, i_b, and i_c):

$$\lambda_a = \lambda_{pma} + \lambda_{ia} = \hat{\lambda}_{pm}\cos(\pi x/\tau_p) + Li_a$$

$$\lambda_b = \lambda_{pmb} + \lambda_{ib} = \hat{\lambda}_{pm}\cos(\pi x/\tau_p - 2\pi/3) + Li_b$$

$$\lambda_c = \lambda_{pmc} + \lambda_{ic} = \hat{\lambda}_{pm}\cos(\pi x/\tau_p - 4\pi/3) + Li_c$$

The inductance is L = 70 mH.
The resistance of the coils is R = 12 Ω.
Other losses than the copper losses are neglected.
In questions a to d, the currents in the translator coils are zero.

10 a Write down Ampere's law and use it to derive an expression the flux density in the air gap below the tooth of phase *a*. Sketch the contour to which you apply Ampere's law; you can use the last page with the cross section. Make your assumptions explicit and explain the difficulties.

In the rest of this question, the amplitude of the flux linkage due to the magnets is $\hat{\lambda}_{pm}$=180 mWb.

2 b Calculate the amplitude of the magnetic flux going through a tooth.

2 c Calculate the amplitude of the magnetic flux density going through a tooth.

3 d Is this a realistic value for the flux density with regard to saturation in the teeth and magnetization of the magnets? Why or why not? What are the typical values for saturation and magnetization of the magnets?

In questions e to g, the translator moves with a speed of 1 m/s, the amplitude of the three sinusoidal phase currents is $\hat{i}$ = 2 A and the phase of the currents is so that the electromagnetic force is maximum.

8 e Calculate the amplitudes of the no-load voltage (the back emf), the voltage drop over the inductance, the voltage drop over the resistance, and the terminal voltage.

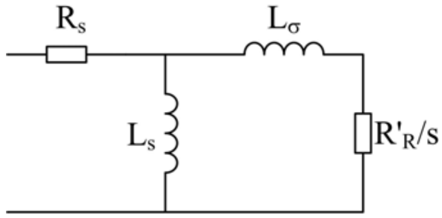
4 f Sketch the phasor diagram.

3 g Calculate the electromagnetic force developed by this motor.

2 h Until now, iron losses have been neglected. Are they present in this motor? Why or why not?

4 i Which kinds of iron losses do you know, how do they depend on frequency and flux density?

44 **Problem 3**



This question deals with a three-phase star-connected squirrel-cage induction machine for an electric car. The equivalent circuit is given.

11 a If this machine is available for testing, how can the parameters of the equivalent circuit be determined in an experimental way? Mention the tests, the quantities that have to be measured and the equations that have to be used to determine the parameters. You can use the assumptions that are normally used.

The parameters of the equivalent circuit are given by
R'_R = 120 mΩ, L_σ = 0.55 mH, L_s = 11 mH.
The stator copper losses are neglected: R_s = 0.
The core losses are also neglected.
The number of poles is p=4.
The nominal line voltage is 400 V.
The nominal electrical input power is P=30 kW.
The machine is driven by a voltage source inverter, which keeps the voltage level proportional to the frequency below the nominal frequency, and constant at the nominal value above the base frequency.
The base frequency is f=200 Hz.

2 b Why is the stator voltage proportional to the stator frequency below the base frequency?
2 c Calculate the no-load speed (in rpm) at the base frequency.
3 d Calculate the nominal no-load current (the magnetizing current) at the base frequency.
4 e Calculate the nominal torque at the base frequency.
6 f Calculate the nominal slip at the base frequency from the given data.
3 g Calculate the angular rotor frequency at which the torque is maximum.

This motor is used to drive an electric vehicle with a mass of 1200 kg including the driver. The wheel radius is 300 mm.
Between the motor and the wheels, there is an ideal gearbox (without losses) with a gear ratio of 10 (the wheel rotates slower than the motor).
It is assumed that the only friction force acting on the vehicle the is air friction, and the air friction force is given by

$$F_{af} = \frac{1}{2} C_d A \rho_{air} v^2 = K_{af} v^2 \quad \text{where } K_{af} = 0.5\ \text{Ns}^2/\text{m}^2.$$

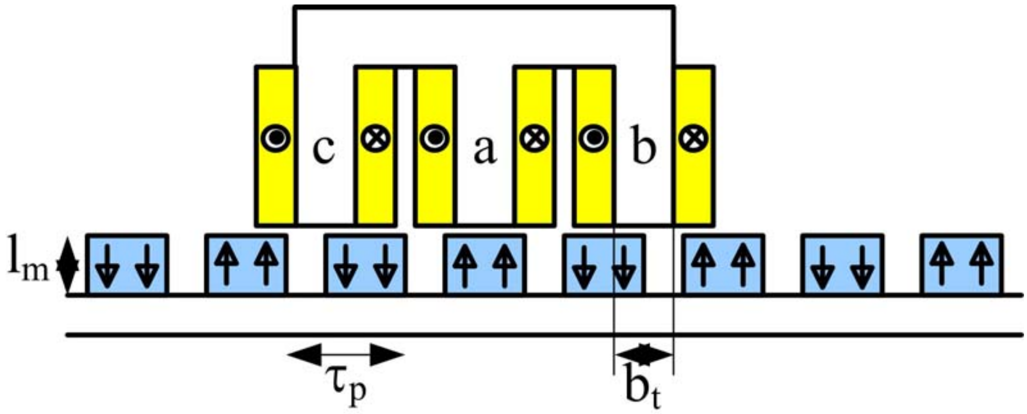
3 h Calculate the acceleration at standstill if the motor develops a torque of 150 Nm for a short time during starting.
2 i Give a good approximation of the speed of the car at the base frequency.
3 j Give a good approximation of the maximum continuous speed of the car.
5 k Sketch a power electronic converter (switches and diodes) that can connect this machine to the battery and that enables regenerative braking.

Examination Electrical Machines and Drives ET4117
Friday, November 9, 2012 from 14.00 to 17.00

Name:

Student number:

Answer to question 2a



Answers to the exam Electrical Machines and Drives ET4117

18 Problem 1

5 a

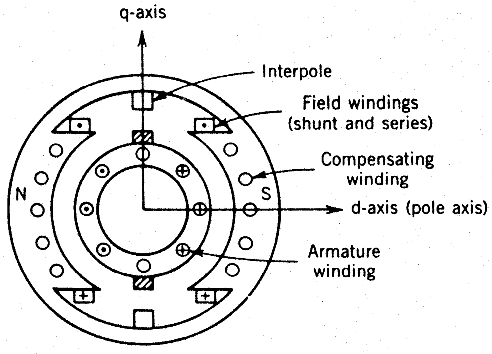


FIGURE 4.10 Schematic diagram of a dc machine.

The direction of the currents in the compensating windings and the currents around the interpoles is opposite to the direction of the current in the armature. Therefore, in the right half of the machine, there should be dots in the compensating windings and around the interpoles, and in the left half of the machine there should be stars.

3 b

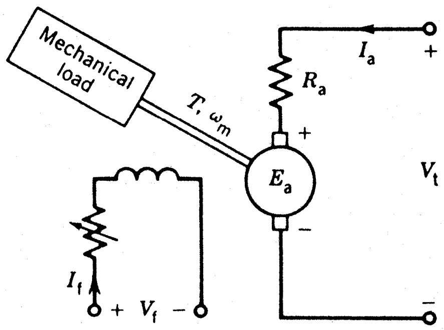


FIGURE 4.50 Separately excited dc motor.

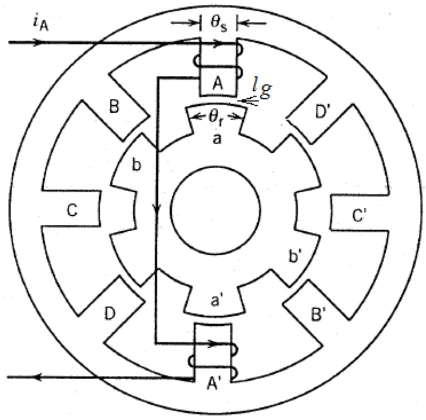
3 c Advantages:

- easy to control with rectifier or chopper

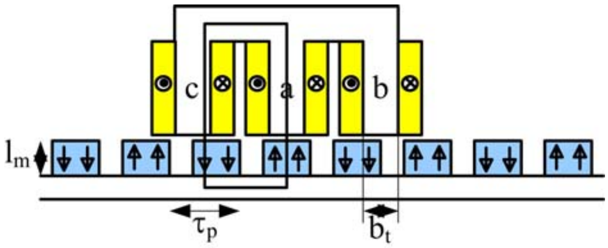
Disadvantages:

- regular maintenance due to brushes,
- runaway when excitation field is removed,
- heavier than induction machines,
- more expensive than induction machines,
- commutation problems may occur (sparking).

7 d The principle of operation is that of magnetic attraction between magnetized steel parts. The stator phases are excited one after the other and then attract the closest rotor teeth.



38 Problem 2



10 a Ampere's law is given by

$$\oint_{C_m} \vec{H}\cdot\vec{\tau}\,\mathrm{d}s = \iint_{S_m} \vec{J}\cdot\vec{n}\,\mathrm{d}A$$

In the figure, a useful contour has been sketched.

We could assume the flux density to cross the air gap perpendicular only on these places where there is a magnet below a tooth. We assume the flux density to be zero if there is no magnet below a tooth.

Application of Ampere's law gives

$$H_{ga}l_{ga} + H_{ma}l_{ma} + H_{gb}l_{gb} + H_{mb}l_{mb} = 0$$

There is a difficulty because the magnetic field intensity in the magnet and the gap below the tooth of phase a may be different from the magnetic field intensity in the magnet and the gap below the teeth of phase b and c.

However, this may be solved by assuming that they are equal because the magnets cover are only half of the surface area of the teeth of phase b and c, so that the sum of the fluxes in tooth b and c is equal to the flux in tooth a.

In this case:

$$2H_g l_g + 2H_m l_m = 0$$

The BH curve of the magnet is given by $B_m = \mu_0\mu_{rm}H_m + B_{rm}$

The BH curve of air is given by $B_g = \mu_0 H_g$

Substituting these BH relations for magnets and air in the result of Amperes law gives:

$$\frac{B_g l_g}{\mu_0} + \frac{(B_m - B_{rm})l_m}{\mu_0} = 0$$

The equation for flux continuity: $\oiint_A \vec{B}\cdot\vec{n}\,\mathrm{d}A = 0$

$$B_g A_g = B_m A_m \Rightarrow B_g = B_m$$

Substitution gives

$$\frac{B_g l_g}{\mu_0} + \frac{(B_g - B_{rm})l_m}{\mu_0} = 0$$

$$B_g = \frac{l_m}{l_g + l_m} B_{rm}$$

2 b $$\hat{\Phi}_{pm} = \frac{\hat{\lambda}_{pm}}{N_s} = 0.3\,\mathrm{mWb}$$

2 c $$B = \frac{\hat{\Phi}_{pm}}{b_t l_s} = 1\ \mathrm{T}$$

3 d Yes, typical remanent flux densities of magnets are up to 1.5 T, so that 1 T in a tooth is realistic, and typical saturation flux densities in teeth are in the order of 1.5 T to 2 T, so 1 T in a tooth is realistic.

8 e The voltage equation of phase a is given by:

$$u_a = Ri_a + \frac{\mathrm{d}\lambda_a}{\mathrm{d}t} = Ri_a + L\frac{\mathrm{d}i_a}{\mathrm{d}t} - \hat{\lambda}_{pm}\frac{\pi}{\tau_p}\sin(\frac{\pi}{\tau_p}x)\frac{\mathrm{d}x}{\mathrm{d}t}\,.$$

The current is in phase with the no-load voltage in order to create maximum force, so

$$i_a = -\hat{i}\sin(\frac{\pi}{\tau_p}x)$$

Substituting this in the voltage equation gives

$$u_a = -\hat{i}R\sin(\frac{\pi}{\tau_p}x) - L\hat{i}\frac{\pi}{\tau_p}\cos(\frac{\pi}{\tau_p}x)\frac{\mathrm{d}x}{\mathrm{d}t} - \hat{\lambda}_{pm}\frac{\pi}{\tau_p}\sin(\frac{\pi}{\tau_p}x)\frac{\mathrm{d}x}{\mathrm{d}t}$$

Therefore, the no-load voltage amplitude is

$$\hat{e}_{pa} = \hat{\lambda}_{pm}\frac{\pi}{\tau_p}\frac{\mathrm{d}x}{\mathrm{d}t} = 47.12\,\mathrm{V}$$

The voltage drop over the resistance is

$$\hat{u}_R = \hat{i}R = 24\,\mathrm{V}$$

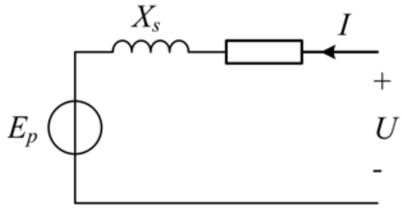
The voltage drop over the inductance is

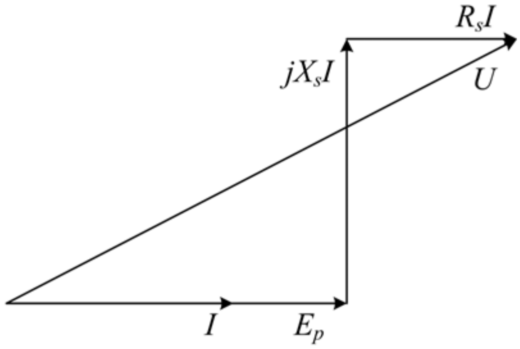
$$\hat{u}_L = L\hat{i}\frac{\pi}{\tau_p}\frac{\mathrm{d}x}{\mathrm{d}t} = 36.65\,\mathrm{V}$$

The terminal voltage is

$$\hat{u}_a = \sqrt{(\hat{e}_{pa} + \hat{u}_R)^2 + u_L^2} = 80.01\,\mathrm{V}$$

4 f





3 g The force developed by this motor is

$$F_{em} = \frac{P_{em}}{v} = \frac{3}{2}\hat{i}\hat{\lambda}_{pm}\frac{\pi}{\tau_p} = 141.4\,\mathrm{N}$$

2 h Yes, the flux density in the translator varies when the translator moves. This results in iron losses in the translator iron.

4 i Hysteresis losses, proportional to frequency and flux density squared and eddy current losses, proportional to frequency squared and flux density squared.

44 Problem 3

11 a ***Resistance measurement***: put a DC voltage on the terminals of two phases and measure the DC current while the machine is at standstill:

$$R_s = \frac{U}{2I}$$

No-load test: The machine runs synchronously ($s = 0$) in no-load at the rated AC voltage. The line voltage U_{line} and the phase current I_s are measured. During this test, $s = 0$, and therefore,

$$R'_{nl} = \frac{P}{3I_s^2}$$

$$L_s = \frac{1}{\omega_s}\sqrt{\frac{U_{line}^2}{3I_s^2} - R_{nl}^2}$$

Short-circuit test: The rotor is blocked ($s = 1$) and the machine is connected to a reduced AC voltage in such a way that the current is about the nominal current. The line voltage U_{line}, the phase current I_s and the three-phase power P are measured. Often, the current through the synchronous inductance is neglected. Therefore,

$$R'_R = \frac{P}{3I_s^2} - R_s$$

$$L_\sigma = \frac{1}{\omega_s}\sqrt{\frac{U_{line}^2}{3I_s^2} - (R_s + R'_R)^2}$$

2 b Below the base frequency, the voltage is proportional to the speed to avoid saturation of the magnetic circuit.

2 c $n = \frac{120f}{p} = 6000\,\text{rpm}$

3 d The magnetizing current through L_s is given by

$$I_m = \frac{U_{phase}}{\omega_{snom} L_s} = \frac{U_{line}}{\sqrt{3}\omega_{snom} L_s} = 16.71\,\text{A}$$

4 e The power balance per phase says:

$$P_\delta = P_{Cu} + P_{mech} = P_{Cu} + \omega_m T$$

The air gap power can be written as:

$$P_\delta = \frac{I_R^2 R'_R}{s} = I_R^2 R'_R + I_R^2 R'_R \frac{1-s}{s}$$

From these equations:

$$\omega_m T = I_R^2 R'_R \frac{1-s}{s}$$

The mechanical speed can be written as

$$\omega_m = \frac{2}{p}(1-s)\omega_s$$

Therefore,

$$\frac{2}{p}(1-s)\omega_s T = I_R^2 R'_R \frac{1-s}{s} \quad \Rightarrow \quad \frac{2}{p}\omega_s T = \frac{I_R^2 R'_R}{s} = P_\delta$$

Therefore, the torque of a three-phase machine can be written as

$$T_{nom} = \frac{pP_{nom}}{2\omega_s} = 47.75\,\text{Nm}$$

6 f In the rated operating point, we know that

$$P_{nom} = \frac{3I_R^2 R_R'}{s} = \frac{U_{line}^2}{\frac{R_R'^2}{s^2} + \omega_s^2 L_\sigma^2} \frac{R_R'}{s} = R_R' \frac{sV_t^2}{R_R'^2 + s^2\omega_s^2 L_\sigma^2}$$

Therefore,

$$P_{nom}\omega_s^2 L_\sigma^2 s^2 - U_{line}^2 R_R' s + P_{nom} R_R'^2 = 0$$

Therefore,

$$s_{nom} = \frac{U_{line}^2 R_R' - \sqrt{U_{line}^4 R_R'^2 - 4\omega_s^2 L_\sigma^2 R_R'^2 P_{nom}^2}}{2\omega_s^2 L_\sigma^2 P_{nom}} = 0.02289$$

3 g $\omega_r = \dfrac{R_R'}{L_\sigma} = 218.2\,\text{rad/s} \quad \Rightarrow \quad f_r = 34.72\,\text{Hz}$.

3 h The gear increases the torque on the wheels to 1500 Nm.
The force on the wheels apply to the car is

$$F = \frac{T}{r} = 5000N$$

The acceleration is

$$a = \frac{F}{m} = 4.167\,\text{m/s}^2$$

3 i The slip is rather low, in the order of 1%. Therefore, neglecting the slip, the speed is

$$v = \omega_w r_w = \frac{\omega_m r_w}{g} = \frac{2\omega_e r_w}{pg} = \frac{4\pi f r_w}{pg} = 18.85\,\text{m/s}$$

3 j The air friction is given by $F = K_f v^2$, so the power to overcome this friction is

$$P = Fv = K_f v^3$$

At speeds above the nominal speed, the machine converter uses field weakening with constant voltage and constant power.
If we neglect all other losses and we also neglect the losses in the machine, the maximum speed can be approximated by

$$v = \sqrt[3]{\frac{P_{nom}}{K_f}} = 39.15\,/\,\text{s}$$

5 k