Examination Electrical Machines and Drives (ET4117)
28 October 2008 from 14.00 to 17.00.
This examination consists of 4 problems on 3 pages.
Page 4 can be used to answer problem 4 question a.
The number before a question indicates how many credits you can earn by answering that question. A partly correct answer may give a part of the credits.
This examination has to be made without using a book, old examinations, notes, dictionaries or programmable calculators; a pocket calculator may be used.

13 Problem 1

When DC machines are loaded, there is armature reaction. The problems due to armature reaction can be reduced by using interpoles and sometimes also compensating windings.

5 a Sketch a cross-section of a DC machine with electrical excitation including interpoles. Indicate the direction of the currents in all the windings with dots and crosses.

In questions b and c, the DC machine with independent electrical excitation is connected to a DC voltage source. Armature reaction is neglected.
The voltage is V_t.
The motor constant is K.
The armature resistance is R.
In questions b and c, the pole flux is $\Phi=\Phi_0$.

3 b Give the (steady-state) voltage equation and the equivalent circuit.
5 c Sketch the torque-speed characteristic including the no-load speed and the stall torque and give an expression for the no-load speed and for the stall torque (the torque at stand still, with blocked rotor).

21 Problem 2

A three-phase synchronous generator is connected to a grid with a line voltage of 20 kV and a frequency of 50 Hz.
It delivers a power of 100 MW.
The power factor is 0.9. The generator delivers a leading current to the grid.
The synchronous reactance is X=4 Ω.
The stator resistance is negligible.
3 a Sketch a cross section of a three-phase synchronous machine with cylindrical rotor.
2 b Draw the equivalent circuit.
9 c Sketch the phasor diagram and calculate the current, the excitation voltage and the load angle.
2 d Sketch the torque speed characteristic of the synchronous machine.
5 e Mention at least 6 important assumptions used in this model and this calculation method other than that Maxwell's equations and the power balance hold.

26 Problem 3

This problem deals with induction machines. First a few general questions.

3 a Sketch the IEEE recommended equivalent circuit of the induction machine.
3 b Sketch the torque-speed characteristic of an induction machine and indicate the three modes of operation.

Questions d to f are about an induction machine with a name plate telling it is a 50 Hz induction machine with a rated speed of 1455 rpm.
2 c What is the rated slip?
2 d What is the synchronous speed if the supply frequency is reduced to 25 Hz?

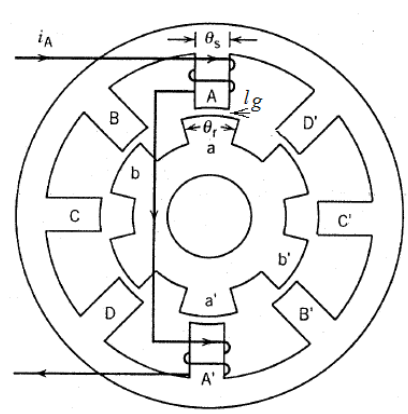
The stator winding of phase *a* has a winding distribution given by

$$n_{sa}(\theta) = \frac{N_s}{2}\sin(\theta)$$

where n_{sa} is the number of conductors per radian, N_s is the number of turns of the phase winding and θ is the angular position.

4 e Derive and expression for the flux density in the air gap as a function of the angular position θ for the case that the current in this winding is i_a.
5 f Explain how the stator of a three-phase induction machine creates a rotating magnetic field in the air gap of the machine. In this explanation, give a mathematical derivation of such a rotating field, starting from expressions for the winding distributions and the currents. The structure of the derivation is more important than the correctness of all the mathematical steps.
Hint: $\cos(x)\cos(y) = \frac{1}{2}\cos(x+y) + \frac{1}{2}\cos(x-y)$
7 g A single-phase induction motor has a pulsating field. Which measures can be taken to start a single-phase induction machine? Explain how these measures work.

30 Problem 4



The figure depicts a cross section of a switched reluctance machine.
The magnetic permeability of iron is infinite.
The axial stack length (in the direction perpendicular to the plane of the drawing) is l_s.
The width of a stator tooth is b_t.
The length of the air gap is l_g.
The resistance of a phase winding (consisting of two coils around two opposite teeth) is R_a.
The number of turns of the coil around one tooth is N.
There is a current i_A flowing through the coil. The currents in the other phases are zero. It may be assumed that the flux crosses the air gap perpendicularly only in the narrow gap where there is overlap between a stator tooth and a rotor tooth.

5 a Give Ampere's law and sketch a contour and a surface in the reluctance machine, in such way that Ampere's law can be applied to contour and surface, to derive an

expression for the air gap flux density. At the end of the exam is a picture added where you can make the sketch.

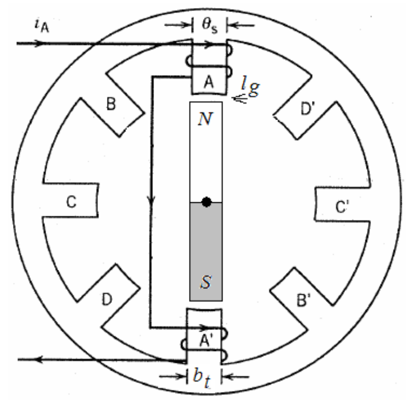
3 b Determine the maximum magnetic flux density in the gap.

3 c Sketch the inductance of phase A as function of the rotor position.

2 d Give an expression for the current, when the machine stands still (as shown in the figure) and a DC voltage U_a is applied to phase A.

2 e If one wants to let the machine rotate in clockwise direction, in which order need the windings to be switched on and off?

3 f Sketch a converter to control this reluctance machine. A sketch of only one converter leg (which can be connected to i.e. the wire as draw here) is sufficient.



Now, the rotor is replaced by a permanent magnet as shown in the figure. The stator remains the same.
The magnetic permeability of iron is infinite.
The axial stack length (in the direction perpendicular to the plane of the drawing) is l_s.
The width of a stator tooth is b_t.
The length of the air gap is l_g.
The width of the magnet is b_t (equal to the width of a stator tooth).
The number of turns of the coil around one tooth is N.
The magnet length in the direction of magnetisation (between the two air gaps) is l_m.
The magnet has a remanent flux density B_{rm} and relative recoil permeability of μ_{rm} and the BH-characteristic is in a straight line in the second quadrant of the BH-plane.
It may be assumed that the flux crosses the air gap perpendicularly only in the narrow gap where there is overlap between a stator tooth and a rotor tooth.
The current i_A is zero in the questions below.

2 g Give an equation describing the BH-characteristic of the magnet in the second quadrant.

4 h Derive an expression for the maximum flux density in the air gap B_g. Hint: use Amperes law.

The maximum flux density in the air gap is B_g.

2 i Give an expression for the maximum flux linkage of phase A.

2 j Sketch the form of the flux linkage of phase A for one turn of the rotor.

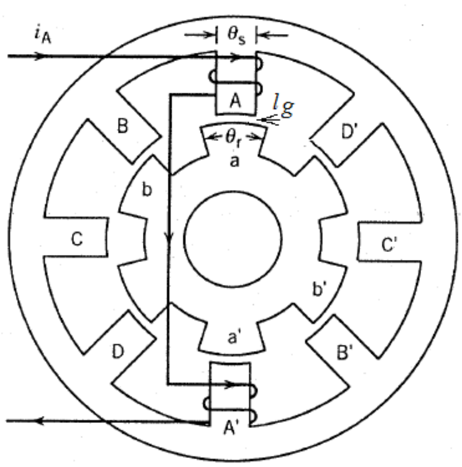
2 k Sketch the form of the voltage induced in phase A, for one turn of the rotor at a constant speed.

Examination Electrical Machines and Drives Et4-117
Tuesday, October 28, 2008 from 14.00 to 17.00

Name:

Student number:

Answer to question 4a)



Answers to the examination Electrical Machines and Drives Et4-117

13 Problem 1

5 a

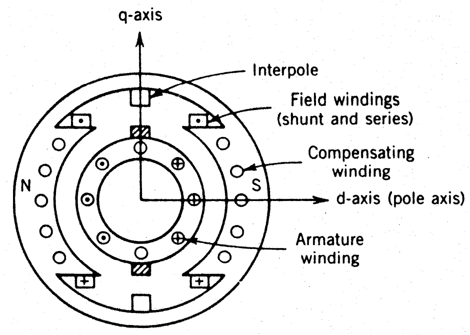


FIGURE 4.10 Schematic diagram of a dc machine.

The direction of the currents in the compensating windings and the currents around the interpoles is opposite to the direction of the current in the armature. Therefore, in the right half of the machine, there should be dots in the compensating windings and around the interpoles, and in the left half of the machine there should be stars.

3 b $V_t = R_a I + E_a$

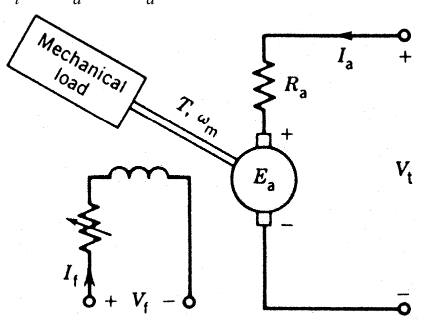
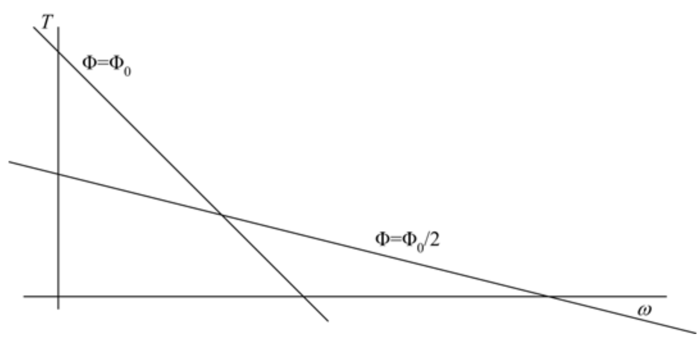


FIGURE 4.50 Separately excited dc motor.

5 c In no-load, the current is zero, therefore $V_t = E_a = K\Phi\omega$

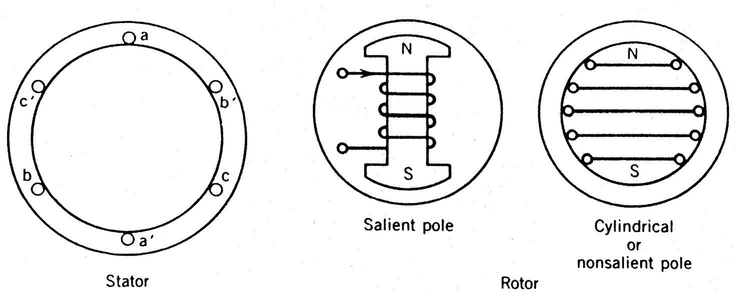
Therefore, $\omega = \dfrac{V_t}{K\Phi}$

The stall torque is the torque when the speed is zero, therefore, $V_t = R_a I$. The torque is then $T = K\Phi I = K\Phi \dfrac{V_t}{R_a}$



21 **Problem 2**

3 a



2 b

9 c The phase voltage is

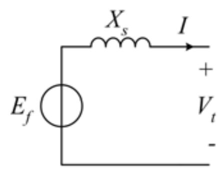
$V_t = \frac{20000}{\sqrt{3}} V = 11547\,\text{V}$

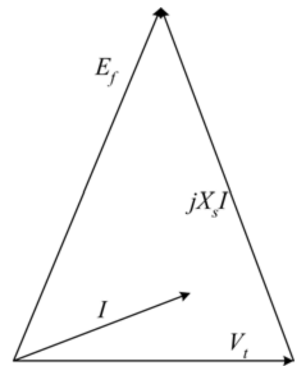
The phase current is

$I = \frac{P}{3V_t \cos\varphi} = 3208\,\text{A}$

The terminal voltage phasor is chosen in the real axis:

V_t=11547V.





The power factor is 0.9 with lagging current.

Therefore, the phasor for the current can be written as

$I = 2887 + j1398\,\text{A}$

The phasor for the excitation voltage can be calculated as

$E_f = V_t + jXI = 11547 - 5592 + j11547\,\text{V} = 5954 + j11547\,\text{V}$

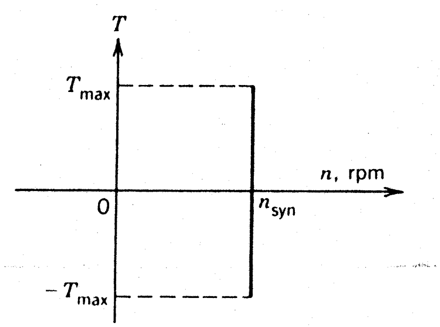
The RMS value of the excitation voltage is

$E_f = \sqrt{5954^2 + 11547^2} = 12992\,\text{V}$

The load angle is

$\delta = \arctan(\frac{11547}{5954}) = 1.095 rad = 62.72°$

2 d



5 e Used assumptions:

- stator windings are sinusoidally distributed
- stator windings have axis which are 120° shifted with respect to each other
- the stator voltages (and the resulting currents) are sinusoidal (with constant amplitude and frequency, steady state)
- the stator voltages (and the resulting currents) have a phase shift of 120° with respect to each other
- the mechanical speed is constant (steady state)
- iron losses are neglected
- stator copper losses are neglected

26 **Problem 3**

3 a See fig. 5.15

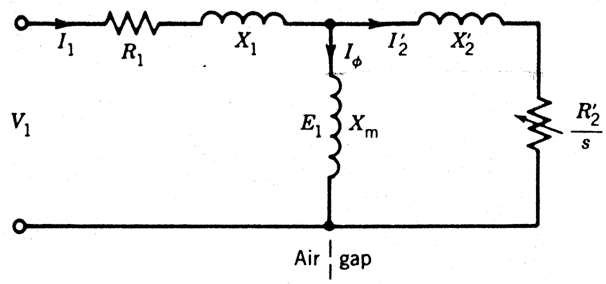


FIGURE 5.15 IEEE-recommended equivalent circuit.

3 b see fig. 5.17

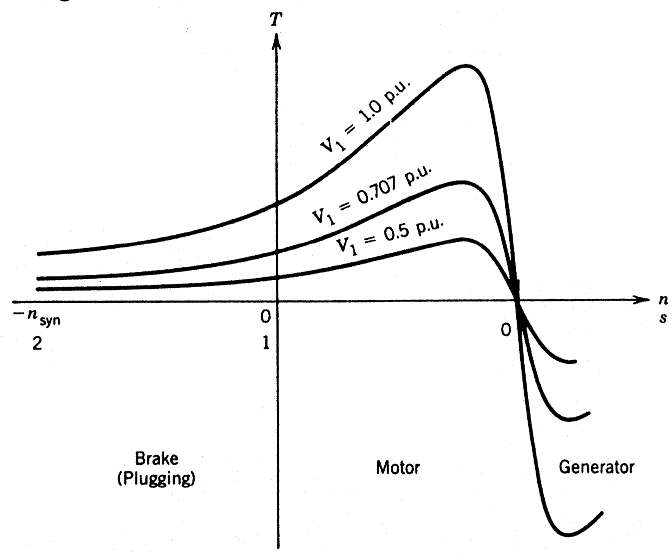


FIGURE 5.17 Torque–speed profile at different voltages.

2 c synchronous speed is 1500 rpm, so rated slip: 3%

2 d synchronous speed at 25 Hz: 750 rpm

4 e Using Amperes law, the flux density in the air gap due to a current in phase a can be calculated:

$$\oint_{C_m} \vec{H} \cdot d\vec{l} = \iint_{S_m} \vec{J} \cdot d\vec{A}$$

$$2l_g H_g(\theta) = \int_{\theta}^{\theta+\pi} n_{sa}(\theta) i_{sa} d\theta = N_s i_{sa} \cos(\theta)$$

$$B_{ga}(\theta) = \mu_0 H_{ga}(\theta) = \mu_0 \frac{N_s}{2l_g} i_{sa} \cos(\theta)$$

5 f The phase windings are sinusoidally distributed with 120 degrees displacement between the axes of the windings:

$$n_{sa}(\theta) = \frac{N_s}{2} \sin(\theta)$$

$$n_{sb}(\theta) = \frac{N_s}{2} \sin(\theta - \frac{2}{3}\pi)$$

$$n_{sb}(\theta) = \frac{N_s}{2} \sin(\theta - \frac{4}{3}\pi)$$

By using Ampere's law, the flux densities due to the currents in the different phases are calculated as

$$B_{ga}(\theta) = \mu_0 \frac{N_s}{2l_g} i_{sa} \cos(\theta)$$

$$B_{gb}(\theta) = \mu_0 \frac{N_s}{2l_g} i_{sb} \cos(\theta - \frac{2}{3}\pi)$$

$$B_{gc}(\theta) = \mu_0 \frac{N_s}{2l_g} i_{sc} \cos(\theta - \frac{4}{3}\pi)$$

The phase currents are sinusoidal currents with 120 degrees phase shift between the three phases:

$$i_{sa} = \hat{i} \cos(\omega t)$$

$$i_{sb} = \hat{i} \cos(\omega t - \frac{2}{3}\pi)$$

$$i_{sb} = \hat{i} \cos(\omega t - \frac{4}{3}\pi)$$

Substitution of the currents in the expression for the flux density gives

$$B_{ga}(\theta) = \mu_0 \frac{N_s \hat{i}}{2l_g} \cos(\theta)\cos(\omega t)$$

$$B_{gb}(\theta) = \mu_0 \frac{N_s \hat{i}}{2l_g} \cos(\theta - \frac{2}{3}\pi)\cos(\omega t - \frac{2}{3}\pi)$$

$$B_{gc}(\theta) = \mu_0 \frac{N_s \hat{i}}{2l_g} \cos(\theta - \frac{4}{3}\pi)\cos(\omega t - \frac{4}{3}\pi)$$

These three waves are standing waves, that can be described as the sum of two traveling waves:

$$B_{ga}(\theta) = \mu_0 \frac{N_s \hat{i}}{4l_g} (\cos(\omega t - \theta) + \cos(\omega t + \theta))$$

$$B_{gb}(\theta) = \mu_0 \frac{N_s \hat{i}}{4l_g} (\cos(\omega t - \theta) + \cos(\omega t + \theta - \frac{4}{3}\pi))$$

$$B_{gc}(\theta) = \mu_0 \frac{N_s \hat{i}}{4l_g} (\cos(\omega t - \theta) + \cos(\omega t + \theta - \frac{8}{3}\pi))$$

If we add these waves, we get

$$B_g(\theta) = \mu_0 \frac{3N_s \hat{i}}{4l_g} \cos(\omega t - \theta)$$

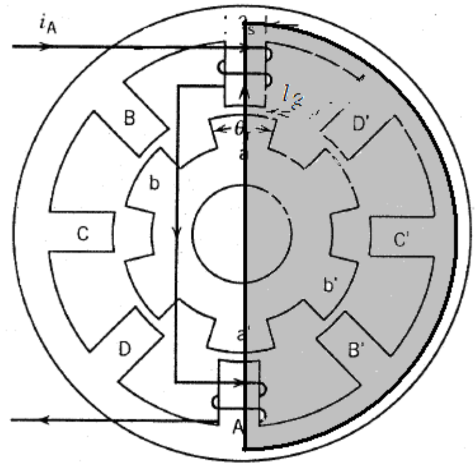
The waves traveling in negative direction cancel each other.

7 g The pulsating magnetic field of a single phase must get a rotating component. This can be done by adding an auxiliary winding. The axis of this winding is shifted 90 degrees with respect to the axis of the main winding. The current in this winding has a phase shift with respect to the current in the main winding because it has a different impedance or because a capacitor is added. Another way of creating a rotating component in a pulsating field is by making a split phase induction motor. Here the stator has an even number of poles with the single phase AC winding around them. Around a part of the pole, there is a short-circuited copper ring, which delays the change of flux compared to the rest of the pole, thus giving a rotating component.

30 Problem 4

5 a $\oint_{C_m} \vec{H} \cdot d\vec{l} = \iint_{S_m} \vec{J} \cdot d\vec{A}$

The contour and the boundary are indicated in the figure.

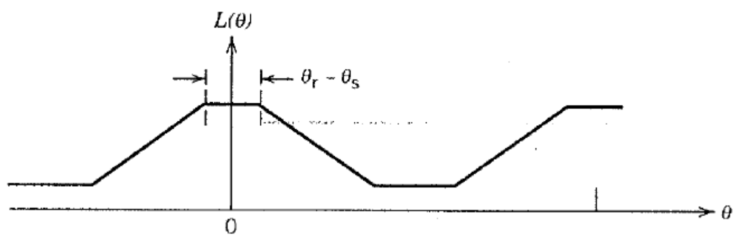


3 b $\oint_{C_m} \vec{H} \cdot d\vec{l} = \iint_{S_m} \vec{J} \cdot d\vec{A}$

$2 l_g H_g = 2 N i_A$

$B_g = \mu_0 H_g = \mu_0 \frac{N i_A}{l_g}$

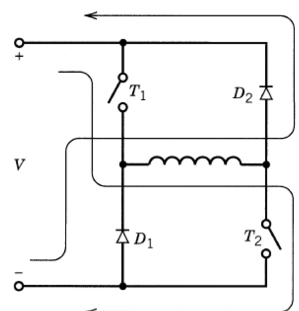
3 c



2 d $I_a = \frac{U_a}{R_a}$

2 e A, B, C, D, A, …

2 f



2 g $B_m = \mu_0 \mu_{rm} H_m + B_{rm}$

4 h $\oint_{C_m} \vec{H} \cdot d\vec{l} = \iint_{S_m} \vec{J} \cdot d\vec{A}$

Therefore: $2 l_g H_g + l_m H_m = 0$

Using the BH-curve of the magnet $B_m = \mu_0 \mu_{rm} H_m + B_{rm}$

The BH-curve of air $B_g = \mu_0 H_g$

And the magnetic flux continuity $B_g = B_m$

gives $B_g = \frac{l_m}{l_m + 2\mu_{rm} l_g} B_{rm}$

2 i $\lambda_{\max} = 2 N l_s b_t B_g$

2 j

2 k