Flight and Orbital Mechanics

Exams

Resit exam AE2104: Flight and Orbital Mechanics (2 February 2012, 09.00 – 12.00)

Please put your name, student number and ALL YOUR INITIALS on your work. Answer all questions and put your name on each page of your exam.

This exam consists of questions: 1, 2, 3, 4a-h, 5a-d, 6a-d and 7a-i

Derive the expressions for each required calculation (unless mentioned in the file '*equations by heart*', for the orbital mechanics part).

The way the answer is obtained should be clearly indicated by visibly substituting the numbers in the formulas. Only mentioning the final answer will NOT result in any credits. Use of pencils to write the exam is NOT permitted. Scrap paper may not be added to your exam work (please take the scrap paper with you after the exam). It is not permitted to have any pre-programmed information on your calculator. The memory of your calculator should be erased prior to the start of the exam. Failure to do so will be seen as fraud.

In total 100 points can be earned. (50 points for flight mechanics and 50 points for orbital mechanics). At least 55 points are required to pass the exam.

Good luck!

Question 1 (Take-off) [10 points]

The **decision speed V_1** is an important operational speed in terms of safety for multi-engine aircraft. The decision speed can depend on several factors, such as the environmental conditions, aircraft weight and airfield elevation.

For a given aircraft, with a take-off weight of 50.000 [N], operating on a concrete runway at sea level conditions (ISA), the decision speed equals 100 knots IAS.

If this aircraft would perform its take off under the same conditions but on a wet runway (with a lower friction coefficient than a concrete runway), will the decision speed be lower, higher or the same? Provide an explanation in your answer which includes the terms **accelerate-stop distance** and **accelerate climb distance**.

Question 2 (Lift drag polar) [10 points]

The parabolic approximation of the lift-drag polar is very useful for aircraft performance calculations. Usually it is written as follows:

$$C_D = C_{D_0} + kC_L^2$$

Sometimes it is necessary to add an extra term to this equation in order to approximate to lift drag polar with sufficient accuracy:

$$C_D = C_{D_0} + k_1 C_L + k_2 C_L^2$$

Derive an equation for the **lift coefficient** at the **minimum power** condition as function of the terms k_1, k_2 and C_{D0}.

Question 3 (Airspeed / equations of motion) [5 points]

An aircraft is performing a climbing flight at a **true airspeed** of 100 [m/s] and a **climb rate** of 20 [m/s]. The aircraft experiences a horizontal headwind of 15 [m/s]. Calculate the ground speed of this aircraft.

Question 4 (Climb) [25 points]

The following equations and data are given to describe the troposphere of the international standard atmosphere (ISA):

$$p = p_0 \left[\frac{T}{T_0} \right]^{-\frac{g_0}{R\lambda}}$$

$$T = T_0 + \lambda H$$

$$p_0 = 101325 \ [\text{N/m}^2]$$

$$g_0 = 9.80665 \ [\text{m/s}^2]$$

$$\lambda = -0.0065 \ [\text{K/m}]$$

$$R = 287.05 \ [\text{m}^2 / \text{s}^2 \text{K}]$$

$$T_0 = 288.15 \ [\text{K}]$$

Additionally, the gas law and the hydrostatic equation are given:

$$p = \rho R T$$

$$\frac{dp}{dH} = -\rho g_0$$

An aircraft is performing a **quasi-rectilinear**, **symmetric** climb with a **constant Mach number** of 0.8 in the troposphere of the international standard atmosphere.

a. Explain what the terms quasi-rectilinear and symmetric mean (2 points)
b. Draw the Free Body Diagram (FBD) and the Kinetic Diagram (KD) visualizing all forces and accelerations that act on the aircraft for this particular flight condition. Draw the aircraft with a certain pitch angle θ, flight path angle γ and angle of attack α. Also indicate the direction of the velocity vector. The angle of attack of the thrust (α_T) can be assumed zero.
c. Derive the corresponding equations of motion for this flight condition using the FBD and KD. Clearly state the assumptions that you make (if any).
d. Derive the power equation by multiplying the equation of motion with the airspeed. (10 points for b, c and d combined)
e. Using the power equation, derive the general relation between the rate of climb in unsteady flight and the rate of climb in steady flight RC/RC_{st}, in terms of the quantity dV/dH. (4 points)
f. If the climb is performed with constant **Mach number**, what is the resulting expression for RC/RC_{st}?
g. The aircraft is flying at 9000 m at Mach 0.8. Calculate the ratio RC/RC_{st} for this condition (6 points for f and g combined)
h. Explain physically why the ratio RC/ RCst is not equal to 1. Hint; you can make use of the answer to question b and the power equation (3 points)

Question 5 (Orbital mechanics) [14 points]

a. Compute the dimension of the Sphere of Influence of the Earth (when the Sun is considered as the perturbing body). The SoI is given by the following general equation (2 points):

$$r_{SoI} = r_{3rd} \left(\frac{M_{main}}{M_{3rd}} \right)^{0.4}$$

b. What is the value of the radial attraction exerted by the Earth at this distance? (if you were unable to make question (a), use a value of 1×10^6 km for this position) (4 points).
c. What is the effective gravitational acceleration by the Sun at this position? Assume that Earth, Sun and this point on the SoI are on a straight line (4 points).
d. What is the relative perturbation of the solar attraction, compared to that of the main attraction of the Earth? (4 points)

Data: 1 AU = 149.6×10^6 km, M_{Sun} = 2.0×10^{30} kg, M_{Earth} = 6.0×10^{24} kg, μ_{Sun} = 1.3271×10^{11} km^3/s^2, μ_{Earth} = 398600 km^3/s^2.

Question 6 (Orbital mechanics) [14 points]

One of the main issues for designing a space mission is the occurrence of eclipses.

a. What is the definition of an eclipse? (3 points)
b. An eclipse has consequences for at least 3 subsystems of the satellite. What are these, and discuss the consequences for each one briefly (about 2 lines each). (4 points)
c. What are the two conditions that determine whether an Earth satellite is in eclipse or not? Give the mathematical conditions in a sketch and discuss each one briefly. (4 points)
d. One of the conditions can translate into the so-called shadow function as given below. Discuss the meaning of the various elements in the equation, and discuss the use of this equation. (3 points)

$$S(\theta) = R_e^2 (1+e\cos\theta)^2 + p^2 (\bar{\alpha}\cos\theta + \bar{\beta}\sin\theta)^2 - p^2$$

Question 7 (Orbital mechanics) [26 points]

Consider a transfer from a circular parking orbit at 185 km and i =5° (i.e., launch from Kourou) to the geostationary orbit (GEO; T=23^{h}56^{m}4^s, e=0, i=0°).

a. Compute the radius of the geostationary orbit. (2 points)
b. Compute the circular velocities in the original orbit and in the target orbit. (2 points)
c. Compute the velocities in the pericenter and the apocenter of a Hohmann transfer orbit (i.e., assuming that the initial and target orbits are coplanar – which they are not in reality). (4 points)
d. Compute the total ΔV that would be required for the orbit raising, assuming that we fly a Hohmann transfer (again, the two orbits would be assumed to be coplanar). (2 points)
e. Compute the ΔV that would be required to only change the inclination of the original parking orbit to that of the GEO. (4 points)
f. Compute the total ΔV if the sequence of maneuvers was (1) full inclination change (only) in initial orbit, and (2) Hohmann transfer to GEO. (2 points)

g. Compute the total ΔV if the sequence of maneuvers was (1) Hohmann transfer to GEO altitude (without changing inclination), and (2) full inclination change at GEO altitude. (2 points)
h. Compute the total ΔV if the full inclination change is combined with the 2nd step of the Hohmann transfer (i.e., the Hohmann transfer orbit still has an inclination of 5°).(4 points)
i. Discuss the results of the computations of total ΔV (i.e., subquestions (f)-(h)). (2 points)

Data: μ_{Earth} = 398600 km^3/s^2, R_{Earth} = 6378.137 km