Flight and Orbital Mechanics

Exams

Exam AE2104: Flight and Orbital Mechanics (27 January 2011)

Remarks:

Please put your name and ALL YOUR INITIALS on your work. Answer all questions and put your name on each page of your exam.

This exam consists of questions: 1a-b, 2a-g, 3a-e and 4a-c, 5a-d, and 6a-f

Derive the expressions for each required calculation.

The way the answer is obtained should be clearly indicated by visibly substituting the numbers in the formulas. Only mentioning the final answer will NOT result in any credits use of pencils to write the exam is NOT permitted. Scrap paper may not be added to your exam work (please take the scrap paper with you after the exam). It is not permitted to have any pre-programmed information on your calculator. The memory of your calculator should be erased prior to the start of the exam. Failure to do so will be seen as fraud.

In total 100 points can be earned. Good luck!

Question 1 (Flight Mechanics) [15 points]

An aircraft is flying at Mach 0.8 at 5000 [m] altitude. The following information is given.

Ratio of specific heats of air:	$\gamma = 1.4$
Specific gas constant of air:	$R = 287.05$ [m^2/s^{2}K]
Air density at 5000 [m]:	$\rho_{5000} = 0.7361$ [kg/m^3]
Temperature at 5000 [m]:	$T_{5000} = 255.65$ [K]

a. (5 points) Calculate the energy height of this aircraft.

Air traffic control asks the aircraft to maintain airspeed and to perform a rate one turn (180 deg/min).

b. (10 points) Calculate the corresponding turn radius and bank angle. You do not have to derive the equations of motion for turning flight. All other equations that you make use of should be derived.

Question 2 (Flight Mechanics) [35 points]

The following data are given for a Boeing 747:

Take-off weight:	$W = 3260$ [kN]]
Wing Surface area:	$S = 510$ [m^2]
Lift drag polar:	$C_D = C_{D_0} + \dfrac{C_L^2}{\pi A e}$
Take-off configuration:	$C_{D_0} = 0.036, \quad A = 6.7, \quad e = 0.7$
Maximum lift coefficient:	$C_{L_{\max}} = 1.8$
Lift coefficient during ground run:	$C_{L_g} = 0.7$
Lift off speed:	$V_{LOF} = 1.2 \cdot V_{\min}$
Maximum thrust for **one** engine:	$T = 165$ [kN]
Number of engines:	$N_{engines} = 4$
Air density:	$\rho = 1.225$ [kg/m^3]

The take-off consists of a ground run and an airborne phase (Figure 1). This aircraft requires 2500 [m] runway for the ground run. The thrust of the engines can be assumed independent of airspeed.

a. (5 points) Derive the equations of motion using the FBD and KD for the aircraft during the airborne phase of the take-off. Clearly indicate all assumptions and draw the airspeed vector, all relevant angles, forces and accelerations in the two diagrams. The thrust vector can be assumed to act in the same direction as the airspeed vector.
b. (5 points) Use the energy method to derive an expression for the horizontal distance covered during the airborne phase of the take-off from lift off until screen height. This expression will be a function of the airspeed at screen height V_{scr}, the lift off speed V_{LOF}, the screen height h_{scr}, the aircraft weight W and the mean excess thrust. The airspeed at screen height V_{scr} is equal to $1.3V_{min}$.
c. (5 points) Calculate the airborne distance from take-off to screen height (10.7 m). At screen height, the airspeed equals $1.3V_{min}$.

Now, assume that **one** engine fails exactly at the moment of take-off. There are two options for the pilot; continue take-off or to abort the take-off.

d. (5 points) Calculate the airborne distance from take-off to screen height (10.7 m) with one failed engine.
e. (5 points) Draw a clear Free Body Diagram (FBD) and Kinetic Diagram (KD) visualizing all forces and accelerations that act on the aircraft during the ground run. Clearly indicate the direction of the velocity and all angles that are relevant for any further calculations.
f. (5 points) Derive the equations of motion for the ground run.
g. (5 points) In case of the aborted take-off, calculate the ground run distance from the moment the engine fails until stand still. Full braking power is applied immediately and engine thrust is reduced to zero. The ground friction coefficient when braking (μ_{brake}) equals 0.4. It can be assumed that the mean deceleration occurs when the airspeed equals $V_{LOF}/\sqrt{2}$

Question 3 (Orbital Mechanics) [15 points]

The gravity potential of the Earth is given by the following equation:

$$U = -\frac{\mu}{r}[1-\sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n(\sin\delta) + \sum_{n=2}^{\infty}\sum_{m=1}^{n} J_{n,m}\left(\frac{R_e}{r}\right)^n P_{n,m}(\sin\delta)\cos(m(\lambda-\lambda_{n,m}))]$$

Here, $P_n(\sin\delta)$ and $P_{n,m}(\sin\delta)$ represent the Legendre polynomials and functions, respectively:

$$P_n(x) = \frac{1}{(-2)^n\, n!}\frac{d^n}{dx^n}(1-x^2)^n$$

$$P_{n,m}(x) = (1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m}$$

a. (2 points) Compute the orbit radius of a geostationary satellite.
b. (3 points) Give the general expression to derive the East-West acceleration from the potential formulation for the gravity field.
c. (6 points) Derive the general equation for the East-West acceleration due to the term (2,2) for an arbitrary satellite (i.e., arbitrary r,δ,λ).
d. (2 points) What is the equation for the East-West acceleration due to $J_{2,2}$ for a geostationary satellite (expressed in numbers, for arbitrary longitude)?
e. (2 points) What are the locations of the equilibrium points?

Data: μ_{Earth} = 398600.4415 km^3/s^2; R_E = 6378.137 km; $T_E = 23^h56^m4^s$; $J_{2,2} = 1.816\times10^{-6}$; $\lambda_{2,2}$ = -14.9°.

Question 4 (Orbital Mechanics) [10 points]

Consider a hypothetical planet X with mass 5×10^{25} kg, orbiting the Sun in a circular orbit with radius 3 AU. The orbital plane coincides with the ecliptic (i.e., the orbital plane of the Earth).

a. (3 points) Make a sketch of the situation when the gravitational attraction of this planet X on satellites around the Earth is largest.
b. (3 points) Idem for the case when this would be smallest.
c. (4 points) Compute the maximum perturbing acceleration due to this planet X, acting on a geostationary satellite (radius orbit is 42200 km). Remember: μ=G×M.

Data: G = 6.673×10^{-20} $km^3/kg/s^2$; 1 AU = 149.6×10^6 km.

(TWO MORE QUESTIONS ON THE NEXT PAGE!)

Question 5 (Orbital Mechanics) [10 points]

One of the main issues for designing a space mission is the occurrence of eclipses.

a. (1 points) What is the definition of an eclipse?
b. (3 points) An eclipse has consequences for at least 3 subsystems of the satellite. What are these, and discuss the consequences for each one briefly (about 2 lines each).
c. (3 points) What are the two conditions that determine whether an Earth satellite is in eclipse or not? Give the mathematical conditions in a sketch and discuss each one briefly.
d. (3 points) One of the conditions can translate into the so-called shadow function as given below. Discuss the meaning of the various elements in the equation, and discuss the use of this equation.

$$S(\theta) = R_e^{\,2}(1 + e\cos\theta)^2 + p^2(\bar{\alpha}\cos\theta + \bar{\beta}\sin\theta)^2 - p^2$$

Question 6 (Orbital Mechanics) [15 points]

Consider the various high-thrust options for a transfer from Earth to Jupiter: a minimum-energy Hohmann transfer, or a faster transfer (demanding more energy).

a. (4 points) Compute the main characteristics of the Hohmann transfer: semi-major axis, eccentricity and transfer time.
b. (2 points) Compute the excess velocity V_∞ (when escaping from Earth).
c. (2 points) Now assume that the launcher is able to give the spacecraft a V_∞ which is 10.5 km/s (so about 20% higher), still in the direction parallel to the heliocentric velocity of Earth itself. What would be the heliocentric velocity of the spacecraft when leaving Earth?
d. (2 points) Compute the semi-major axis and the eccentricity of the new transfer orbit. If question (c) could not be answered by you, use a value of 40 km/s for this heliocentric velocity.
e. (2 points) Compute the value for the true anomaly θ, when arriving at Jupiter.
f. (3 points) Given the relations tan(E/2)=sqrt[(1-e)/(1+e)]*tan(θ/2), M=E-e*sin(E) and $M=n(t-t_0)$, compute the value for the eccentric anomaly E, the mean anomaly M and the travel time.

Data: $\mu_{Sun}=1.3271\times10^{11}$ km^3/s^2; distance Earth-Sun = 1 AU; distance Jupiter-Sun = 5.2 AU; 1 AU = 149.6×10^6 km.